



University of
Zurich ^{UZH}

Agent-based Financial Economics

Lesson 5: Stocks

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“What I cannot create, I do not understand.”

- Richard Feynman

Today

- Discussion of exercise 4
- Stocks
- Market making
- The Bancor protocol
- Exercise 5: stocks



Exercise 4 - Ranking

Rank	Agent	Utility	Source	Version
1	team005-Farmer (capitalist)	8.30201613593182	source	julsto93 on 2017-10-19T17:45:49Z
2	team001-Farmer (capitalist)	8.284380658174962	source	yzioer on 2017-10-19T18:40:07Z
3	team007-Farmer (capitalist)	8.11524640251214	source	LWNHRZ on 2017-10-19T21:20:23Z
4	team003-Farmer (capitalist)	8.092664685090991	source	DESKTOP-CRT5F4J\\Nico-PC on 2017-10-19T20:01:41Z
5	team010-Farmer (capitalist)	8.080064519428033	source	Robin Stohler on 2017-10-19T21:32:11Z
6	team002-Farmer (capitalist)	8.055526470789205	source	paneton on 2017-10-19T20:31:22Z
7	team002-Farmer (worker)	5.311970727146201	source	paneton on 2017-10-19T20:31:22Z
8	team001-Farmer (worker)	5.255061752072779	source	yzioer on 2017-10-19T18:40:07Z
9	team007-Farmer (worker)	5.137287765420637	source	LWNHRZ on 2017-10-19T21:20:23Z
10	team005-Farmer (worker)	4.960819727111132	source	julsto93 on 2017-10-19T17:45:49Z
11	team010-Farmer (worker)	4.945600635302069	source	Robin Stohler on 2017-10-19T21:32:11Z
12	team003-Farmer (worker)	4.944675452986779	source	DESKTOP-CRT5F4J\\Nico-PC on 2017-10-19T20:01:41Z

Exercise 4 - Theory

Formal problem looks complicated, with lots of variables and unknowns...

Formal problem:

$$\max \sum_{i=1}^{500} U(x_{p,i}, h_{l,i}) \quad \text{s.t.} \quad \begin{array}{l} \text{leisure time} \quad \text{work time} \\ h_{l,i} + h_{w,i} = 24 \quad \text{and} \quad h_{w,i} = 0 \quad \forall i > 400 \\ \text{potatoe consumption} \end{array}$$

$$\underbrace{\sum_{i=1}^{400} h_{w,i} \cdot p_{h,i}}_{\text{life-time income } w} + \underbrace{\sum_{i=1}^{500} d_i}_{\text{life-time spending}} \geq \underbrace{\sum_{i=1}^{500} p_{p,i} \cdot x_{p,i}}_{\text{life-time spending}}$$

$$\sum_{i=1}^k h_{w,i} p_{h,i} + \sum_{i=1}^k d_i \geq \sum_{i=1}^k p_{p,i} \cdot x_{p,i} \quad \forall k$$

Negative balances not allowed.

That's why I suggested to assume constant prices and wages, reducing the problem to:

$$\max \sum_{i=1}^{500} u(x_{p,i}) \quad \text{subject to the budget constraint} \quad \sum_{i=1}^{500} p x_{p,i} = \sum_{i=1}^{400} w_i = 400w \quad (\text{without dividends for now})$$

→ You will find that it is optimal to smooth consumption, and to consume the same number of potatoes every day. But what if prices can change?

Exercise 4 - Theory

Still assuming a given total life-time income W , how do we optimally allocate our budget when prices change?

→ Distribute budget evenly, spend the same amount every day.

Income effect: when potatoes are cheap, I should buy more of them.

Substitution effect: when potatoes are cheap, I save money by spending less on them.

