FUNKCIONÁLNÍ DERIVACE

$$\frac{d}{d\lambda}\Big|_{\lambda=0} F(u+\lambda\,\delta u) = \int dx'\, \frac{\delta F}{\delta u(x')}\,\delta u(x')$$

Gateaux

ve směru δu

$$v^p(u(x)) = \int dx'\, v^p(u(x'))\,\delta(x-x')$$

$$\frac{\delta v^p(u(x))}{\delta u^i(x')} = \frac{\partial v^p}{\partial u^i}(x)\,\delta(x-x')$$

Poissonova závorka dvou funkcionálů A, B

$$\{A,B\} = \int dx \int dy\, \frac{\delta A}{\delta u^i(x)}\, L^{ij}\, \frac{\delta B}{\delta u^j(y)}$$

Poissonův bivektor

$$\mathbf{L} = L^{ij}\,\frac{\partial}{\partial u^i}\otimes\frac{\partial}{\partial u^j} = L^{ij}\,\frac{\partial v^a}{\partial u^i}\,\frac{\partial v^b}{\partial u^j}\,\frac{\partial}{\partial v^a}\otimes\frac{\partial}{\partial v^b}$$

$$L^{pq} \equiv \{v^p(u(x)),\, v^q(u(y))\} = \int dx' \int dy'\, \underbrace{\frac{\delta v^p(u(x))}{\delta u^i(x')}}_{\frac{\partial v^p}{\partial u^i}(x')\,\delta(x-x')}\, L^{i(x')\,j(y')}\, \underbrace{\frac{\delta v^q(u(y))}{\delta u^j(y')}}_{\frac{\partial v^q}{\partial u^j}(y')\,\delta(y-y')}$$

$$= \frac{\partial v^p}{\partial u^i}(x)\,\frac{\partial v^q}{\partial u^j}(y)\,\{u^i(x),\, u^j(y)\}$$

speciální bivektor $\quad L^{ij}(x,y) = \{u^i(x), u^j(y)\} = g^{ij}\,\delta'(x-y) - g^{is}\,\Gamma^j_{sk}\,\frac{\partial u^k}{\partial x}\,\delta(x-y)$

$$\{A,B\} = \int dx \int dy\, \frac{\delta A}{\delta u^i(x)}\left[g^{ij}\,\delta'(x-y) - g^{is}\,\Gamma^j_{sk}\,\frac{\partial u^k}{\partial x}\,\delta(x-y)\right]\frac{\delta B}{\delta u^j(y)}$$

per partes

$$= -\int dx \int dy \left\{\frac{\partial}{\partial x}\left[\frac{\delta A}{\delta u^i(x)}\, g^{ij}(u(x))\right]\delta(x-y) - g^{is}\,\Gamma^j_{sk}\,\frac{\partial u^k}{\partial x}\,\delta(x-y)\,\frac{\delta A}{\delta u^i(x)}\right\}\frac{\delta B}{\delta u^j(y)}$$

$$= -\int dx \left\{\frac{\partial}{\partial x}[\;\;] - g\,\Gamma\,\frac{\partial u^k}{\partial x}\,\frac{\delta A}{\delta u^i(x)}\right\}\frac{\delta B}{\delta u^j(x)}$$

per partes

$$\boxed{\{A,B\} = \int dx\, \frac{\delta A}{\delta u^i(x)}\left[g^{ij}(u(x))\,\frac{\partial}{\partial x}\frac{\delta B}{\delta u^j(x)} - g^{is}\,\Gamma^j_{sk}\,\frac{\partial u^k}{\partial x}\,\frac{\delta B}{\delta u^j(x)}\right]}$$

$$\boxed{\{A,B\} = \int dx \frac{\delta A}{\delta u^i(x)} \left[g^{ij}(u(x)) \frac{\partial}{\partial x} \frac{\delta B}{\delta u^j(x)} - g^{is} \Gamma^j_{sk} \frac{\partial u^k}{\partial x} \frac{\delta B}{\delta u^j(x)} \right]}$$

Jacobiho identita ?

