Week3 Question 7 to 11

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1 Question 7

1. **Exercise 3.1.1, sections a-g**

   a. True

   Because $27 = 3 \times 9$, 27 is an integer multiple of 3. Therefore, 27 is an element in set A. The statement is true.

   b. False

   Because there is no integer y such that $27 = y \times y$. Therefore, 27 is not an element in set B. The statement is False.

   c. True

   Because $100 = 10 \times 10$. Therefore, 100 is an element in the set B. The statement is True.

   d. False

   Because $3 \in E$ but $3 \notin C$ Therefore,$E \nsubseteq C$. Because $4 \in C$ but $4 \notin E$. Therefore, $C \nsubseteq E$. Overall, the statement is False.

   e. True

   Because 3, 6, 9 are multiples of 3.Then, 3, 6, and 9 are all in set A. Therefore, the statement is True.

   f. False

   There exists an integer 27, which is an element in set A but not in set E. Therefore, the statement is False.

   g. False

   Because set A only includes integer elements. But E is a set, E cannot be an element in A. Therefore, the statement is False.

2. **Exercise 3.1.2, sections a-e**

   a. False

   15 is an integer not an set. Therefore, the statement is False.

   b. True

   set $\{15\}$ only has one element 15. And this element is also an element in set A. And there exists an integer 3 that is an element in set A but not in set $\{15\}$. Therefore, the statement is True.

   c. True

   The empty set is the proper subset of every set that is not an empty set. Therefore, the statement is True.

   d. True

   Any set can be the subset of itself. Therefore, the statement is True.

   e. False

   Set B only includes integer elements. But the empty set is a set. Therefore, the empty set cannot be an element in set B. The statement is False.

3. **Exercise 3.1.5, sections b, d**

   b. $\{ 3, 6, 9, 12, ... \}$

   B = $\{x \in N$: x is an integer multiple of 3$\}$

   The set is **infinite**.

   The set includes all the positive integers that are multiples of 3. Therefore, the set is infinite.

   d. $\{ 0, 10, 20, 30, ..., 1000 \}$

   D = $\{$x $\in Z : 0 \leq x \leq 1000$ and x is an integer multiple of 10$\}$

   $|D| = 101$

   The cardinality of the set is 101.

   Apparently the set includes all the non-negative integers from 0 to 1000 that are the multiples of 10. Therefore, there exist $(1000-0) \div 10+1 = 101$ elements in the set.

4. Exercise 3.2.1, sections a-k

   a. True

   2 is an element in set X. Therefore, the statement is True.

   b. True

   2 is an element in set X. Therefore, the set $\{2\}$ is a subset of X. The statement is True.

   c. False

   set$\{2\}$ is not an element in set X. Therefore, the statement is False.

   d. False

   3 is not an element in set X. $\{3\}$ is an element in the set X. Therefore, the statement is False.

   e. True

   The element $\{1, 2\}$ is an element in set X. Therefore, the statement is True.

   f. True

   Because both 1 and 2 are elements in the set X. Therefore, set$\{1, 2\}$ is a subset of set X.The statement is True.

   g. True

   Because 2 and 4 are both elements in the set X. Therefore, set$\{2, 4\}$ is a subset of set X. The statement is True.

   h. False

   Because $\{2, 4\}$ is not an element in the set X. Therefore, the statement is False.

   i. False

Because 3 is not an element in the set X. Therefore, the set $\{2\ ,3\}$cannot be a subset of the set X. The statement is False.

j. False

The set $\{2, 3\}$ is not an element in the set X. The statement is False.

k. False

Because X only has 6 elements. Therefore, the statement is False.

2 Question 8

Exercise 3.2.4 section b

A = {1, 2, 3}

Because the set X must have an element 2. And the set X must be the subset of P(A).

$\therefore \{X \in P(A) : 2 \in X\} = \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$

3 Question 9

1. **Exercise 3.3.1, sections c-e**

   c. $A \cap C$ = <u>{-3, 1, 17}</u>

   d. $A \cup (B \cap C)$ = <u>{-5, -3, 0, 1, 4, 17}</u>

   e. $A \cap B \cap C$ = <u>{1}</u>

2. **Exercise 3.3.3, sections a, b, e, f**

   a. Because n = 5, i = 2.

   And $A_2 = \{1, 2, 4\}$

   $A_3 = \{1, 3, 9\}$

   $A_4 = \{1, 4, 16\}$

   $A_5 = \{1, 5, 25\}$

   The union of sequence of sets = <u>{1}</u>

   b. cause n = 5, i = 2.

   And$A_2 = \{1, 2, 4\}$

   $A_3 = \{1, 3, 9\}$

   $A_4 = \{1, 4, 16\}$

   $A_5 = \{1, 5, 25\}$

   The union of sequence of sets = <u>{1, 2, 3, 4, 5, 9, 16, 25}</u>

   e. from the question we know that

   $C_1 = \{x \in R : -1 \leq x \leq 1\}$

   $C_2 = \{x \in R : -\frac{1}{2} \leq x \leq \frac{1}{2}\}$

   $C_3 = \{x \in R : -\frac{1}{3} \leq x \leq \frac{1}{3}\}$

   ...

   $C_{100} = \{x \in R : -\frac{1}{100} \leq x \leq \frac{1}{100}\}$

   Therefore, apparently the union of sequence of sets

   $= C_{100}$

   $\underline{= \{\text{x} \in R : -\frac{1}{100} \leq x \leq \frac{1}{100}\}}$

   f. from the question we know that

   $C_1 = \{x \in R : -1 \leq x \leq 1\}$

   $C_2 = \{x \in R : -\frac{1}{2} \leq x \leq \frac{1}{2}\}$

   $C_3 = \{x \in R : -\frac{1}{3} \leq x \leq \frac{1}{3}\}$

   ...

