The field content of the Standard Model (SM)

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Vector bosons		Field	Adj.repr.	Charge	Kinetic term	Covariant derivative	Field tensor	
$U(1)_{Y}$	$B(\gamma)$	$B_{\mu}, \ (B_{\mu} = B_{\mu}^{\dagger})$	1 , 1 , 0	Weak hypercharge, Y	$\left(\begin{array}{cc} -rac{1}{4}B_{\mu u}B^{\mu u} \end{array}\right)$	$D_{\mu} = \partial_{\mu} + \dots + Y \mathrm{i} g' B_{\mu}$	$B_{\mu\nu} \equiv 2\partial_{[\mu}B_{\nu]}$	
$\mathrm{SU}(2)_{\mathrm{L}}$	$ \begin{array}{ c c } \hline W^{\pm}, \\ W^3 (Z^0) \end{array} $	$W^{i}_{\mu}, i = 1, 2, 3$ $(W^{i}_{\mu} = W^{\dagger i}_{\mu})$	1,3 ,0	Weak isospin, T^3	$ \begin{array}{c} -\frac{1}{4}W_{\mu\nu}^{i}W^{i\mu\nu} = \\ -\frac{1}{2}\operatorname{Tr}(\boldsymbol{W}_{\mu\nu}\boldsymbol{W}^{\mu\nu}) \end{array} $	$D_{\mu} = \partial_{\mu} + \dots + ig \mathbf{W}_{\mu}$ $[D_{\mu}, D_{\nu}] = \dots + ig \mathbf{W}_{\mu\nu}$	$W_{\mu\nu} \equiv 2\partial_{[\mu}W_{\nu]} + ig[W_{\mu}, W_{\nu}]$ $W_{\mu\nu}^{i} = 2\partial_{[\mu}W_{\nu]}^{i} - g\epsilon^{ijk}W_{\mu}^{j}W_{\nu}^{k}$	$egin{aligned} egin{aligned} oldsymbol{W}_{\mu} &\equiv W_{\mu}^i t^i \ oldsymbol{W}_{\mu u} &\equiv W_{\mu u}^i t^i \end{aligned}$
$SU(3)_{C}$	g	$G^{a}_{\mu}, \ a = 1,, 8$ $(G^{a}_{\mu} = G^{\dagger a}_{\mu})$	8,1,0	Color	$ \begin{array}{c} -\frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu} = \\ -\frac{1}{2}\operatorname{Tr}(\boldsymbol{G}_{\mu\nu}\boldsymbol{G}^{\mu\nu}) \end{array} $	$D_{\mu} = \partial_{\mu} + \dots + ig_{s} \mathbf{G}_{\mu}$ $[D_{\mu}, D_{\nu}] = \dots + ig_{s} \mathbf{G}_{\mu\nu}$	$G_{\mu\nu} = 2\partial_{[\mu}G_{\nu]} + ig_s[G_{\mu}, G_{\nu}]$ $G^a_{\mu\nu} = 2\partial_{[\mu}G^a_{\nu]} - g_s f^{abc}G^b_{\mu}G^c_{\nu}$	$G_{\mu} \equiv G_{\mu}^{a} T^{a}$ $G_{\mu\nu} \equiv G_{\mu\nu}^{a} T^{a}$

The SM Lagrangian = Sum of the kinetic terms

+ the Higgs potential

Chirality projection:

Dirac conjugate:

Charge

Trading R-chiral for the charge conjugated

 $\Psi_{\bar{u}} = (\Psi_u)^{\mathsf{C}} = \begin{pmatrix} \psi_{\alpha} \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$

i.e. $\bar{u}_L = (u_R)^C$

breaks the EW symmetry.

L-chiral fields:

+ the Yukawa interaction terms

The $SU(n)$ generators							
SU(2)	$\left(egin{array}{c} t^i \equiv rac{1}{2} au^i \end{array} ight)$	$\begin{bmatrix} [t^i, t^i] &= \mathrm{i}\epsilon^{ijk} t^k \\ \mathrm{Tr}(t^i t^j) &= \frac{1}{2} \delta^{ij} \end{bmatrix}$					
SU(3)	$T^a \equiv \frac{1}{2}\lambda^a$	$T^{a}, T^{b}] = if^{abc} T^{c}$ $Tr(T^{a}T^{b}) = \frac{1}{2}\delta^{ab}$					