$$\frac{\delta \{\ \}^{ij}}{\delta u^m}$$

$$\left\{ \{u^i(x), u^j(y)\}, u^k(z) \right\} = \int d\xi \ \frac{\delta}{\delta u^m(\xi)} \left[g^{ij}(u(x)) \frac{\partial}{\partial x} \delta(x-y) - g^{is}_{(x)} \Gamma^j_{sr}(x) \frac{\partial u^r}{\partial x} \delta(x-y) \right] \cdot$$

$$\cdot \underbrace{\left[g^{mn}(u(\xi)) \frac{\partial}{\partial \xi} \underbrace{\frac{\delta u^k(z)}{\delta u^n(\xi)}}_{\delta^k_r \delta(z-\xi)} - g^{mt} \Gamma^n_{te} \frac{\partial u^e}{\partial \xi} \underbrace{\frac{\delta u^k(z)}{\delta u^n(\xi)}}_{\delta^k_r \delta(z-\xi)} \right]}_{L^{mn} \frac{\delta u^k}{\delta u^n}} =$$

$$= \left[\frac{\partial g^{ij}}{\partial u^m(x)} \frac{\partial}{\partial x} \delta(x-y) + \frac{\partial b^{ij}_r}{\partial u^m} \frac{\partial u^r}{\partial x} \delta(x-y) \right] \left[g^{mk}(u(x)) \frac{\partial}{\partial x}(z-x) + b^{mk}_e \frac{\partial u^e}{\partial x} \delta(z-x) \right] -$$

$$- \frac{\partial}{\partial x} \left[b^{ij}_m(u(x)) \delta(x-y) \left(g^{mk}(x) \frac{\partial}{\partial x} \delta(z-x) + b^{mk}_e \frac{\partial u^e}{\partial x} \delta(z-x) \right) \right] =$$

$$= g^{ij}{}_{,m} g^{mk} \delta'(x-y)\delta'(z-x) + g^{ij}{}_{,m} b^{mk}_e u^e_x \delta'(x-y)\delta(z-x) +$$
$$+ b^{ij}_{r,m} u^r_x g^{mk} \delta(x-y)\delta'(z-x) + b^{ij}_{r,m} u^r_x b^{mk}_e u^e_x \delta(x-y)\delta(z-x)$$
$$- b^{ij}_{m,t} u^t_x g^{mk} \delta(x-y)\delta'(z-x) - b^{ij}_{m,t} u^t_x b^{mk}_e u^e_x \delta(x-y)\delta(z-x)$$
$$- b^{ij}_m g^{mk} \delta'(x-y)\delta'(z-x) - b^{ij}_m b^{mk}_e u^e_x \delta'(x-y)\delta(z-x)$$
$$- b^{ij}_m g^{mk}{}_{,r} u^r_x \delta(x-y)\delta'(z-x) - b^{ij}_m g^{mk} \delta(x-y)\delta''(z-x)$$
$$- b^{ij}_m b^{mk}_{e,r} u^e_x u^r_x \delta(x-y)\delta(z-x) - b^{ij}_m b^{mk}_e u^e_{xx} \delta(x-y)\delta(z-x)$$
$$- b^{ij}_m b^{mk}_e u^e_x \delta(x-y)\delta'(z-x)$$

0,0
0,1
1,0
1,1
0,2

0,0 $\left(b^{ij}_{r,m} b^{mk}_e u^r_x u^e_x - b^{ij}_{m,t} b^{mk}_e u^t_x u^e_x - b^{ij}_m b^{mk}_{e,r} u^e_x u^r_x - b^{ij}_m b^{mk}_e u^e_{xx} \right) \delta(x-y)\delta(z-x)$

0,2 $-b^{ij}_m g^{mk} \delta(x-y)\delta''(z-x)$

0,1 $\left(b^{ij}_{r,m} u^r_x g^{mk} - b^{ij}_{m,t} u^t_x g^{mk} - b^{ij}_m g^{mk}{}_{,r} u^r_x - b^{ij}_m b^{mk}_e u^e_x \right) \delta(x-y)\delta'(z-x)$

1,0 $\left(g^{ij}{}_{,m} b^{mk}_e u^e_x - b^{ij}_m b^{mk}_e u^e_x \right) \delta'(x-y)\delta(z-x)$

1,1 $\left(g^{ij}{}_{,m} g^{mk} - b^{ij}_m g^{mk} \right) \delta'(x-y)\delta'(z-x)$

