Homework No.1

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1 Question One

A. Converting the following numbers to their decimal representation. Show your work.

1. $10011011_2 = 1\times2^7+0\times2^6+0\times2^5+1\times2^4+1\times2^3+0\times2^2+1\times2^1+1\times2^0 = 128+16+8+2+1 = 155_{10}$

2. $456_7 = 4\times7^2+5\times7+6\times7^0 = 4\times49+5\times7+6\times1 = 196+35+6 = 237_{10}$

3. $38A_{16} = 3\times16^2+8\times16+10\times16^0 = 768+128+10 = 906_{10}$

4. $2214_5 = 2\times5^3+2\times5^2+1\times5+4\times5^0 = 250+50+5+4 = 309_{10}$

B. Converting the following numbers to their binary representation.

1. $69_{10} = 1000101_2$
$69 \div 2 = 34\ R\ 1$
$34 \div 2 = 17\ R\ 0$
$17 \div 2 = 8\ R\ 1$
$8\ \ \div 2 = 4\ R\ 0$
$4\ \ \div 2 = 2\ R\ 0$
$2\ \ \div 2 = 1\ R\ 0$
$1\ \ \div 2 = 0\ R\ 1$
$69_{10} = 1000101_2$

2. $485_{10} = 111100101_2$
$485 \div 2 = 242\ R\ 1$
$242 \div 2 = 121\ R\ 0$
$121 \div 2 = 60\ R\ 1$
$60 \div 2 = 30\ R\ 0$
$30 \div 2 = 15\ R\ 0$
$15 \div 2 = 7\ R\ 1$
$7 \div 2 = 3\ R\ 1$
$3 \div 2 = 1\ R\ 1$
$1 \div 2 = 0\ R\ 1$
$485_{10} = 111100101_2$

3. $6D1A_{16} = 110110100011010_2$
$6_{16} = 110_2$
$D_{16} = 1101_2$
$1_{16} = 1_2 = 0001_2$
$A_{16} = 1010_2$
$6D1A_{16} = 110110100011010_2$

C. Converting the following numbers to their hexadecimal representation:

1. $1101011_2 = 6B_{16}$
$110_2 = 6_{16}$
$1011_2 = B_{16}$
$1101011_2 = 6B_{16}$

2. 895_{10}
$895 \div 16 = 55\ R\ 15(F)$
$55 \div 16 = 3\ R\ 7$
$3 \div 16 = 0\ R\ 3$
$895_{10} = 37F_{16}$

2 Question Two

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 =$

$6 \times 8^0 + 5 \times 8^0 = 1 \times 8^1 + 3 \times 8^0$

$6 \times 8^1 + 1 \times 8^1 + 1 \times 8^1 = 1 \times 8^2$

$5 \times 8^2 + 5 \times 8^2 + 1 \times 8^2 = 1 \times 8^3 + 3 \times 8^2$

$7 \times 8^3 + 4 \times 8^3 + 1 \times 8^3 = 1 \times 8^4 + 4 \times 8^3$

$7566_8 + 4515_8 = 1 \times 8^4 + 4 \times 8^3 + 3 \times 8^2 + 3 \times 8^0 = \mathbf{14303_8}$

2. $10110011_2 + 1101_2 =$

2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	1	**0**	**1**	**1**	**0**	**0**	**1**	**1**
+					**1**	**1**	**0**	**1**
	1	**1**	**0**	**0**	**0**	**0**	**0**	**0**

$10110011_2 + 1101_2 = 11000000_2$

3. $7A66_{16} + 45C5_{16} =$

16^4	16^3	16^2	16^1	16^0
	7	**A**	**6**	**6**
+	**4**	**5**	**C**	**5**
	C	**0**	**2**	**B**

$= C02B_{16}$

4. $3022_5 - 2433_5 =$

3^4	3^3	3^2	3^1	3^0
	3	**0**	**2**	**2**
-	**2**	**4**	**3**	**3**
			3	**4**

$= 34_5$

3 Question Three

A.Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} =$

$124 \div 2 = 62\ R\, 0$

$62 \div 2 = 31\ R\, 0$

$31 \div 2 = 15\ R\, 1$

$15 \div 2 = 7\ R\, 1$

$7 \div 2 = 3\ R\, 1$

$3 \div 2 = 1\ R\, 1$

$1 \div 2 = 0\ R\, 1$

$124_{10} = 1111100_2 = 01111100_{8\ bit\ 2's\ comp}$

2. -124_{10}

from (1) we know:

$124_{10} = 1111100_2$

1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
-	**0**	**1**	**1**	**1**	**1**	**1**	**0**	**0**
	1	**0**	**0**	**0**	**0**	**1**	**0**	**0**

-124$_{10} = 10000100_{8\ bit\ 2's\ comp}$

3. 109_{10}

$109 \div 2 = 54\ R\, 1$

$54 \div 2 = 27\ R\, 0$

$27 \div 2 = 13\ R\, 1$

$13 \div 2 = 6\ R\, 1$

$6 \div 2 = 3\ R\, 0$

$3 \div 2 = 1\ R\, 1$

$1 \div 2 = 0\ R\, 1$

$109_{10} = 1101101_2 = 01101101_{8\ bit\ 2's\ comp}$

4. -79_{10}

First find out the binary number of 79_{10}**:**

$79 \div 2 = 39\ R\, 1$

$39 \div 2 = 19\, R\, 1$

$19 \div 2 = 9\, R\, 1$

$9 \div 2 = 4\, R\, 1$

$4 \div 2 = 2\, R\, 0$

$2 \div 2 = 1\, R\, 0$

$1 \div 2 = 0\, R\, 1$

$79_{10} = 1001111_2$

$79_{10} = 01001111_{8\ bit\ 2's\ comp}$

Then find -79_{10} to their 8-bits two's complement representation:

1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
-	**0**	**1**	**0**	**0**	**1**	**1**	**1**	**1**
	1	**0**	**1**	**1**	**0**	**0**	**0**	**1**

$-79_{10} = 10110001_{8\ bit\ 2's\ comp}$

B. Convert the following numbers(represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8\ bit\ 2's\ comp} =$

Form the first number, we know that it is a positive integer.

$1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 = 30_{10}$

$00011110_{8\ bit\ 2's\ comp} = 30_{10}$

2. $11100110_{8\ bit\ 2's\ comp} =$

Form the first number, we know that it is a negative integer.

1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
-	**1**	**1**	**1**	**0**	**0**	**1**	**1**	**0**
	0	**0**	**0**	**1**	**1**	**0**	**1**	**0**

$1 \times 2^4 + 1 \times 2^3 + 1 \times 2^1 = 26_{10}$

$11100110_{8\ bit\ 2's\ comp} = -26_{10}$

3. $00101101_{8\ bit\ 2's\ comp} =$

Form the first number, we know that it is a positive integer.

$1 \times 2^5 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0 = 45_{10}$

$00101101_{8\ bit\ 2's\ comp} = 45_{10}$

4. $10011110_{8\ bit\ 2's\ comp} =$

Form the first number, we know that it is a negative integer.

1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
-	**1**	**0**	**0**	**1**	**1**	**1**	**1**	**0**
	0	**1**	**1**	**0**	**0**	**0**	**1**	**0**

$1 \times 2^6 + 1 \times 2^5 + 1 \times 2^1 = 98_{10}$

$10011110_{8\ bit\ 2's\ comp} = -98_{10}$

4 Question Four

1. **Exercise 1.2.4, sections b, c**

   (a) **section b: write a truth table for** $\neg(p \vee q)$

p	q	$(p \vee q)$	$\neg(p \vee q)$
T	**T**	**T**	**F**
T	**F**	**T**	**F**
F	**T**	**T**	**F**
F	**F**	**F**	**T**

   (b) **section c: write a truth table for**$r \vee (p \wedge \neg q)$

p	q	r	$\neg q$	$p \wedge \neg q$	$r \vee (p \wedge \neg q)$
T	**T**	**T**	**F**	**F**	**T**
T	**T**	**F**	**F**	**F**	**F**
T	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**T**
F	**T**	**T**	**F**	**F**	**T**
F	**T**	**F**	**F**	**F**	**F**
F	**F**	**T**	**T**	**F**	**T**
F	**F**	**F**	**T**	**F**	**F**

2. **Exercise 1.3.4, sections b,d**

   (a) **section b**

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**
F	**T**	**T**	**F**	**F**
F	**F**	**T**	**T**	**T**

   (b) **section d**

p	**q**	$\neg p$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	**T**	**F**	**T**	**F**	**T**
T	**F**	**T**	**F**	**T**	**T**
F	**T**	**F**	**F**	**T**	**T**
F	**F**	**T**	**T**	**F**	**T**