Fermions		Field	Repr.	$Q = T^3 +$		+ Y	Kinetic term	Covariant derivative
Quarks	$u_{L},(c_{L},t_{L})$	$oldsymbol{\Psi}_Q = egin{pmatrix} \Psi_u^{L} \ \Psi_d^{L} \end{pmatrix}, \; \Psi_Q^{\dagger} ,$	$oxed{egin{array}{c} oxed{3,2,+rac{1}{6}} \end{array}}$	+2/3	+1/2	+1/6	$\overline{m{\Psi}_Q} \; \mathrm{i} \gamma^\mu D_\mu m{\Psi}_Q$	$D_{\mu}\Psi_{Q} = \left(\partial_{\mu} + ig_{s}G_{\mu} + igW_{\mu} + \frac{1}{6}ig'B_{\mu}\right)\Psi_{Q}$
	$d_{ extsf{L}},(s_{ extsf{L}},b_{ extsf{L}})$	$\Psi_u^{L} = P_{L}\Psi_u, \dots$		$\begin{bmatrix} -1/3 \end{bmatrix}$	$\left[\begin{array}{c} -1/2 \end{array}\right]$	+1/6		
	$\left(u_{R}, \left(c_{R}, t_{R} ight) ight)$	$egin{pmatrix} \Psi_u^{ extsf{R}} = P_{ extsf{R}} \Psi_u, \ \Psi_u^{ extsf{R}\dagger} \end{pmatrix}$	$oxed{egin{array}{c} oxed{3,1,+rac{2}{3}} \end{array}}$	+2/3	0	(+2/3)	$\left(egin{array}{c} \overline{\Psi_u^{\scriptscriptstyle extsf{R}}} \ \mathrm{i} \gamma^\mu D_\mu \Psi_u^{\scriptscriptstyle extsf{R}} \end{array} ight)$	$D_{\mu}\Psi_{u}^{R} = \left(\partial_{\mu} + \mathrm{i}g_{s}G_{\mu} + \boxed{\mathrm{i}gW_{\mu}} + \frac{2}{3}\mathrm{i}g'B_{\mu}\right)\Psi_{u}^{R}$
	$\left(d_{R}, \left(s_{R}, b_{R} ight) ight)$	$\Psi_d^{ extsf{R}} = P_{ extsf{R}} \Psi_d, \; \Psi_d^{ extsf{R}\dagger}$	$oxed{egin{array}{c} oldsymbol{3,1,-rac{1}{3}} \end{array}}$	$\left[-1/3\right]$	0	$\left[-1/3\right]$	$\left(egin{array}{c} \overline{\Psi_d^{ t R}} \ { m i} \gamma^\mu D_\mu \Psi_d^{ t R} \end{array} ight)$	$D_{\mu}\Psi_{d}^{R} = \left(\partial_{\mu} + \mathrm{i}g_{s}\boldsymbol{G}_{\mu} + \boxed{\mathrm{i}g\boldsymbol{W}_{\mu}} - \frac{1}{3}\mathrm{i}g'B_{\mu}\right)\Psi_{d}^{R}$
	$\left(ar{u}_{L}=(u_{R})^{C}, ight)$	$egin{pmatrix} \Psi^{ t L}_{ar{u}} = P_{ t L}\Psi_{ar{u}}, \Psi^{ t L\dagger}_{ar{u}} \end{pmatrix}$	$\left[egin{array}{c} \overline{f 3}, {f 1}, -rac{2}{3} \end{array} ight]$	$\left[-2/3\right]$	0	$\left(-2/3\right)$	$\left(egin{array}{c} \overline{\Psi^{ extsf{L}}_{ar{u}}} \ \mathrm{i} \gamma^{\mu} D_{\mu} \Psi^{ extsf{L}}_{ar{u}} \end{array} ight)$	$D_{\mu}\Psi_{\bar{u}}^{L} = \left(\partial_{\mu} - \mathrm{i}g_{s}G_{\mu}^{*} - \boxed{\mathrm{i}gW_{\mu}^{*}} + \frac{2}{3}\mathrm{i}g'B_{\mu}\right)\Psi_{\bar{u}}^{L}$
	$egin{pmatrix} ar{d}_{ extsf{L}} = (d_{ extsf{R}})^{ extsf{C}}, \end{pmatrix}$	$\Psi_{ar{d}}^{ t L} = P_{ t L} \Psi_{ar{d}}, \; \Psi_{ar{d}}^{ t L\dagger}$	$oxed{\overline{f 3},{f 1},+rac{1}{3}}$	+1/3	0	+1/3	$\left(egin{array}{c} \overline{\Psi}^{ extsf{L}}_{ar{d}} \ \mathrm{i} \gamma^{\mu} D_{\mu} \Psi^{ extsf{L}}_{ar{d}} \end{array} ight)$	$D_{\mu}\Psi_{\bar{d}}^{L} = \left(\partial_{\mu} - \mathrm{i}g_{s}G_{\mu}^{*} - \boxed{\mathrm{i}gW_{\mu}^{*}} - \frac{1}{3}\mathrm{i}g'B_{\mu}\right)\Psi_{\bar{d}}^{L}$