(0,0) s cyklickou záměnou

$$\left(b^{ij}_{r,m} b^{mk}_{e} u^r_x u^e_x - b^{ij}_{m,r} b^{mk}_{e} u^r_x u^e_x - b^{ij}_{m} b^{mk}_{e,r} u^r_x u^e_x - b^{ij}_{m} b^{mk}_{e} u^e_{xx}\right) \delta(x-y)\, \delta(z-x)$$

$$\left(b^{jk}_{r,m} b^{mi}_{e} u^r_x u^e_x - b^{jk}_{m,r} b^{mi}_{e} u^r_x u^e_x - b^{jk}_{m} b^{mi}_{e,r} u^r_x u^e_x - b^{jk}_{m} b^{mi}_{e} u^e_{xx}\right) \delta(y-z)\, \delta(x-y)$$

$$\left(b^{ki}_{r,m} b^{mj}_{e} u^r_x u^e_x - b^{ij}_{m,r} b^{mj}_{e} u^r_x u^e_x - b^{ki}_{m} b^{mj}_{e,r} u^r_x u^e_x - b^{ki}_{m} b^{mj}_{e} u^e_{xx}\right) \delta(z-x)\, \delta(y-z)$$

(0,2)

$$- b^{ij}_{m} g^{mk}(x)\ \delta(x-y)\, \delta''(z-x) - b^{jk}_{m}(y)\, g^{mi}(y)\, \delta(y-z)\, \delta''(x-y)$$

$$- b^{ki}_{m}(z)\, g^{mj}(z)\ \delta(z-x)\, \delta''(y-z)$$

(0,1)

$$\left(b^{ij}_{r,m} g^{mk} - b^{ij}_{m,r} g^{mk} - b^{ij}_{m} g^{mk}_{\ ,r} - b^{ij}_{m} b^{mk}_{r}\right)\Big|_x u^r_x\ \delta(x-y)\, \delta'(z-x)$$

$$+\left(b^{jk}_{r,m} g^{mi} - b^{jk}_{m,r} g^{mi} - b^{jk}_{m} g^{mi}_{\ ,r} - b^{jk}_{m} b^{mi}_{r}\right)\Big|_y u^r_y\ \delta(y-z)\, \delta'(x-y)$$

$$+\left(b^{ki}_{r,m} g^{mj} - b^{ki}_{m,r} g^{mj} - b^{ki}_{m} g^{mj}_{\ ,r} - b^{ki}_{m} b^{mj}_{r}\right)\Big|_z u^r_z\ \delta(z-x)\, \delta''(y-z)$$

(1,0)

$$\left(g^{ij}_{\ ,m} b^{mk}_{r} - b^{ij}_{m} b^{mk}_{r}\right)\Big|_x u^r_x\ \delta'(x-y)\, \delta(z-x)$$

$$+\left(g^{jk}_{\ ,m} b^{mi}_{r} - b^{jk}_{m} b^{mi}_{r}\right)\Big|_y u^r_y\ \delta'(y-z)\, \delta(x-y)$$

$$+\left(g^{ki}_{\ ,m} b^{mj}_{r} - b^{ki}_{m} b^{mj}_{r}\right)\Big|_z u^r_z\ \delta'(z-x)\, \delta(y-z)$$

(1,1)

$$\left(g^{ij}_{\ ,m} g^{mk} - b^{ij}_{m} g^{mk}\right)\Big|_x \delta'(x-y)\, \delta'(z-x)$$

$$+\left(g^{jk}_{\ ,m} g^{mi} - b^{jk}_{m} g^{mi}\right)\Big|_y \delta'(y-z)\, \delta'(x-y)$$

$$+\left(g^{ki}_{\ ,m} g^{mj} - b^{ki}_{m} g^{mj}\right)\Big|_z \delta'(z-x)\, \delta'(y-z)$$

teď to musím jenom vypozorovat

idea?

$$f(y)\,\delta'(x-y) = f(x)\,\delta'(x-y) + f'(x)\,\delta(x-y)$$

z $(1,1)$ můžu udělat $(0,2)$?

$$\left(g^{ij}{}_{,m}\, g^{mk} - b^{ij}_{m}\, g^{mk}\right)\Big|_x \delta'(x-y)\,\delta'(z-x)$$

$$+ \left(g^{jk}{}_{,m}\, g^{mi} - b^{jk}_{m}\, g^{mi}\right)\Big|_y \delta'(y-z)\,\delta'(x-y) = \left(\ \right)\Big|_x \delta'(x-z)\,\delta'(x-y) +$$

$$+ \left(g^{ki}{}_{,m}\, g^{mj} - b^{ki}_{m}\, g^{mj}\right)\Big|_z \delta'(z-x)\,\delta'(y-z) \qquad + \partial_x\left(\ \right)\delta''(x-z)\,\delta(x-y)$$