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2009 Phys. Med. Biol. 54 2037

(http://iopscience.iop.org/0031-9155/54/7/012)

Phys. Med. Biol. **54** (2009) 2037–2047 doi:10.1088/0031-9155/54/7/012

Efficient gamma index calculation using fast Euclidean distance transform

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Received 17 October 2008, in final form 13 February 2009
Published 12 March 2009
Online at stacks.iop.org/PMB/54/2037

Abstract

The gamma index is a tool for dose distribution comparison. It combines both dose difference (DD) and distance to agreement (DTA) into a single quantity. Though it is an effective measure, making up for the inadequacy of DD or DTA alone, its calculation can be very time-consuming. For a k-D space with N quantization levels in each dimension, the complexity of the exhaustive search is $O(N^{2k})$. In this work, we proposed an efficient method that reduces the complexity from $O(N^{2k})$ to $O(N^kM)$, where M is the number of discretized dose values and is comparable to N. More precisely, by embedding the reference dose distribution in a $(k+1)$-D spatial-dose space, we can use fast Euclidean distance transform with linear complexity to obtain a table of gamma indices evaluated over a range of the $(k+1)$-D spatial-dose space. Then, to obtain gamma indices for the test dose distribution, it requires only table lookup with complexity $O(N^k)$. Such a table can also be used for other test dose distributions as long as the reference dose distribution is the same. Simulations demonstrated the efficiency of our proposed method. The speedup for 3D gamma index calculation is expected to be on the order of tens of thousands (from $O(N^6)$ to $O(N^3M)$) if N is a few hundreds, which makes clinical usage of the 3D gamma index feasible. A byproduct of the gamma index table is that the gradient of the gamma index with respect to either the spatial or dose dimension can be easily derived. The gradient can be used to identify the main causes of the discrepancy from the reference distribution at any dose point in the test distribution or incorporated in treatment planning and machine parameter optimization.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In radiotherapy we often need to compare two dose distributions, measured or calculated. Dose comparison tools include the dose difference (DD) tool and the distance-to-agreement

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(DTA) tool. However, DD or DTA alone are insufficient. In fact, they are complementary to each other (Van Dyk *et al* 1993). DD is suitable for low dose gradient regions, but not for high dose gradient regions. DTA is the opposite. The gamma index introduced in Low *et al* (1998) provides another dose comparison tool. It combines both DD and DTA into a single quantity normalized by the acceptance criteria. For convenience, in this paper, the ideal or the standard dose distribution is referred to as the reference distribution, and the other one to be compared with the reference, the test distribution. The gamma index can be regarded as a distance measure for every dose point in the test distribution from the reference distribution. Note that the gamma index is not symmetric with respect to these two distributions, and in the literature (Low *et al* 1998, Depuydt *et al* 2002, Low and Dempsey 2003), the roles of the test and reference are reversed in the above statement, which is due to the concern of different spatial resolutions (Ju *et al* 2008). Since the introduction of the gamma index, it has become a widely accepted dose quality assurance (DQA) tool (Agazaryan *et al* 2003). However, its computation can be very time-consuming. The reported methods of gamma index calculation include the exhaustive search, limited search and adaptive search (Stock *et al* 2005, Wendling *et al* 2007, Ju *et al* 2008). The exhaustive search calculates for every dose point in the test distribution its minimal distance from the reference distribution by visiting every reference dose point. The limited search only searches those reference points within certain spatial distance from test points, because beyond that, gamma values are too high and they would simply be labeled as 'fail' instead (Harms *et al* 1998). The adaptive search calculates the point-to-point distance between the test and reference distributions but the reference dose points are searched one spatial layer at a time until the spatial distance alone is larger than the spatial-dose distances from previous layers. The complexity of the exhaustive search is $O(N^{2k})$ for a k-D space with N quantization levels in each dimension. The complexity of the limited search is $O(r^k N^{2k})$, where r is the ratio of the number of samples in acceptable DTA to N in each dimension. The calculation time may still be long for high dimensions depending on the ratio r, and the result may become partially binary though representing high gamma values in binary is often sufficient clinically. The complexity of the adaptive search is $O(N^k)$ for the best case scenario, e.g. when the test distribution is very similar or identical to the reference distribution. But the complexity becomes $O(N^{2k})$ for the worst case scenario, e.g. when the two distributions are complementary. Thus, calculation of the gamma index can take up to minutes or hours for 2D distributions and days for 3D distributions, which is clinically unacceptable.