Leptons	$\left(otag $	$egin{pmatrix} oldsymbol{\Psi}_L = egin{pmatrix} \Psi_{ u_e}^{L} \ \Psi_{ u}^{L} \end{pmatrix}, \; oldsymbol{\Psi}_L^{\dagger} , \end{cases}$	$oxed{1,2,-rac{1}{2}}$	0	+1/2	$\left[-1/2\right]$	$\overline{m{\Psi}_L} \; \mathrm{i} \gamma^\mu D_\mu m{\Psi}_L$	$D_{\mu}\Psi_{L} = (\partial_{\mu} + ig\mathbf{W}_{\mu} - \frac{1}{2}ig'B_{\mu})\Psi_{L}$
	$\left(e_{ extsf{L}}, \left(\mu_{ extsf{L}}, au_{ extsf{L}} ight)$	$\Psi_{\nu_e}^{L} = P_{L} \Psi_{\nu_e}, \dots$		$\begin{bmatrix} -1 \end{bmatrix}$	$\begin{bmatrix} -1/2 \end{bmatrix}$	$\begin{bmatrix} -1/2 \end{bmatrix}$		
	$\left(u_{e\mathtt{R}}, \left(u_{\mu\mathtt{R}}, u_{ au\mathtt{R}} ight) ight)$	$\left(egin{array}{c} \Psi_{ u_e}^{ extsf{R}} = P_{ extsf{R}}\Psi_{ u_e}, \ \Psi_{ u_e}^{ extsf{R}\dagger} \end{array} ight)$	$\begin{bmatrix} 1, 1, 0 \end{bmatrix}$	0	0	0	$\left(egin{array}{c} \overline{\Psi^{ extsf{R}}_{ u_e}} \ \mathrm{i} \gamma^\mu D_\mu \Psi^{ extsf{R}}_{ u_e} \end{array} ight)$	$D_{\mu}\Psi_{\nu_{e}}^{L} = \left(\partial_{\mu} + \underbrace{ig_{s}G_{\mu} + igW_{\mu} + ig'YB_{\mu}}\right)\Psi_{\nu_{e}}^{L}$
	$\left(e_{R},\left(\mu_{R}, au_{R} ight) ight)$	$egin{pmatrix} \Psi_e^{ extsf{R}} = P_{ extsf{R}} \Psi_e, \ \Psi_e^{ extsf{R}\dagger} \end{pmatrix}$	$\begin{bmatrix} 1, 1, -1 \end{bmatrix}$	$\begin{bmatrix} -1 \end{bmatrix}$	0	$\begin{bmatrix} -1 \end{bmatrix}$	$\left(egin{array}{c} \overline{\Psi_e^{R}} \ \mathrm{i} \gamma^\mu D_\mu \Psi_e^{R} \end{array} ight)$	$ \left(D_{\mu} \Psi_{e}^{R} = \left(\partial_{\mu} + \underbrace{\mathrm{i} g_{s} G_{\mu} + \mathrm{i} g W_{\mu}} \right) - \mathrm{i} g' B_{\mu} \right) \Psi_{e}^{R} $
	$egin{pmatrix} ar{ u}_{eL} = (u_{eR})^{C}, \end{pmatrix}$	$\boxed{ \Psi^{L}_{\bar{\nu}_e} = P_{L} \Psi_{\bar{\nu}_e}, \ \Psi^{L\dagger}_{\bar{\nu}_e} }$	$\boxed{ 1, 1, 0}$	0	0	0	$\left(egin{array}{c} \overline{\Psi^{ t L}_{ar{ u}_e}} \ { m i} \gamma^\mu D_\mu \Psi^{ t L}_{ar{ u}_e} \end{array} ight)$	$D_{\mu}\Psi_{\bar{\nu}_{e}}^{L} = \left(\partial_{\mu} - \underbrace{ig_{s}G_{\mu}^{*} - igW_{\mu}^{*} + ig'YB_{\mu}}\right)\Psi_{\bar{\nu}_{e}}^{L}$
	$ (\bar{e}_{L} = (e_{R})^{C}, \dots) $	$\boxed{ \Psi_{\bar{e}}^{L} = P_{L} \Psi_{\bar{e}}, \ \Psi_{\bar{e}}^{L\dagger} }$	$\left[egin{array}{c} oldsymbol{1},oldsymbol{1},+1 \end{array} ight]$	+1	0	+1	$\left(egin{array}{c} \overline{\Psi_{ar{e}}^{ t L}} \ \mathrm{i} \gamma^{\mu} D_{\mu} \Psi_{ar{e}}^{ t L} \end{array} ight)$	$ D_{\mu}\Psi_{\bar{e}}^{L} = (\partial_{\mu} - \overrightarrow{\mathrm{i}g_{s}G_{\mu}^{*}} \overrightarrow{\mathrm{i}g}W_{\mu}^{*} + ig'B_{\mu})\Psi_{\bar{e}}^{L} $

