Homework8

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1 Question 7

a. Exercise 6.1.5 b-d

b

The hand is a three of a kind. A three of a kind has 3 cards of the same rank. the other two cards do not have the same rank as each other and do not have the same rank as the three with the same rank.

1. Number of 5-card hands with 3 cards of the same rank. There are $\binom{13}{1}$ ways to select the ranks for the three cards from {2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A}.

2. There are $\binom{4}{3}$ ways to select the suits for three cards from the set{Diamonds, Hearts, Clubs, Spades}.

3. The second last card can be any card whose rank is the not same as the 3 cards. There are 52 - 4 = 48 choices.

4. The last card can be any card whose rank is no the same as the second last card and the 3 cards. There are 52 - 4 - 4 = 44 choices.

5. The probability that it is a three of a kind is $\binom{4}{3} \times \binom{13}{1} \times 48 \times 44 \div 2! \div \binom{52}{5} \approx \mathbf{0.021128}$

c

The number of outcomes for all 5 cards have $\heartsuit$ is $\binom{13}{5}$

The number of outcomes for all 5 cards have $\spadesuit\clubsuit\diamondsuit$ are all $\binom{13}{5}$

The number of outcomes for a 5-card hand is $\binom{52}{5}$

Therefore, the possibility that all 5 cards have the same suit is
$\binom{13}{5} \times 4 \div \binom{52}{5} \approx \mathbf{0.00198}$

d

1. Number of 5-card hands with one pair. There are $\binom{13}{1}$ ways to select the ranks for the pair from {2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A}.

2.There are $\binom{4}{2}$ ways to select the suits for one pair from the set {Diamonds, Hearts, Clubs, Spades}.

3. The third card can be any card whose rank is not the same as the pair. There are 52 - 4 = 48 choices.

4. The fourth card can be any card whose rank is not the same as the three cards mentioned above. There are 52 - 4 - 4 = 44 choices.

5. The last card can be any card in a different rank. There are 52 - 4 - 4 - 4 = 40 choices.

6. The probability that a random hand is a two of a kind is $\binom{13}{1}\binom{4}{2} \times 48 \times 44 \times 40 \div 3! \div \binom{52}{5} \approx \mathbf{0.42257}$

b. Exercise 6.2.4, sections a-d

a The hand has at least one club

p(the hand has no club) $= \binom{39}{5} \div \binom{52}{5} \approx 0.22153$

Therefore, p(the hand has at least one club) = 1 - p(the hand has no club) $\approx$ **0.778466**

b The hand has at least two cards with the same rank

The total outcome of the hand has no two cards with the same rank $= 52 \times 48 \times 44 \times 40 \times 36 \div 2 \div 3 \div 4 \div 5 = 1317888$

The number of the total outcome $= \binom{52}{5} = 2598960$

p(the hand has no two cards with the same rank) = The total outcome of the hand has no two cards with the same rank $\div$ The number of the total outcome $= 1317888 \div 2598960 \approx 0.50708$

Therefore, p(the hand has at least two cards with the same rank) = 1 - p(the hand has no two cards with the same rank) $\approx$ **0.492917**

c The hand has exactly one club or exactly one spade

p(the hand has exactly one club) $= \binom{13}{1} \times \binom{39}{4} \div \binom{52}{5} \approx 0.37260$

p(the hand has exactly one spade) $= \binom{13}{1} \times \binom{39}{4} \div \binom{52}{5} \approx 0.37260$

p(the hand has exactly one spade and one club) $= \binom{13}{1} \times \binom{13}{1} \times \binom{26}{3} \div \binom{52}{5} \approx 0.1691$

Therefore, p(the hand has exactly one club or exactly one spade) $\approx 0.37260 \times 2 - 0.1691 \approx \mathbf{0.5761}$

d The hand has at least one club or at least one spade

p(the hand has no club nor spade) $= \binom{26}{5} \div \binom{52}{5} \approx 0.02531$

p(The hand has at least one club or at least one spade) = 1 - p(the hand has no club and no spade) $\approx 1 - 0.02531 = \mathbf{0.97469}$

2 Question 8

1. **Exercise 6.3.2, sections a-e**

   a

   There are 6! different permutations for event "the letter b falls in the middle".

   There are 7! different permutations for the event - "the letters a, b, c, d, e, f, g are put in a random order".

   There are $(1 + 2 + 3 + 4 + 5 + 6 = 21) \times 5! = 2520$ different permutations for the event - "the letter c appears to the right of b".

   There are 5! different permutations for event - the letters "def" occur together in that order.

   Therefore, p(A) $= 6! \div 7! = \frac{1}{7}$

   P(B) $= 2520 \div 7! = 0.5$

   P(C) $= 5! \div 7! = \frac{1}{42}$

   b

   P$(A|C) = |A \cap C| \div |C|$

   A∩C = The letter b falls in the middle and the letters "def" occur together in that order

   the number of permutations in $|A \cap C| = 3! \times 2 = 12$

   the number of permutations in C $= 5! = 120$

   Therefore, P$(A|C) = 12 \div 120 = \mathbf{0.1}$

   c

   B = The letters "def" occur together in that order and the letter c appears to the right of b.

   the number of permutation of B∩C is $(1 + 2 + 3 + 4) \times 3! = 60$

   the number of permutation of C is $5! = 120$

   Therefore P(B|C) = **0.5**

   d

   A∩B = The letter b falls in the middle and the letter c appears to the right of b

   the number of permutation of A∩B $= 5! \times 3 = 360$

   the number of permutation of B = 2520

   P(A|B) $= 360 \div 2520 = \frac{1}{7}$

   e

From the textbook, we know that two events are independent if conditioning on on event does not change the probability of the other event.

if P(E|F) = P(E)

Then events E and F are independent.

