



University of
Zurich ^{UZH}

Agent-based Financial Economics

Lesson 7: Equality

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“What I cannot create, I do not understand.”

- Richard Feynman

Today



- Principle of positive expressions
- Correction: savings rule
- Discussion of exercise 6
- Gini coefficient
- Club of Rome model
- Exercise 7: Equality

Principle of Positive Expressions

- Why “equality” and not “inequality”?
 - Human brain is bad at parsing negatives.
 - Compare:
“If the street is not wet, there cannot be rain.” vs. “If there is rain, the street is wet.”
Logically equivalent, but the latter is much easier to understand.
 - Compare:
if (!list.isEmpty()) { doSomething() };
if (list.hasSome()) { do Something() };
 - Compare:
“Do you mind if I sit here?” (People usually say “Yes”, when they actually don’t mind...)
“May I sit here?” (A yes is always a yes.)
- Using positive expressions makes your programs more readable and helps with human interaction.

Exercise 5 - corrections

To behave optimally, the agent should spend an equal share of his life-time wealth W_{tot} every day.

We use the dividend yield to derive the right discount rate, because that's how much we get from saving. Here, Gordon works. We disregard capital gains.

Dividend yield farm: 1%
→ Discount rate $r = 0.99$

Handwritten mathematical derivation on grid paper:

day to retirement

$$W_L = \sum_{i=0}^{T_{\text{ret}}} w \cdot r^i = w \underbrace{\left(\frac{1-r^{T+1}}{1-r} \right)}_{=k}$$
$$W_{\text{tot}} = W_L + W_d = w \cdot k + \frac{d}{1-r}$$
$$c = \frac{W_{\text{tot}}}{T_{\text{left}}} = \frac{wk + d/(1-r)}{T_{\text{left}}} \Rightarrow wk = cT_{\text{left}} - \frac{d}{1-r}$$

consumption L days left

Exercise 5 - corrections

- Corrected rule, with two approximative examples for day 0 and day 300.
- One can nicely see that initially, only 20% of work income is consumed.
- Actually, it would be optimal to consume even less on the first day as this was made under the assumption of constant work amounts.

$$\begin{aligned}
 r &= 0 \text{ in retirement} \Rightarrow \frac{d}{1-r} = c_{\text{left}} \\
 wk &= c_{\text{left}} - \frac{d}{1-r} \\
 w &= \frac{c_{\text{left}} - d/(1-r)}{k} \\
 s + c &= w + d \\
 s &= \frac{c_{\text{left}} - d/(1-r)}{k} + d - c \\
 \text{On day 0: } s + c &= \frac{500c - 100d}{100} + d \\
 s + c &= 5c - d + d \\
 s &= 4c \\
 \text{On day 300: } s + c &= \frac{200c - 100d}{100 \cdot 2/3} + d \\
 s + c &= 3c - \frac{3}{2}d + d \\
 s &= 2c + \frac{1}{2}d
 \end{aligned}$$

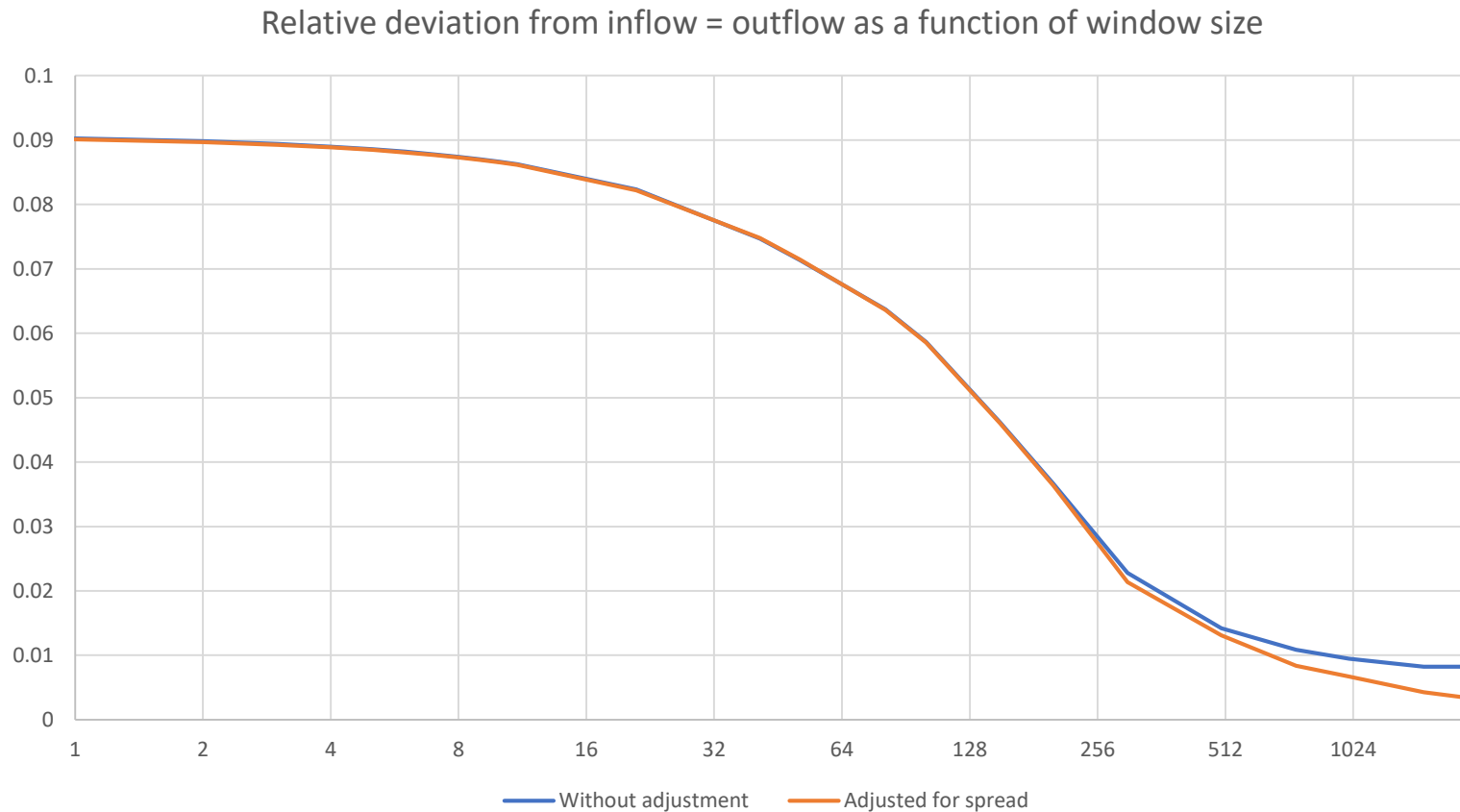
Reinvesting dividends – optimal rule

```
@Override
public void managePortfolio(IStockMarket stocks) {
    boolean retired = isRetired();
    int daysLeft = getMaxAge() - getAge() + 1;
    if (retired) {
        double proceeds = getPortfolio().sell(stocks, this, 1.0d / daysLeft);
        listeners.notifyDivested(this, proceeds); // notify listeners for inflow / outflow statistics
    } else {
        int daysToRetirement = getRetirementAge() - getAge();
        double dividends = getPortfolio().getLatestDividendIncome();
        double constantFactor = Numbers.geometricSum(DISCOUNT_RATE, daysToRetirement);
        double consumption = getDailySpending();
        double optimalSavings = (consumption * (daysLeft - 1) - dividends / (1 - DISCOUNT_RATE)) / constantFactor + dividends - getDailySpending();
        double actualInvestment = getPortfolio().invest(stocks, this, optimalSavings);
        listeners.notifyInvested(this, actualInvestment); // notify listeners for inflow / outflow statistics
    }
}
```

Improves the utility from 8.077 to 8.330.

