1 Information Theory

1.1 Entropy and Probability Distributions

a) The marginal distribution of X =

$$\{(\frac{1}{8}+\frac{1}{16}+\frac{1}{16}+\frac{1}{4}),(\frac{1}{16}+\frac{1}{8}+\frac{1}{16}+0),(\frac{1}{32}+\frac{1}{32}+\frac{1}{16}+0),(\frac{1}{32}+\frac{1}{32}+\frac{1}{16}+0)\}$$

$$= \{\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{8}\}$$

The marginal distribution of Y =

$$\{(\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{32}),(\frac{1}{16}+\frac{1}{8}+\frac{1}{32}+\frac{1}{32}),(\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}),(\frac{1}{4}+0+0+0)\}$$

$$= \{\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}\}$$

b) The Entropy of X, H(X) =

$$= -\frac{1}{2}log_2\frac{1}{2} - \frac{1}{4}log_2\frac{1}{4} - \frac{1}{8}log_2\frac{1}{8} - \frac{1}{8}log_2\frac{1}{8}$$

$$= .5 + .5 + .375 + .375$$

$$= 1.75\, bits$$

The Entropy of Y, H(Y) =

$$= 4 * (-\frac{1}{4}log_2\frac{1}{4})$$

$$= 4 * .5$$

$$= 2\, bits$$

c)

$$H(X|Y) = \sum_{i=1}^{4} p(Y=i)H(X|Y=i)$$

$$= \frac{1}{4}H(\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{8}) + \frac{1}{4}H(\frac{1}{4},\frac{1}{2},\frac{1}{8},\frac{1}{8}) + \frac{1}{4}H(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}) + \frac{1}{4}H(1,0,0,0)$$

$$= \frac{1}{4} * \frac{7}{4} + \frac{1}{4} * \frac{7}{4} + \frac{1}{4} * 2 + \frac{1}{4} * 0$$

$$= 1.375\, bits$$

$$H(Y|X) = H(X|Y) - H(X) + H(Y)$$

$$= 1.375 - 1.75 + 2$$

$$= 1.625\, bits$$

$$H(X,Y) = H(Y|X) + H(X)$$
$$= 1.625 + 1.75$$
$$= 3.375\, bits$$

d) The mutual information I(X;Y)

$$I(X;Y) = H(X) - H(X|Y) \tag{1}$$
$$= 1.75 - 1.375$$
$$= 0.375\, bits$$

If we calculate I(X;Y) as follows:

$$I(X;Y) = H(Y) - H(Y|X) \tag{2}$$
$$= 2 - 1.625$$
$$= 0.375\, bits$$

So, (1) and (2) are generating the same value which authenticates the validity of the symmetry property of the mutual information.