University of Zurich[UZH]

Agent-based Financial Economics
Lesson 9: Capital

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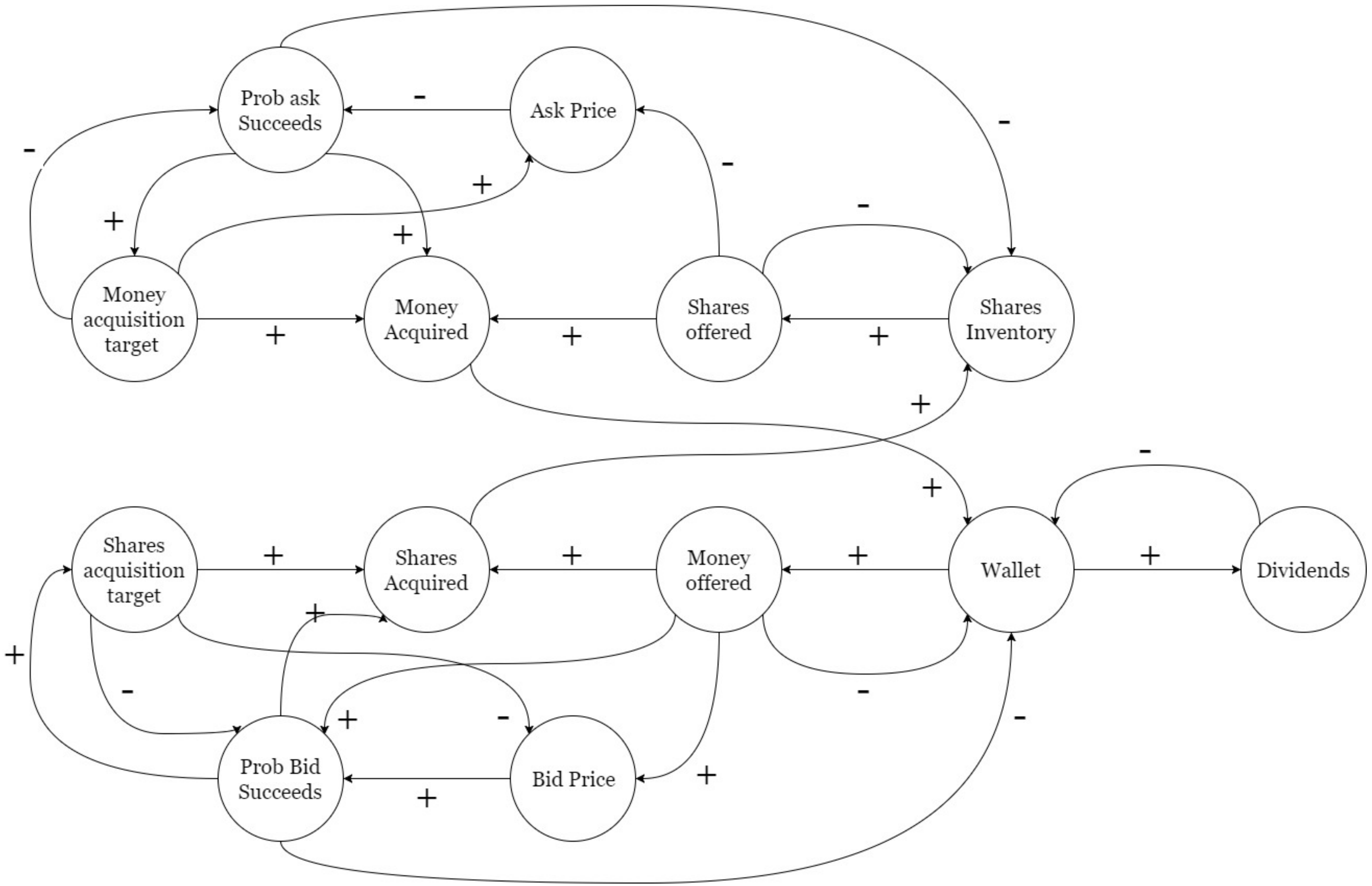
"What I cannot create, I do not understand."