Relevant version tagged as “ex5-optimal-rule”.

Exercise 6 – Flow – Task 1



$$\frac{1}{\delta} \sum_{i=t}^{t+\delta} inflow_i \approx \frac{1}{\delta} \sum_{i=t}^{t+\delta} outflow_i$$

The larger the window of the rolling average, the smaller the relative difference between inflow and outflow.

In the long run, inflow = outflow holds pretty well, especially when adjusting for the spread, which effectively is a fee paid to the market maker.

Exercise 6 – Flow – Task 1

Task 1: Does inflow and outflow add up?

Including the market maker, inflow = outflow holds in the model (it must by definition). However, excluding the market maker, the inflow exceeds the outflows by about 1% in the long run.

Where does that money go?

- Note that this 1% corresponds to the spread, the missing money is the trading profits of the market maker!
- Inflows should be adjusted downwards by 1% as this money should be considered «trading fees» paid to the market maker.

(In fact, the actual difference is more like 0.8%, as the market maker also can make trading losses when price exhibit momentum.)

Exercise 6 – Flow – Task 2

Could simply regress:

$$P_{t+1} = a + b P_t + c \text{ inflow} + d \text{ outflow}$$

This yields an excellent t-stat (over 1000!). It says that the price today is an excellent predictor for the price tomorrow. Is this useful?

Not very, if we want to make money, we actually want to know about the returns, and not the absolute prices!

Also, note that approximately $c = -d$, so it seems it is only the difference that matters. Let's analyze the following hypothesis:

$$r_t = a (\text{inflow} - \text{outflow})$$

Exercise 6 – Flow – Task 2

Especially for larger windows of the rolling average, net inflow correlates well with returns.

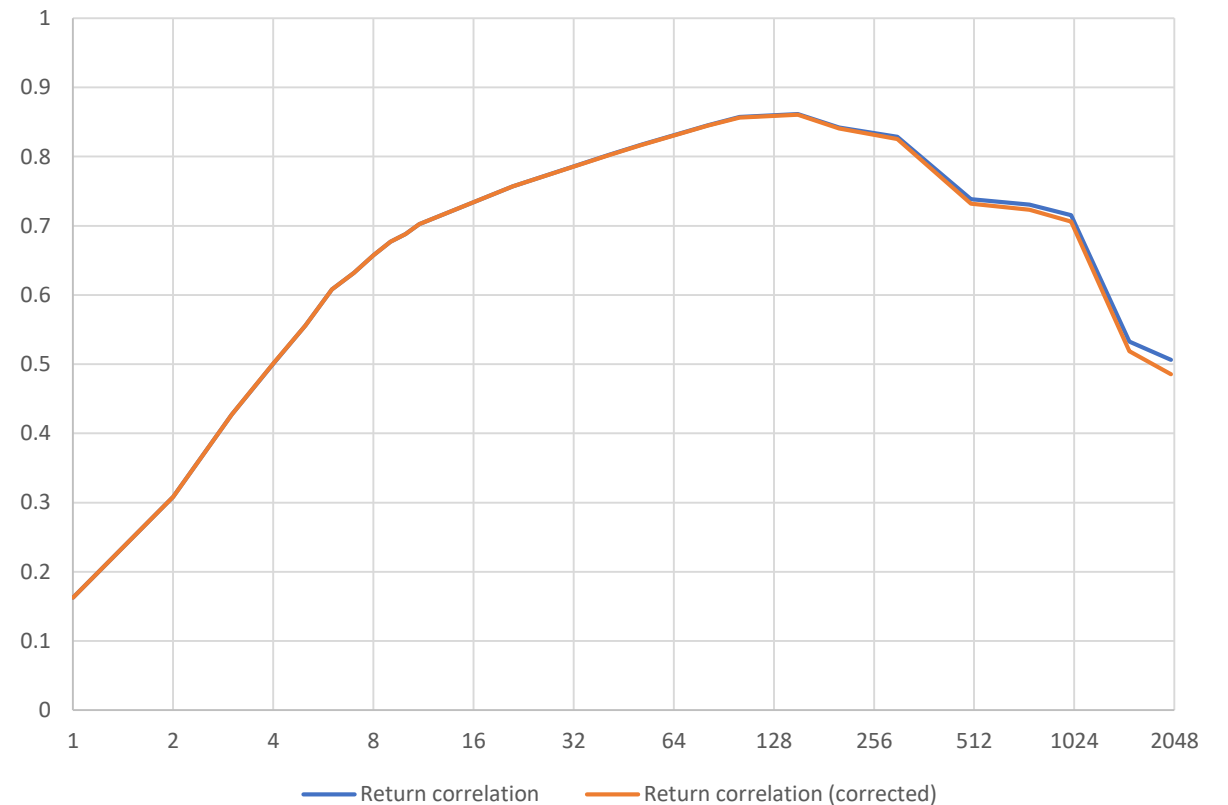
Note that when having a single variable, correlation corresponds to R-squared.

For a rolling window size of 50, we get a t-stat of about 100! Without using a rolling average window, the t-stat is “only” about 13.

→ Using a rolling window average can help in reducing noise.

Also note that having a simulation, we could generate arbitrary amounts of data, thereby pushing up the t-stat as high as we want as long as there is a non-zero correlation.

Corr(log return, inflow - outflow) as a function of window size



Exercise 6 – Flow – Task 2

To conclude, the following relation seems to hold:

$$r_t = a(inflow_t - outflow_t)$$

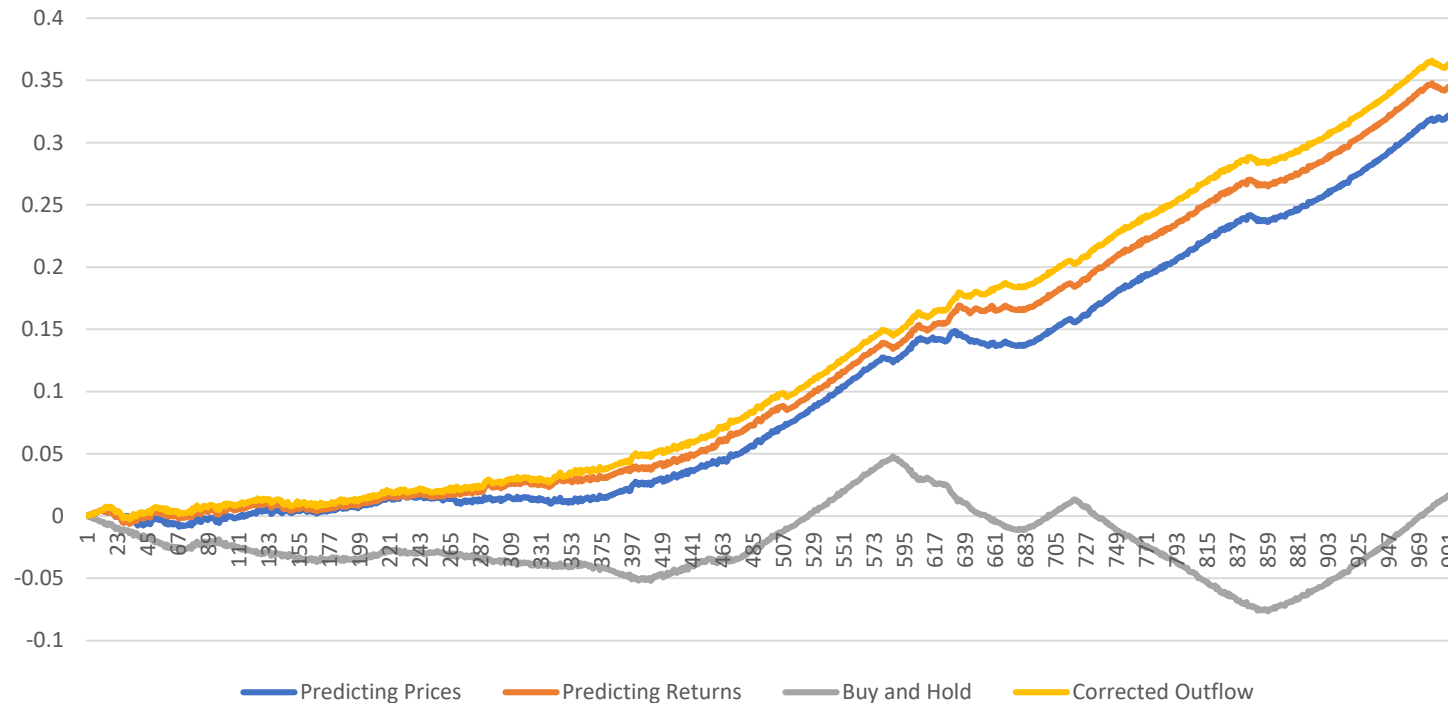
Or, alternatively, one could say that we observe:

$$\frac{dp}{dt} \propto (in_t - out_t)$$

Exercise 6 – Flow

Can we leverage this insight on our toy stock market? Yes!

Out of sample performance



Yes, **disregarding dividends**, we can nicely outperform the buy and hold strategy.

“Predicting prices” strategy goes long whenever the direct regression on prices predicts an increase, and short otherwise.

“Predicting returns” does the same when the return regression predicts a positive return.

“Corrected outflow” does the same using the a metric of inflow that is adjusted for the market maker’s spread and known prices.

Note that the rule with the t-stat of 1000 does not perform better than the rule with the t-stat of 13.
Might be interesting to also run a probit regression on whether to go long or short.

Paper Idea

“Momentum as an artefact of traditional market making.”

Momentum means: assets that went up in the past, tend to go up further.

The momentum puzzle: momentum is hard to reconcile with efficient markets, but there is overwhelming evidence that momentum actually exists.

<https://www.aqr.com/cliffs-perspective/fama-on-momentum>

Where does it come from? Who pays for these momentum gains?

Maybe it is the market makers! When the price goes up, they have to buy more. And when the price goes down, they have to sell more. Maybe the 0.2% they did not make (they made about 0.8% when the spread is actually 1%) is the amount that can be harvested with a momentum strategy?

If that hypothesis is true, then there should be less observable momentum for stocks with a Bancor-style market maker.

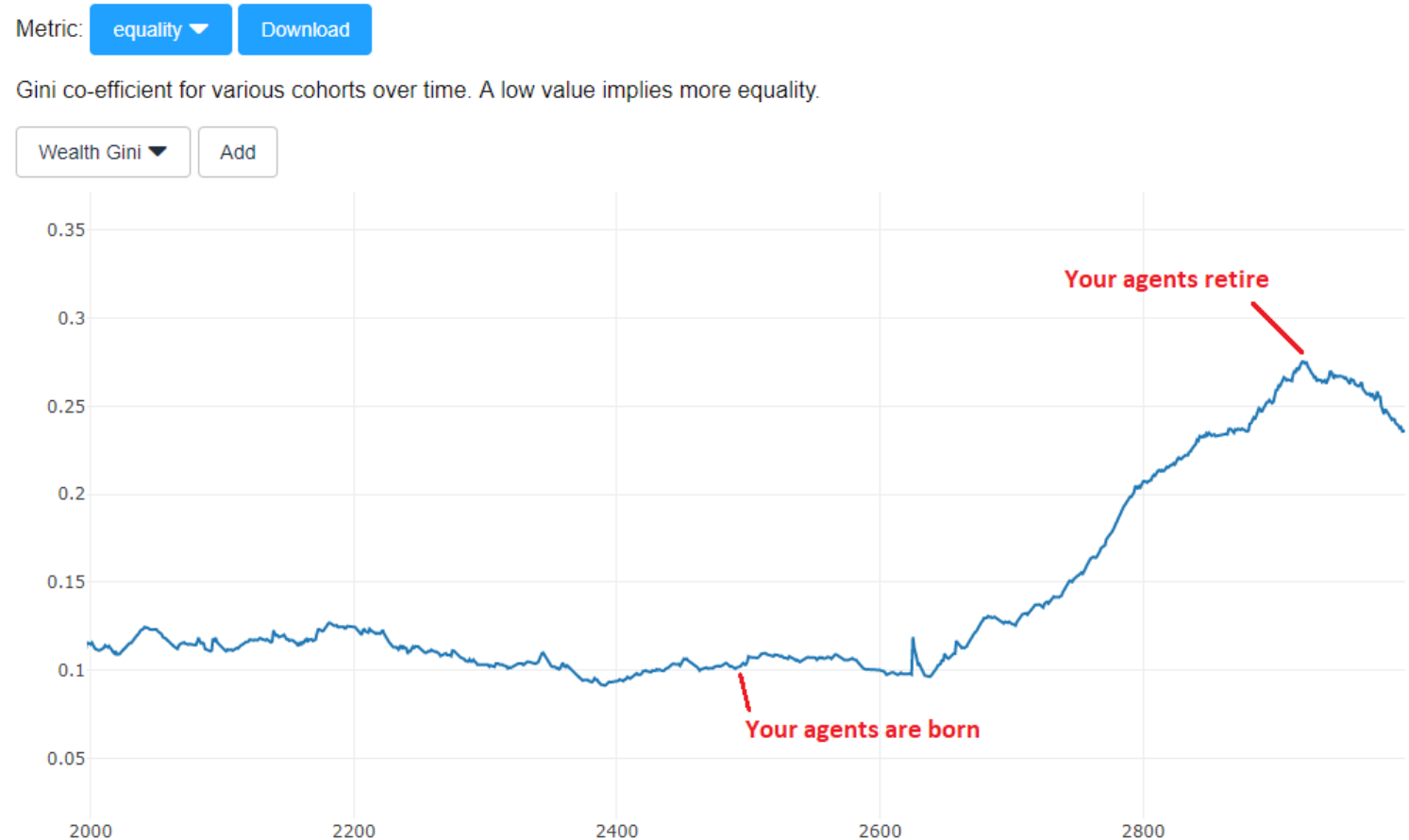
→ Very preliminary idea. Just wanted to tell you because I like it. 😊

New Metric: GINI Coefficient

Wealth gini coefficient for exercise 5: shows increasing inequality as your agents outwit all the others and accumulate a lot of wealth.

