Leverage causes fat tails and clustered volatility

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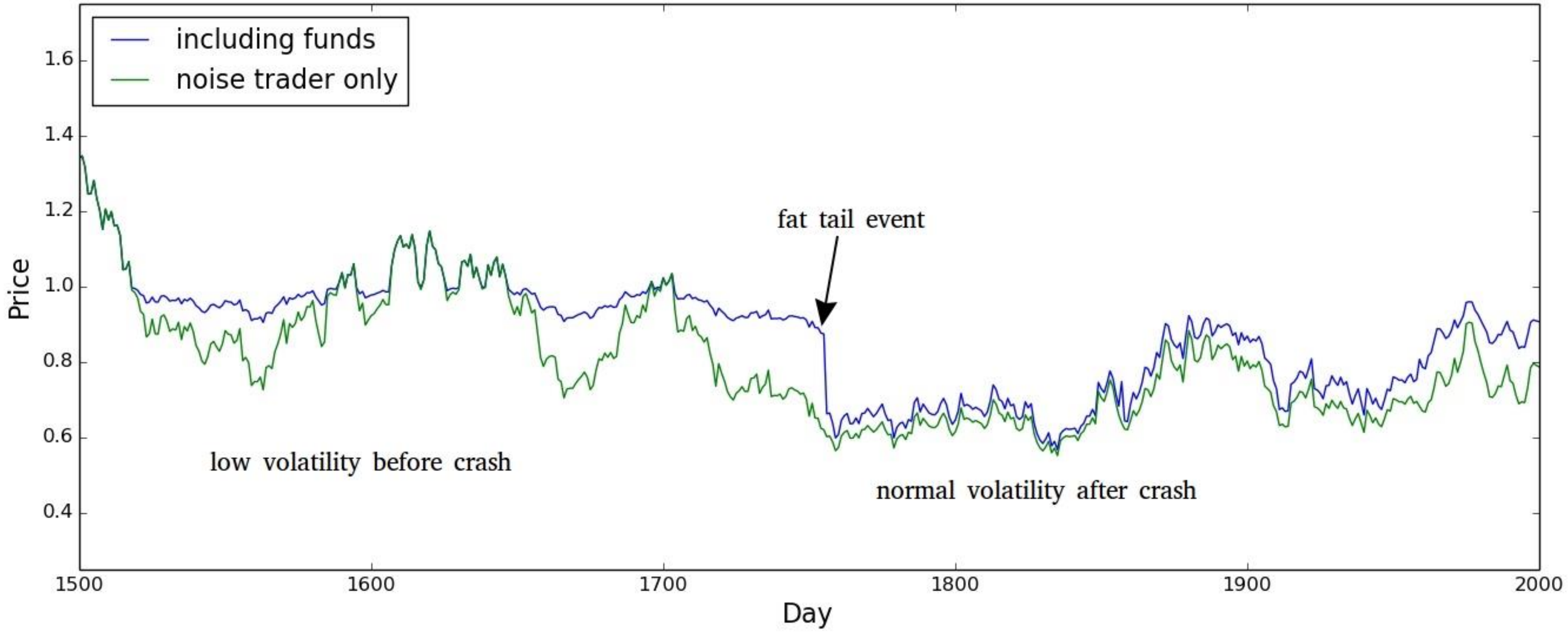
Single Slide Summary

“The market can stay irrational longer than you can stay solvent.” - Keynes

- Leverage can cause fat tail events through cascade of margin calls.
- Two active types of investors:
    - Noise traders
    - Leveraged, fundamentalist funds



Context

- Fat tails and clustered volatility are commonly observed in price fluctuations (Mandelbrot (1963), Engle (1982))
- Real-life example: LTCM (fund by Merton, Scholes and Meriwether), took long-term bets and collapsed due to short-term volatility and a cascade of margin calls. Lost billions.

- Classic literature assumes normal distribution, no endogenous fat tails. (Bachelier 1964, Black and Scholes 1973, etc.)
- Many financial market simulations create endogenous fat tails by introducing trend-followers that destabilize the market (Palmer et al. 1994, Arthur et al. 1997, Brock and Hommes 1997, Lux and Marcesi 1999, Giardina and Bouchaud 2003, etc.)
- Key achievement: no irrational trend followers, but leveraged (more rational) fundamentalists.

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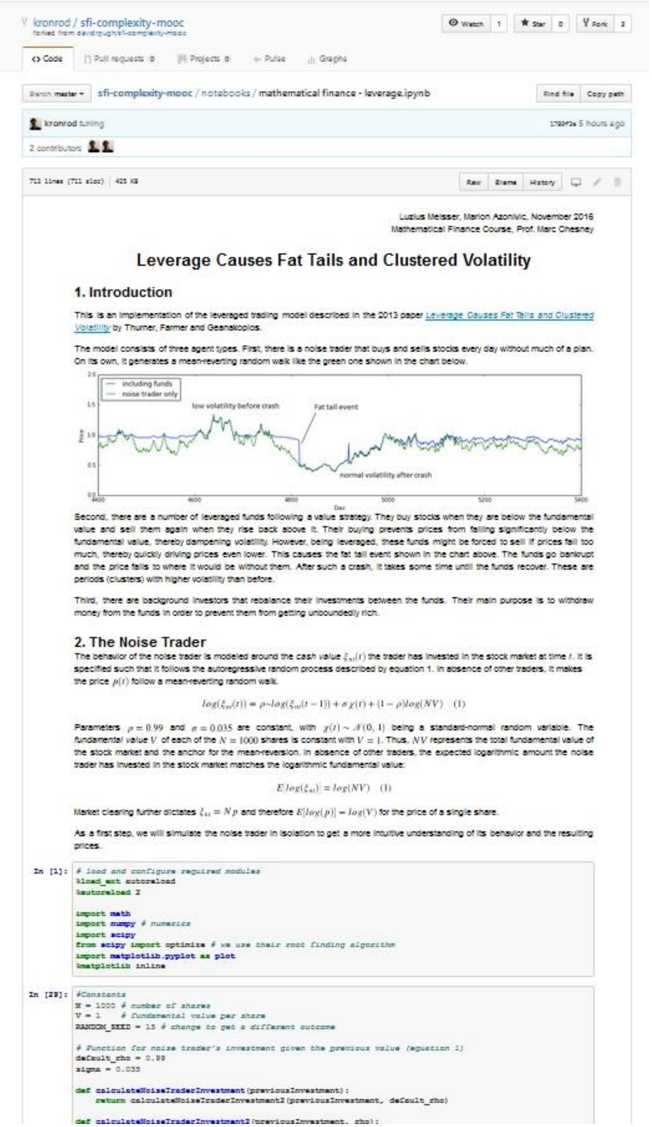
What we did