With log-utility, these two effects cancel out and the spending stays the same, regardless of the price.

$$\begin{aligned} \max \sum U(x_{p,i}) \quad \text{s.t.} \quad \sum x_{p,i} p_{p,i} &= W \\ &\quad \text{life-time income/wealth} \\ &= \sum w_i \\ &\quad \text{wealth spent on day } i \end{aligned}$$

$$\max \sum \log \frac{w_i}{p_i} \quad \text{s.t.} \quad \sum w_i = W$$

Observe that $\log \frac{w_i}{p_i} = \log w_i - \underbrace{\log p_i}_{\text{constant?}} \Rightarrow \text{prices have no influence on optimal } w_i$

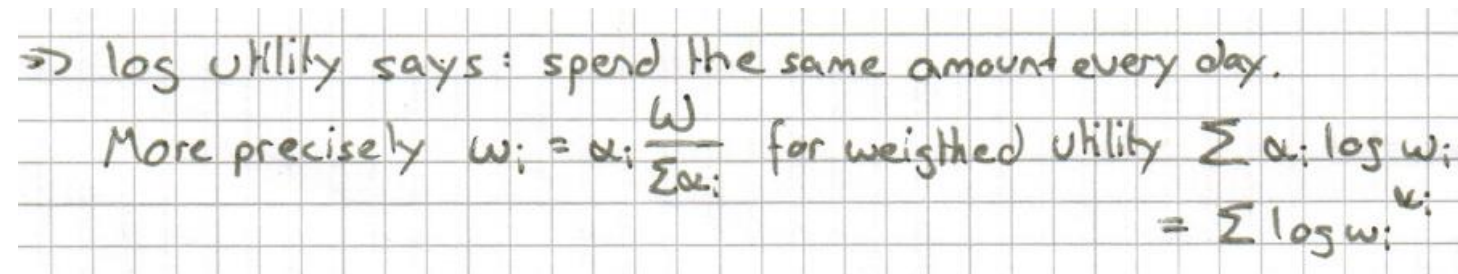
$$\begin{aligned} \max \sum \log \frac{w_i}{p_i} &= \max \sum \log w_i - \sum \log p_i \quad \text{s.t.} \quad \sum w_i = W \\ &= \max \log \underbrace{\left(W - \sum_{i=2}^{500} w_i \right)}_{w_1} + \sum_{i=2}^{500} \log w_i - \sum \log p_i \end{aligned}$$

$$\frac{d(\dots)}{dw_i} = \frac{-1}{W - \sum_{j=2}^{500} w_j} + \frac{1}{w_i} \stackrel{!}{=} 0$$

$$\Rightarrow w_i = W - \sum_{j=2}^{500} w_j = w_1 \Rightarrow \text{all } w_i \text{ are the same!}$$

Exercise 4 – Theory (side remark)

More precisely: with log utility, optimality implies spending according to the weights given in the utility function. In ours, all the weights are 1, so it does not matter. But more generally, it looks like this:



⇒ log utility says: spend the same amount every day.
More precisely $w_i = \alpha_i \frac{w}{\sum \alpha_i}$ for weighted utility $\sum \alpha_i \log w_i$
 $= \sum \log w_i^{\alpha_i}$

Exercise 4 – Theory: But what about interest?

Adding interest rates does not change anything either.

The income effect tells me: “Save money today, so you can spend even more on potatoes tomorrow.”

The substitution effect tells me: “You can spend more today, thanks to interest your money will grow back.”

→ Both effects cancel out, and I still decide to spend the same amount today.

(More precisely, if I previously spent 100 on day one and 100 on day two, introducing an interest rate of 10% does not affect my spending on day one, but I will spend 110 on day two.)

What about interest?

Consider simple two-period model

$$U(x_1, x_2) = \log x_1 + \log \overbrace{(1+r)x_2}^{\text{interest}}$$
$$\text{s.t. } x_1 + x_2 = W$$
$$\Rightarrow U(x_1, x_2) = \log x_1 + \log x_2 + \underbrace{\log(1+r)}_{\text{constant?}}$$

⇒ still spend the same amount on day 1.

Exercise 4 – Retiree decision heuristic

These considerations lead us to a very simple, but also very effective decision heuristic for retirees:

Simply spend $1/d$ of your wealth today if you have d days left to live.

This heuristic is robust against:

- Nominal and real price changes
- Inflation / deflation
- Changes in nominal and real interest rate
- Dividends (work like interests), when stocks can be sold

Caveats:

- It only works so nicely thanks to assuming log-utility.
- It is not entirely correct when stocks cannot be sold.
- It is only efficient if the stocks have the right price on the stock market (discounted value of all future dividends).