- Richard Feynman

Today

- Discussion of exercise 8
- The market maker's problem
- Capital: optimal firm count
- Capital: land developer
- Externalities
- Farm: decision heuristics
- Code walk-through

Exercise 8 – System Dynamics



Model Extension to Capital

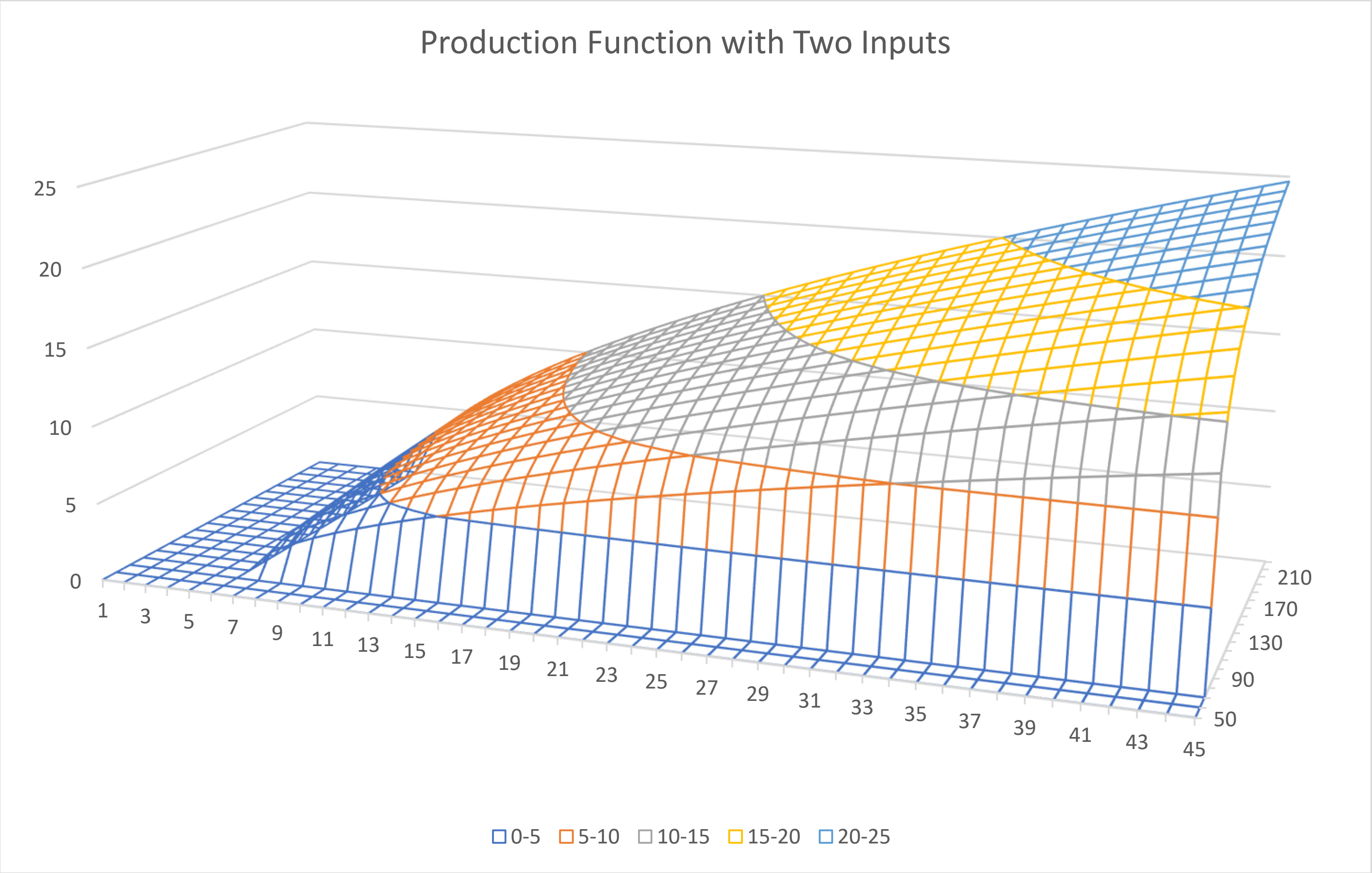
1. Allow land to be traded. → Changes the optimal number of firms and the aggregate production function
2. Allow land to be "produced". Think of building roads to make additional land accessible, removing forests, draining swamps, etc.
3. A new agent "Land Developer" produces new land and acts as a market maker for land.

Unrealistic assumptions in the model: land is fungible (it does not matter which square meter you get), real estate agents hold land on their own accounts instead of just brokering deals.

Consequences:

1. As land becomes tradable, the aggregate production function changes
2. Farms need to start intertemporal optimization (so far it was mostly intratemporal), finding a good path towards the optimal level of capital

