Homework6

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1 Question 5

Use the definition of θ in order to show the following:

a. $5n^3 + 2n^2 + 3\text{n} = \theta(n^3)$

According to the definition of θ:

Let f(n) and g(n) be two functions mapping positive integers to positive real numbers.

We say that f(n) = $\theta(g(n))$ if there exist positive real constants c_1, c_2 and a positive integer constant n_0, such that c_2g(n) ≤ f(n) ≤ c_1g(n) for all n ≥ n_0

In this task, f(n) = $5n^3 + 2n^2 + 3n = \theta(g(n)) = \theta(n^3)$

Proof:

First of all, we need to find c_1, c_2, and n_0.

$5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^2 \times n + 3n \times n^2 = 5n^3 + 2n^3 + 3n^3 = 10n^3$

Therefore, $c_1 = 10$

We know that f(n) and g(n) be two functions mapping positive integers to positive real numbers.

Therefore, $n > 0$, $2n^2 + 3n > 0$.

$5n^3 + 2n^2 + 3n$ is greater than $5n^3$. It means that when $n > 0$, $5n^3 \leq 5n^3 + 2n^2 + 3n$.

Take $c_1 = 10$

$c_2 = 5$

$n_0 = 0$

Then for all n ≥ n_0 we have: $5n^3 \leq 5n^3 + 2n^2 + 3n \leq 10n^3$

Therefore, $5n^3 + 2n^2 + 3n = \theta(n^3)$

b. $\sqrt{7n^2 + 2n - 8} = \theta(n)$

According to the definition of θ:

Let f(n) and g(n) be two functions mapping positive integers to positive real numbers.

We say that f(n) = $\theta(g(n))$ if there exist positive real constants c_1, c_2 and a positive integer constant n_0, such that c_2g(n) ≤ f(n) ≤ c_1g(n) for all n ≥ n_0

In this task, f(n) = $\sqrt{7n^2 + 2n - 8}$, g(n) = n.

Proof:

First of all, let's find c_1, c_2, and n_0.

$\sqrt{7n^2 + 2n - 8} < \sqrt{7n^2 + 2n} \leq \sqrt{7n^2 + 2n * n} = \sqrt{9n^2} = 3n$

Therefore, 3 is a good fit for c_1.

When $2n - 8 \geq 0$, $\sqrt{7n^2} = \sqrt{7} \times n \leq \sqrt{7n^2 + 2n - 8}$. It means that when n≥ 4, $\sqrt{7} \times n \leq \sqrt{7n^2 + 2n - 8}$.

Therefore, $\sqrt{7}$ is a good fit for c_2. And 4 is a good fit for n_0.

if we take $c_1 = 3$

$c_2 = \sqrt{7}$

$n_0 = 4$

Then for all $n \geq n_0$ we have: $\sqrt{7} \times n \leq \sqrt{7n^2 + 2n - 8} \leq 3n$.

Therefore, $\sqrt{7n^2 + 2n - 8} = \theta(n)$