1 $\mathcal{O}$-Notation

$$f, g : \mathbb{N} \mapsto \mathbb{R}^*$$

$$\mathcal{O}(g(n)) = \{f(n) \mid \exists c \in \mathbb{R}^*, n_o \in \mathbb{N} : \forall n \geq n_0 : f(n) \leq c \cdot g(n)\}$$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} < \infty$$

$$2 = \frac{\log_{10} 2}{\log_{10} 10}$$

$$3 = 2^{\operatorname{ld} 3}$$

$$a(n) \in \mathcal{O}(b(n)) \wedge b(n) \in \mathcal{O}(c(n)) \to a(n) \in \mathcal{O}(c(n))$$

$$n^n = 2^n \cdot \operatorname{ld} n = \left(2^{\operatorname{ld} n}\right)^n = n^n$$

1.1 Upper boundary $\mathcal{O}$

$$\mathcal{O}(g(n)) = \left\{f(n) \mid \exists c \in \mathbb{R}^+, n_0 \in \mathbb{N} : f(n) \leq c \cdot g(n) \forall n \geq n_0\right\}$$

$$f(n) = \mathcal{O}(g(n)) \Leftrightarrow f(n) \in \mathcal{O}(g(n))$$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} < \infty \Rightarrow f(n) = \mathcal{O}(g(n))$$

1.2 Lower boundary Ω

$$f, g : \mathbb{N} \to \mathbb{R}^+$$

$$\Omega(g(n)) = \left\{f(n) \mid \exists c \in \mathbb{R}^+, n_0 \in \mathbb{N} : f(n) \geq c \cdot g(n) \forall n \geq n_0\right\}$$

$$f(n) = \Omega(g(n)) \Leftrightarrow f(n) \in \Omega(g(n))$$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} > 0 \Rightarrow f(n) = \Omega(g(n))$$

1.3 Exact boundary Θ

$$f, g : \mathbb{N} \to \mathbb{R}^+$$

$$\Theta(g(n)) = \left\{f(n) \mid \exists c_1, c_2 \in \mathbb{R}^+, n_0 \in \mathbb{N} : c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n) \forall n \geq n_0\right\}$$

$$f(n) = \Theta(g(n)) \Leftrightarrow \Theta(g(n)) \Leftrightarrow f(n) \in \Theta(g(n))$$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = K > 0 \Rightarrow f(n) = \Theta(g(n))$$

1.4 Rules

$$\mathcal{O}(1), \mathcal{O}(\log n), \underbrace{\mathcal{O}(n^c)}_{0<c<1}, \mathcal{O}(n), \mathcal{O}(n \log n), \underbrace{\mathcal{O}(n^c)}_{c>1}, \underbrace{\mathcal{O}(c^n)}_{c>1}, \mathcal{O}(n!)$$

1.4.1 Transitivity

$$\begin{array}{ccccc} f(n) = \mathcal{O}(g(n)) & \wedge & g(n) = \mathcal{O}(h(n)) & \Rightarrow & f(n) = \mathcal{O}(h(n)) \\ f(n) = \Omega(g(n)) & \wedge & g(n) = \Omega(h(n)) & \Rightarrow & f(n) = \Omega(h(n)) \\ f(n) = \Theta(g(n)) & \wedge & g(n) = \Omega(h(n)) & \Rightarrow & f(n) = \Theta(h(n)) \end{array}$$

1.4.2 Reflexivity

$$f(n) = \mathcal{O}(f(n)) \quad f(n) = \Omega(f(n)) \quad f(n) = \Theta(f(n))$$

1.4.3 Symmetry

$$\begin{array}{ccc} f(n) = \mathcal{O}(g(n)) & \Leftrightarrow & g(n) = \Omega(f(n)) \\ f(n) = \Theta(g(n)) & \Leftrightarrow & g(n) = \Theta(f(n)) \end{array}$$

$$f(n) = \Theta(g(n)) \Leftrightarrow f(n) = \mathcal{O}(g(n)) \wedge f(n) = \Omega(g(n))$$

$$\mathcal{O}(f(n)) + \mathcal{O}(g(n)) = \mathcal{O}(\max\{f(n), g(n)\})$$

$$\mathcal{O}(f(n)) \cdot \mathcal{O}(g(n)) = \mathcal{O}(f(n) \cdot g(n))$$

$$\mathcal{O}(f(n)) - \mathcal{O}(f(n)) = \mathcal{O}(f(n))$$

1.4.4 Recursion

- possibly better to understand and implement
- possibly worse memory and runtime behavior