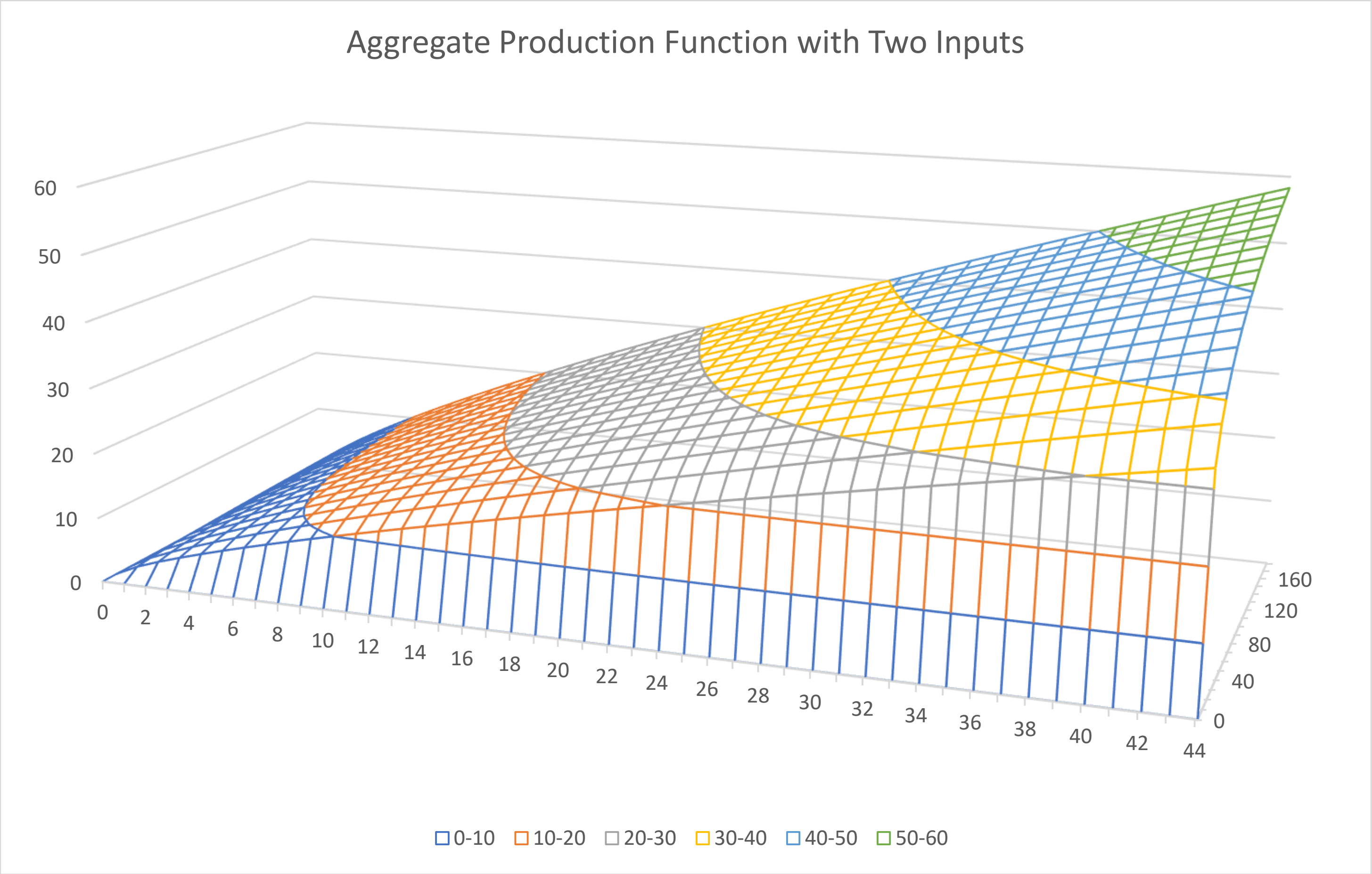
Optimal number of firms



As land becomes tradable, we need to consider both inputs when optimizing production.

→ The production maximization problem becomes «two-dimensional»

Optimal number of firms



Aggregate production function is now a bent surface with a straight line through the origin.
Doubling both inputs double the output.

How can we be sure that equal allocation to each firm is optimal?

If we have 10 man-hours and 10 units of land, wouldn't it be conceivable that it is optimal to give firm A 6 man-hours and 4 units of land, and firm B the rest?
Does it have to be 5 each for each firm?

→ Sketch quick geometric idea of proof.

Optimal number of firms

Optimal number of firms k with $x(h,L) = A_x (h-c)^{\alpha} L^{\gamma}$ as firm prod. fun.

Previously: $1 = \alpha \frac{h}{h-kc_h}$ $\Rightarrow$ optimal $k^* = \frac{h}{c}(1-\alpha)$

Now: $1 = \alpha \frac{h}{h-kc_h} + \gamma$ $\Rightarrow$ optimal $k^* = \frac{h}{c}\left(\frac{1-\alpha-\gamma}{1-\gamma}\right)$

See lesson 2.
(Here: using gamma instead of beta so we can use the letter beta for the discount rate as usual.)