Peak inequality is at the day of retirement, before your agents start consuming their amassed wealth.

$$G = \frac{\sum_{i=1}^n \sum_{j=1}^n |x_i - x_j|}{2 \sum_{i=1}^n \sum_{j=1}^n x_j} = \frac{\sum_{i=1}^n \sum_{j=1}^n |x_i - x_j|}{2n \sum_{i=1}^n x_i}$$



Metric:

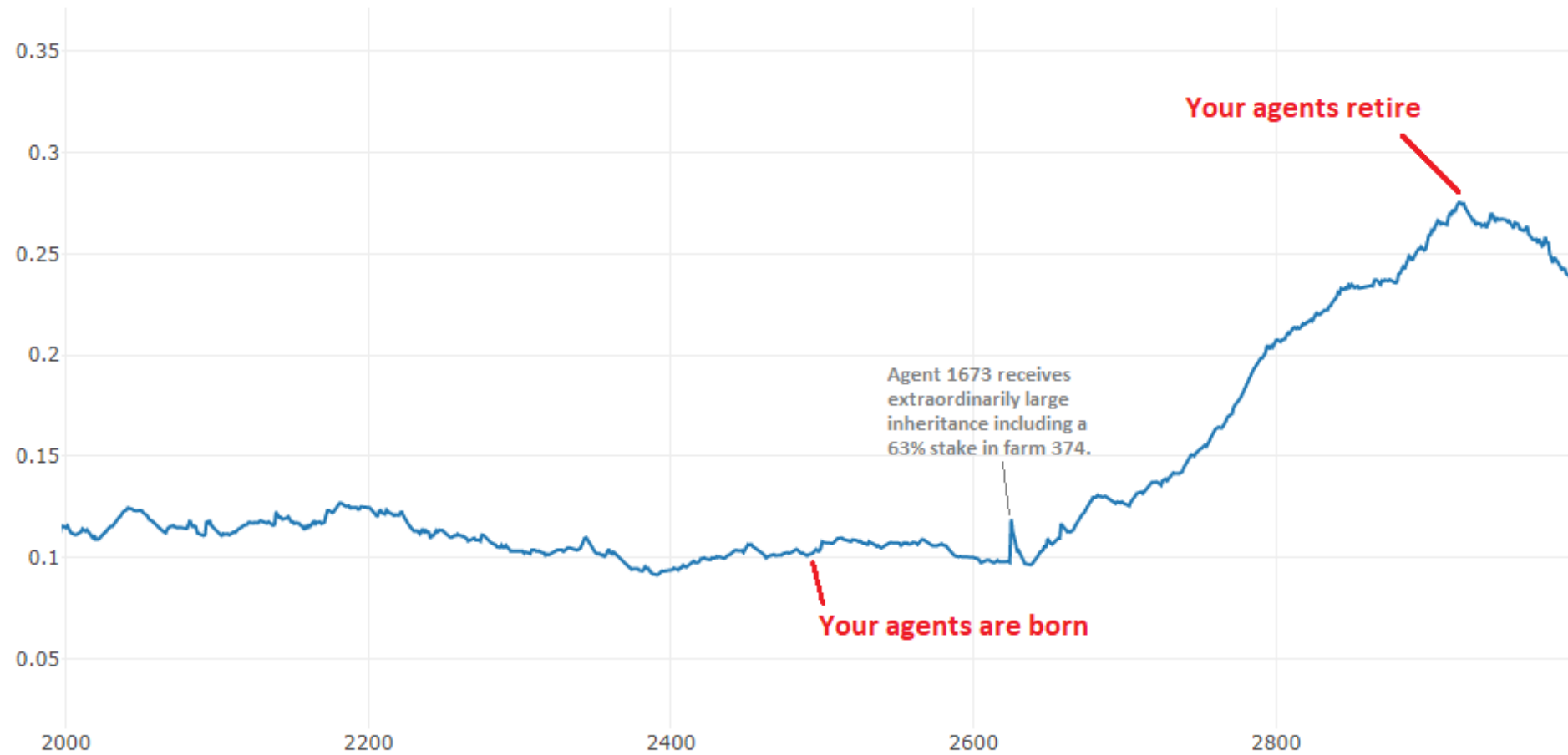
equality ▼

Download

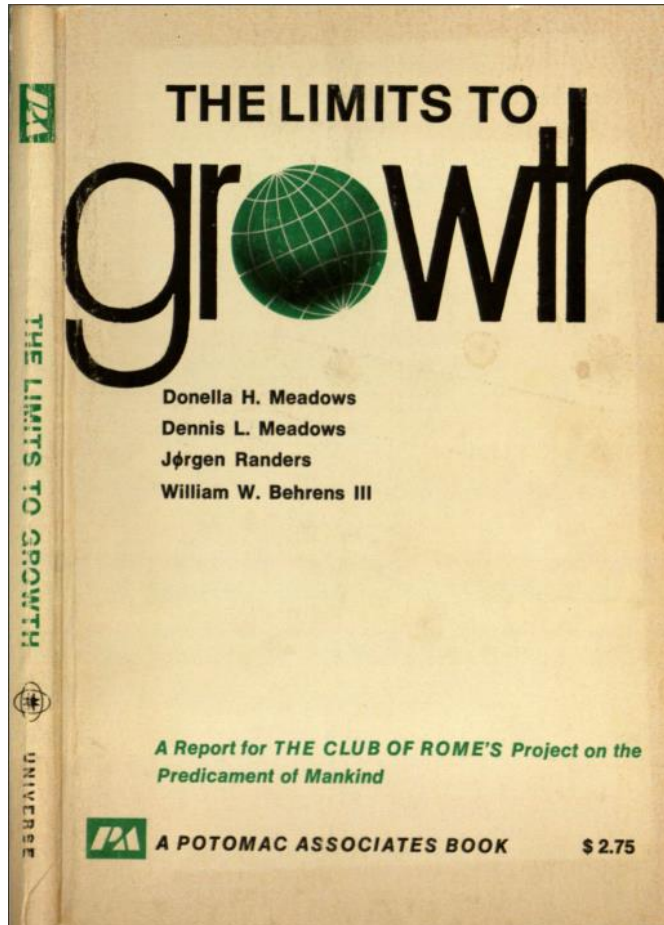
Gini co-efficient for various cohorts over time. A low value implies more equality.

Wealth Gini ▼

Add



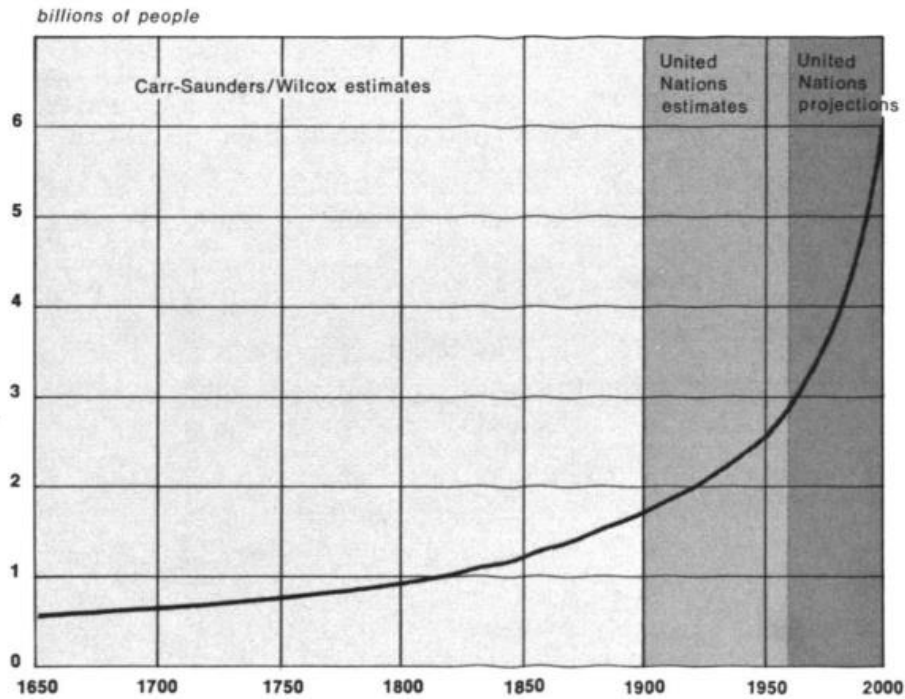
Club of Rome: Limits to Growth



- Hugely influential book from 1972
- Based on System Dynamics (not agent-based, but also exhibits non-linear endogenous dynamics)
- Start of the green movement: recycling, outlawing DDT, etc.
- Pessimistic predictions
- PDF available from:
www.clubofrome.org/report/the-limits-to-growth

