The field content of the Standard Model (SM)

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The SM Lagrangian = Sum of the kinetic terms
+ the Higgs potential
+ the Yukawa interaction terms

Vector bosons		Field	Adj.repr.	Charge	Kinetic term	Covariant derivative	Field tensor	
$\mathrm{U}(1)_\mathrm{Y}$	B (γ)	B_μ, $(B_\mu = B_\mu^\dagger)$	$\mathbf{1},\mathbf{1},0$	Weak hypercharge, Y	$-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}$	$D_\mu = \partial_\mu + \ldots + Y\,\mathrm{i}g' B_\mu$	$B_{\mu\nu} \equiv 2\partial_{[\mu}B_{\nu]}$	
$\mathrm{SU}(2)_\mathrm{L}$	$W^\pm$, W^3 (Z^0)	W^i_μ, $i = 1,2,3$ $(W^i_\mu = W^{\dagger i}_\mu)$	$\mathbf{1},\mathbf{3},0$	Weak isospin, T^3	$-\frac{1}{4}W^i_{\mu\nu}W^{i\,\mu\nu} = -\frac{1}{2}\,\mathrm{Tr}(\boldsymbol{W}_{\mu\nu}\boldsymbol{W}^{\mu\nu})$	$D_\mu = \partial_\mu + \ldots + \mathrm{i}g\boldsymbol{W}_\mu$; $[D_\mu, D_\nu] = \ldots + \mathrm{i}g\boldsymbol{W}_{\mu\nu}$	$\boldsymbol{W}_{\mu\nu} \equiv 2\partial_{[\mu}\boldsymbol{W}_{\nu]} + \mathrm{i}g[\boldsymbol{W}_\mu, \boldsymbol{W}_\nu]$; $W^i_{\mu\nu} = 2\partial_{[\mu}W^i_{\nu]} - g\epsilon^{ijk}W^j_\mu W^k_\nu$	$\boldsymbol{W}_\mu \equiv W^i_\mu t^i$; $\boldsymbol{W}_{\mu\nu} \equiv W^i_{\mu\nu}t^i$
$\mathrm{SU}(3)_\mathrm{C}$	g	G^a_μ, $a = 1,\ldots,8$ $(G^a_\mu = G^{\dagger a}_\mu)$	$\mathbf{8},\mathbf{1},0$	Color	$-\frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} = -\frac{1}{2}\,\mathrm{Tr}(\boldsymbol{G}_{\mu\nu}\boldsymbol{G}^{\mu\nu})$	$D_\mu = \partial_\mu + \ldots + \mathrm{i}g_s\boldsymbol{G}_\mu$; $[D_\mu, D_\nu] = \ldots + \mathrm{i}g_s\boldsymbol{G}_{\mu\nu}$	$\boldsymbol{G}_{\mu\nu} = 2\partial_{[\mu}\boldsymbol{G}_{\nu]} + \mathrm{i}g_s[\boldsymbol{G}_\mu, \boldsymbol{G}_\nu]$; $G^a_{\mu\nu} = 2\partial_{[\mu}G^a_{\nu]} - g_s f^{abc}G^b_\mu G^c_\nu$	$\boldsymbol{G}_\mu \equiv G^a_\mu T^a$; $\boldsymbol{G}_{\mu\nu} \equiv G^a_{\mu\nu}T^a$

The SU(n) generators		
SU(2)	$t^i \equiv \frac{1}{2}\tau^i$	$[t^i, t^j] = \mathrm{i}\epsilon^{ijk}\,t^k$; $\mathrm{Tr}(t^i t^j) = \frac{1}{2}\delta^{ij}$
SU(3)	$T^a \equiv \frac{1}{2}\lambda^a$	$[T^a, T^b] = \mathrm{i}f^{abc}\,T^c$; $\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$