More generally, if both inputs would have fixed costs:

What if both inputs have fixed costs?

$$1 = \alpha \frac{h}{h-kc_h} + \beta \frac{L}{L-kc_L}$$

$$\Rightarrow \quad k = \frac{(1-\beta)c_h L + (1-\alpha)c_L h \pm \sqrt{((\beta-1)c_h L)^2 + ((\alpha-1)c_L h)^2 + 2(\alpha+1)(\beta+1)c_h c_L L h}}{2 c_h c_L}$$

Optimal number of firms

Optimal number of firms k with $x(h, L) = A_x (h-c)^{\alpha} L^{\gamma}$ as firm prod. fun.

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Aggregate production function:

$$x^*(h, L) = \underbrace{c^{\alpha+\gamma-1}(1-\alpha-\gamma)^{1-\alpha-\gamma}(1-\gamma)^{\gamma-1}\alpha^{\alpha}}_{\text{constant } = A^*} \underbrace{h^{1-\gamma} L^{\gamma}}_{\text{constant returns to scale}}$$

(Set gamma = 0 to get the previous aggregate production function before land was tradable.)

The Land Production Function

- All land developers share one common production function
- The production function has a memory, each additional square meter of land is more expensive to produce.

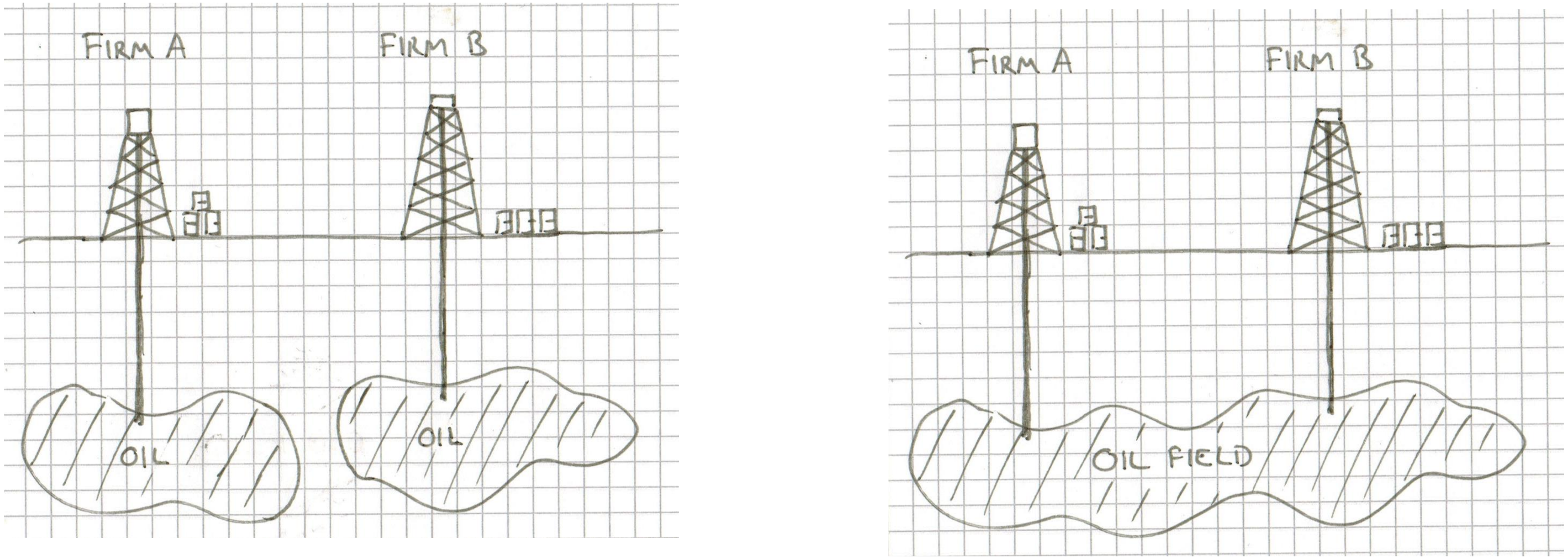
→Aggregate production function with decreasing returns to scale

→The production decision of one firm influences what the other firms can produce. Economists call this “externalities”.

Why does the replication argument not hold?
We are assuming that earth cannot be simply copied. There is a natural limit that also holds in the aggregate.

Externalities: Oil Example

How does a firm’s optimal behavior change between the two scenarios?



It depends. ☺

Externalities: Overfishing

"Tragedy of the commons."

Scenario: two fisheries fishing in the same lake.

How much fish should they catch? If they catch two much, the fish population collapses.

See overfishing.xlsx

$$x_{t+1} = \overbrace{r x_t (1 - x_t)}^{\text{logistic growth}} - \overset{\text{fishing}}{f}$$

Choose x to maximize f such with $x_{t+1} = x_t = x$

$$x = rx(1-x) - f$$

$$\Rightarrow f(x) = rx(1-x) - x$$

$$f'(x) = r(1-x) + rx(-1) - 1 = 0$$

$$\Rightarrow r - rx = 1 + rx \quad \Rightarrow 2rx = r - 1 \quad \Rightarrow x^* = \frac{r-1}{2r}$$

Problem: the agent which cares the least about the future gets the most fish!