In this work, we proposed a high efficient method that utilizes the fast Euclidean distance transform (EDT) algorithm. In fact, our method builds a table of gamma indices over a range of the (k+1)-D space and the gamma index for the test distribution can be obtained via table lookup. The complexity of the construction of the gamma index table is $O(N^k M)$, where M is the number of dose values determined by the dose range and resolution, and is comparable to N. The gamma index table is only associated with the reference distribution and independent of the test distribution. Therefore, the same gamma index table can be used for arbitrary test distributions. The complexity of table lookup for an arbitrary test distribution is $O(N^k)$. In addition, from the gamma index table we can easily obtain its derivatives, which are not available via other approaches.

2. Theory

2.1. Review of the gamma index

Let us start by reviewing the definition of the gamma index and the choice of normalization factors. Let $\{D_R(\boldsymbol{x})\}$ denote the reference dose distribution, and $\{D_T(\mathbf{y})\}$, the test dose

distribution. The gamma index defines for every point in the test distribution a distance measure from the reference distribution:

$$\gamma_{D_T,D_R}(\boldsymbol{y}) = \min_{\boldsymbol{x}} \sqrt{\left(\frac{\boldsymbol{y}-\boldsymbol{x}}{\delta}\right)^2 + \left(\frac{D_T(\boldsymbol{y}) - D_R(\boldsymbol{x})}{\Delta}\right)^2}, \tag{1}$$

where δ and Δ are normalization factors for positions and dose, respectively. The normalization factors can also be regarded as weighting factors for DTA and DD, and should be proportional to the measurement errors for measured distributions or bin size for calculated distributions. In radiotherapy applications, the common acceptance for DQA is that DTA be less than 3 mm and DD less than 3% of the maximal dose (Low *et al* 1998). The convention is to define the normalization factors using the acceptance, e.g. $\delta = 3$ mm and $\Delta = 3\%$ of the maximal dose. Then, dose quality is considered acceptable if $\gamma_{D_T,D_R}(\boldsymbol{y}) < 1$. Note that the definition in (1) reverses the roles of the reference and the test dose distributions compared with the gamma index defined in the literature (Low *et al* 1998, Depuydt *et al* 2002, Low and Dempsey 2003). However, the main concern for choosing one versus the other as the searched space is the spatial resolution (Ju *et al* 2008). On one hand, if the searched space has high resolution, the search time will be increased. On the other hand, if the searched space has low resolution, it will not reflect the distance accurately. Here, we choose to designate the reference distribution as the searched space because (i) the reference distribution usually means a gold standard with fine resolution and (ii) it is natural to ask how far a dose point is from the 'reference'. Other than that, they are just dummies like 'distribution 1' and 'distribution 2'. In our approach, we in fact apply interpolation on the reference distribution so that it has the finest resolution in all spatial and dose dimensions.

2.2. EDT and its fast algorithm

In a k-D binary image, every point has a value of either 0 or 1, and is referred to as a background or feature point, respectively (Maurer *et al* 2003). Suppose that a metric is defined on the image. Then distance transform (DT) of the binary image finds for every point in the image its distance from the set of feature points. When the metric is Euclidean distance, the corresponding DT is called EDT. Exact EDT can be calculated by the exhaustive search with complexity $O(N^{2k})$ if the image has N points in each dimension, which is extremely time consuming. However, algorithms for EDT with linear complexity $O(N^k)$ (Breu *et al* 1995, Lee *et al* 1997, Cuisenaire and Macq 1999) have been developed since last decade. In this paper, we use Maurer's algorithm (Maurer *et al* 2003) for EDT. Maurer's algorithm is based on dimension reduction, which is briefly described as follows. Let $S_n(\boldsymbol{x})$ denote the n-D subspace, in which every point has the same last $k-n$ coordinates as $\boldsymbol{x}$. Let $F_n(\boldsymbol{x})$ denote the closest feature point of $\boldsymbol{x}$ for the search space $S_n(\boldsymbol{x})$. The goal is to find $F_k(\boldsymbol{x})$. The essence of dimension reduction is that to find $F_{n+1}(\boldsymbol{x})$, one only needs to check through the closest feature points found previously in one lower dimension $\{F_n(\boldsymbol{x}')|\boldsymbol{x}' \in R_{n+1}(\boldsymbol{x})\}$, where $R_{n+1}(\boldsymbol{x})$ is the line in $S_{n+1}(\boldsymbol{x})$, passes through $\boldsymbol{x}$ and is perpendicular to $S_n(\boldsymbol{x})$. In fact, by first screening $\{F_n(\boldsymbol{x}')|\boldsymbol{x}' \in R_{n+1}(\boldsymbol{x})\}$ to reduce it to a subset whose Voronoi partitions intersect $R_n(\boldsymbol{x})$, the check-through becomes linear in complexity.