Exercise 4 – Retiree decision heuristic

Thus, the implementation for the retiree could look as follows:

```
@Override
public void managePortfolio(IStockMarket stocks) {
    boolean retired = isRetired();
    if (retired) {
        int daysLeft = getMaxAge() - getAge() + 1;
        double proceeds = getPortfolio().sell(stocks, this, 1.0d / daysLeft);
        listeners.notifyDivested(this, proceeds); // notify listeners for statistics
    } else {
```

For shareholders that cannot sell their shares, this is not perfectly correct.

In theory, future dividends should be discounted with the interest rate. At the same time, dividends are growing in magnitude. In our configuration, both effects cancel each other out, so we can ignore them for now.

Exercise 4 – Worker decision heuristic

In order to spend the same amount every day, about 1/5 of the daily work income needs to be saved, and 4/5 can be spent on potatoes. In other words: if daily spendings ignoring dividends are 100, an amount of 25 should go into savings.

```
@Override
public void managePortfolio(IStockMarket stocks) {
    boolean retired = isRetired();
    if (retired) {
        int daysLeft = getMaxAge() - getAge() + 1;
        double proceeds = getPortfolio().sell(stocks, this, 1.0d / daysLeft);
        listeners.notifyDivested(this, proceeds); // notify listeners for statistics
    } else {
        double dividends = getPortfolio().getLatestDividendIncome(); // how much dividends did we get today?
        double workFraction = 1.0d / getMaxAge() * getRetirementAge(); // 80%
        double retirementFraction = 1 - workFraction; // 20%
        double toInvest = (getDailySpendings() - dividends) / workFraction * retirementFraction;
        double actualInvestment = getPortfolio().invest(stocks, this, toInvest);
        listeners.notifyInvested(this, actualInvestment); // notify listeners for statistics
    }
}
```

Exercise 4 – Theory: worker

But should workers work harder to save for retirement?

I.e. should a worker that has maxAge=400 work less than one with maxAge=500?

→ Yes. The longer you spend in retirement, the harder you should work to accumulate retirement savings.

Nicely, this happens automatically in the trade function when hiding the right amount.

What about income? Should I work more to have more as retiree?
I.e. should I work harder when young if I spend more time in retirement?

Yes!

Consider
$$\sum_{i=1}^{500} U(x_p, h_t) = \sum_{i=1}^{500} \log x_p + \sum_{i=1}^{400} \log h_t + \underbrace{\sum_{i=401}^{500} \log 24}_{\text{constant}}$$

$$= 500 \log x_p + 400 \log h_t$$

⇒ looks like weighted utility function

Before: work $\frac{1}{2} \cdot 24 = 12$ hours per day (if no dividends)

Now: work $\frac{5}{9} 24 = 13\frac{1}{3}$ hours per day to have more in retirement (about 10% more)

Should you adjust the heuristic accordingly?

No need to, "trade" method is smart enough to do this automatically when setting aside (inv.hide(...)) the right amount.

Exercise 4 – Price changes

When solving the exercise, we assumed constant prices and wages.

What if this does not hold?

What about changing wages?

Two-period example: $\max \log x_1 + \log(24-h_1) + \log x_2 + \log(24-h_2)$
 s.t. $\underbrace{p_{p,1}}_{\text{potatoes on day 1}} x_1 + \underbrace{p_{p,2}}_{\text{work on day 1}} x_2 = d + p_{h,1} h_1 + p_{h,2} h_2$

Lagrangian:

$$L = \log x_1 + \log x_2 + \log(24-h_1) + \log(24-h_2) + \lambda(d + p_{h,1} h_1 + p_{h,2} h_2 - x_1 - p_{p,2} x_2)$$

$$\frac{\partial L}{\partial x_1} = \frac{1}{x_1} - \lambda \stackrel{!}{=} 0 \quad \Rightarrow \quad \lambda = \frac{1}{x_1}$$

$$\frac{\partial L}{\partial x_2} = \frac{1}{x_2} - \lambda p_{p,2} = 0 \quad \Rightarrow \quad \frac{1}{x_2} = \frac{1}{x_1} p_{p,2} \Rightarrow x_1 = x_2 p_{p,2}$$

spend same amount on both days

$$\frac{\partial L}{\partial h_1} = \frac{-1}{24-h_1} + \lambda p_{h,1} = 0 \quad \Rightarrow \quad \frac{1}{x_1} p_{h,1} = \frac{1}{24-h_1} \Rightarrow (24-h_1) p_{h,1} = x_1$$

$$\frac{\partial L}{\partial h_2} = \frac{-1}{24-h_2} + \lambda p_{h,2} = 0 \quad \Rightarrow \quad (24-h_2) p_{h,2} = x_1 = (24-h_1) p_{h,1}$$