→ We reproduced the simulation in a Jupyter notebook (Python), all charts in this presentation generated by us.
→ Results of the paper confirmed
→ Found a way to simplify the model

If you ever write a paper based on a simulation: publish the source code so others can reproduce it with reasonable effort.
It also helps understanding the details. This paper, for example, is ambiguous about the exact order of events.



kronrod / sfi-complexity-mooc
forked from davidrpugh/sfi-complexity-mooc

Code | Pull requests 0 | Projects 0 | Pulse | Graphs

Branch: master ▾ sfi-complexity-mooc / notebooks / mathematical finance - leverage.ipynb

kronrod tuning — 5 hours ago

2 contributors

712 lines (711 sloc) | 425 KB

Luzius Meisser, Marlon Azonivic, November 2016
Mathematical Finance Course, Prof. Marc Chesney

Leverage Causes Fat Tails and Clustered Volatility

1. Introduction

This is an implementation of the leveraged trading model described in the 2013 paper Leverage Causes Fat Tails and Clustered Volatility by Thurner, Farmer and Geanakoplos.

The model consists of three agent types. First, there is a noise trader that buys and sells stocks every day without much of a plan. On its own, it generates a mean-reverting random walk like the green one shown in the chart below.

Second, there are a number of leveraged funds following a value strategy. They buy stocks when they are below the fundamental value and sell them again when they rise back above it. Their buying prevents prices from falling significantly below the fundamental value, thereby dampening volatility. However, being leveraged, these funds might be forced to sell if prices fall too much, thereby quickly driving prices even lower. This causes the fat tail event shown in the chart above. The funds go bankrupt and the price falls to where it would be without them. After such a crash, it takes some time until the funds recover. These are periods (clusters) with higher volatility than before.

Third, there are background investors that rebalance their investments between the funds. Their main purpose is to withdraw money from the funds in order to prevent them from getting unboundedly rich.

2. The Noise Trader

The behavior of the noise trader is modeled around the cash value $\xi_{nt}(t)$ the trader has invested in the stock market at time t. It is specified such that it follows the autoregressive random process described by equation 1. In absence of other traders, it makes the price $p(t)$ follow a mean-reverting random walk.

$$log(\xi_{nt}(t)) = \rho \cdot log(\xi_{nt}(t-1)) + \sigma\chi(t) + (1-\rho)log(NV) \quad (1)$$

Parameters $\rho = 0.99$ and $\sigma = 0.035$ are constant, with $\chi(t) \sim \mathcal{N}(0,1)$ being a standard-normal random variable. The fundamental value V of each of the $N = 1000$ shares is constant with $V = 1$. Thus, NV represents the total fundamental value of the stock market and the anchor for the mean-reversion. In absence of other traders, the expected logarithmic amount the noise trader has invested in the stock market matches the logarithmic fundamental value:

$$E[log(\xi_{nt})] = log(NV) \quad (1)$$

Market clearing further dictates $\xi_{nt} = Np$ and therefore $E[log(p)] = log(V)$ for the price of a single share.

As a first step, we will simulate the noise trader in isolation to get a more intuitive understanding of its behavior and the resulting prices.

In [1]:
```
# load and configure required modules
%load_ext autoreload
%autoreload 2

import math
import numpy # numerics
import scipy
from scipy import optimize # we use their root finding algorithm
import matplotlib.pyplot as plot
%matplotlib inline
```

In [29]:
```
#Constants
N = 1000 # number of shares
V = 1    # fundamental value per share
RANDOM_SEED = 13 # change to get a different outcome

# Function for noise trader's investment given the previous value (equation 1)
default_rho = 0.99
sigma = 0.035

def calculateNoiseTraderInvestment(previousInvestment):
    return calculateNoiseTraderInvestment2(previousInvestment, default_rho)

def calculateNoiseTraderInvestment2(previousInvestment, rho):
```

Model, Ingredient 1: Noise Trader

- Noise traders make the stock price follow a mean-reverting random walk around fundamental value V.
- Noise trader demand $\xi_{nt}(t)$ (in terms of cash invested in asset) is given by an autoregressive process of order 1:

$$\log(\xi_{nt}(t)) = \rho\log(\xi_{nt}(t-1)) + (1-\rho)\log(VN) + \sigma_x$$

where:

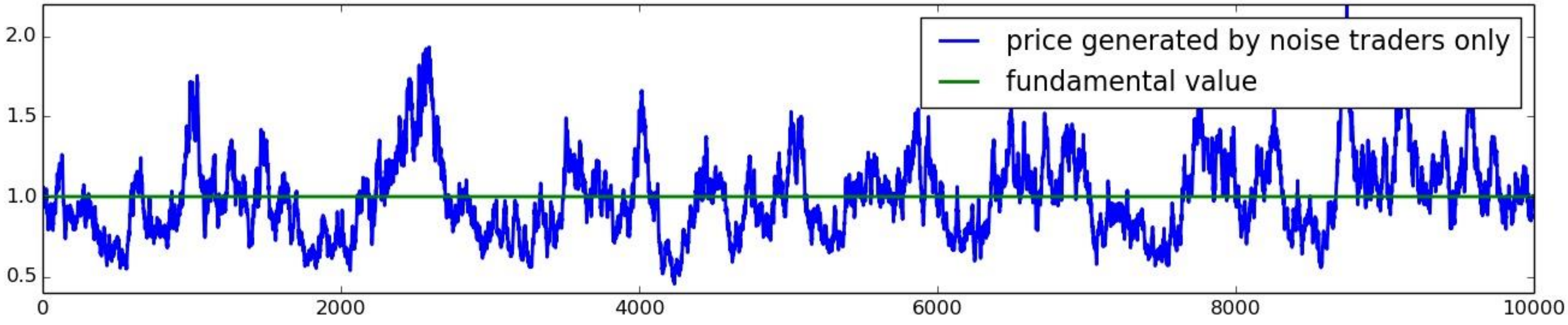
- $V = 1$: fundamental value
- $\rho = 0.99$: constant to be mean reverting
- $N = 1000$: number of shares
- $t \geq 1$: time period, discrete
- $\sigma_x \sim N(0,0.035)$: normally distributed random variable