	Fermions	Field	Repr.	Q	$= T^3$	$+ Y$	Kinetic term	Covariant derivative
Quarks	$u_\mathrm{L}, (c_\mathrm{L}, t_\mathrm{L})$	$\boldsymbol{\Psi}_Q = \begin{pmatrix}\Psi^\mathrm{L}_u \\ \Psi^\mathrm{L}_d\end{pmatrix}$, $\boldsymbol{\Psi}^\dagger_Q$, $\Psi^\mathrm{L}_u = P_\mathrm{L}\Psi_u$, ...	$\mathbf{3},\mathbf{2},+\frac{1}{6}$	$+2/3$	$+1/2$	$+1/6$	$\overline{\boldsymbol{\Psi}_Q}\ \mathrm{i}\gamma^\mu D_\mu\,\boldsymbol{\Psi}_Q$	$D_\mu\boldsymbol{\Psi}_Q = (\partial_\mu + \mathrm{i}g_s\boldsymbol{G}_\mu + \mathrm{i}g\boldsymbol{W}_\mu + \frac{1}{6}\,\mathrm{i}g'B_\mu)\boldsymbol{\Psi}_Q$
	$d_\mathrm{L}, (s_\mathrm{L}, b_\mathrm{L})$			$-1/3$	$-1/2$	$+1/6$		
	$u_\mathrm{R}, (c_\mathrm{R}, t_\mathrm{R})$	$\Psi^\mathrm{R}_u = P_\mathrm{R}\Psi_u$, $\Psi^{\mathrm{R}\dagger}_u$	$\mathbf{3},\mathbf{1},+\frac{2}{3}$	$+2/3$	0	$+2/3$	$\overline{\Psi^\mathrm{R}_u}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{R}_u$	$D_\mu\Psi^\mathrm{R}_u = (\partial_\mu + \mathrm{i}g_s\boldsymbol{G}_\mu + \cancel{\mathrm{i}g\boldsymbol{W}_\mu} + \frac{2}{3}\,\mathrm{i}g'B_\mu)\Psi^\mathrm{R}_u$
	$d_\mathrm{R}, (s_\mathrm{R}, b_\mathrm{R})$	$\Psi^\mathrm{R}_d = P_\mathrm{R}\Psi_d$, $\Psi^{\mathrm{R}\dagger}_d$	$\mathbf{3},\mathbf{1},-\frac{1}{3}$	$-1/3$	0	$-1/3$	$\overline{\Psi^\mathrm{R}_d}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{R}_d$	$D_\mu\Psi^\mathrm{R}_d = (\partial_\mu + \mathrm{i}g_s\boldsymbol{G}_\mu + \cancel{\mathrm{i}g\boldsymbol{W}_\mu} - \frac{1}{3}\,\mathrm{i}g'B_\mu)\Psi^\mathrm{R}_d$
	$\bar{u}_\mathrm{L} = (u_\mathrm{R})^\mathrm{c}$, ...	$\Psi^\mathrm{L}_{\bar u} = P_\mathrm{L}\Psi_{\bar u}$, $\Psi^{\mathrm{L}\dagger}_{\bar u}$	$\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3}$	$-2/3$	0	$-2/3$	$\overline{\Psi^\mathrm{L}_{\bar u}}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{L}_{\bar u}$	$D_\mu\Psi^\mathrm{L}_{\bar u} = (\partial_\mu - \mathrm{i}g_s\boldsymbol{G}^*_\mu - \cancel{\mathrm{i}g\boldsymbol{W}^*_\mu} + \frac{2}{3}\,\mathrm{i}g'B_\mu)\Psi^\mathrm{L}_{\bar u}$
	$\bar{d}_\mathrm{L} = (d_\mathrm{R})^\mathrm{c}$, ...	$\Psi^\mathrm{L}_{\bar d} = P_\mathrm{L}\Psi_{\bar d}$, $\Psi^{\mathrm{L}\dagger}_{\bar d}$	$\bar{\mathbf{3}},\mathbf{1},+\frac{1}{3}$	$+1/3$	0	$+1/3$	$\overline{\Psi^\mathrm{L}_{\bar d}}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{L}_{\bar d}$	$D_\mu\Psi^\mathrm{L}_{\bar d} = (\partial_\mu - \mathrm{i}g_s\boldsymbol{G}^*_\mu - \cancel{\mathrm{i}g\boldsymbol{W}^*_\mu} - \frac{1}{3}\,\mathrm{i}g'B_\mu)\Psi^\mathrm{L}_{\bar d}$