("spend" same amount on leisure time)

Exercise 4 – Price changes

$$\Rightarrow 24p_{h,2} - h_2p_{h,2} = 24p_{h,1} - h_1p_{h,1}$$

$$\Rightarrow h_1p_{h,1} = 24p_{h,1} - 24p_{h,2} + h_2p_{h,2}$$

$$\Rightarrow h_1 = (24p_{h,1} - 24p_{h,2} + h_2p_{h,2})/p_{h,1} = 24 - (24 - h_2)\frac{p_{h,2}}{p_{h,1}}$$

\Rightarrow Work more on the day with the higher wage.

Or: invest less in leisure time when it is more expensive.

In a static world with constant productivity and no growth, it is optimal to work the same amount every day.

$\Rightarrow p_{h,2} \neq p_{h,1}$. If central bank manages price poorly, people work too much or not enough on the wrong days.

- \rightarrow Unlike the potatoe price, which did not affect savings at all, the wage can make a difference.
- \rightarrow A rational consumer works harder when he gets a higher salary.
- \rightarrow The efficient outcome is only reached if the central bank prints the right amount of money.

Exercise 4 – Open questions

There are a number of interesting questions that we will skip in this course. However, I still want you to know that they exist:

- Correct estimation and discounting of future income. Affects savings schedule.
 - Calculation of the efficient equilibrium. This can get hairy when it is unclear how to aggregate the agents.
 - How should the central bank set its interest rate to attain the efficient equilibrium? What is the right interest rate?
 - If there are overlapping generations: how should their utilities be weighted relative to each other? If we can make 100 young better off by making 50 old worse off, should we do so?
 - What's the effect of a pay-as-you-go (AHV-like) pension system financed by taxing salaries?
- Lots of interesting question. We move on to the stock market.

Stocks

- So far: no stock market, farms passed on to successor after death
 - Similar to feudalism
 - New: allow trading of stocks
 - Savings should now be invested into stocks instead of money
 - And stocks should all be sold over the course of retirement
- > Completely different dynamics.



Figure 1.2: **Fragmentation of farmland in Laxton.** Allocation made in order to give a fraction of each type of land to each family. Reprinted from *The Open Fields*, by C. S. and C. S. Orwin, 1954, Oxford Clarendon Press.

Terms 1/3

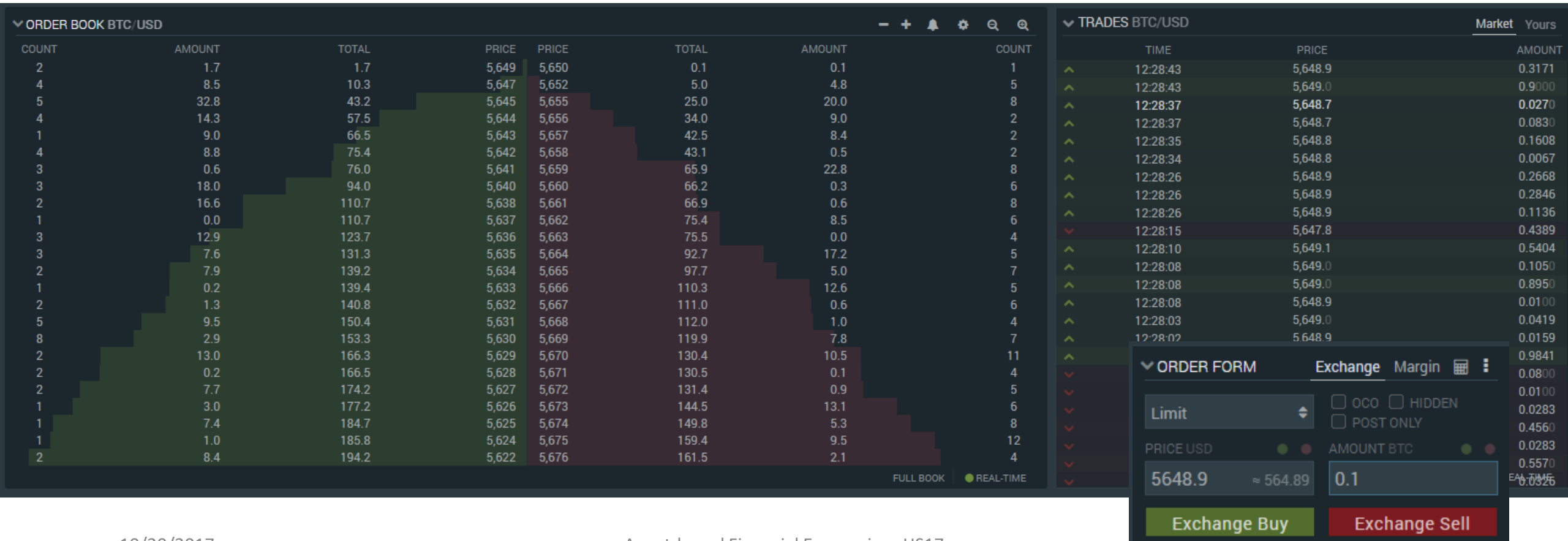
- Stocks, shares: units of account to keep track of who owns which percentage of a company. In our simulation, each company has exactly 100 shares. So 1 share represents 1% of the company. Unlike in reality, our shares are infinitely divisible.
- Dividends: money that is paid from the firm to its shareholders, in amounts proportional to their holdings.
- Dividend yield: the “interest rate” of stocks. $\text{Dividend yield} = (\text{dividends per share}) / (\text{price per share})$
- Raising capital: creating and selling new shares, with the proceeds going to the firm.
- Outstanding shares: all the shares that are not owned by the company itself.
- Market capitalization: the total value of a firm, calculated by multiplying the number of outstanding shares with their price.
- Bankruptcy: a firm deciding to stop its business. The leftovers are distributed among the shareholders.