Model, Ingredient 1: Noise Trader

Price is calculated to clear market:

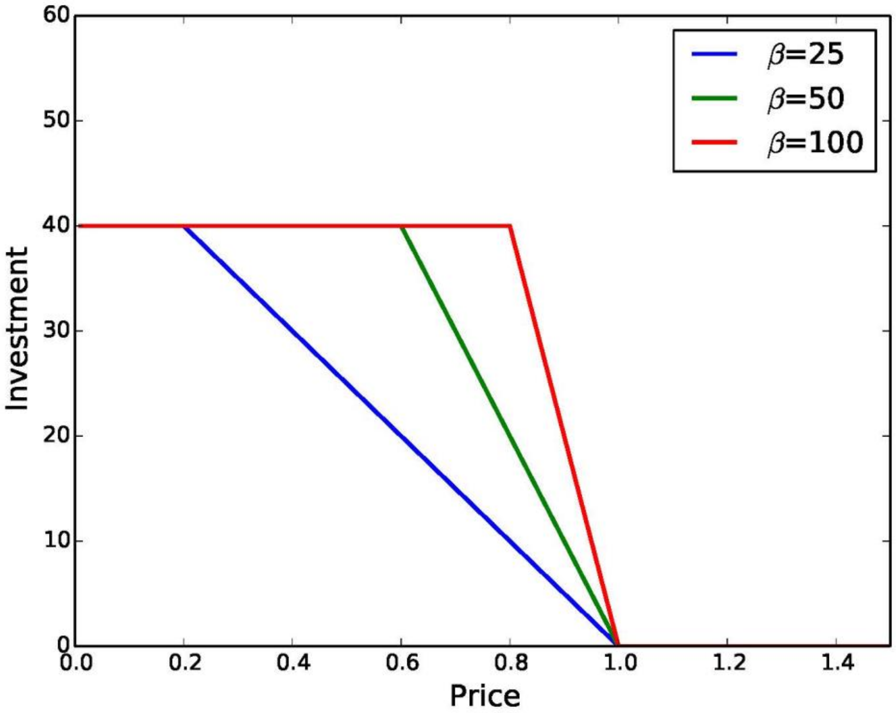
$$p(t) = \frac{\xi_{nt}(t)}{N}$$

Price follows an mean-reverting AR(1) process based on normally distributed log-returns.



Model, Ingredient 2: Funds

- Funds know the fundamental value V=1
- No demand if p>V
- If p<V, the lower p and the more aggressive the fund, the more they buy
- Multiple funds of varying aggressiveness competing



- Here: wealth $W_h(t) = 2$
- Max leverage $\lambda_{MAX} = 20$
- Aggression parameter: β

Model, Ingredient 2: Funds

- The funds are value investors and base their demand $\xi_h(t,p)$ on a mispricing signal $m(p) \coloneqq V - p$.

$$\xi_h(t,p) = \begin{cases} 0,\ m(p) < 0 \\ \beta_h m(p) W_h(t), 0 \leq m(t) < m_h^{crit} \\ \lambda_{MAX} W_h(t), m \geq m_h^{crit} \end{cases}$$

- The wealth of fund h is calculated by $W_h(t) = D_h(t)p(t) + C_h(t)$. Where The funds initial wealth is set to $W_h(0) = 2$. $C_h(t)$ is net cash of fund h (negative if fund borrows).

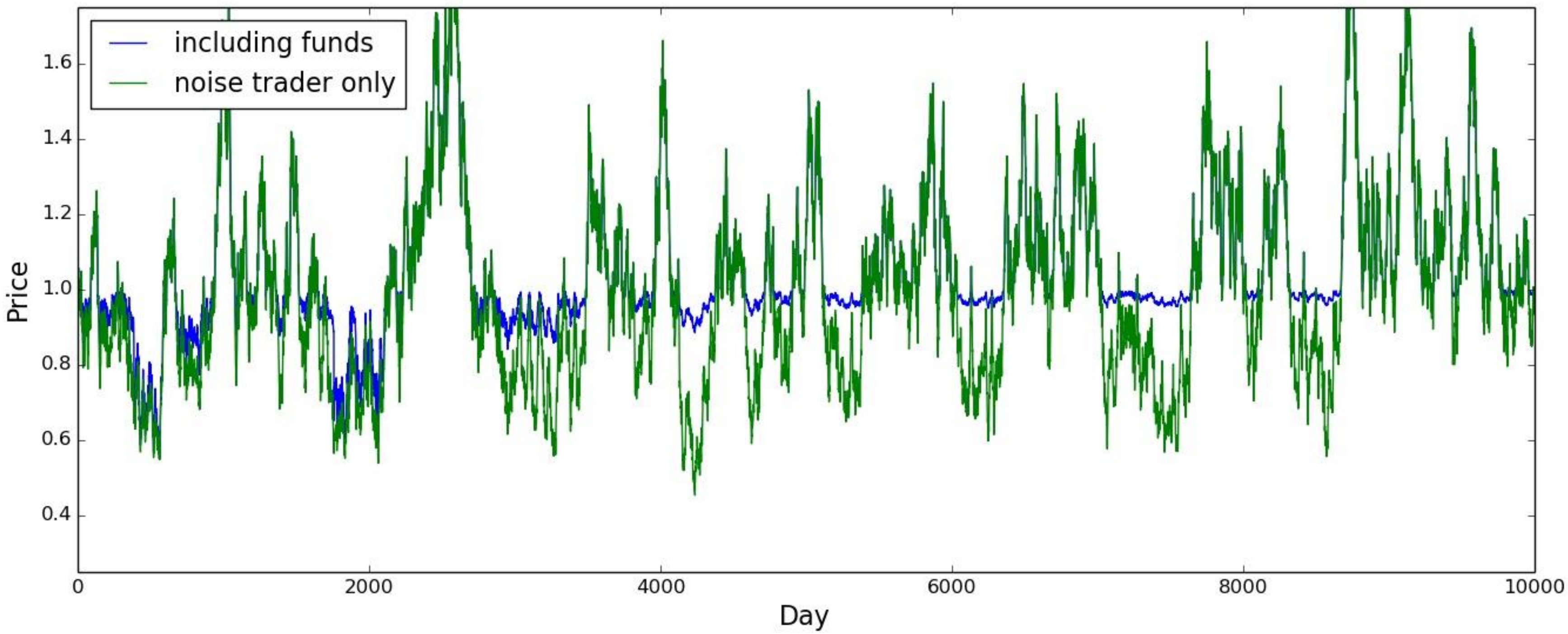
- The leverage λ_h is defined: $\lambda_h = \frac{\xi_h(t)}{\xi_h(t)+C_h(t)}$