2.3. Relation between gamma index and EDT

Now, we want to describe our viewpoint and approach. A k-D dose distribution $\{D(\boldsymbol{x})\}$ can be regarded as a hyper-surface $\{(\boldsymbol{x}, d) \in \mathbf{R}^{k+1} | d = D(\boldsymbol{x})\}$ in the $(k+1)$-D spatial-dose space. In

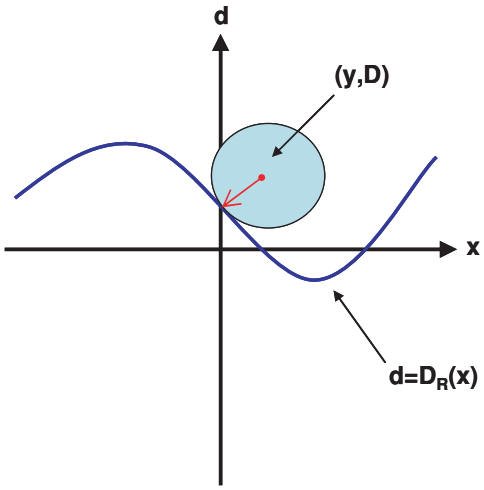

Figure 1. Illustration of the gamma index as the distance measured from the reference dose distribution in the spatial-dose representation. The radius indicated by the arrow is the gamma index of $(\boldsymbol{y}, D)$ with respect to the reference $d = D_R(\boldsymbol{x})$. It is the closest distance of the dose point $(\boldsymbol{y}, D)$ from the reference distribution $d = D_R(\boldsymbol{x})$, which is regarded as a set of feature points in the spatial-dose space.

this spatial-dose representation, the gamma index defined in (1) can be extended to any dose point $(\boldsymbol{y}, d)$ in the $(k+1)$-D spatial-dose space:

$$\Gamma_{D_R}(\boldsymbol{y}, d) = \min_{\boldsymbol{x}} \sqrt{\left(\frac{\boldsymbol{y} - \boldsymbol{x}}{\delta}\right)^2 + \left(\frac{d - D_R(\boldsymbol{x})}{\Delta}\right)^2}. \tag{2}$$

In other words, the restriction of Γ_{D_R} to the test dose distribution $\{D_T(\boldsymbol{y})\}$ returns (1):

$$\gamma_{D_T}, D_R(\boldsymbol{y}) = \Gamma_{D_R}(\boldsymbol{y}, D_T(\boldsymbol{y})). \tag{3}$$

In this viewpoint, the embedded reference distribution can be regarded as a set of feature points in the $(k + 1)$-D space and the extended gamma index (2) is the EDT over a range of the $(k + 1)$-D space, where the Euclidean distance for any two dose points $(\boldsymbol{x}, d)$ and $(\boldsymbol{y}, D)$ is defined as

$$\sqrt{\left(\frac{\boldsymbol{x} - \boldsymbol{y}}{\delta}\right)^2 + \left(\frac{d - D}{\Delta}\right)^2}. \tag{4}$$

Figure 1 illustrates that the gamma index of a dose point $(\boldsymbol{y}, D)$ with respect to the reference distribution $d = D_R(\boldsymbol{x})$ is the closest distance of $(\boldsymbol{y}, D)$ from the feature set $d = D_R(\boldsymbol{x})$ in the spatial-dose representation.