Terms 2/3

- Position: a number of shares owned by a specific investor. I.e. “I have a large position in Nestlé.”
- Portfolio: a collection of positions. Ideally, the positions in a portfolio are chosen such that they complement each other, such that the portfolio as a whole has a better risk-reward structure than each position on its own.
- Portfolio theory: the art/science of managing portfolios.
- Diversification: gets you better returns for the same “risk budget”. One of the very few things you get for free when investing.
- Order book: stock markets keep a book with all open orders for each stock.
- Bid (Geld): an order to buy shares
- Ask (Brief): an order to sell shares
- Spread: the difference between the highest bid and the lowest ask.
- Limit order: an order to buy for at most X \$ per share, or to sell for at least Y \$ per share.
- Market order: a bid or ask without limit, to be executed immediately regardless of the price.
- Liquidity: the extent to which shares can be bought and sold without moving the price much

Order book example

Bitfinex order book. Knowing the pending order allows to estimate the price impact of a larger trade. However, there are also “hidden” orders that don’t show up in the order book.

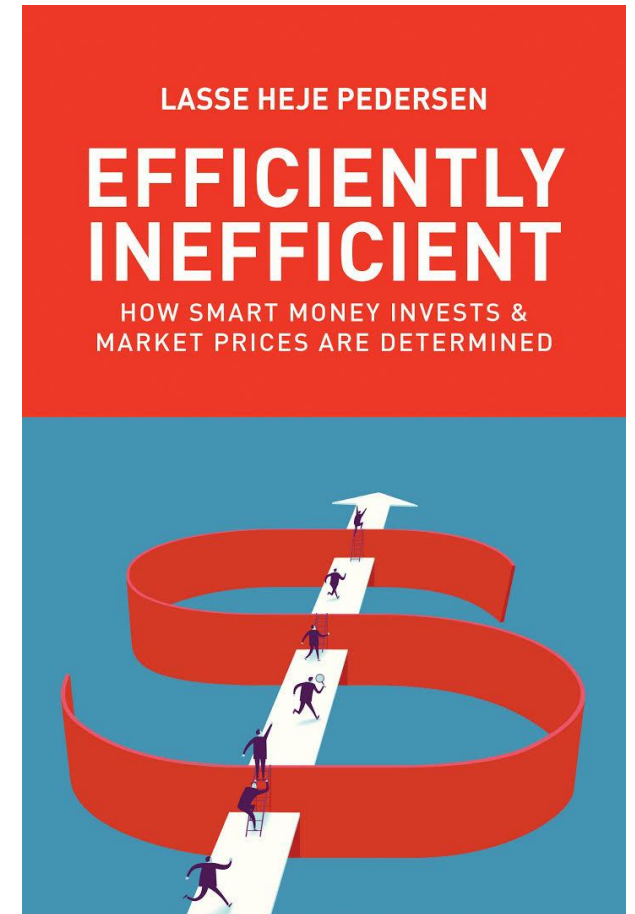


Terms 3/3

- Fund: a firm that owns a portfolio and has no “real” productive business.
- Index fund: a fund that manages its portfolio such that it reflects the whole market.
- Passive strategy: a portfolio management strategy that follows strict, mechanic rules and often only requires a minimal amount of trading.