5 Question Five

1. **Exercise 1.2.7, section b,c**

   (a) **section b**
   $(B \wedge D) \vee (B \wedge M) \vee (D \wedge M) \vee (B \wedge D \wedge M)$

   (b) **section c**
   $B \vee (D \wedge M)$

2. **Exercise 1.3.7, sections b-e**

   b. $(s \vee y) \rightarrow p$

   c. $p \rightarrow y$

   d. $p \leftrightarrow (s \wedge y)$

   e. $p \rightarrow (s \vee y)$

3. **Exercise 1.3.9, sections c,d**

   c. $c \rightarrow p$

   d. $c \rightarrow p$

6 Question Six

1. **Exercise 1.3.6, sections b-d**

   b. If Joe is eligible for the honors program, then he maintained a B average.

   c. If Rajiv can go on the roller coaster, then he is at least four feet tall.

   d. If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. **Exercise 1.3.10, sections c-f**

   c.
   $p \vee r$ = **True**
   $q \wedge r$ = **unknown**
   $(p \vee r) \leftrightarrow (q \wedge r)$ = **unknown**

   d.
   $p \wedge r$ = **unknown**
   $q \wedge r$ = **unknown**
   $(p \wedge r) \leftrightarrow (q \wedge r)$ = **unknown**

   e.
   p = True
   $r \vee q$ = **unknown**
   $p \rightarrow (r \vee q)$ = **unknown**

   f.
   $p \wedge q$ = **False**
   $(p \wedge q) \rightarrow r$ = **True**

7 Question Seven

Solve Exercise 1.4.5, sections b-d

b.

1. **Logical Expressions:**
   $\neg j \to (l \lor \neg r)$
   $(r \land \neg l) \to j$

2. **Truth table for $\neg j \to (l \lor \neg r)$**

l	r	j	$\neg j$	$\neg r$	$l \lor \neg r$	$\neg j \to (l \lor \neg r)$
T	**T**	**T**	**F**	**F**	**T**	**T**
T	**T**	**F**	**T**	**F**	**T**	**T**
T	**F**	**T**	**F**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**T**	**T**
F	**T**	**T**	**F**	**F**	**T**	**T**
F	**T**	**F**	**T**	**F**	**F**	**F**
F	**F**	**T**	**F**	**T**	**T**	**T**
F	**F**	**F**	**T**	**T**	**T**	**T**

3. **Truth Table for $(r \land \neg l) \to j$**

l	r	j	$\neg l$	$r \land \neg l$	$(r \land \neg l) \to j$
T	**T**	**T**	**F**	**F**	**T**
T	**T**	**F**	**F**	**F**	**T**
T	**F**	**T**	**F**	**F**	**T**
T	**F**	**F**	**F**	**F**	**T**
F	**T**	**T**	**T**	**T**	**T**
F	**T**	**F**	**T**	**T**	**F**
F	**F**	**T**	**T**	**F**	**T**
F	**F**	**F**	**T**	**F**	**T**

4. **According to the tables, these two expressions are logically equivalent.**

c.

1. **Logical Expressions:**
   $j \to \neg l$
   $\neg j \to l$

2. **Truth table for** $j \to \neg l$

j	**l**	$\neg l$	$j \to \neg l$
T	**T**	**F**	**F**
T	**F**	**T**	**T**
F	**T**	**F**	**T**
F	**F**	**T**	**T**

3. **Truth table for** $\neg j \to l$

j	**l**	$\neg j$	$\neg j \to l$
T	**T**	**F**	**T**
T	**F**	**F**	**T**
F	**T**	**T**	**T**
F	**F**	**T**	**F**

4. **According to the tables, these two expressions are not logically equivalent.**

d. Logical Expressions:

$(r \vee \neg l) \to j$

$j \to (r \wedge \neg l)$

These two are not logically equivalent.

When r = T, l = T, j = F, the first expression = False But, the second expression = True.

Therefore, these two are not logically equivalent.