  Note: for $C_h(t) < 0$, λ_h a strictly decreasing in $\xi_h(t)$. If price falls, leverage increases.

Model, Ingredient 2: Funds

- If $W_h(t) < 0$: fund defaults and is replaced with a fresh one
- Current price p_t is found by solving the market clearing condition:

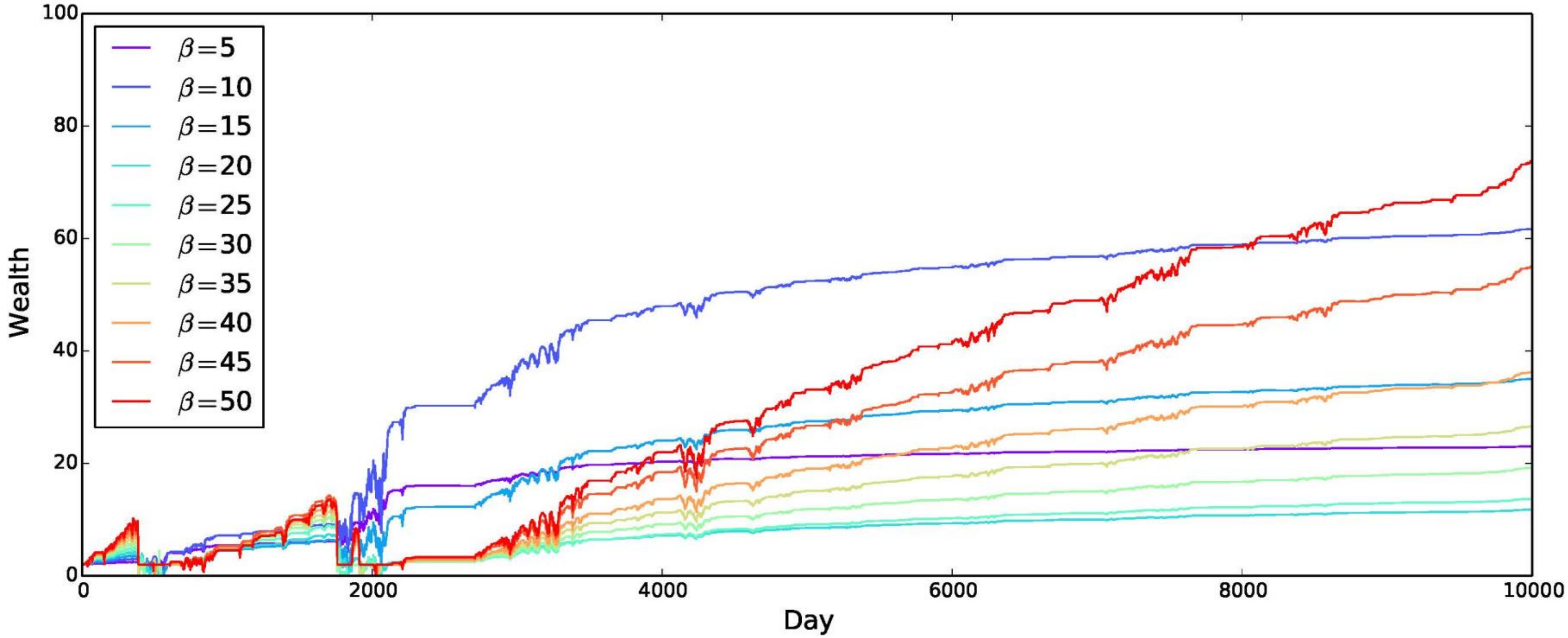
$$\xi_{nt}(t) + \sum_h \xi_h(t, p_t) = Np_t)$$