Club of Rome: Limits to Growth

Figure 5 WORLD POPULATION



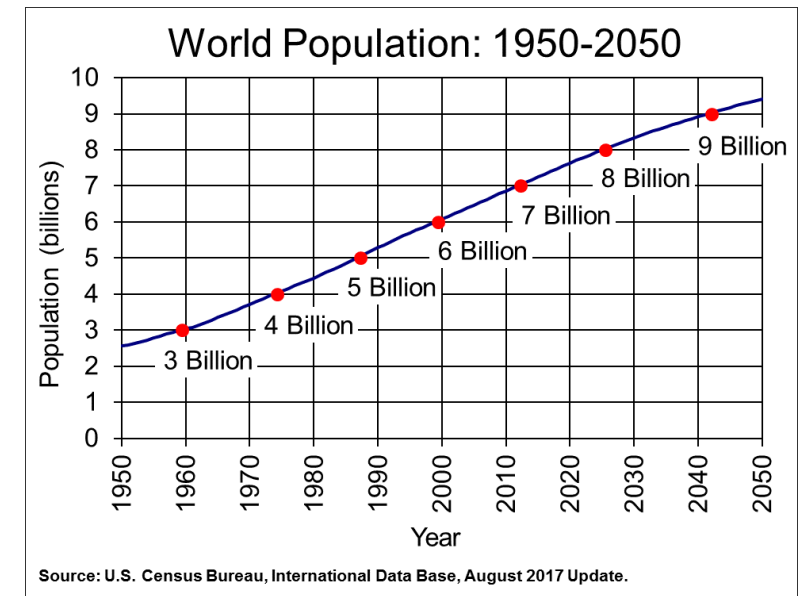
World population since 1650 has been growing exponentially at an increasing rate. Estimated population in 1970 is already slightly higher than the projection illustrated here (which was made in 1958). The present world population growth rate is about 2.1 percent per year, corresponding to a doubling time of 33 years.

SOURCE: Donald J. Bogue, *Principles of Demography* (New York: John Wiley and Sons, 1969).

Some estimates have been excellent.

Prediction for world population in the year 2000 has been spot on.

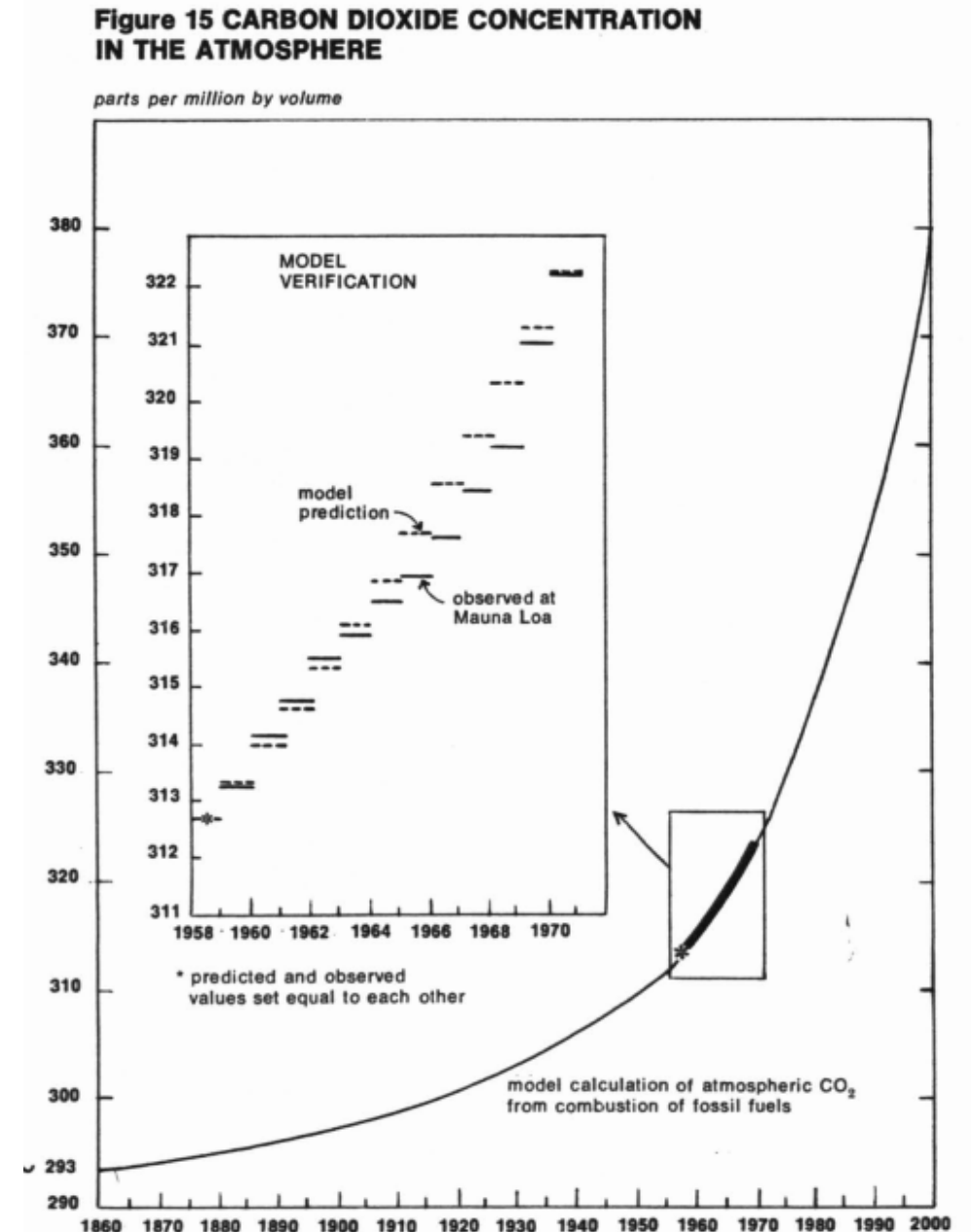
Current outlook



Club of Rome

Also prediction for CO₂ concentration in atmosphere was excellent.