- Active strategy: a portfolio management strategy in which one actively seeks to exploit market inefficiencies. This only works to the extent markets are not efficient and the manager succeeds in identifying the resulting opportunities.
- Efficient market hypothesis: the hypothesis that markets are efficient and thus prices always “right”. The only valid source of price changes is the arrival of new information, e.g. about a particular firm or the economy. There is no room for behavioral errors.

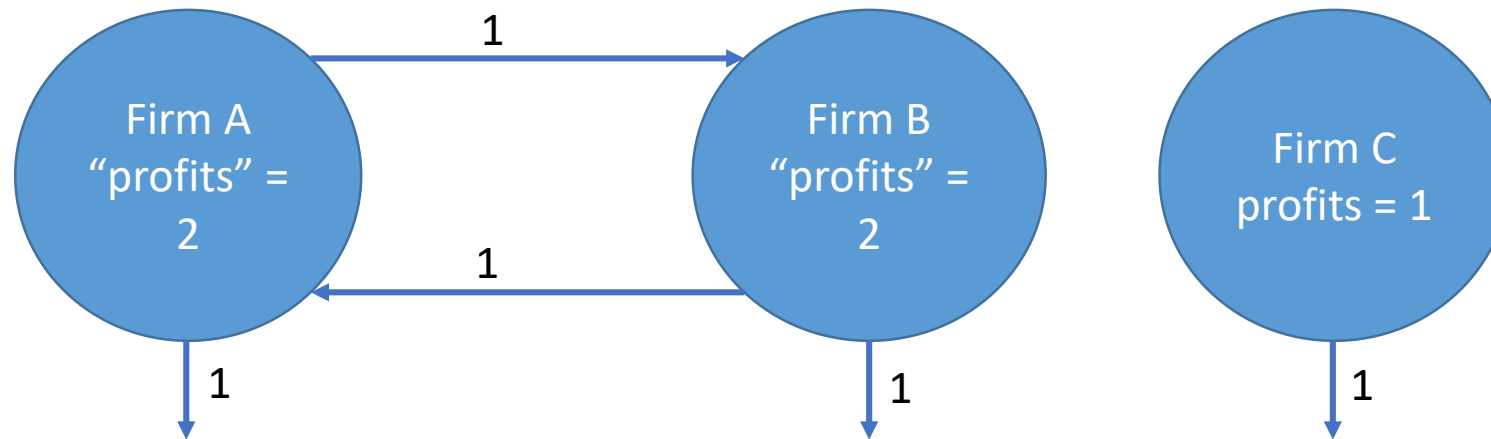
Interesting paper: Grossman, S.J. and Stiglitz, J.E., 1980. On the impossibility of informationally efficient markets. *The American economic review*, 70(3), pp.393-408.

It says that markets cannot be fully efficient when it is costly to obtain information about what prices are right. Most up to date book on the topic: Efficiently Inefficient by Pedersen.



What should firms maximize?

- Usual assumption: firms maximize real profits
- In our model so far: firms maximize nominal profits
(This should be equivalent to maximizing real profits as long as the central bank does its job well.)
- However, consider the following setup:



All firms make 1\$ profits from real business per year, but A and B own 50% of each other, so they can add the dividends they receive from each other to their profits. When the flows are in equilibrium, the A and B have doubled their profits in comparison to C!

→ We should rank firms not by the profits they made, but by the dividends they paid to consumers!

What should firms maximize?