Leptons	$\nu_{e\mathrm{L}}, (\nu_{\mu\mathrm{L}}, \nu_{\tau\mathrm{L}})$	$\boldsymbol{\Psi}_L = \begin{pmatrix}\Psi^\mathrm{L}_{\nu_e} \\ \Psi^\mathrm{L}_e\end{pmatrix}$, $\boldsymbol{\Psi}^\dagger_L$, $\Psi^\mathrm{L}_{\nu_e} = P_\mathrm{L}\Psi_{\nu_e}$, ...	$\mathbf{1},\mathbf{2},-\frac{1}{2}$	0	$+1/2$	$-1/2$	$\overline{\boldsymbol{\Psi}_L}\ \mathrm{i}\gamma^\mu D_\mu\,\boldsymbol{\Psi}_L$	$D_\mu\boldsymbol{\Psi}_L = (\partial_\mu + \cancel{\mathrm{i}g_s\boldsymbol{G}_\mu} + \mathrm{i}g\boldsymbol{W}_\mu - \frac{1}{2}\,\mathrm{i}g'B_\mu)\boldsymbol{\Psi}_L$
	$e_\mathrm{L}, (\mu_\mathrm{L}, \tau_\mathrm{L})$			-1	$-1/2$	$-1/2$		
	$\nu_{e\mathrm{R}}, (\nu_{\mu\mathrm{R}}, \nu_{\tau\mathrm{R}})$	$\Psi^\mathrm{R}_{\nu_e} = P_\mathrm{R}\Psi_{\nu_e}$, $\Psi^{\mathrm{R}\dagger}_{\nu_e}$	$\mathbf{1},\mathbf{1},0$	0	0	0	$\overline{\Psi^\mathrm{R}_{\nu_e}}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{R}_{\nu_e}$	$D_\mu\Psi^\mathrm{L}_{\nu_e} = (\partial_\mu + \cancel{\mathrm{i}g_s\boldsymbol{G}_\mu + \mathrm{i}g\boldsymbol{W}_\mu + \mathrm{i}g'YB_\mu})\Psi^\mathrm{L}_{\nu_e}$
	$e_\mathrm{R}, (\mu_\mathrm{R}, \tau_\mathrm{R})$	$\Psi^\mathrm{R}_e = P_\mathrm{R}\Psi_e$, $\Psi^{\mathrm{R}\dagger}_e$	$\mathbf{1},\mathbf{1},-1$	-1	0	-1	$\overline{\Psi^\mathrm{R}_e}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{R}_e$	$D_\mu\Psi^\mathrm{R}_e = (\partial_\mu + \cancel{\mathrm{i}g_s\boldsymbol{G}_\mu + \mathrm{i}g\boldsymbol{W}_\mu} - \mathrm{i}g'B_\mu)\Psi^\mathrm{R}_e$
	$\bar{\nu}_{e\mathrm{L}} = (\nu_{e\mathrm{R}})^\mathrm{c}$, ...	$\Psi^\mathrm{L}_{\bar\nu_e} = P_\mathrm{L}\Psi_{\bar\nu_e}$, $\Psi^{\mathrm{L}\dagger}_{\bar\nu_e}$	$\mathbf{1},\mathbf{1},0$	0	0	0	$\overline{\Psi^\mathrm{L}_{\bar\nu_e}}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{L}_{\bar\nu_e}$	$D_\mu\Psi^\mathrm{L}_{\bar\nu_e} = (\partial_\mu - \cancel{\mathrm{i}g_s\boldsymbol{G}^*_\mu - \mathrm{i}g\boldsymbol{W}^*_\mu + \mathrm{i}g'YB_\mu})\Psi^\mathrm{L}_{\bar\nu_e}$
	$\bar{e}_\mathrm{L} = (e_\mathrm{R})^\mathrm{c}$, ...	$\Psi^\mathrm{L}_{\bar e} = P_\mathrm{L}\Psi_{\bar e}$, $\Psi^{\mathrm{L}\dagger}_{\bar e}$	$\mathbf{1},\mathbf{1},+1$	$+1$	0	$+1$	$\overline{\Psi^\mathrm{L}_{\bar e}}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{L}_{\bar e}$	$D_\mu\Psi^\mathrm{L}_{\bar e} = (\partial_\mu - \cancel{\mathrm{i}g_s\boldsymbol{G}^*_\mu - \mathrm{i}g\boldsymbol{W}^*_\mu} + \mathrm{i}g'B_\mu)\Psi^\mathrm{L}_{\bar e}$