Model, Ingredient 2: Funds

Problem: Value strategy “wins” against noise traders

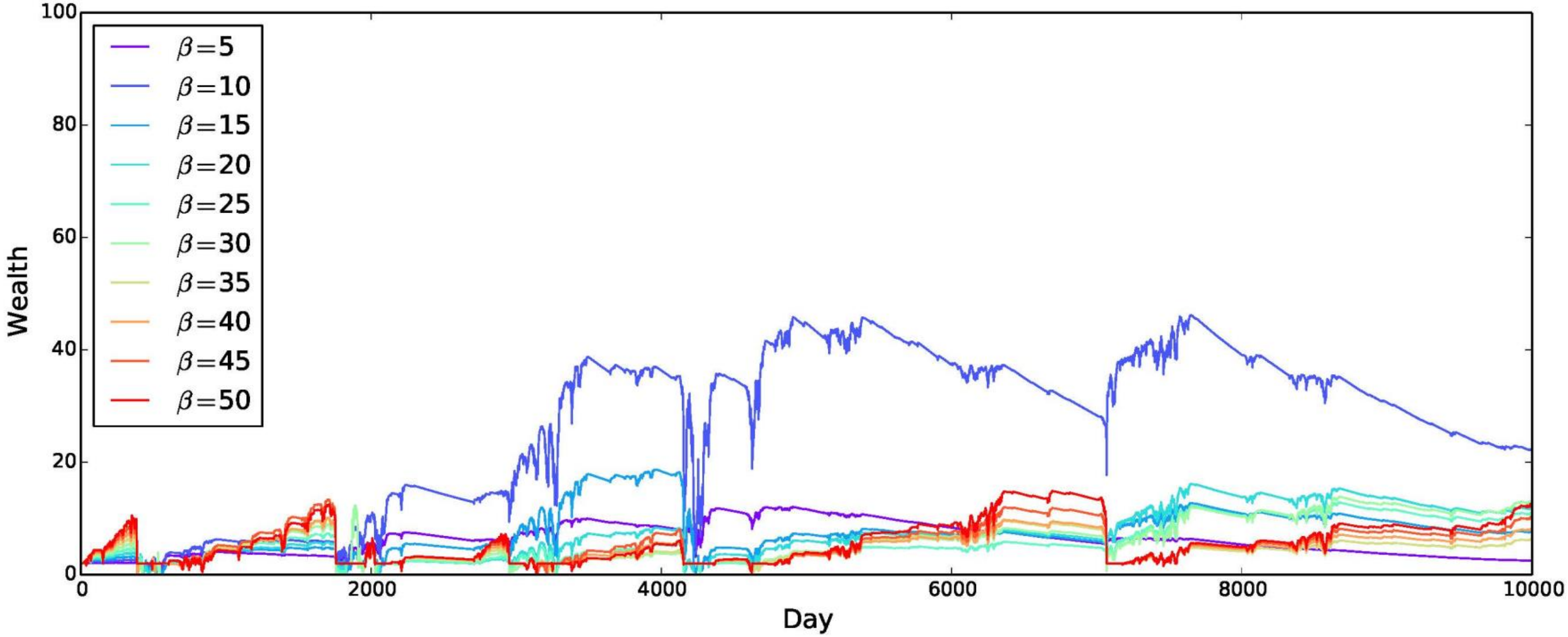
→ Funds get unboundedly wealth



Model, Ingredient 3: Background Investor

- Withdraws money from each fund if performance below benchmark
- Adds money if performance above benchmark

→ Limits the funds‘ wealth



Model, Ingredient 3: Background Investor

- Rate of return provided by fund h is given by:

$$r_h(t) = \frac{D_h(t-1)(p(t) - p(t-1))}{W_h(t-1)}$$

- The background investor decides whether to invest in the fund based on $r_h^{perf}(t)$, an exponential moving average of the funds performances defined as:

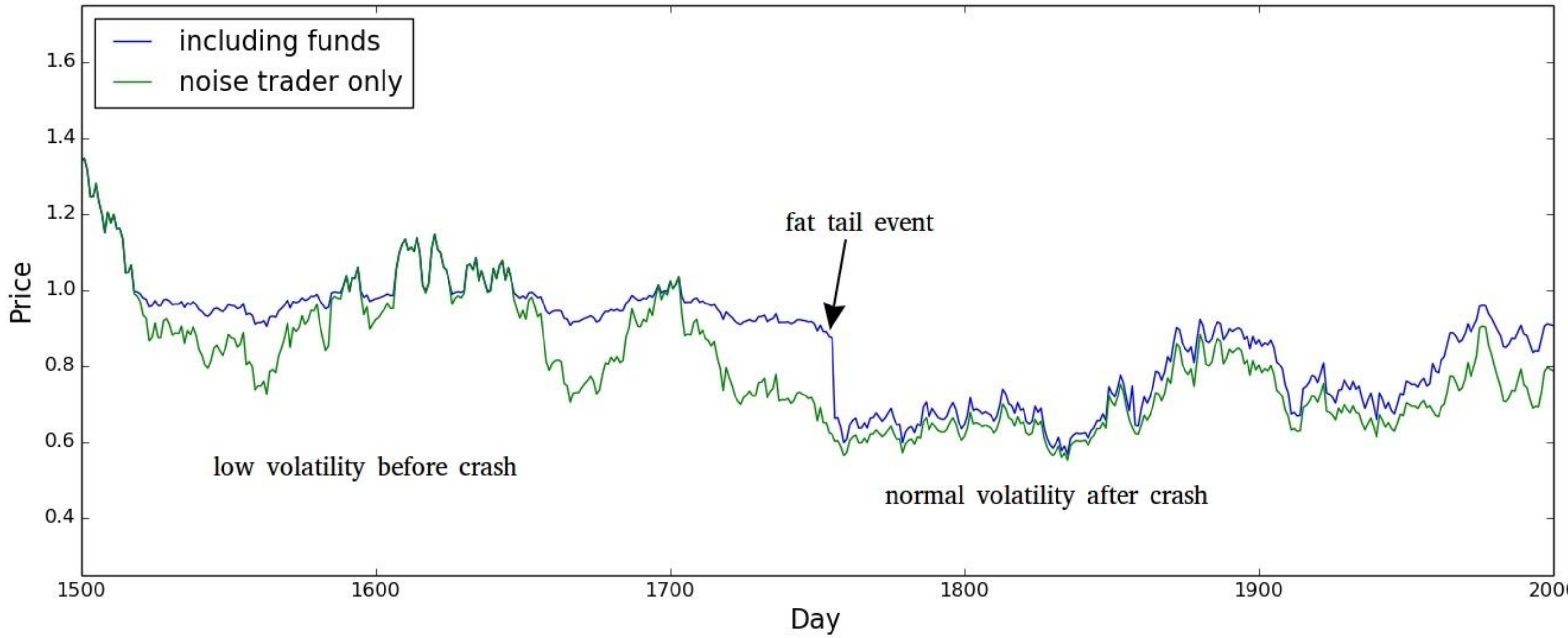
$$r_h^{perf}(t) = 0.9\, r_h^{perf}(t-1) + 0.1\, r_h(t)$$

- The flow of capital in (or out of) the fund is given by:

$$F_h(t) = b\big(r_h^{perf}(t) - r^b\big)\tilde{w}(t)$$

with sensitivity parameter b (mostly 0.15 in the paper, we use 0.10 in the charts), benchmark return $r^b = 0.005\,.\, \tilde{w}(t)$ is wealth before trading.