Externalities: Overfishing

How to address externalities?

Generally, the problem of externalities can be solved by “internalizing” them, i.e. making sure that the one who causes the externality also bears its costs.

- Private ownership: if the profit-maximizing fisheries are rational, it is in their best interest to not overfish the lake. They could even agree to certain quotas in a private contract (looks like a “cartel”, but here it is in the best interest of the public).

  Potential problem 1: irrational short-term profit maximization (manager optimizing his bonus). Market failure due to agency problem.
  Potential problem 2: rational short-term profit maximization (weak property rights due to political uncertainty or limited term fishing licenses).
- Regulation: fishing licenses and quotas. This is how it is often done.
- Bad idea: limited time fishing license with no strings attached.

Capital: household

- Chapter 3 in “Economic foundations for finance” by Thorsten Hens and Sabine Elmiger
- Households’ decision problem does not change, as it is the firm that accumulate capital
- To the extent firms can use capital productively, interest rate r gets higher.

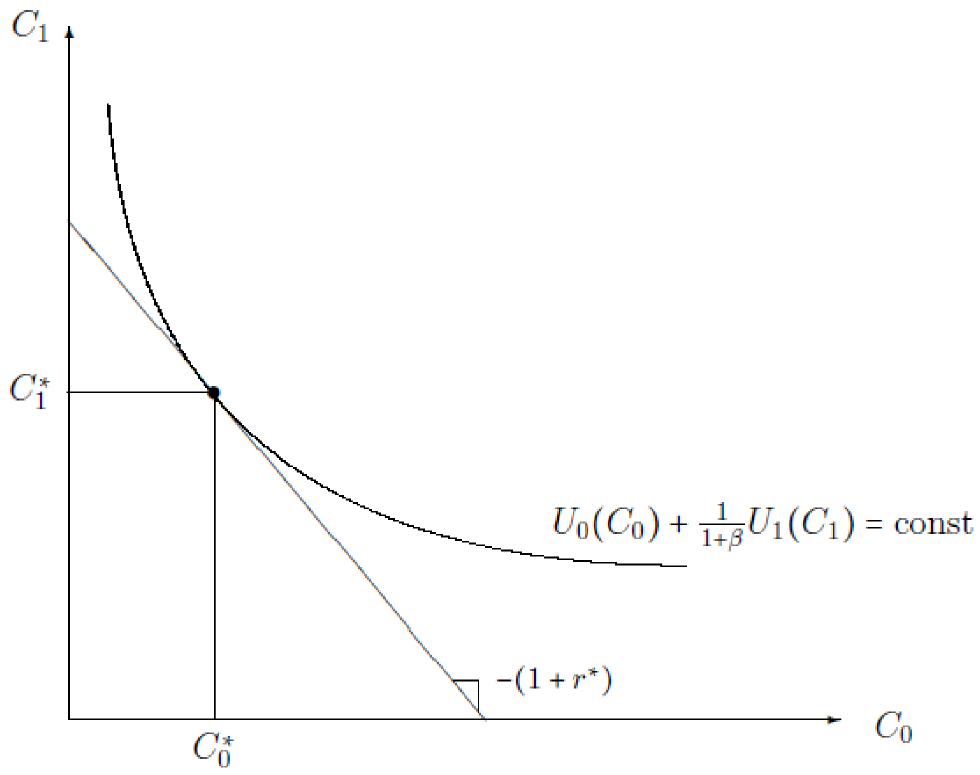


Figure 3.3: **Decision problem of the household.** Optimal consumption allocation of the household. The budget constraint is a line with slope $-(1+r^*)$. In the optimal point, the slope of the indifference curve is $-(1+r^*)$ as well, i.e., equation 3.1 holds.

From: Economic Foundations of Finance

Capital: firm

- In this chart, the capital good and the consumption good are the same
- Generally: firms need to decide how much to spend on production today versus how much to spend on accumulating capital