Current level: around 400 ppm



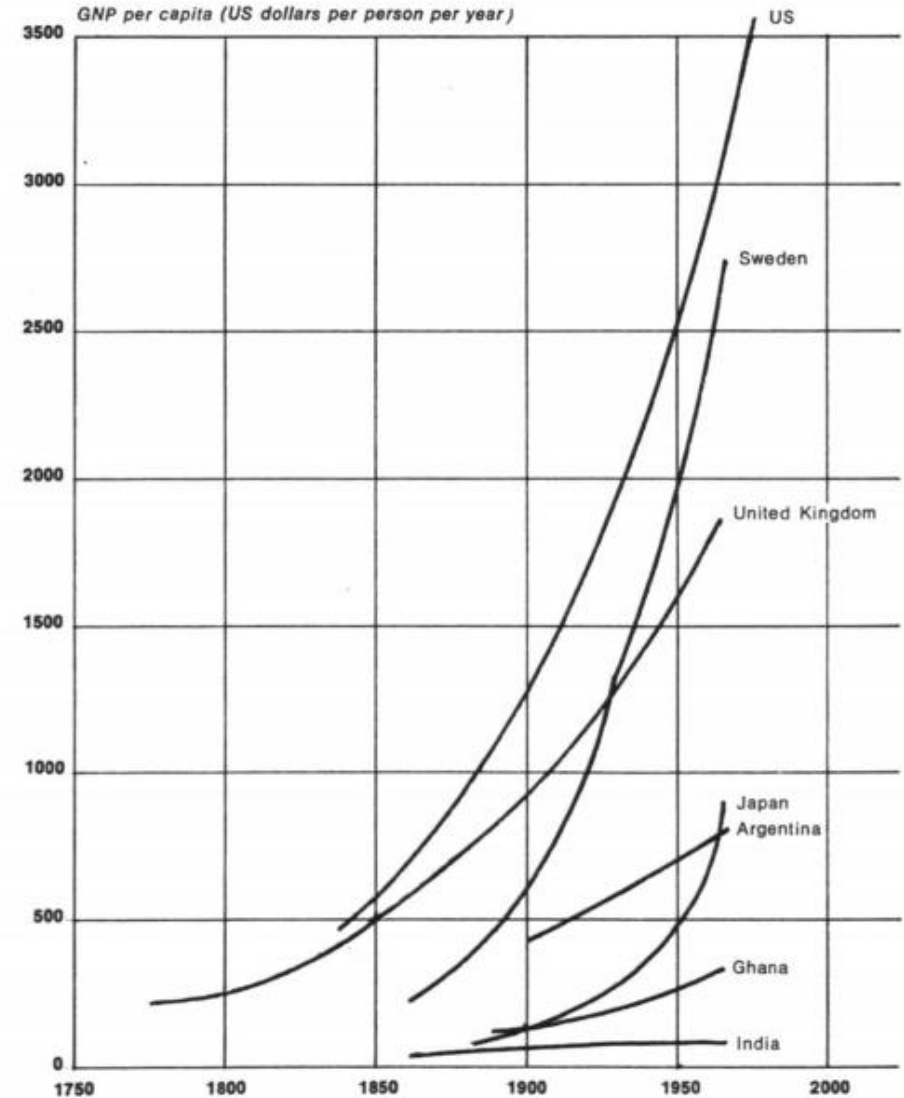
Club of Rome

Basic observation: things are growing exponentially.

Table 2 ECONOMIC AND POPULATION GROWTH RATES

Country	Population (1968) (million)	Average annual growth rate of population (1961-68) (% per year)	GNP per capita (1968) (US dollars)	Average annual growth rate of GNP per capita (1961-68) (% per year)
People's Republic of China *	730	1.5	90	0.3
India	524	2.5	100	1.0
USSR *	238	1.3	1,100	5.8
United States	201	1.4	3,980	3.4
Pakistan	123	2.6	100	3.1
Indonesia	113	2.4	100	0.8
Japan	101	1.0	1,190	9.9
Brazil	88	3.0	250	1.6
Nigeria	63	2.4	70	— 0.3
Federal Republic of Germany	60	1.0	1,970	3.4

Figure 7 ECONOMIC GROWTH RATES



The economic growth of individual nations indicates that differences in exponential growth rates are widening the economic gap between rich and poor countries.

SOURCE: Simon Kuznets, *Economic Growth of Nations* (Cambridge, Mass.: Harvard University Press, 1971).

Club of Rome

Basic observation: things are growing exponentially.

What if we extrapolate this?

Table 3 EXTRAPOLATED GNP FOR THE YEAR 2000

<i>Country</i>	<i>GNP per capita (in US dollars *)</i>
People's Republic of China	100
India	140
USSR	6,330
United States	11,000
Pakistan	250
Indonesia	130
Japan	23,200
Brazil	440
Nigeria	60
Federal Republic of Germany	5,850

* Based on the 1968 dollar with no allowance for inflation.

1 USD from 1968 corresponds to 7 USD from 2017.

USD estimate is okayish (57k vs 77k). Others are way off.

Actual vs Club of Rome estimate:

China: 8k vs 0.7k → Underestimated China

Russia: 9k vs 42k → Overestimated Russia

Japan: 39k vs 160k

Nigeria: 2.2k vs 0.4k

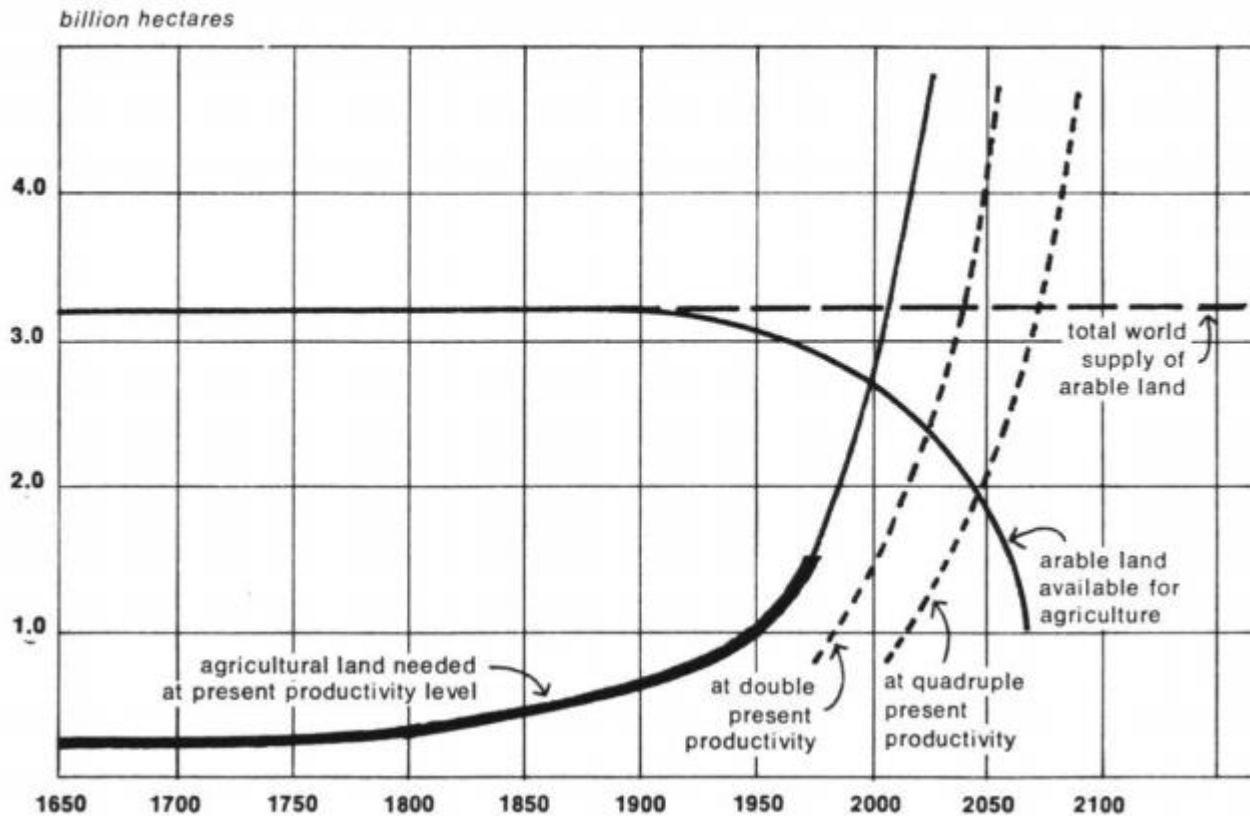
Germany: 42k vs 42k

Brazil: 8.6k vs 3k

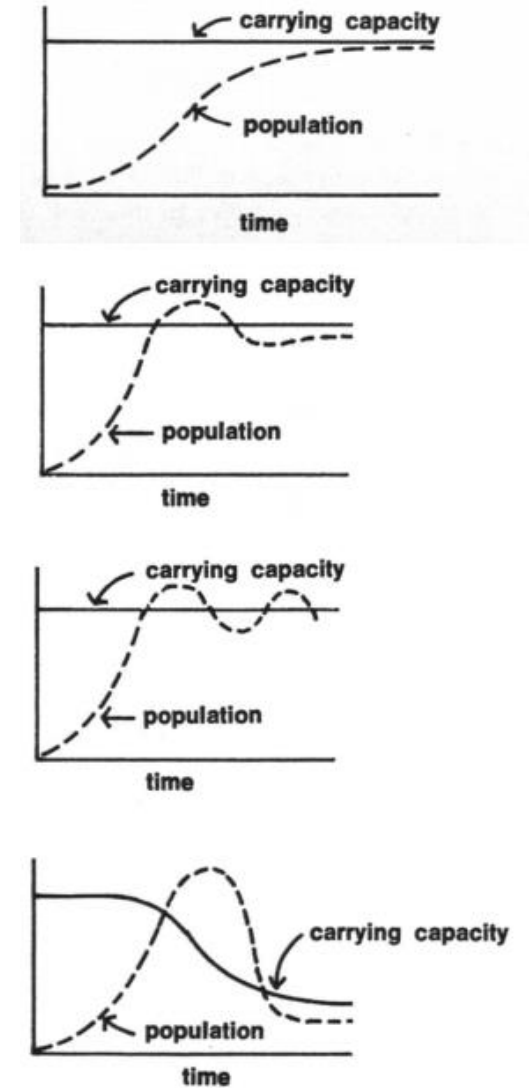
Indonesia (now 3.5k) overtook Pakistan (now 1.5k)

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Figure 10 ARABLE LAND



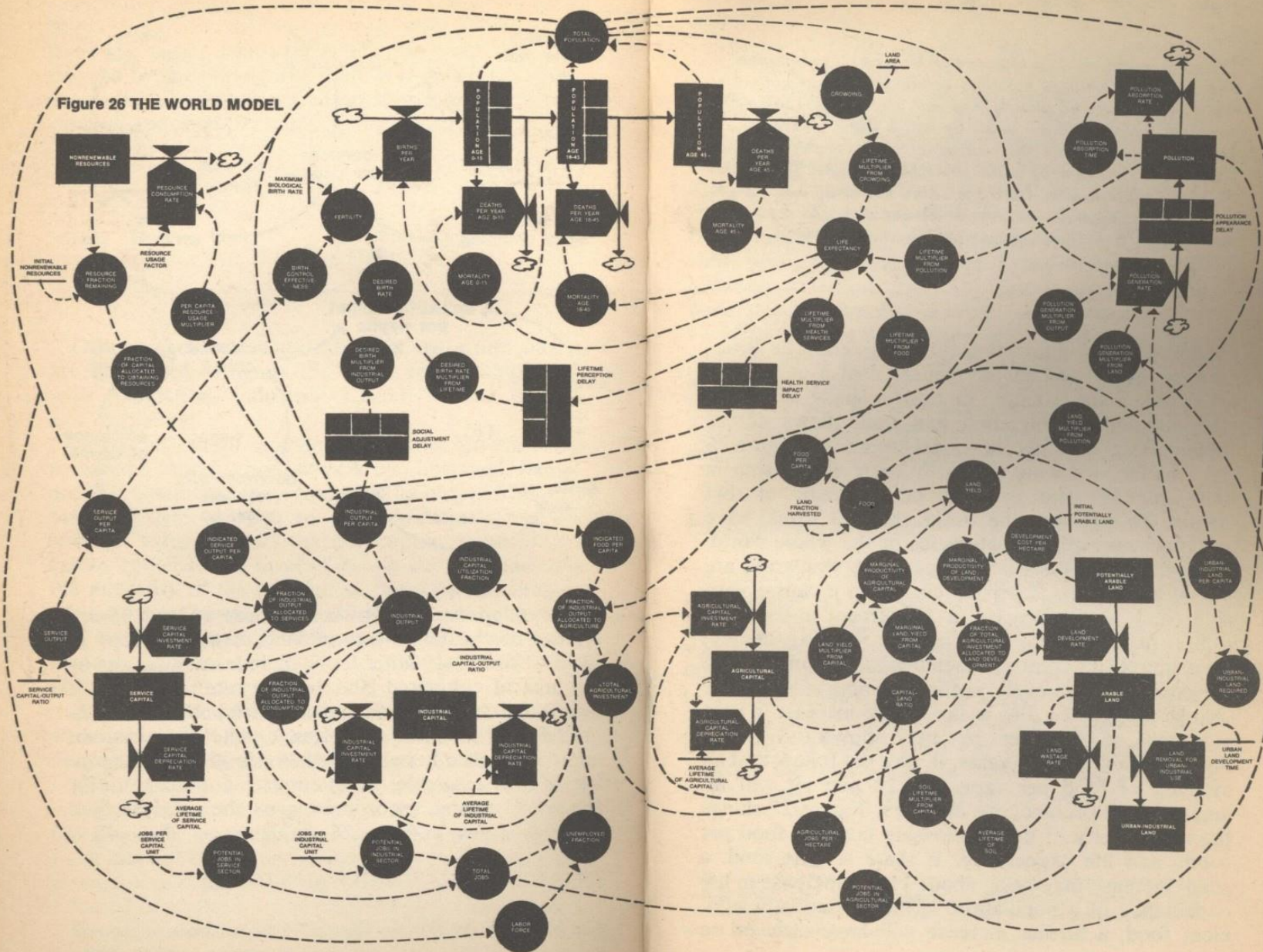
Club of Rome warning:
Regardless of how accurate our predictions are, with exponential growth, we will hit some natural limits sooner or later! This cannot go on forever!



Types of dynamics.