According to questions a, b, c and d, all events does not qualify the requirement for the independent events. Therefore, there are no pair of events among A,B, and C are independent.

2. **Exercise 6.3.6, sections b,c**

   b

   P(the first 5 flips come up heads) $= \frac{1}{3}^5$

   P(the last 5 flips come up tails) $= \frac{2}{3}^5$

   $$\mathbf{P} = \frac{2^5}{3^{10}}$$

   c

   P(the first flip comes up head) $= \frac{1}{3}$

   P(the rest of the flips come up tails) $= \frac{2}{3}^9$

   $$\mathbf{p} = \frac{2^9}{3^{10}}$$

3. **Exercise 6.4.2 section a**

   Let F be the event that you selects the fair die and $\overline{F}$ be the event that you select the biased die.

   Let X be the event that you choose a die at random, and roll it six times, getting the values 4, 3, 6, 6, 5, 5.

   P(X|F) $= \frac{1}{6^6}$

   P(X|$\overline{F}$) $= 0.25^2 \times 0.15^4 = 0.00003164$

   P(F|X) $= \frac{P(X|F)P(F)}{p(X|F)p(F)+p(X|\overline{F})p(\overline{F})} \approx 0.4$

3 Question 9

1. **Exercise 6.5.2 sections a, b**

   a

   Let the random variable A denote the number of aces in the hands.

   the range of A = {0, 1, 2, 3, 4}

   b

   The number of permutations that there is no A = $\binom{48}{5}$ = 1712304

   The number of permutations for all = $\binom{52}{5}$ = 2598960

   The number of permutations that there is one A = $4 \times \binom{48}{4}$ = 778320

   The number of permutations that there are two As =$\binom{48}{3} \times \binom{4}{2}$ = 103776

   The number of permutations that there are three As = $\binom{48}{2} \times \binom{4}{3}$ = 4512

   The number of permutations that there are four As = $\binom{48}{1} \times \binom{4}{4}$ = 48

   <u>The distribution over A</u> $=\{(0, 0.65884), (1, 0.29947), (2, 0.03993), (3, 0.00174), (4, 0.000018469\}$

2. **Exercise 6.6.1 section a**

   Because it is a two student council chosen at random from a group of 10 people. Therefore, the number of permutations is $\binom{10}{2}$ = 45

   p(G = 1) = $\binom{7}{1} \times \binom{3}{1} \div \binom{10}{2} \approx 0.467$

   p(G = 2) = $\binom{7}{2} \div \binom{10}{2} \approx 0.4667$

   E[G] $= 1 \times p(G = 1) + 2 \times p(G = 2) + 0 \times p(G = 0) = 1 \times 0.467 + 2 \times 0.467 + 0 \approx$ 1.40

3. **Exercise 6.6.4 section a,b**

   (a)

   There are 6 different outcomes {1, 4, 9, 16, 25, 36}

   E[x] $= 1 \times 1/6 + 4 \times 1/6 + 9 \times 1/6 + 16 \times 1/6 + 25 \times 1/6 + 36 \times 1/6 \approx$ **15.167**

   (b)

   E[Y] $= 0 \times 1/8 + 9 \times 1/8 + 1 \times 3/8 + 4 \times 3/8 =$ **3**

4. **Exercise 6.7.4, section a**

   Probability = 0.1 for different X_i

   E[X] $= 10 \times 0.1 =$ **1**

4 Question 10

1. **Exercise 6.8.1, sections a-d**

   (a)

   p(100 circuit boards made exactly 2 have defects) = b(k;n,p) = b(2;100,0.01) $= \binom{100}{2} \times (0.01)^2(0.99)^{98}$

   (b)

   p(at least 2 have defects) = 1 - p(no one have defect) - p(only 1 have defect) = 1 - 0.99^{100} - $\binom{100}{1} \times (0.01)^1 \times (0.99)^{99}$

   (c) What is the expected number of circuit boards with defects out of the 100 made?

   E[k] = np = $0.01 \times 100 = 1$

   (d)

   p(out of 50 batches at least 2 have defects) = 1 - p(no one have defects) = 1 - $(0.99)^{50}$

   E[k] = $100 \div 2 \times 0.01 \times 2 = 1$

   Compared with the situation that each circuit board is made separately, p is different while expected number of circuit boards with defects is the same.

2. **Exercise 6.8.3, section b**

   (b)

   p(incorrect decision given coin is biased)

   $=1 - \binom{10}{0} \times (0.3)^0 \times (0.7)^{10} - \binom{10}{1} \times (0.3)^1 \times (0.7)^9 - \binom{10}{2} \times (0.3)^2 \times (0.7)^8 - \binom{10}{3} \times (0.3)^3 \times (0.7)^7$

   =0.3504