   $C_{100} = \{x \in R : -\frac{1}{100} \leq x \leq \frac{1}{100}\}$

   Therefore, apparently the union of sequence of sets

$= C_1$

$\underline{= \{\text{x} \in R : -1 \leq x \leq 1 \ \}}$

3. **Exercise 3.3.4, sections b, d**

   b: $P(A \cup B)$

   $A \cup B = \{a, b, c\}$

   $P(A \cup B) = \underline{\{\emptyset, \{a\}.\{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}}$

   d:$P(A) \cup P(B)$

   $P(A) = \{\emptyset, \{a\}.\{b\}, \{a, b\}\}$

   $P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

   $P(A) \cup P(B) = \underline{\{\emptyset, \{a\}.\{b\}, \{c\}, \{a, b\}, \{b, c\}\}}$

4 Question 10

1. **Exercise 3.5.1, sections b, c**
   b. an element from the set $B \times A \times C$ can be:

   (foam, tall, non-fat)

   c. $B \times C =$ **{(foam, non-fat), (foam, whole), (no-foam, non-fat), (non-foam, whole)}**

2. **Exercise 3.5.3, sections b, c, e**

   b. True

   Z = integers

   R = real numbers

   $\therefore Z \subseteq R$

   $\therefore Z^2 \subseteq R^2$

   $\therefore$ the statement is True.

   c. True

   Z^2 are all ordered 2-tuples.

   Z^3 are all ordered 3-tuples.

   $\therefore$, there is no element that can be at Z^2 and Z^3 at the same time. $\therefore$ $Z^2 \cap Z^3 = \emptyset$

   e. True

   Because $A \subseteq B$, then all the elements in set A can be found in set B. Therefore, $A \times C \subseteq B \times C$. The statement is True.

3. **Exercise 3.5.6, sections d, e**

   d. {xy: where $x \in \{0\} \cup \{0\}^2$ and $y \in \{1\} \cup \{1\}^2$}

   $x \in \{0, 00\}$

   $y \in \{1, 11\}$

   $\underline{\{xy\} = \{01, 001, 011, 0011\}}$

   e. $\{xy : x \in \{aa, ab\}$ and $y \in \{a\} \cup \{a\}^2\}$

   $x \in \{aa, ab\}$

   $y \in \{a, aa\}$

   {xy} = {aaa, aaaa, aba, abaa}

4. **Exercise 3.5.7, sections c, f, g**

   c. $(A \times B) \cup (A \times C)$

   $A \times B = \{ab, ac\}$

   $A \times C = \{aa, ab, ad\}$

$(A \times B) \cup (A \times C) = \{\text{aa, ab, ac, ad}\}$

f. $P(A \times B)$

$A \times B = \{ab, ac\}$

$P(A \times B) = \{\emptyset, \{\text{ab}\}, \{\text{ac}\}, \{\text{ab, ac}\}\}$

g. $P(A) \times P(B)$. Use ordered pair notation for elements of the Cartesian product.

$P(A) = \{\emptyset, \{a\}\}$

P(B) = $\{\emptyset, \{\text{b}\}, \{\text{c}\}\ \ \{\text{b, c}\}\}$

$P(A) \times P(B) = \{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\}), (\emptyset, \{b, c\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\})\ (\{a\}, \{b, c\})\}$

5 Question 11

1. **Exercise 3.6.2, sections b, c**

   b.

	$(B \cup A) \cap (\overline{B} \cup A)$	
1.	$= A \cup (B \cap \overline{B})$	Distributive Laws
2.	$= A \cup \emptyset$	Complement laws
3.	= A	Identity laws

   1. According to the Distributive laws, $(B \cup A) \cap (\overline{B} \cup A)$ is $= A \cup (B \cap \overline{B})$.

   2. According to the Complement laws, $A \cup (B \cap \overline{B}) = A \cup \emptyset$

   3.According to the Identity laws, $A \cup \emptyset =$ A.

   Therefore, $(B \cup A) \cap (\overline{B} \cup A) = A$

   c.

	$\overline{A \cap \overline{B}}$	
1.	$= \overline{A} \cup \overline{\overline{B}}$	De Morgan's Law
2.	$= \overline{A} \cup B$	Double Complement Law

   Therefore, the set identity $\overline{A \cap \overline{B}} = \overline{A} \cup B$ is true.

2. **Exercise 3.6.3, sections b, d**

   b. When A = B = $\{1, 2\}$,

   $B \cap A = \{1, 2\}$.

   Therefore, $A - (B \cap A) = \emptyset$,

   $A \neq \emptyset$.

   The set equation given below is not a set identity.

   d. When$B = \{1, 2, 3, 4, 5\}$,

   $A = \{2\}$, $B - A = \{1, 3, 4, 5\}$,

   $(B - A) \cup A = \{1, 2, 3, 4, 5\} \neq A$

   Therefore, the set equation given below is not a set identity.

3. **Exercise 3.6.4, sections b, c**

   b)

	$A \cap (B - A)$	
1.	$= A \cap (B \cap \overline{A})$	Set subtraction law
2.	$= A \cap \overline{A} \cap B$	Associative law
3.	$= \emptyset \cap B$	Complement law
4.	$= \emptyset$	Domination law

$\therefore A \cap (B - A) = \emptyset$

c.

	$A \cup (B - A)$	
1.	$= A \cup (B \cap \overline{A})$	Set subtraction law
2.	$= (A \cup B) \cap (A \cup \overline{A})$	Distributive law
3.	$= (A \cup B) \cap U$	Complement law
4.	$= A \cup B$	Identity law

$\therefore A \cup (B - A) = A \cup B$