Model, Ingredient 3: Background Investor



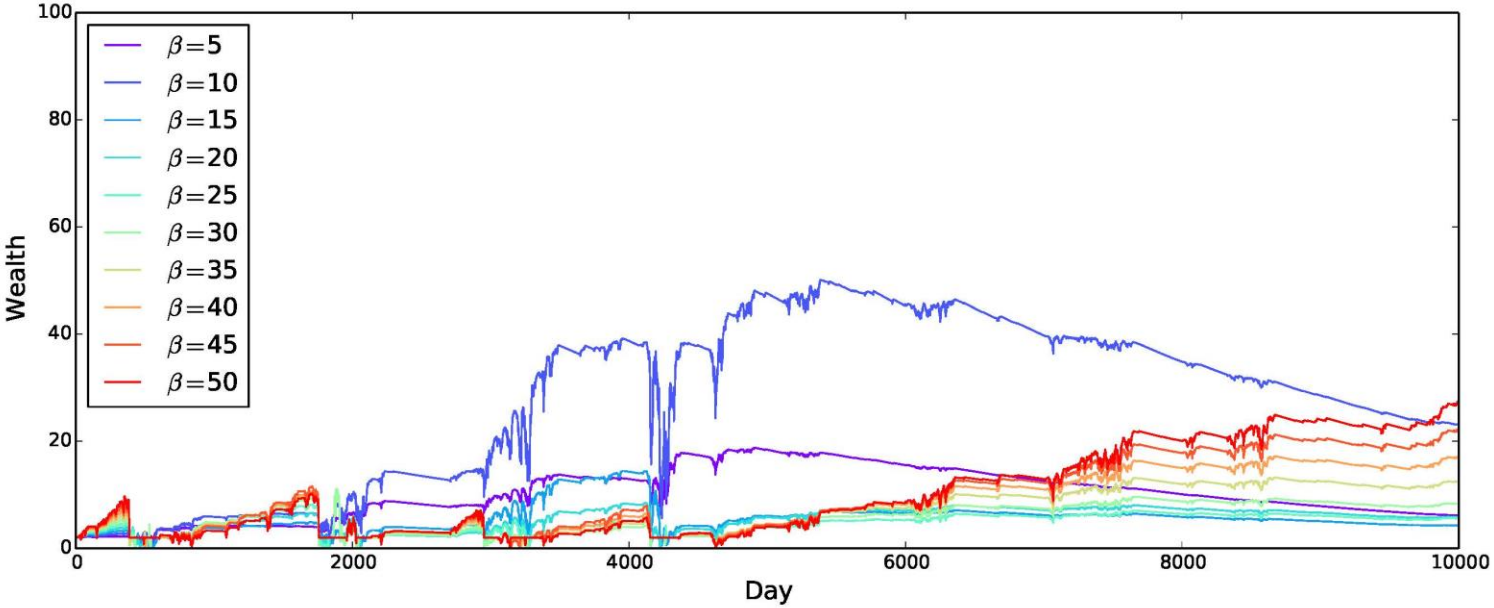
Considering the behavior of the background investor, is this really a model without trend followers?

Authors: *"We are confident that the wealth dynamics of the investors is not the source of the heavy tails."*