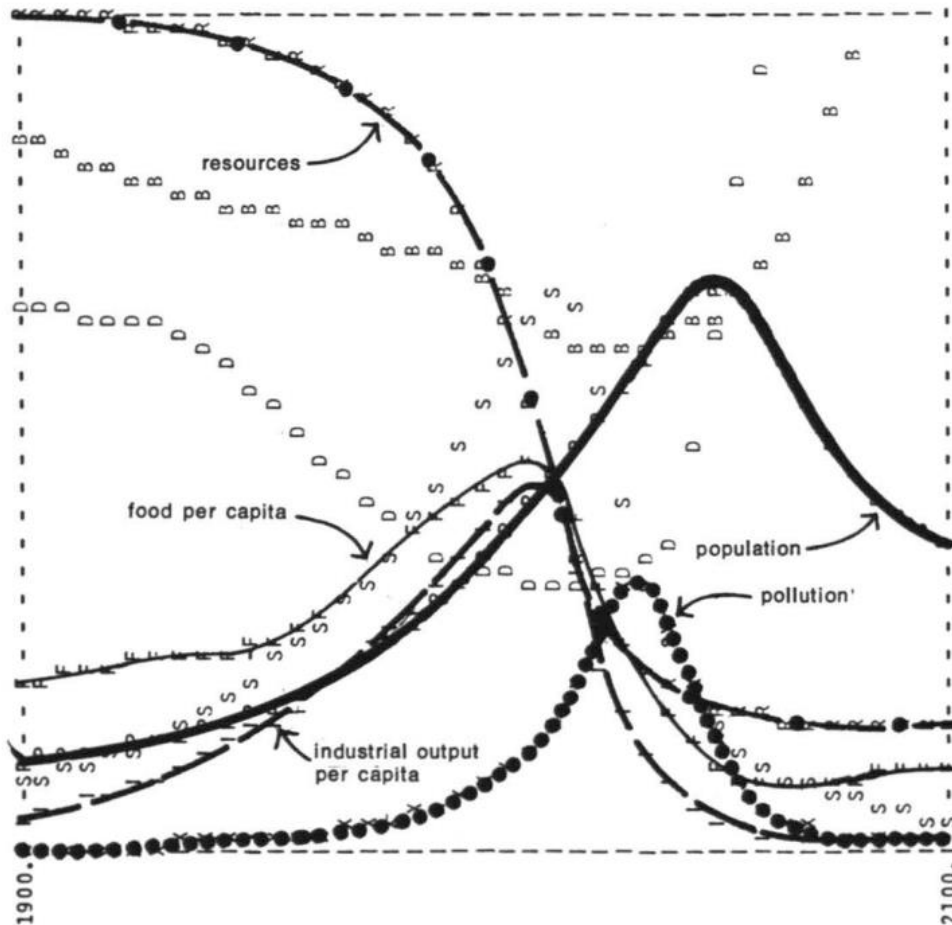
The "Limits to Growth" world model.

Figure 26 THE WORLD MODEL



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Figure 35 WORLD MODEL STANDARD RUN



The “standard” world model run assumes no major change in the physical, economic, or social relationships that have historically governed the development of the world system. All variables plotted here follow historical values from 1900 to 1970. Food, industrial output, and population grow exponentially until the rapidly diminishing resource base forces a slowdown in industrial growth. Because of natural delays in the system, both population and pollution continue to increase for some time after the peak of industrialization. Population growth is finally halted by a rise in the death rate due to decreased food and medical services.

→ Turned out to be overly pessimistic. Underestimated inventiveness of firms and free innovation, i.e. adjustment to less resource usage as they got more expensive. Did not foresee the “digital age”. Instead, they called for the creation of “supranational institutions” to manage population and capital growth...

You can play with the model online on:
insightmaker.com/insight/1954/The-World3-Model-A-Detailed-World-Forecaster

Current Setting: Consumer

→ Look at code

Current Setting: Market Maker

→ Look at code

Exercise 7 – Equality

See exercise 7 on github:

<https://github.com/meisser/course/blob/master/exercises/journal/exercise07-task.md>

Administrative Note

managed-server@hetzner.de

14:10 (vor 1 Stunde)

an mich

Sehr geehrter Herr Meisser,

die Plattenkapazität Ihres Servers ist am Ende. Aktuell sind auf der usr Partition 100% belegt. Bitte kümmern Sie sich darum. Wir haben ein Hardquota auf den Account auf dem Server gesetzt.

Daher würden wir Sie bitten ältere, nicht mehr benötigte Daten zu archivieren oder zu löschen.

Falls dies nicht möglich ist sollte

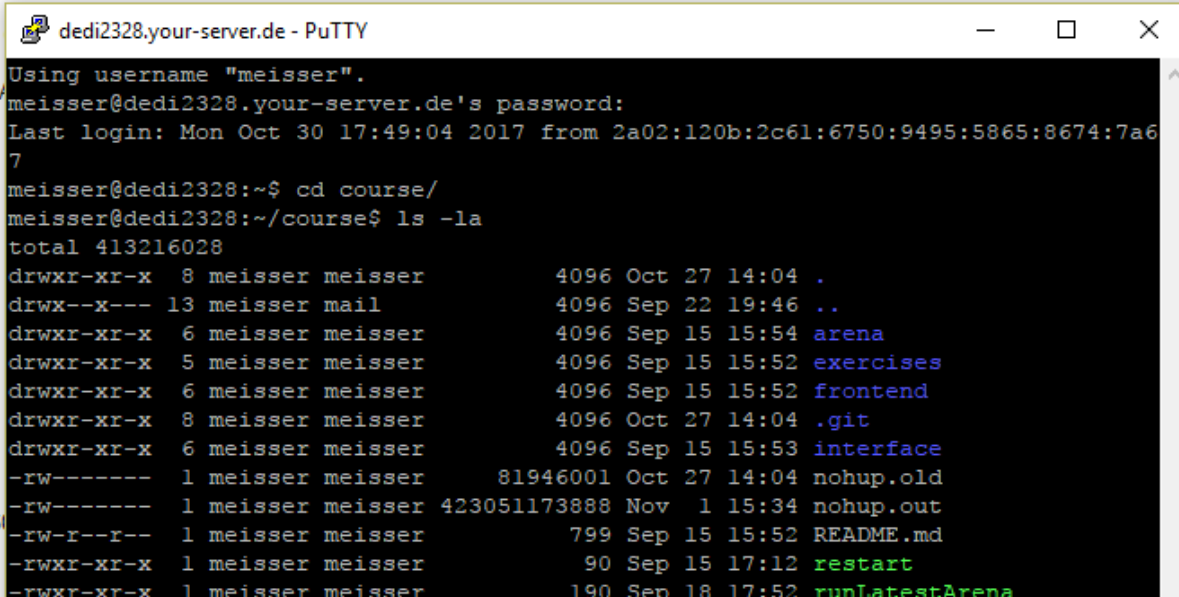
Sollten Sie weitere Fragen oder

Mit freundlichen Grüßen

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<http://www.hetzner.de>

Registergericht Ansbach, HRB 6
Geschäftsführer: Martin Hetzner



```
dedi2328.your-server.de - PuTTY
Using username "meisser".
meisser@dedi2328.your-server.de's password:
Last login: Mon Oct 30 17:49:04 2017 from 2a02:120b:2c61:6750:9495:5865:8674:7a67
meisser@dedi2328:~$ cd course/
meisser@dedi2328:~/course$ ls -la
total 413216028
drwxr-xr-x  8 meisser meisser          4096 Oct 27 14:04 .
drwx--x--- 13 meisser mail            4096 Sep 22 19:46 ..
drwxr-xr-x  6 meisser meisser          4096 Sep 15 15:54 arena
drwxr-xr-x  5 meisser meisser          4096 Sep 15 15:52 exercises
drwxr-xr-x  6 meisser meisser          4096 Sep 15 15:52 frontend
drwxr-xr-x  8 meisser meisser          4096 Oct 27 14:04 .git
drwxr-xr-x  6 meisser meisser          4096 Sep 15 15:53 interface
-rw-----  1 meisser meisser      81946001 Oct 27 14:04 nohup.old
-rw-----  1 meisser meisser 423051173888 Nov  1 15:34 nohup.out
-rw-r--r--  1 meisser meisser          799 Sep 15 15:52 README.md
-rwxr-xr-x  1 meisser meisser          90 Sep 15 17:12 restart
-rwxr-xr-x  1 meisser meisser        190 Sep 18 17:52 runLatestArena
```

→ Please remove all “System.out” statements from your Code before pushing it to github!

→ Also, it would be great if you could remove them from your old agents.