Chirality projection: $\Psi \equiv \begin{pmatrix}\chi_\alpha \\ \bar\psi^{\dot\alpha}\end{pmatrix}$, $\boxed{\Psi^\mathrm{L} \equiv P_\mathrm{L}\Psi} = \begin{pmatrix}\chi_\alpha \\ 0\end{pmatrix}$, $\boxed{\Psi^\mathrm{R} \equiv P_\mathrm{R}\Psi} = \begin{pmatrix}0 \\ \bar\psi^{\dot\alpha}\end{pmatrix}$

Dirac conjugate: $\boxed{\overline{\Psi} \equiv \Psi^\dagger\beta}$, $\beta \equiv \begin{pmatrix}0 & \delta^\alpha{}_\beta \\ \delta_{\dot\alpha}{}^{\dot\beta} & 0\end{pmatrix}$, $\Psi^\dagger = (\bar\chi^{\dot\alpha}\ \ \psi_\alpha)$, $\overline{\Psi} = (\psi_\alpha\ \ \bar\chi^{\dot\alpha})$

Charge conjugate: $\boxed{\Psi^\mathrm{c} \equiv C_0\Psi^*}$, $C_0 \equiv \begin{pmatrix}0 & \varepsilon_{\alpha\beta} \\ \varepsilon^{\dot\alpha\dot\beta} & 0\end{pmatrix}$, $\Psi^\mathrm{c} = \begin{pmatrix}\psi_\alpha \\ \bar\chi^{\dot\alpha}\end{pmatrix}$

$(D_\mu\Psi)^\mathrm{c} = C_0(D_\mu\Psi)^* = D^*_\mu\Psi^\mathrm{c}$

$^\intercal$ transpose, * complex conjugate, † hermitian conjugate
$\alpha, \beta, \ldots$ L-type 2-component spinor indices (dotted = R-type)

Trading R-chiral for the charge conjugated L-chiral fields:

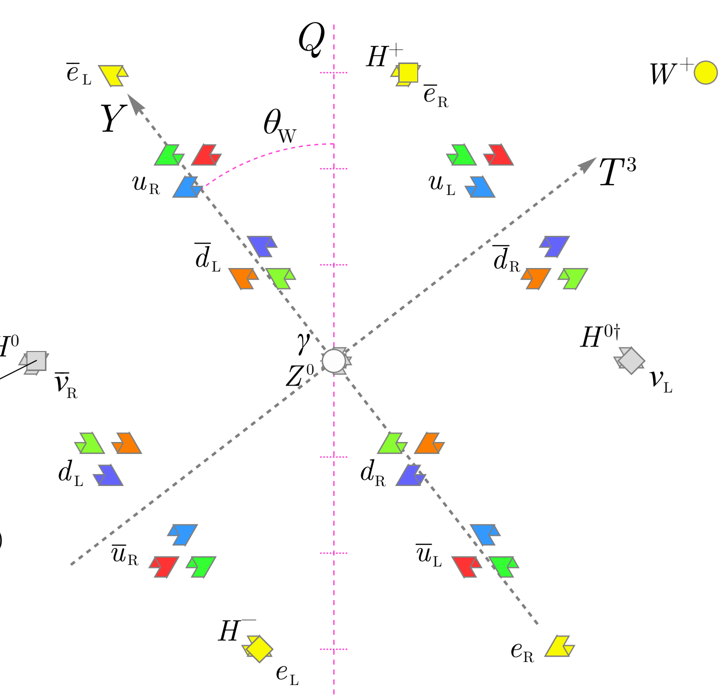
$\Psi_{\bar u} = (\Psi_u)^\mathrm{c} = \begin{pmatrix}\psi_\alpha \\ \bar\chi^{\dot\alpha}\end{pmatrix}$,

$\Psi^\mathrm{L}_{\bar u} = P_\mathrm{L}\Psi_{\bar u} = P_\mathrm{L}(\Psi_u)^\mathrm{c} = \begin{pmatrix}\psi_\alpha \\ 0\end{pmatrix}$,

$(\Psi^\mathrm{R}_u)^\mathrm{c} = \begin{pmatrix}\psi_\alpha \\ 0\end{pmatrix} \implies \Psi^\mathrm{L}_{\bar u} = (\Psi^\mathrm{R}_u)^\mathrm{c}$,

i.e. $\bar u_\mathrm{L} = (u_\mathrm{R})^\mathrm{c}$

The electroweak charges



The neutral Higgs field which breaks the EW symmetry. ($H^\pm, H^{0\dagger}$ are eaten by $W^\pm, Z^0$)