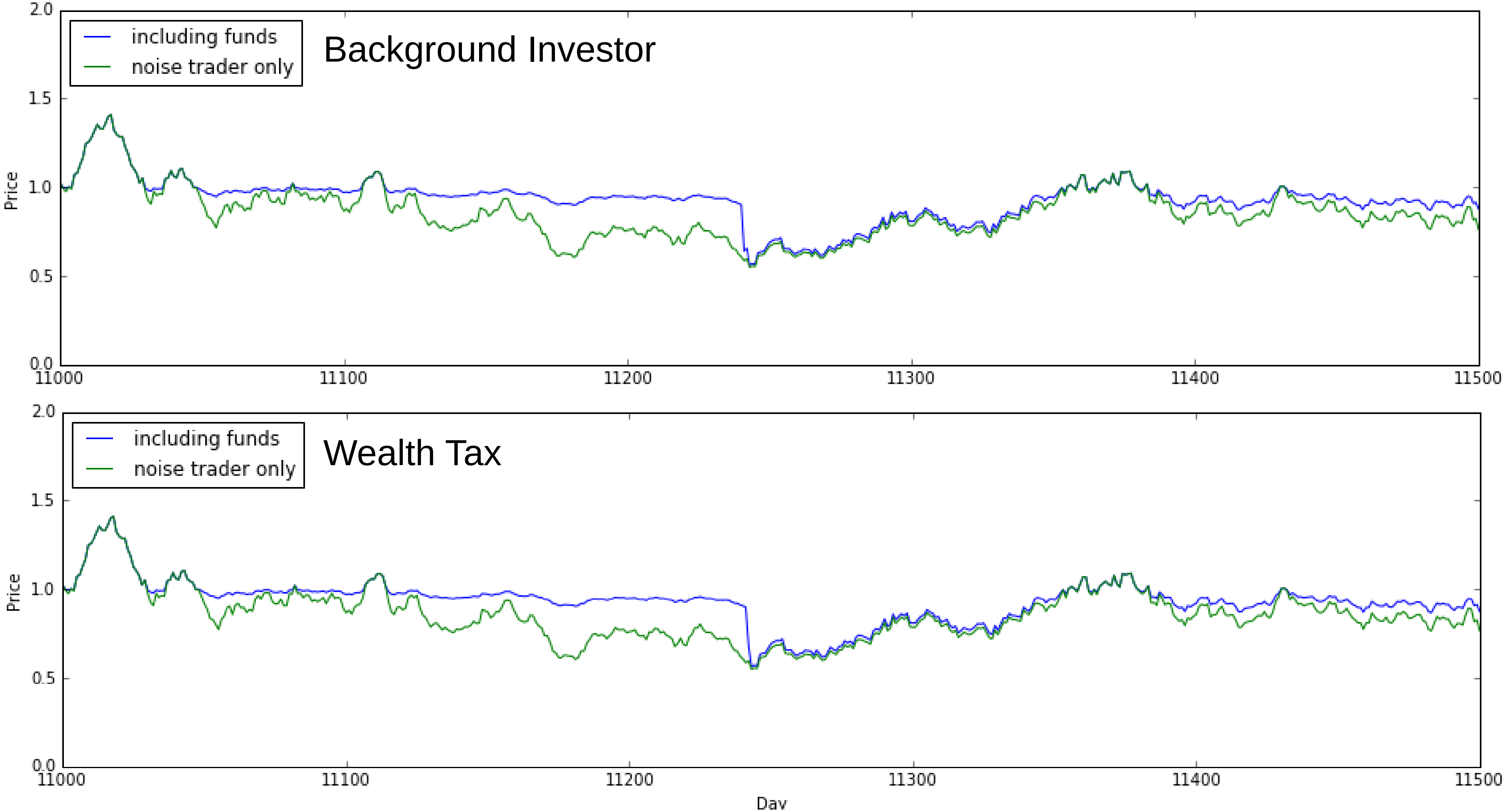
Alternative ingredient: wealth tax

To verify these claims, we removed the background investor and introduced a daily wealth tax of 0.04%.

Effect of wealth tax on fund wealth:

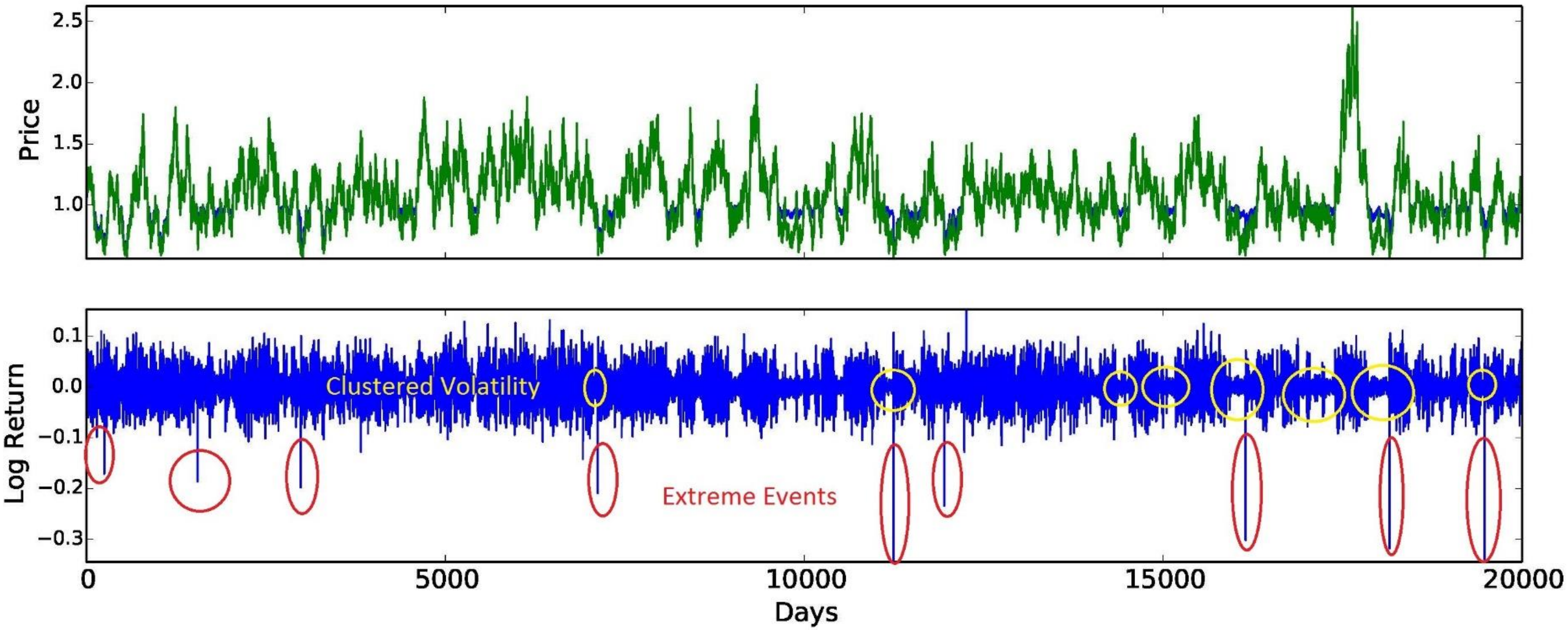


Background Investor vs Wealth Tax



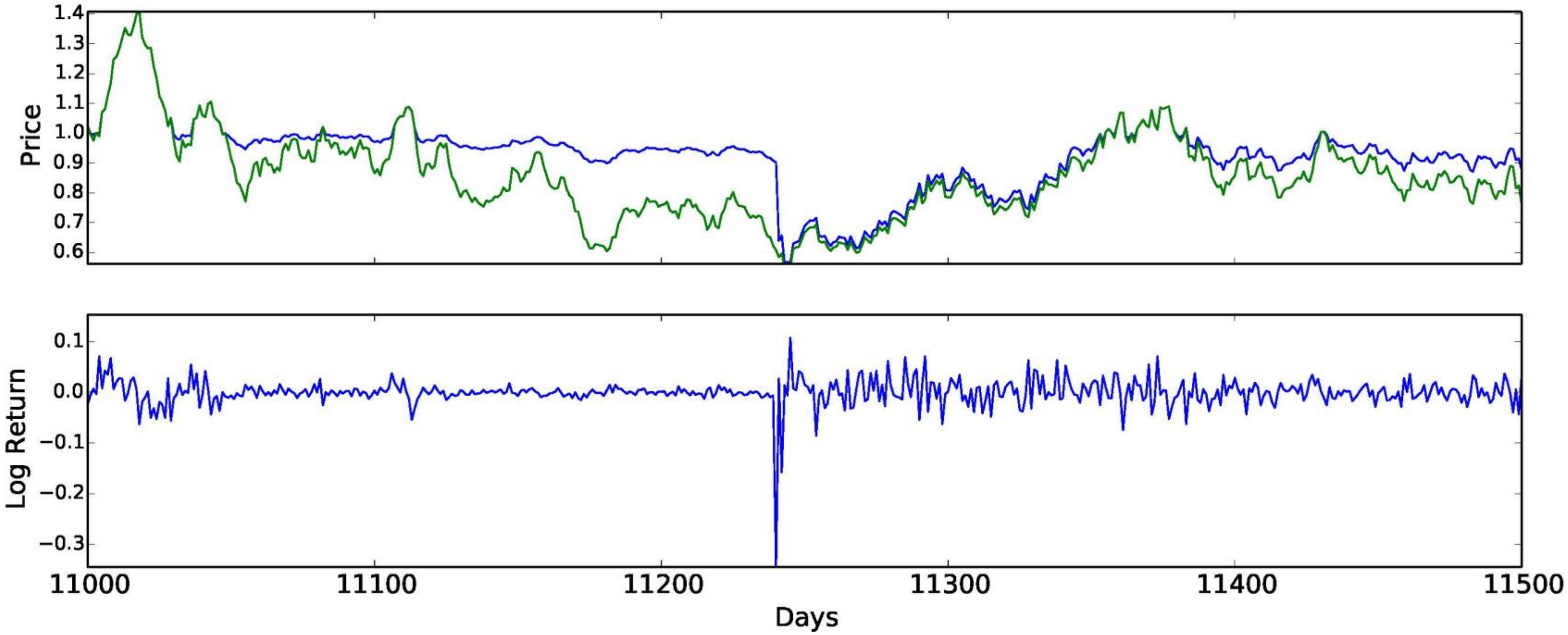
Clustered Volatility

Leveraged funds induce a clustered volatility:



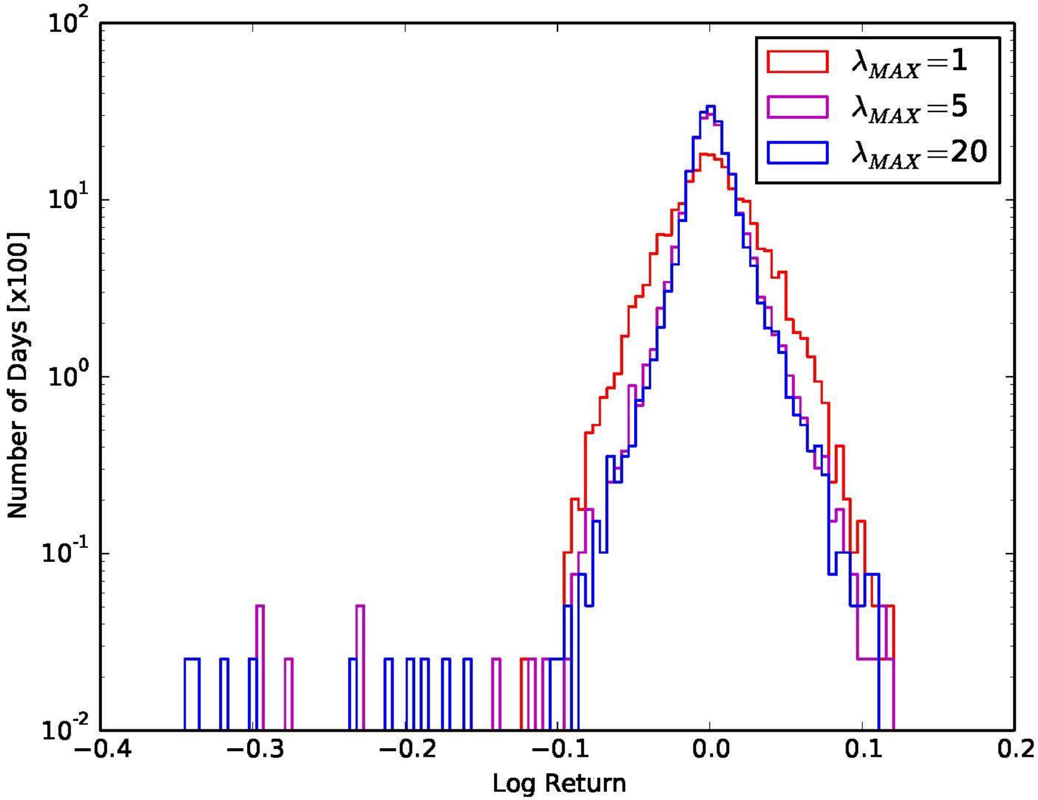
Clustered Volatility

A closer look (leveraged case):



Fat Tails

Leveraged funds lead to a heavy tailed distribution of log returns.



Conditioned on p(t)<V.
i.e. the days at which funds invested.

Findings (according to the model)

- Leverage can create clustered volatility and fat tailed returns
- No trend followers needed
- Risk of sudden fall is highest when volatility is lowest!
- Prices never fall lower than they would have fallen without leverage (we think this is unrealistic)
- Mark-to-market can be dangerous: If the true value V=1 is known, mark-to-market is pointless and allowing funds to do book-value accounting would stabilize the systen. Closely related to „continuity fallacy“.
- Suddenly limiting leverage in times when everyone is fully leveraged can trigger a crisis.