3. Implementation

3.1. Quantization in the dose dimension

The spatial components are already quantized whether the dose distributions are measured or calculated. But we also need to quantize the dose dimension for our implementation. We choose the quantization level so that dose resolution is the same as spatial resolution after normalization. Therefore, quantization-induced errors are no larger than the intrinsic noise associated with the normalized spatial resolution.

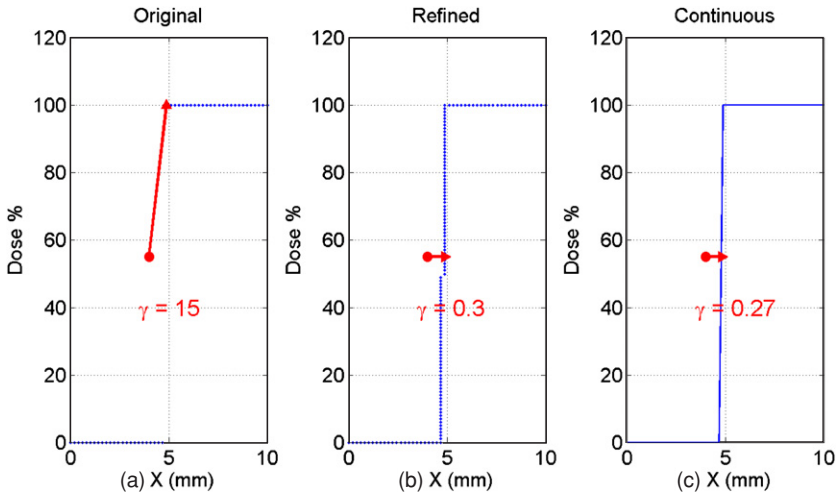

Figure 2. Illustration of the inaccuracy of the gamma index due to a high dose gradient. Dose points are added to the reference distribution to increase the resolution of the reference in the dose dimension and thus the accuracy of the gamma index calculation. (a) The gamma index of the test dose point, indicated by the big solid dot, w.r.t. the original reference distribution is 15; (b) the gamma index of the same test dose point becomes 0.3 when using the refined reference; (c) the true gamma index is 0.27, which is calculated using the continuous reference. Note that the gamma index of the test dose point from the refined reference is much closer to the true value. In fact, the difference is within the quantization error. The added dose points to the reference are obtained by interpolating linearly between adjacent dose points of the original reference and rounding off the spatial components to the nearest spatial grids.

3.2. Refining the reference distribution

The gamma values would not be accurate if the reference has low resolutions either spatially or dosimetrically. Figure 2(a) is an example that shows the gamma index of a test dose point near the region of high dose gradient, i.e. low dose resolution. The inaccuracy of the gamma index due to low spatial resolution of the reference can be pictured similar to figure 2(a). The true reference distribution should be continuous in all spatial and dose dimensions; but, due to finite spatial resolution, the measured distribution has discontinuity in dose dimension in high gradient regions, which causes inaccuracy of the gamma index. To overcome the problem of low spatial resolution, we interpolate the reference distribution so that it is defined on every spatial grid point. To overcome the problem of high dose gradient, along each spatial dimension, we add dose points by linear interpolation of adjacent dose points and rounding off the spatial components to the nearest spatial grids. As a result of the refinement, there may be more than one dose points at a spatial location. For such refined distribution, the DD of adjacent grid points along each spatial dimension is less than or equal to the bin size of dose. Figure 2(b) illustrates the refined reference, and the gamma index with respect to it. More dose points would be added to the high dose gradient region than the low dose gradient region. In the example, the gamma index of the test dose point is 15 with respect to the original reference, 0.3 with respect to the refined reference and 0.27 with respect to the continuous reference in figure 2(c), where the last value is considered most accurate. Thus, using the refined reference reduces the inaccuracy of the gamma index to within the noise level. The refined reference that has an increased number of dose points results in increased calculation time of gamma index for other search methods. Ju *et al* bypassed the refining process and found the gamma

