MATH 141 TEST STUDY GUIDE

In this guide we will list questions in three categories:

I Category One will consist of beginner level questions. All students should be able to complete all of these exercises with no difficulty. These are basically warm-up questions and are too easy to appear on an exam.

II Category Two will consist of exam level questions. These are questions which most students should be able to solve on their own. These questions make take a little longer than Category One questions, and some students may need to seek help, but these are still meant to be solvable within 10-15 minutes each by the majority of students without outside help.

III Category Three will consist of questions that only a few students will be able to solve without outside help. It is my strong recommendation that each student try to solve every question in this section without outside help first. These problems may require an hour or more or concentration, but the reward for being able to do these without help will be that a student will be capable of facing more and more difficult problems with ease. These questions are highly recommended for mathematics and computer science majors.

In addition to these questions, I will provide several questions which cross over several topics within the discrete math sequence. These may not necessarily be category three, but meant to show that certain problems have multiple methods of solution. Without Further ado, here are the problems:

Category One:
Section 3.3: 2,3, 13-19, 21
Section 4.5: 1-9
Section 4.8: 1-7
Section 5.1: 1-16,62-76
Section 5.2: 3,4,5
Section 5.3: 2,3,6,7
Section 5.4: 1-7
Section 5.5: 2,4,6
Section 5.6: 1-8
Section 5.7: 3-15
Section 5.8: 1-6
Section 5.9: 1,2
Section 6.1: 1-16
Section 6.2: 1-5
Section 6.3: 1-21
Section 6.4: 1-3
Section 8.1: 1,3,5,8
Section 8.2: 1-8,12,16
Section 8.3: 2,4,6,8,9
Section 8.4: 1,3,5,6,8,9,10,13
Section 8.5: 2,3,5,7,10,11
Section 11.1: 1-14
Section 11.2: 1,2,3,10,11,13
Section 11.3: 1-5
Section 11.4: 1-9,18,19
Section 11.5: 1-4
Section 12.1: 1-6
Section 12.2: 1
Section 12.3: $\emptyset$

Category Two:
Section 3.3: 24-28, 33-39, 41
Section 4.5: 10,13,17-29
Section 4.8: 13-18,21,23
Section 5.1: 20,21,28,36,50-52,80
Section 5.2: 6,8,10,11,13-17,19,23,28,29
Section 5.3: 8-20,22,23,28,33
Section 5.4: 9,13,16,19,22,25
Section 5.5: 8
Section 5.6: 15,17,21,22,26-32
Section 5.7: 22,23,43,48,49
Section 5.8: 7,8,12,15,16
Section 5.9: 5-11
Section 6.1: 17-32
Section 6.2: 6-19,25,26,29,
Section 6.3: 29-45
Section 6.4: 10-15
Section 8.1: 13-18,20
Section 8.2: 18-42
Section 8.3: 12,14,15,16,22,27,29,32,33
Section 8.4: 14-18,31-35
Section 8.5: 12,20,31,35
Section 11.1: 15-26
Section 11.2: 4-9,12,14,16-25,40-43,51,52
Section 11.3: 6-26
Section 11.4: 13-17,21-24,27,29,40,42
Section 11.5: 7-16
Section 12.1: 7-30
Section 12.2: 2-11,20-28
Section 12.3: 1-7

Category Three:
Section 3.3: 43, 55-59
Section 4.5: 30
Section 4.8: 24,25,29
Section 5.1: 88-91
Section 5.2: 18,30,31,33-35
Section 5.3: 21,30,31,36,38-40
Section 5.4: 27-32
Section 5.5: 10,12
Section 5.6: 34,42,44
Section 5.7: 52-54
Section 5.8: 18,20,22,24
Section 5.9: 20-24
Section 6.1: 33,36,37
Section 6.2: 36-41
Section 6.3: 26,46,52
Section 6.4: 19,20,24,25
Section 8.1: 23,24
Section 8.2: 51-56
Section 8.3: 42,43,44,47
Section 8.4: 41-43
Section 8.5: 42-50
Section 11.1: 27,28
Section 11.2: 27,44-46,48,57,58,59
Section 11.3: 27,32,35,36-43
Section 11.4: 41,43,46-49,52,54-56
Section 11.5: 24-26
Section 12.1: 37-41
Section 12.2: 48-54
Section 12.3: 8

Mixed Problems:
Bernoulli Polynomials (induction, algorithmic order):

Sums of the form

$$\sum_{j=1}^{N} j^k$$

are called Bernoulli polynomials. However, k need not be an integer. For any $k > 0$ show the following:

$$\sum_{j=1}^{N} j^k \in O(N^{k+1})$$

and

$$\sum_{j=1}^{N} j^k \in \Theta(N^{k+1})$$

Now armed with that information we can actually compute the polynomials using induction:

$$\sum_{j=1}^{N} j^k = a_{k+1}N^{k+1} + \cdots + a_1 N + a_0$$

Use induction to compute the Bernoulli polynomials.

For example:

$$\sum_{j=1}^{N} j = a_2 N^2 + a_1 N + a_0$$

We already know that $a_2 = 1/2$ and $a_1 = 1/2$ and $a_0 = 0$.

We have shown by mathematical induction how to show divisibilities in some of the following forms:

- $6|(7^n - 1)$
- $4|(6^n - 2^n)$
- $13|(18^n - 5^n)$

Show the following by mathematical induction:

$$\forall r \in \mathbb{Z}^+, r \neq 1, (r-1)|(r^n - 1)$$

In this case, this divisibility works even for rational numbers.

Now setting $r = x/y$ show

$$(x-y)|(x^n - y^n)$$

Now show this fact using mathematical induction. This should be easy now that you've shown it to be true in general.

Finally, let's solve this a third way... Using Modular arithmetic show

$$(x-y)|(x^n - y^n)$$

For all integers x, y and for all positive (or nonnegative) integers n.

Stirling's Approximation:

We will give some weaker forms of the approximation:

$$\lim_{n\to\infty} \frac{n!}{\sqrt{2\pi n}(n/e)^n} = 1$$

First off, try showing the following inequality

$$\ln(n) < \sum_{j=1}^{n} \frac{1}{j} < \ln(n) + 1$$

Now try showing the following inequality

$$\frac{n^n}{3^n} < n! < \frac{n^n}{2^n}$$

If we continue on this series of inequalities we can find a b so that

$$\frac{n^n}{(b+\epsilon)^n} < n! < \frac{n^n}{(b-\epsilon)^n}$$

Show that $b = e$.
Now show

- $e^n \in O(n!)$
- $n! \in O(n^n)$
- $n! \in \Theta(n^{n+1/2}e^{-n})$

Use (some of) this information to derive the useful version of Stirling's approximation.

$$\ln(n!) \approx n\ln(n) - n$$

And the give good approximations for the following $\ln(1000!)$, $\ln(10^9!)$, and $\ln(((9^9)!)^9)$

Stirling's approximation comes up frequently (that is an understatement) in Statistical Mechanics (quantum thermodynamics, statistical quantum mechanics, etc) when counting roughly the number of configurations of molecules in a substance. For example, a mole is roughly 6.02×10^{23}. So consider a kilogram of carbon dioxide. How many molecules are in this substance? How many ways can these molecules be arranged?