8 Question Eight

1. **Exercise 1.5.2, sections c,f,i**

   - **c. Use the laws of propositional logic to prove** $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

     $(p \to q) \wedge (p \to r)$
     $\equiv (\neg p \vee q) \wedge (\neg p \wedge r)$
     $\equiv \neg p \vee (q \wedge r)$
     $\equiv p \to (q \wedge r)$
     Therefore$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

   - **f. Use the laws of propositional logic to prove** $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

     $\neg(p \vee (\neg p \wedge q))$
     $\equiv \neg p \wedge \neg(\neg p \wedge q)$
     $\equiv \neg p \wedge (\neg\neg p \vee \neg q)$
     $\equiv \neg p \wedge (p \vee \neg q)$
     $\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$
     $\equiv F \vee (\neg p \wedge \neg q)$
     $\equiv \neg p \wedge \neg q$

   - **i. Use the laws of propositional logic to prove** $(p \wedge q) \to r \equiv (p \wedge \neg r) \to \neg q$

     $(p \wedge q) \to r$
     $\equiv \neg(p \wedge q) \vee r$
     $\equiv (\neg p \vee \neg q) \vee r$
     $\equiv \neg p \vee r \vee \neg q$
     $\equiv \neg(p \vee \neg r) \vee \neg q$
     $\equiv (p \wedge \neg r) \to \neg q$

2. **Exercise 1.5.3, sections c, d**

- **c.** $\neg r \vee (\neg r \to p)$

  $\equiv \neg r \vee (\neg\neg r \vee p)$

  $\equiv \neg r \vee (r \vee p)$

  $\equiv (\neg r \vee r) \vee p$

  $\equiv T \vee p$

  $\equiv T$

  Therefore, $\neg r \vee (\neg r \to p)$ **is a tautology.**

- **d. Use the laws of propositional logic to prove that** $\neg(p \to q) \to \neg q$ **is a tautology.**

  $\neg(p \to q) \to \neg q$

  $\equiv \neg(\neg p \vee q) \to \neg q$

  $\equiv (\neg\neg(\neg p \vee q)) \vee \neg q$

  $\equiv (\neg p \vee q) \vee \neg q$

  $\equiv \neg p \vee (q \vee \neg q)$

  $\equiv \neg p \vee T$

  $\equiv T$

  Therefore, $\neg(p \to q) \to \neg q$ **is a tautology.**

9 Question Nine

1. **Exercise 1.6.3, sections c,d**

   - **c.** $\exists x \in R(x = x^2)$
   - **d.** $\forall x \in R(x \leq (x^2 + 1)$

2. **Exercise 1.7.4, sections b-d**

   - **b.** $\forall x(\neg S(x) \wedge W(x))$
   - **c.** $\forall x(S(x) \rightarrow \neg W(x))$
   - **d.** $\exists x(S(x) \wedge W(x))$

10 Question Ten

1. **Exercise 1.7.9, sections c-i**

   - **c. True**
     When x = a, P(x) = True. $\exists x((x = c) \to P(x))$
     Therefore, the whole expression evaluates to True.
   - **d. True**
     When x = e, Q(x) = R(x) = True.
     Q(x) $\wedge$ R(x) = True.
     Therefore, the whole expression evaluates to true.
   - **e. True**
     according to the table, Q(a) $\wedge$ P(d) = True.
   - **f. True**
     When $x \neq b$**, all Q(x) = True.**
   - **g False**
     When x = c, P(x) = R(x) = False. $P(x)eeR(x)$ **= False.**
   - **h. True**
     No matter what value is x, $R(x) \to P(x) =$ **True.**
     Therefore, the whole expression evaluates to true.
   - **i. True**
     When x = e, Q(x) = R(x) = True. Q(x) $\vee$ R(x) = True.

2. **Exercise 1.9.2, sections b-i**

   - **b. True**
     When x = 2, Q(x,y) always equal to True.
   - **c. True**
     When y = 1, P(x,y) always equal to True.
   - **d. False**
     For all x and y, S(x,y) = False.
   - **e. False**
     When x = 1, Q(x,y) always equal to False.

- **f. True**
  When x = 1, P(1,1) = True; When x = 2, P(2,3) = True; when x = 3, P(3,1) = True.
- **g. False**
  When x = y = 2, P(x,y) = False.
- **h. True**
  When x = y = 2, Q(x,y) = True.
- **i. True**
  For all x and y, S(x,y) = False. ¬ S(x,y) = True.