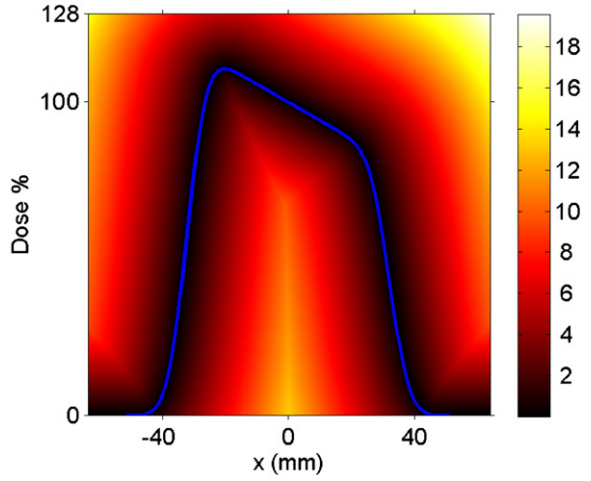

Figure 3. An example of 1D dose distribution overlaid on the gamma index table.

index by solving quadratic equations based on simplicial interpolation without increasing the number of reference points (Ju *et al* 2008). However, the refined reference does not change the order of complexity of our approach, because the order of complexity of the fast EDT algorithm is independent of the number of feature points.

3.3. Gamma index table and derivatives of the gamma index

Once the function Γ is calculated via fast EDT, the gamma index for the test dose distribution $\{D_T(\boldsymbol{y})\}$ can be obtained from table lookup. The function Γ is also referred to as the gamma index table. Figure 3 shows an example of 1D dose distribution overlaid on the gamma index table. The spatial range is $-64 \leqslant x \leqslant 64$ mm, the spatial resolution is 0.5 mm, the dose range is $0\% \leqslant d \leqslant 128\%$ and the dose resolution is 0.5% of the maximal dose. The image intensity indicates the gamma index value. Figure 4(a) is a test distribution overlaid on that gamma index table. Figure 4(b) is another test distribution overlaid on the same gamma index table. Figures 4(c) and (d) are the evaluation of gamma for the test distributions in (a) and (b), respectively, via table lookup. The same table can be used for both test distributions because their references are the same. The gamma index table can also be used to derive derivatives $\partial\Gamma/\partial\boldsymbol{x}$ or $\partial\Gamma/\partial D$, simply by subtracting the gamma index from its shift along one of the spatial dimensions or dose dimension followed by dividing by the respective resolutions. In order not to bias to either side, the derivatives are implemented using both left and right shifts:

$$\begin{aligned} \left.\frac{\partial\Gamma}{\partial x}\right|_{(x,D)} &= \frac{\Gamma(x+\Delta x, D) - \Gamma(x-\Delta x, D)}{2\cdot\Delta x}, \\ \left.\frac{\partial\Gamma}{\partial D}\right|_{(x,D)} &= \frac{\Gamma(x, D+\Delta D) - \Gamma(x, D-\Delta D)}{2\cdot\Delta D}. \end{aligned} \tag{5}$$

The utility of the gamma derivatives includes identifying the dominant causes for the deviations from the reference distribution. The gamma index angle referred in Stock *et al* (2005) can also be used to determine dominant factors in the gamma index. Let us look at the examples used in figures 3 and 4. The regions of high gamma indices (>3) for test1 have higher $\partial\Gamma/\partial\boldsymbol{x}$ than $\partial\Gamma/\partial D$, as can be seen in figure 5. This implies that moving test1 left or right (depending

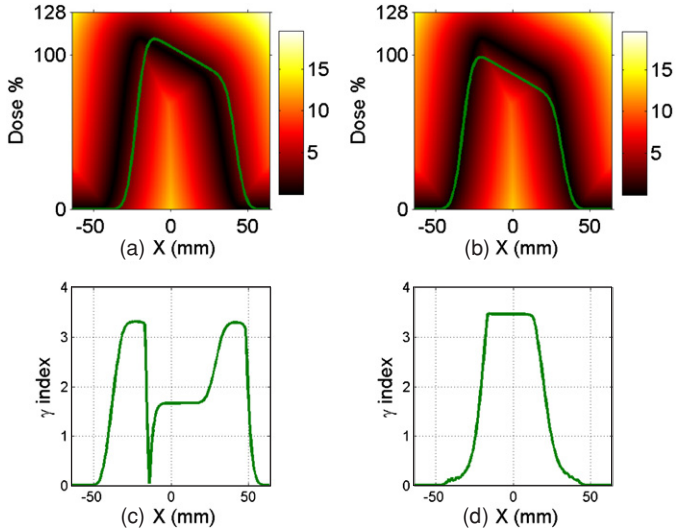

Figure 4. (a) The gamma index table as in figure 3 overlaid by a test distribution (indicated by a curve). (b) The same gamma index table overlaid by another test distribution (indicated by a curve). (c) and (d) are the gamma index of the test distribution in (a) and (b), respectively, obtained via table lookup.