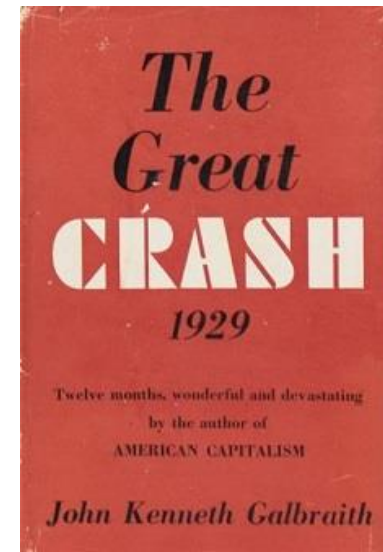
→ We should rank firms not by the profits they made, but by the dividends they paid to consumers!

In economic theory, we do not do that, and we do not need to as long as firms are not allowed to buy each other's shares.

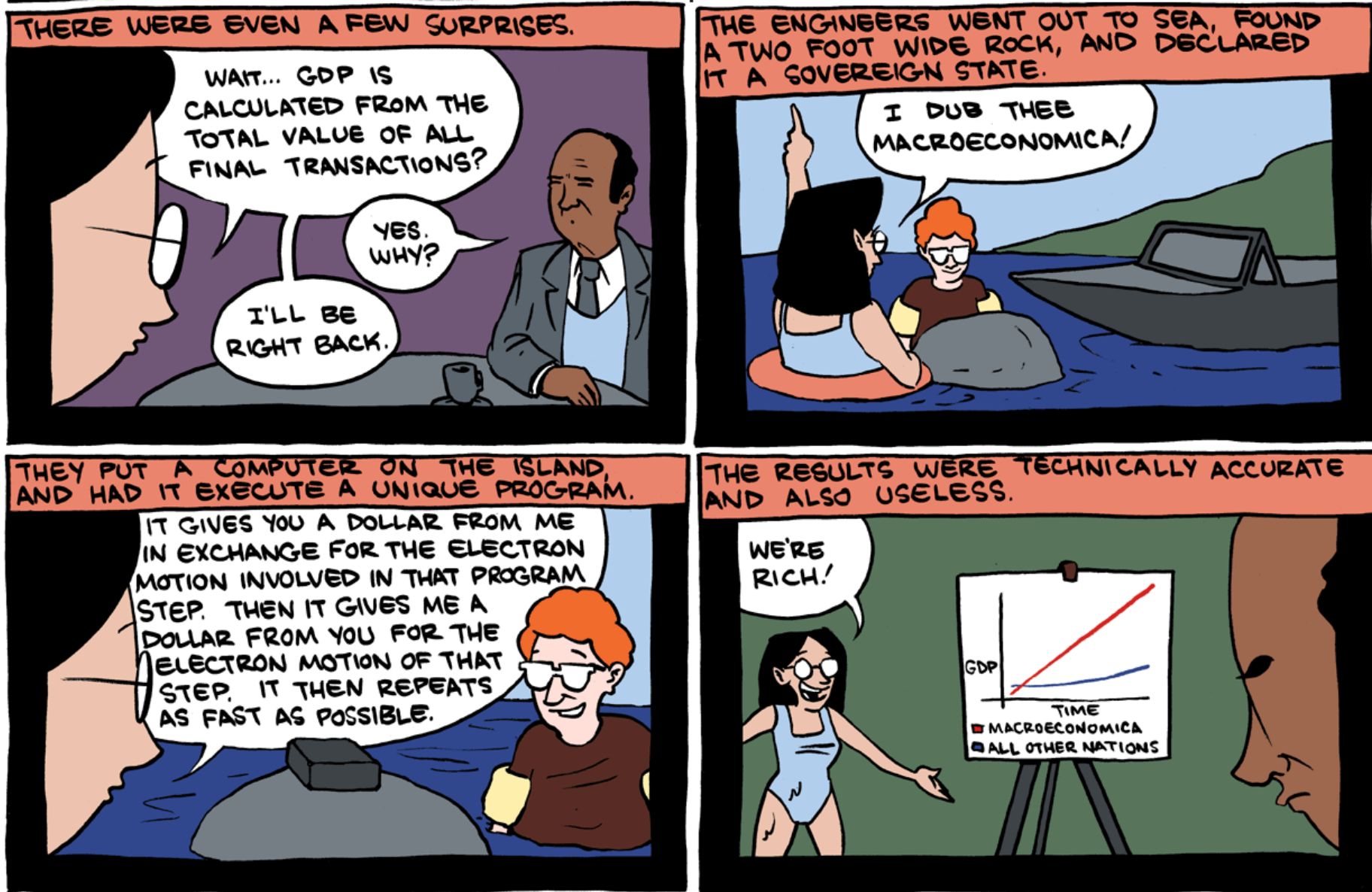
In reality, we do not do that either, leading to double accounting and contributing to the bubble and crash of 1929.