11 Question Eleven

1. **Exercise 1.10.4, sections c-g**
   - **c.** $\exists x \exists y((x \neq y) \land (x + y = x \times y)$
   - **d.** $\forall x \forall y((x > 0, y > 0) \to (\frac{x}{y} > 0))$
   - **e.** $\forall x((x > 0) \land (x < 1) \to \frac{1}{x} > 1)$
   - **f.** $\neg \exists x \forall y(x < y)$
   - **g.** $\forall x \exists y((x \neq 0) \land x \times y = 1)$
2. **Exercise 1.10.7, sections c-f**
   - **c.** $\exists x(N(x) \land D(x))$
   - **d.** $\forall x(D(x) \to P(Sam, x))$
   - **e.** $\exists x \forall y(N(x) \land P(x, y))$
   - **f.** $\exists x(N(x) \land D(x)) \land \forall y((x \neq y) \to (\neg N(y) \lor \neg D(y))$
3. **Exercise 1.10.10, sections c-f**
   - **c.** $\forall x \exists y((y \neq Math101) \land T(x, y))$
   - **d.** $\exists x \forall y((y \neq Math101) \land T(x, y))$
   - **e.** $\forall x \exists y \exists z((x \neq Sam) \to ((y \neq z) \land T(x, y) \land T(x, z)))$
   - **f.** $\exists x \exists y \exists z((x \neq y) \land T(Sam, x) \land T(Sam, y) \land (((z \neq x) \land (z \neq y)) \to \neg T(Sam, z)))$

12 Question Twelve

1. **Exercise 1.8.2, sections b-e**

   - **b.** $\forall x(D(x) \vee P(x))$
     Negation: $\neg\forall x(D(x) \vee P(x))$
     Applying De Morgan's law: $\exists x(\neg D(x) \wedge \neg P(x))$
     English: There is a patient that did not given the medication and not given the placebo.
   - **c.** $\exists x(D(x) \wedge P(x))$ **Negation:** $\neg\exists x(D(x) \wedge P(x))$ **Applying De Morgan's law:** $\forall x(\neg D(x) \vee \neg P(x))$
     English: Every patient was either not given the medication or not given the placebo (or both)
   - **d.** $\forall x(P(x) \to M(x))$
     Negation: $\neg\forall x(P(x) \to M(x))$
     Applying De Morgan's law and the conditional identity: $\neg\forall x(P(x) \to M(x)) \equiv \neg\forall x(\neg P(x) \vee M(x)) \equiv \exists x(\neg\neg P(x) \wedge \neg M(x)) \equiv \exists x(P(x) \wedge \neg M(x))$
     English: There is a patient who was given the placebo but did not have migrains.
   - **e.** $\exists x(M(x) \wedge P(x))$
     Negation: $\neg\exists x(M(x) \wedge P(x))$
     Applying De Morgan's law: $\forall x(\neg M(x) \vee \neg P(x))$
     English: Every patient neither did not have migrains or was not given the placebo (or both).

2. **Exercise 1.9.4, sections c-e**

   - **c.** $\exists x\forall y(P(x,y) \to Q(x,y))$
     Negation:
     $\neg\exists x\forall y(P(x,y) \to Q(x,y))$
     $\equiv \forall x\exists y\neg(\neg P(x,y) \vee Q(x,y))$
     $\equiv \forall x\exists y(\neg\neg P(x,y) \wedge \neg Q(x,y))$
     $\equiv \forall x\exists y(P(x,y) \wedge \neg Q(x,y))$

- **d.** $\exists x \forall y (P(x,y) \leftrightarrow P(y,x))$

  Negation: $\neg \exists x \forall y (P(x,y) \leftrightarrow P(y,x))$

  $\equiv \forall x \exists y (\neg(P(x,y) \leftrightarrow P(y,x)))$

  $\equiv \neg(\exists x \forall y ((P(x,y)) \rightarrow P(y,x)) \wedge (P(y,x) \rightarrow P(x,y)))$

  $\equiv \neg(\exists x \forall y ((\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y))))$

  $\equiv \forall x \exists y (\neg(\neg P(x,y) \vee P(y,x)) \vee \neg(\neg P(y,x) \vee P(x,y)))$

  $\equiv \forall x \exists y ((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y)))$

- **e.** $\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y)$

  Negation: $\neg(\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y))$

  $\equiv \neg(\exists x \exists y P(x,y)) \vee \neg(\forall x \forall y Q(x,y))$

  $\equiv \forall x \forall y \neg P(x,y) \vee \exists x \exists y \neg Q(x,y)$