on whether $\partial\Gamma/\partial \boldsymbol{x} > 0$ or $\partial\Gamma/\partial \boldsymbol{x} < 0$) in the $\boldsymbol{x}$ component is more effective in reducing the deviation of test1 from the reference. In other words, the deviation of the test distribution from the reference is more likely due to positioning error. On the other hand, the regions of high gamma indices (>3) for test2 have higher $\partial\Gamma/\partial D$ than $\partial\Gamma/\partial \boldsymbol{x}$. This implies that the deviation of the test from the reference is more likely due to dosimetric error, e.g. error due to linac output changes. Another possible application of the derivatives of the gamma index is that they can be used in optimization for treatment planning and machine parameters, which is worth further investigation.

3.4. Closest reference dose point and gamma index

The gamma index calculated by EDT would have rounding errors due to quantization, though the errors are within the noise level. To reduce the discrepancy of the gamma index from the conventional search using non-discretized dose values, we do not use the gamma index table directly. Instead, we obtain the closest reference dose point, which is recorded in the EDT calculation as the closest feature point, and calculate the gamma index using non-discretized dose values.

3.5. Implementation complexity

The proposed method was implemented in C++ and tested using a personal computer with the Window XP operation system. The code was written in a way that it can be used to evaluate the gamma index for 1D, 2D, 3D, etc. The complexity of our proposed gamma evaluation method is the same as that of the fast EDT, which has linear complexity (Maurer *et al* 2003). Suppose there are N voxels along each of the k spatial dimensions and the dose values from 0% to $(100 + x)$% of the maximal dose are discretized into M values. Then, there are totally

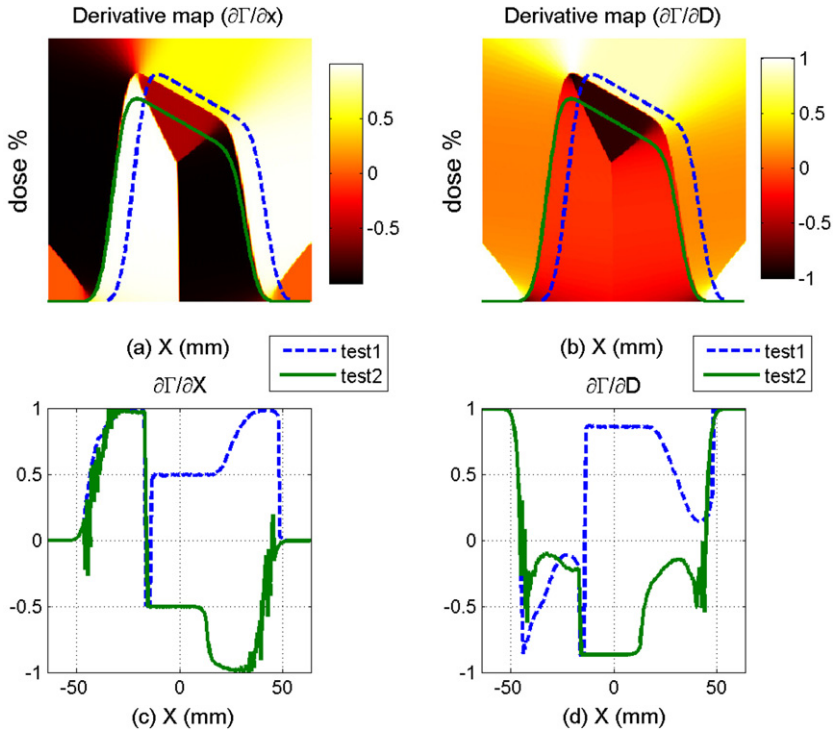

Figure 5. Derivatives of the gamma index. (a) $\partial\Gamma/\partial\boldsymbol{x}$, derivative of the gamma index w.r.t. the spatial component. (b) $\partial\Gamma/\partial D$, derivative of gamma w.r.t. the dose component; (c) and (d) are the evaluation of $\partial\Gamma/\partial\boldsymbol{x}$ and $\partial\Gamma/\partial D$, respectively, over the test distributions.