Third quarter ^{[12][note 1]}	
	Apple Inc. ▲791,726
	Alphabet Inc. ▲664,550 ^[13]
	Microsoft ▲568,965
	Amazon.com ▼459,435
	Berkshire Hathaway ▲451,840 ^[14]

Most of the market cap of Berkshire Hathaway, the 5th largest public firm in the world, comes from owning stocks of other listed companies, e.g. Coca Cola. So a part of Coca Cola's value is counted twice when calculating the total value of all US stocks.



As Galbraith describes, the bubble and the crash of 1929 was partially driven by funds buying each other's shares, creating an illusion of value that quickly deflated once the funds were forced to sell and sold each other's shares again, forcing them to sell even more.




Market Making

The problem: individual investors often just want to buy and sell stocks, not having to wait until another investor appears for a counter trade.

Solution: market makers temporarily buy and sell stocks, bridging the needs of the other investors and providing liquidity.

By buying stocks without knowing at what price they can sell them again, market makers take a small risk. They are compensated for that risk by earning some money from the spread. The larger the spread, the more money they make by buying and selling a stock again.

Many high frequency trading firms do or claim to do market making, e.g. IMC.

A screenshot of the IMC Europe website. The header features the IMC logo (a grid of blue and green squares) and the text "imc | EUROPE". To the right are two blue navigation buttons labeled "WHAT WE DO" and "CAREERS". The main content area has a light gray background and contains a paragraph of text about IMC's history and global operations, with horizontal lines separating the sentences.

imc | EUROPE

WHAT WE DO CAREERS

For more than 25 years, IMC has set the pace for the evolution of market making. When we launched in 1989 in Amsterdam we were quick to recognise the role of technology in making markets more efficient. Today we operate globally from offices in Europe, the US and Asia Pacific. And we continue to invest and grow.

The “International Marketmaking Company” describing itself.

Market Making

Problem with market making today: “penny stocks” (firms with a low market capitalization) often are not very liquid, with daily trading volume in the millions.

For example, Swiss electrician Burkhalter only traded 4445 stocks today by lunch time, worth about 500k CHF.

How can you get rid of your stocks if you own shares worth 5 million?

Takes a long time.

Regulation that aimed at making the markets safer by restricting banks from trading for their own accounts made markets for such stocks even less liquid than they used to be! (And therefore markets more volatile and less safe.)

Datum	Zeit	Valorennummer	Veränderung (%)
20-10-2017	12:20:18	21'225'580	▲ 1.10 (0.93 %)
Volumen	Vortag	Eröffnung	Letzter
4'445	118.00	118.00	119.10 CHF
Vol. Geld	Vol. Brief	Geldkurs 12:20:26	Briefkurs 12:21:20
126	30	119.00	119.40
52 Wochen Hoch	52 Wochen Tief	Tages Hoch	Tages Tief
158.40	118.00	120.00	118.00
Titelart	Dividend	Datum (ex-Dividende)	Derivate auf
Namenaktie	5.50 CHF	24-05-2017	Call / Put / KO / Map



Market Making Innovation: Bancor

Bancor provides a blockchain-based smart contract that acts as market maker.

- That contracts holds a fraction of F of the market capitalization as a trading reserve.
- To buy shares, anyone can send money to the contract and gets newly printed shares in return
- To sell shares, anyone can send shares to the contract and receive money in return

The price is always adjusted such that the invariant F continues to hold.

github.com/meisserecon/www/blob/gh-pages/assets/abfe-lesson5-bancor.pdf



$$T = S_0 \left(\left(1 + \frac{E}{R_0} \right)^F - 1 \right)$$

E : ether (money) sent to contract

T : tokens (shares) received

F : reserve ratio

R, S : constant parameters

Exercise 5: Stock market

- Invest in the stock market instead of saving
- How to choose the right stocks?

github.com/meisser/course/blob/master/exercises/journal/exercise05-task.md