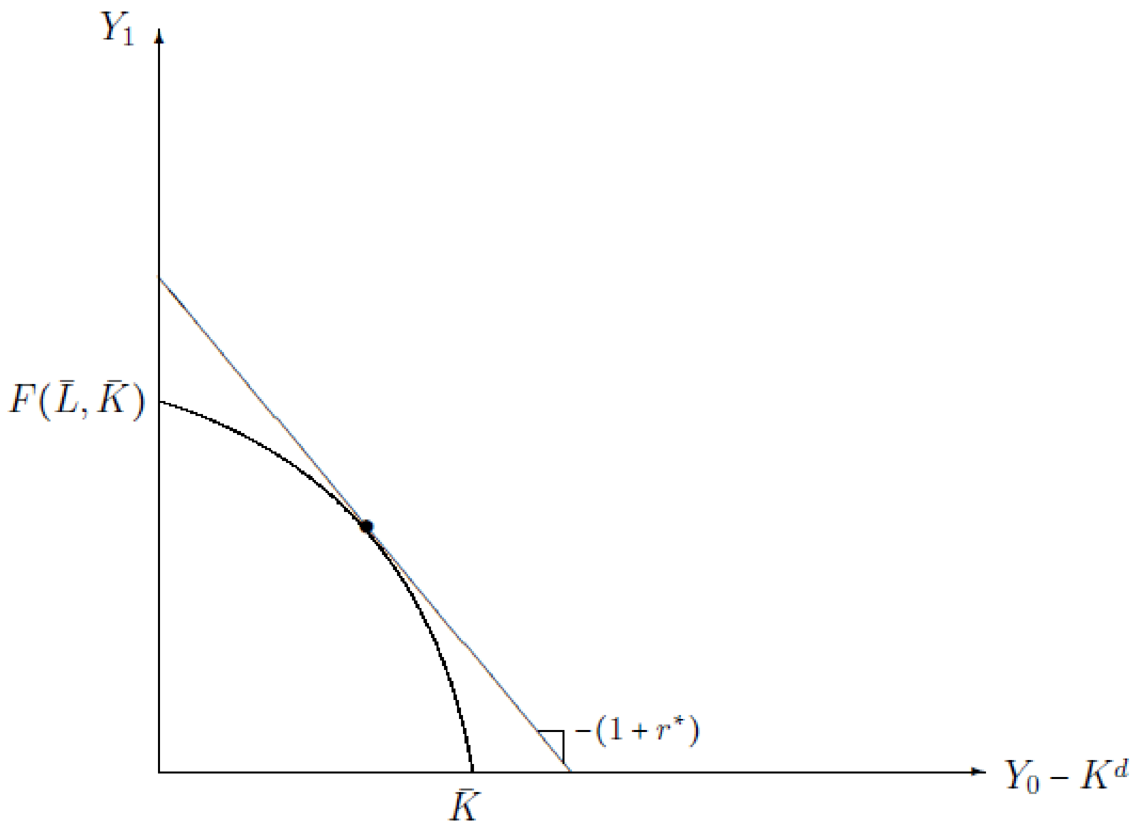


Figure 3.4: **Decision problem of the firm.** Optimal investments of the firm.

From: Economic Foundations of Finance

Capital: combined

Combining the two curves reveals the efficient market equilibrium where:

C1 = Y1

C0 = (Y0 – I)

C is consumption

Y is output

I is investment

r is the interest rate

What is the interest rate in our simulation?