N^kM points in the $(k + 1)$-D spatial-dose space and the evaluation of equation (2) and table lookup for any test dose distribution have complexity $O(N^kM)$ and $O(N^k)$, respectively.

4. Results

In this section, we tested our method on simulated dose distributions, which are 2D distributions of dimension 400×400. For comparison, we also implemented the exhaustive search approach with the same programming language on the same machine. The reference and test distributions here are similar to those used in Low and Dempsey (2003) and Ju *et al* (2008). Figure 6(a) is the reference distribution, which simulates a square radiation field. Figures 6(b) and (c) are two test distributions A and B, respectively. The test distribution A has four quadrants with each quadrant simulating shifts from the reference dosimetrically (the second quadrant), spatially (the third quadrant) or both (the fourth quadrant), except the first quadrant, which is identical to that of the reference. The test distribution B was obtained from A by adding 3% of noise. Our approach first constructed a 3D table of gamma indices. Dose values were discretized into 256 equally spaced numbers from 0% to 128%. It took ∼16 s to pre-calculate the table and less than 20 ms for table lookup using a PC with a clock speed of 3 GHz. In contrast, it took ∼2700 s for the exhaustive search. Therefore, to evaluate the gamma index for two test distributions, it took ∼16 s in total using our approach and ∼5400 s using the exhaustive search.

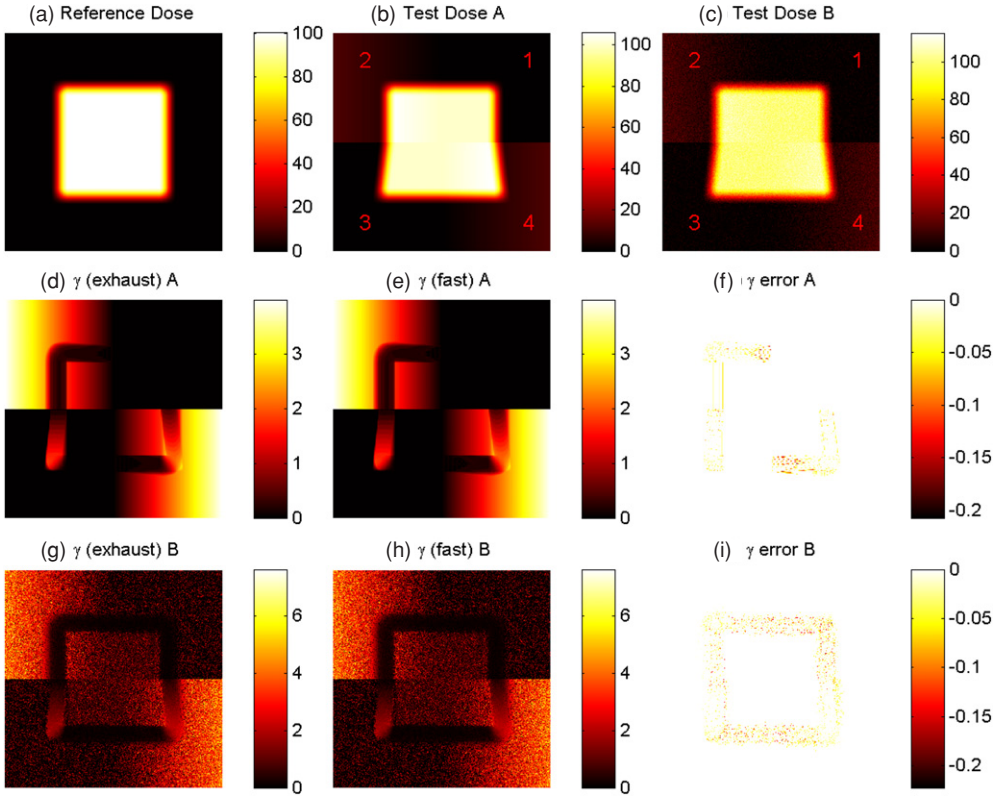

Figure 6. Simulations of 2D dose distributions. (a) The reference distribution that simulates a square radiation field; (b) the test distribution *A* with four quadrants simulating different shifts from the reference: (1) no shift, (2) dose shift, (3) spatial shift, (4) dose and spatial shift; (c) the test distribution *B* obtained from *A* by adding 3% of noise; (d) gamma index of the test distribution *A* calculated by the exhaustive search; (e) gamma index of the test distribution *A* obtained by the proposed method; (f) difference of (d) and (e) ((d) minus (e)); (g), (h) and (i) are similar to (d), (e) and (f), but for the test distribution *B*.