	Scalar boson	Field	Repr.	Q	T^3	Y	Kinetic term	Covariant derivative
Higgs	H^+, H^-	$\boldsymbol{\Phi} = \begin{pmatrix}H^+ \\ H^0\end{pmatrix}$, $\boldsymbol{\Phi}^\dagger$, $\boldsymbol{\Phi}^\dagger = (H^-\ \ H^{0\dagger})$	$\mathbf{1},\mathbf{2},+\frac{1}{2}$	$+1$	$+1/2$	$+1/2$	$(D_\mu\boldsymbol{\Phi})^\dagger D^\mu\boldsymbol{\Phi}$	$D_\mu\boldsymbol{\Phi} = (\partial_\mu + \cancel{\mathrm{i}g_s\boldsymbol{G}_\mu} + \mathrm{i}g\boldsymbol{W}_\mu + \frac{1}{2}\,\mathrm{i}g'B_\mu)\boldsymbol{\Phi}$
	H^0, $H^{0\dagger}$			0	$-1/2$	$+1/2$		$(D_\mu\boldsymbol{\Phi})^* = (\partial_\mu - \cancel{\mathrm{i}g_s\boldsymbol{G}^*_\mu} - \mathrm{i}g\boldsymbol{W}^*_\mu - \frac{1}{2}\,\mathrm{i}g'B_\mu)\boldsymbol{\Phi}^*$

The representation of $\widetilde{\boldsymbol{\Phi}}$ is $(\mathbf{1}, \bar{\mathbf{2}}, -\frac{1}{2})$.

The Higgs potential

$$\mathcal{L}^\mathrm{H} = -\mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$$

For **SSB**: $\mu^2 < 0$ and $\lambda > 0$

vev: $\left\langle\boldsymbol{\Phi}^\dagger\boldsymbol{\Phi}\right\rangle_\mathrm{min} = \frac{v^2}{2}$, $\quad v = \sqrt{\frac{-\mu^2}{\lambda}} > 0$,

unitary g.: $\boldsymbol{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v + \sigma(x)\end{pmatrix}$, $\boldsymbol{\Phi}_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v\end{pmatrix}$

The interaction terms

The interactions between the gauge bosons and the other fields (fermions and Higgs) all arise from the gauge covariant derivatives. The self-interactions of the nonabelian gauge bosons are all contained in their kinetic terms. The interactions between the fermions and the Higgs field are all given by Yukawa terms (on SSB, these terms generate the fermion masses).

The Yukawa interaction terms

$$-\Big\{\, y^{IJ}_d\,(\overline{\boldsymbol{\Psi}^\mathrm{L}_{Q_I}}\,\boldsymbol{\Phi})\,\Psi^\mathrm{R}_{d_J} + y^{IJ}_u\,(\overline{\boldsymbol{\Psi}^\mathrm{L}_{Q_I}}\,\widetilde{\boldsymbol{\Phi}})\,\Psi^\mathrm{R}_{u_J} + y^{IJ}_\ell\,(\overline{\boldsymbol{\Psi}^\mathrm{L}_{L_I}}\,\boldsymbol{\Phi})\,\Psi^\mathrm{R}_{\ell_J} + y^{IJ}_\nu\,(\overline{\boldsymbol{\Psi}^\mathrm{L}_{L_I}}\,\widetilde{\boldsymbol{\Phi}})\,\Psi^\mathrm{R}_{\nu_J} + \mathrm{h.c.}\,\Big\},$$

where $\widetilde{\boldsymbol{\Phi}} \equiv (-\mathrm{i}\tau^2)^\intercal\boldsymbol{\Phi}^*$, (Note: $\boldsymbol{\Phi}_d = \boldsymbol{\Phi}$, $\boldsymbol{\Phi}_u = \widetilde{\boldsymbol{\Phi}}$)
and the indices $I, J = 1,2,3$ run over generations

$$\begin{pmatrix}W^+_\mu \\ W^-_\mu\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -\mathrm{i} \\ \mathrm{i} & 1\end{pmatrix}\begin{pmatrix}W^1_\mu \\ W^2_\mu\end{pmatrix}, \quad \begin{pmatrix}Z_\mu \\ A_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_\mathrm{W} & -\sin\theta_\mathrm{W} \\ \sin\theta_\mathrm{W} & \cos\theta_\mathrm{W}\end{pmatrix}\begin{pmatrix}W^3_\mu \\ B_\mu\end{pmatrix}$$

The Weinberg angle: $\tan\theta_\mathrm{W} \equiv \frac{g'}{g}$, $\quad e = g'\cos\theta_\mathrm{W} = g\sin\theta_\mathrm{W}$,

$\sin\theta_\mathrm{W} \equiv \frac{g'}{\sqrt{g^2 + g'^2}}$, $\quad \cos\theta_\mathrm{W} \equiv \frac{g}{\sqrt{g^2 + g'^2}}$