Learning Guide Unit 6

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Book: Learning Guide Unit 6

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Description

Learning Guide Unit 6

Table of contents

Overview

Unit 6: Linear Programming and Reductions

Topics:

- Linear Programming
- Simplex method

Learning Objectives:

By the end of this Unit, students will be able to:

1. Understand linear programming as an algorithm pattern and be able t0 articulate the characteristics of the problems it is best suited to solve
2. Understand the operation of the simplex method and be able to implement the simplex method as an algorithm

Tasks:

- Peer assess Unit 5 Programming Assignment
- Read the Learning Guide and Reading Assignments
- Participate in the Discussion Assignment (post, comment, and rate in the Discussion Forum)
- Make entries to the Learning Journal
- Take the Graded Quiz
- Take the Self-Quiz

Introduction

Throughout this course we have been focused on understanding different types of algorithms and one of the key ways that we have used to understand the operation and characteristics of different algorithms is through asymptotic analysis. In many cases the efficiency and subsequently the applicability of a particular algorithm or algorithm "pattern" (in this context by pattern we refer to Brute Force, Branch and Bound, Backtracking, and Divide and Conquer as patterns) is dependent upon the input size of the problem. In these cases the issue of efficiency becomes relevant as the size of n (or the input size of the problem being solved) grows large.

In this and the subsequent unit, we will be taking a break from efficiency and asymptotic analysis and will be looking at a form of algorithm development that is known a linear programming.

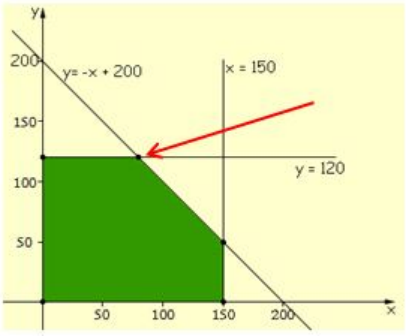
Linear programming is the process of taking various linear inequalities relating to some situation, and finding the "best" value obtainable under those conditions. Linear programming is often used in situations where efficiency is required. For example imagine that you were trying to determine the maximum of a particular product (or products) to produce or could be produced based upon some set of constraints. Another example might be one where you were trying to maximize profits by minimizing cost.

We just used two very important words that have great meaning in the context of linear programming: Maximize and Minimize. Linear programming is an approach that to either maximize or minimize some goal.

In "real life", linear programming is part of a very important area of mathematics called "optimization techniques". This field of study (or at least the applied results of it) are used every day in the organization and allocation of resources. These "real life" systems can have dozens or hundreds of variables, or more.

The basic idea behind linear programming is that you develop a model by translating the problem (often known as a goal) that you want to solve into a mathematical formula or function. To this model you add the various constraints. The graph of the goal function coupled with the constraints will reveal the set of possible options and can be used to reveal the most optimal solution available.

Consider the following graph.



In this example we see a constraint function which is identified as y = x + 200. We also see a couple of additional constraints the first of which is x≤150 and the other is y≤120. When each of these function have been plotted they define a region in the graph (colored in green) and this region represents all of the solutions that are possible.

Imagine that y in this model represents units of Baseball mitts manufactured and x represents the number of Baseballs manufactured. We can see that a maximum of 120 Mits can be manufactured and that a maximum of 150 baseballs can be manufactured. Finally we have a constraint that indicates that our total production of units (balls and mitts) cannot exceed 200. The question is how many of each should we manufacture to maximize our profit if Mitts sell for $20 and balls sell for $5?

The red arrow visually shows us the point of maximum profit. We can tell this because it is the point that maximizes the number of units produced. There are a number of different methods to implement linear programming algorithms.

By the way, please remember that an algorithm is defined as a well-defined, finite, set of instructions used to accomplish a specific task, typically a computation. Although we do apply the term algorithm to the instructions executed by a computer the implementation of an algorithm isn't limited to computers.

One well known such algorithm (which often is referred to as a method) is the simplex method, as illustrated in this PDF document. The simplex algorithm is a well-defined and finite set of instructions that can be used to solve a linear programming problem for either maximization or minimization. The following section will walk through an example of a linear programming problem that has been designed for the 'human' computer to process. This unit is focused on the simplex method. The discussion question for this unit will require that the manual simplex process be followed, documented, and explained. This process will help you to understand the simplex procedure well enough to complete the development project which will be to automate the simplex procedure by developing a java program that calculate the simplex problem provided in the assignment.