Figures 6(d) and (e) are the results of gamma index calculation for the test dose distribution *A* using the exhaustive search and our proposed method, respectively. Figure 6(f) is their difference ((d) minus (e)). Figure 6(g), (h) and (i) are similar to (d), (e) and (f), but for the test dose distribution *B*. Note that there is slight difference in gamma index values of these two approaches, mainly in high dose gradient region. The difference arises because our approach uses discrete dose values in distance transform to find the closest feature points and distances and causes rounding errors. In fact, the gamma index found via distance transform is always larger than or equal to that found via the exhaustive search, shown as non-positive values in figures 6(f) and (i). However, these rounding errors are no larger than the intrinsic noise associated with the normalized spatial resolution and should have little effect on the clinical implication of gamma index evaluations.

5. Discussion

In our proposed gamma index calculation method, we discretize dose values, and that causes rounding errors. However, these rounding errors are within the intrinsic noise level. In

implementation, we refine the reference distribution, which does not increase complexity of the algorithm but reduces artifact in the regions of high dose gradient significantly.

Although we only tested our method for 1D and 2D dose distribution cases, the method can be applied to any dimension. In our test, we compared the proposed method with the exhaustive search. The speedup for gamma index evaluation was on the order of 100 for a single 2D test dose distribution. The speedup for multiple (L) test dose distributions with a fixed reference dose distribution was $L \times 100$, since table lookup took orders less time than construction of the table. The speedup for 3D dose distributions is expected to be even more dramatic, from $O(N^6)$ to $O(N^3M)$, which is on the order of 10^4–10^5 when N is a few hundreds. It is expected that the construction of the gamma index table takes few minutes for a typical 3D reference dose distribution and gamma index evaluation for any test dose distribution with the same reference takes sub-second to seconds. However, the limited search or adaptive search could still be faster than our method for many cases in practice, when only two distributions are compared and they are similar. But our approach is relatively fast, especially when there are multiple test distributions for the same reference regardless if the distributions are similar or not. On the other hand, the proposed method has the spatial complexity of $O(N^3M)$ as well. For a region of 256^3 voxels, the spatial complexity is on the order of $256^4 = 4$G, which is too big for a 32 bit machine with the maximal address space of 3G unless some parallel algorithms can be implemented in applying the distance transform. Fortunately, the fast EDT is capable of parallel implementation as discussed in Maurer *et al* (2003).

The gamma index defined in (2) can be extended to accommodate locally defined, or spatially varying, normalization factors (Bakai *et al* 2003, Jiang *et al* 2006):

$$\Gamma_{D_R}(\boldsymbol{y}, d) = \min_{\boldsymbol{x}} \sqrt{\left(\frac{\boldsymbol{y} - \boldsymbol{x}}{\delta(\boldsymbol{x})}\right)^2 + \left(\frac{d - D_R(\boldsymbol{x})}{\Delta(\boldsymbol{x})}\right)^2}. \tag{6}$$

Our approach could be extended to such gamma index with locally defined normalization factors by modifying Maurer's EDT algorithm. However, this is still under investigation and will be reported elsewhere.

6. Conclusions

We proposed a high efficient method for gamma index calculation. For a k-D reference dose distribution, construction of the gamma index table is in linear complexity $O(N^kM)$ and the table can be used repeatedly to evaluate the gamma index with complexity $O(N^k)$ for any test dose distribution if the reference is fixed. In addition, the gradient of the gamma index with respect to either the spatial dimensions or dose dimension can be easily derived from the gamma index table. The gradient can be used to identify the dominant factors, whether it is spatial or DD, contributing most to the deviation of the test from the reference distribution. After the construction of the gamma table, the ultra-fast gamma index and its gradient evaluations make the inclusion of the gamma index feasible in treatment planning and machine parameter optimization.

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