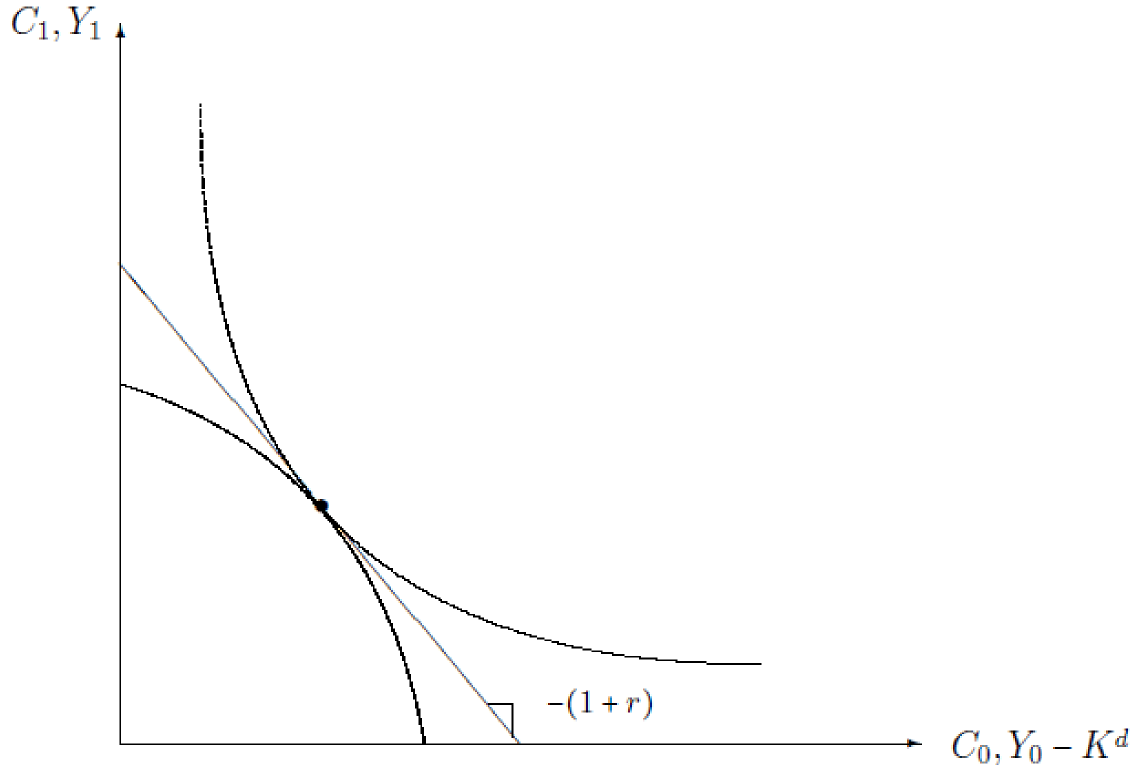


Figure 3.5: **Market equilibrium.** Graphical illustration of the First Welfare Theorem. For a detailed explanation see Box 3.4.

From: Economic Foundations of Finance

Capital: Euler Equation

Optimal Capital Allocation: aggregate view

production is a function of capital

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{with} \quad c_t = \overbrace{y_t}^{\text{production}} - \underbrace{i_t}_{\text{investments}} = f(k_t) - \underbrace{(k_{t+1} - \delta k_t)}$$

investment is the difference between capital tomorrow and depreciated existing capital

$$\Rightarrow u(c_t) = u(f(k_t) - (k_{t+1} - \delta k_t))$$

$$\frac{dU}{dk_T} = \frac{d(\beta^{T-1} u(f(k_{T-1}) - (k_T - \delta k_{T+1})) + \beta^T u(f(k_T) - (k_{T+1} - \delta k_T))}{dk_T}$$

take the derivative for a specific day T

there are actually only two terms in $\sum u(c_t)$ that contain k_T

$$\frac{dU}{dk_T} = \beta^{T-1} u'(c_{T-1})(-1) + \beta^T u'(c_T)(f'(k_T) + \delta) \overset{!}{=} 0$$

$$\Rightarrow \beta(f'(k_T) + \delta) = \frac{u'(c_{T-1})}{u'(c_T)} \Leftarrow \text{Euler equation}$$

Capital: Euler Equation

Simple case: no discounting $\beta = 1$ and no depreciation $\delta = 1$:

$$f'(k_T) + 1 = \frac{u'(c_{T-1})}{u'(c_T)}$$

Under Inada conditions (e.g. log utility): marginal utility tomorrow is lower than today $\Rightarrow$ consume more tomorrow than today.

With log utility: $u'(c) = \frac{1}{c}$

$$\Rightarrow f'(k_T) + 1 = \frac{c_T}{c_{T-1}}$$

$\Rightarrow$ if increasing capital by one unit increases production by 3%, you should plan to consume 3% more tomorrow.

$$\underbrace{\beta(f'(k_T) + \delta)}_{=1 \text{ if there is a steady-state equilibrium and capital does not go to infinity}} = \frac{c_T}{c_{T-1}}$$

$\Rightarrow$ Steady-state capital level: $f'(k^*) = \frac{1}{\beta} - \delta$

Course Outlook

November 10^{th}: testing, last exercise: market maker

November 17^{th}: presentation setup, the three agent types

November 24^{th}: control theory, learning

December 1^{st}: leverage

December 8^{th}: starting at 14:00, presentations: land developer, team 7, team 5
team 1, team 10 on investment fund

December 15^{th}: presentations: farm with capital, team 3, team 2

December $22^{nd:}$: blockchain talk

(Dropped topic: endogenous technology.
Maybe topics: heterogeneous discount rates)

Final Exercise – Land Developer

- Acts as a market maker for land, i.e. as a real estate agent
- “Produces” new land with a production function
- Production function has a memory:

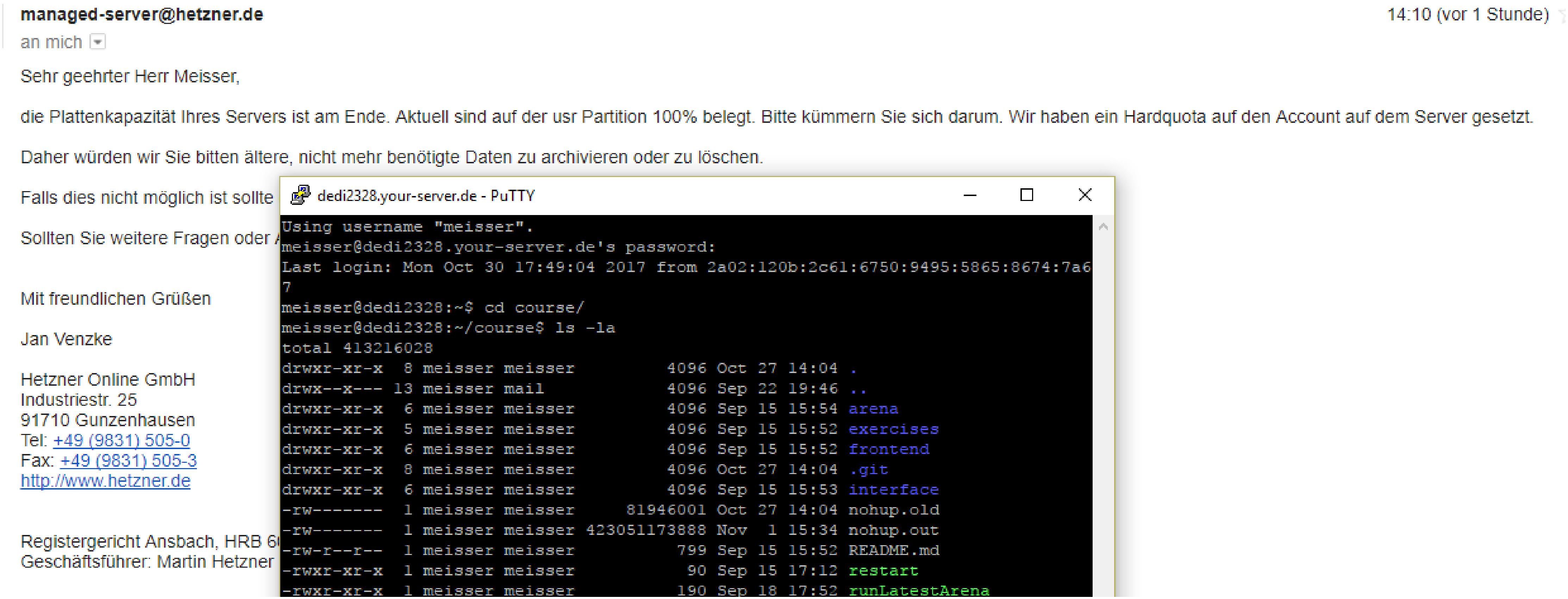
Final Exercise – Farm with Land

- Like before, but can now buy additional land
- How much should it spend on land?
- When should it sell it?

Final Exercise – Investment Fund

- Should be able to beat the average consumer at investing
- If it does so and thus can afford to pay a higher dividend than everyone else, consumers will start to buy its shares
- If successful, it ends up owning everything ☺
- Should raise capital when its shares are overvalued
- Should buy back its own shares when undervalued

Administrative Note



managed-server@hetzner.de 14:10 (vor 1 Stunde)

an mich

Sehr geehrter Herr Meisser,

die Plattenkapazität Ihres Servers ist am Ende. Aktuell sind auf der usr Partition 100% belegt. Bitte kümmern Sie sich darum. Wir haben ein Hardquota auf den Account auf dem Server gesetzt.

Daher würden wir Sie bitten ältere, nicht mehr benötigte Daten zu archivieren oder zu löschen.

Falls dies nicht möglich ist sollte

Sollten Sie weitere Fragen oder

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Geschäftsführer: Martin Hetzner

dedi2328.your-server.de - PuTTY

```
Using username "meisser".
meisser@dedi2328.your-server.de's password:
Last login: Mon Oct 30 17:49:04 2017 from 2a02:120b:2c61:6750:9495:5865:8674:7a6
7
meisser@dedi2328:~$ cd course/
meisser@dedi2328:~/course$ ls -la
total 413216028
drwxr-xr-x  8 meisser meisser         4096 Oct 27 14:04 .
drwx--x--- 13 meisser mail            4096 Sep 22 19:46 ..
drwxr-xr-x  6 meisser meisser         4096 Sep 15 15:54 arena
drwxr-xr-x  5 meisser meisser         4096 Sep 15 15:52 exercises
drwxr-xr-x  6 meisser meisser         4096 Sep 15 15:52 frontend
drwxr-xr-x  8 meisser meisser         4096 Oct 27 14:04 .git
drwxr-xr-x  6 meisser meisser         4096 Sep 15 15:53 interface
-rw-------  1 meisser meisser     81946001 Oct 27 14:04 nohup.old
-rw-------  1 meisser meisser 423051173888 Nov  1 15:34 nohup.out
-rw-r--r--  1 meisser meisser          799 Sep 15 15:52 README.md
-rwxr-xr-x  1 meisser meisser           90 Sep 15 17:12 restart
-rwxr-xr-x  1 meisser meisser          190 Sep 18 17:52 runLatestArena
```

→ Please remove all “System.out” statements from your Code before pushing it to github!

→ Also, it would be great if you could remove them from your old agents.