Reading Assignment

Topic 1: Linear Programming
Chapter 7 Linear Programming in Algorithms by S. Dasgupta, C.H. Papadimitriou, and U.V. Vazirani available at http://www.cs.berkeley.edu/~vazirani/algorithms/chap7.pdf

Topic 2: Simplex Method and Simplex Algorithm
Chapter 7 Linear Programming in Algorithms by S. Dasgupta, C.H. Papadimitriou, and U.V. Vazirani available at http://www.cs.berkeley.edu/~vazirani/algorithms/chap7.pdf

Supplemental Materials
PatrickJMT Free Math Video Tutorials – Simplex Method http://www.youtube.com/user/patrickJMT/videos?query=simplex

Unit 6 Optional Video Lectures
The following video lectures are optional resources that have been made available to students who can take advantage of them. These lectures are strictly optional resources. All of the information in these lectures is available in other learning resources within the course. These lectures are provided for those students who have sufficient network bandwidth and technology capabilities to take advantage of video content. These lectures cannot be used instead of the required assigned resources and there is no information that is not contained in the assigned resources. These lectures simply present some of the information in a different format.

- Unit 6 Lecture 1: Linear Programming
- Unit 6 Lecture 2: Simplex Method and Simplex Algorithm

Discussion Assignment

In your own words describe the operation of the simplex algorithm to implement a linear programming solution to the real-world problem described below.

Include one or two examples to explain your thought process to show what is occurring and how the methodology works. Demonstrate your understanding of the intricacies of the algorithm. Use APA citations and references for any sources used.

Problem Statement
A store has requested a manufacturer to produce pants and sports jackets. The manufacturer has one week to fill the order. For materials, the manufacturer has 1,000m2 of cotton textile and 950m2 of polyester. Every pair of pants (1 unit) needs .5m2 of cotton and 2m2 of polyester. Every jacket needs 3m2 of cotton and 2m2 of polyester.

Each item produced must go through a final inspection process and the factory can only inspect a maximum of 500 units per week.

The price of the pants is fixed at $40 and the jacket at $75.

What is the number of pants and jackets that the manufacturer must give to the store so that these items obtain a maximum sale?

For this assignment we will use a specific grading rubric:
For each element from the following list that is represented in the assignment, the specified points (in red) will be awarded.

- If the simplex process is described
- If the goal and constraint equations are included
- If the tableau showing the reductions is included
- If the solution in terms of the number of pants and jacket to produce is provided including the total amount of sales that will be generated

Learning Journal

The Learning Journal is a tool for self-reflection on the learning process. In addition to completing directed tasks, you should use the Learning Journal to document your activities, record problems you may have encountered and to draft answers for Discussion Forums and Assignments. The Learning Journal should be updated regularly (on a weekly basis), as the learning journals will be assessed by your instructor as part of your Final Grade.

Your learning journal entry must be a reflective statement that considers the following questions:

- Describe what you did. This does not mean that you copy and paste from what you have posted or the assignments you have prepared. You need to describe what you did and how you did it.
- Describe your reactions to what you did
- Describe any feedback you received or any specific interactions you had. Discuss how they were helpful
- Describe your feelings and attitudes
- Describe what you learned

Another set of questions to consider in your learning journal statement include:

- What surprised me or caused me to wonder?
- What happened that felt particularly challenging? Why was it challenging to me?
- What skills and knowledge do I recognize that I am gaining?
- What am I realizing about myself as a learner?
- In what ways am I able to apply the ideas and concepts gained to my own experience?

Your Learning Journal should be a minimum of 500 words

Self-Quiz

The Self-Quiz gives you an opportunity to self-assess your knowledge of what you have learned so far.

The results of the Self-Quiz do not count towards your final grade, but the quiz is an important part of the University's learning process and it is expected that you will take it to ensure understanding of the materials presented. Reviewing and analyzing your results will help you perform better on future Graded Quizzes and the Final Exam.

Please access the Self-Quiz on the main course homepage; it will be listed inside the Unit.

Graded Quiz

The Graded Quiz will test your knowledge of all the materials learned thus far. The results of the quiz will count towards your final grade.

Please access the Graded Quiz on the main course homepage; it will be listed inside the Unit. After you click on it, the quiz's introduction will inform you of any time or attempt limits in place.

Good luck!

Checklist

- [ ] Peer assess Unit 5 Programming Assignment
- [ ] Read the Learning Guide and Reading Assignments
- [ ] Participate in the Discussion Assignment (post, comment, and rate in the Discussion Forum)
- [ ] Make entries to the Learning Journal
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