Q T^3 Repr. YKinetic term Covariant derivative +1/2+1+1/2 $D_{\mu}\mathbf{\Phi} = (\partial_{\mu} + \overrightarrow{igs}\mathbf{G}_{\mu}) + ig\mathbf{W}_{\mu} + \frac{1}{2}ig'B_{\mu})\mathbf{\Phi}$ $(D_{\mu}\mathbf{\Phi})^{\dagger}D^{\mu}\mathbf{\Phi}$ $1, 2, +\frac{1}{2}$ $(D_{\mu}\boldsymbol{\Phi})^{*} = \left(\partial_{\mu} - \left[ig_{s}\boldsymbol{G}_{\mu}^{*}\right] - ig\boldsymbol{W}_{\mu}^{*} - \frac{1}{2}ig'B_{\mu}\right)\boldsymbol{\Phi}^{*}$ -1/2+1/2

The representation of $\widetilde{\Phi}$ is $(1, \overline{2}, -\frac{1}{2})$.

The interaction terms

 H^{+}, H^{-}

 $H^0, H^{0\dagger}$

Scalar boson

Higgs

The interactions between the gauge bosons and the other fields (fermions and Higgs) all arise from the gauge covariant derivatives. The self-interactions of the nonabelian gauge bosons are all contained in their kinetic terms. The interactions between the fermions and the Higgs field are all given by Yukawa terms (on SSB, these terms generate the fermion masses).

Field

 $\mathbf{\Phi}^{\dagger} = (H^{-} H^{0\dagger})$

The Yukawa interaction terms

 $-\left\{y_d^{IJ}\left(\overline{\boldsymbol{\Psi}_{Q_I}^{\mathsf{L}}}\,\boldsymbol{\Phi}\right)\boldsymbol{\Psi}_{d_I}^{\mathsf{R}}+y_u^{IJ}\left(\overline{\boldsymbol{\Psi}_{Q_I}^{\mathsf{L}}}\,\widetilde{\boldsymbol{\Phi}}\right)\boldsymbol{\Psi}_{u_J}^{\mathsf{R}}\right.$ $+ \ y_{\ell}^{IJ} \left(\overline{\boldsymbol{\Psi}_{L_{I}}^{\text{L}}} \, \boldsymbol{\Phi} \right) \boldsymbol{\Psi}_{\ell_{I}}^{\text{R}} + y_{\nu}^{IJ} \left(\overline{\boldsymbol{\Psi}_{L_{I}}^{\text{L}}} \, \widetilde{\boldsymbol{\Phi}} \right) \boldsymbol{\Psi}_{\nu_{J}}^{\text{R}} + \text{h.c.} \, \Big\},$ where $\widetilde{\mathbf{\Phi}} \equiv (-i\tau^2)^{\mathsf{T}} \mathbf{\Phi}^*$, (Note: $\mathbf{\Phi}_d = \mathbf{\Phi}, \mathbf{\Phi}_u = \widetilde{\mathbf{\Phi}}$) and the indices I, J = 1, 2, 3 run over generations

The Higgs potential

 $\mathcal{L}^{H} = -\mu^{2} \Phi^{\dagger} \Phi - \lambda (\Phi^{\dagger} \Phi)^{2}$

For **SSB**:
$$\mu^2 < 0$$
 and $\lambda > 0$

$$\text{vev:} \quad \left\langle \mathbf{\Phi}^\dagger \mathbf{\Phi} \right\rangle_{\min} = \frac{v^2}{2}, \quad v = \sqrt{\frac{-\mu^2}{\lambda}} > 0,$$

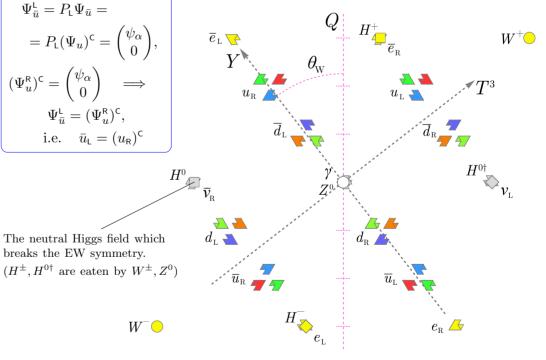
$$\text{unitary g.:} \quad \mathbf{\Phi} = \frac{1}{\sqrt{2}} \binom{0}{v + \sigma(x)}, \quad \mathbf{\Phi}_0 = \frac{1}{\sqrt{2}} \binom{0}{v}$$

$$\Psi \equiv \begin{pmatrix} \chi_{\alpha} \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}, \quad \boxed{\Psi^{\rm L} \equiv P_{\rm L} \Psi} = \begin{pmatrix} \chi_{\alpha} \\ 0 \end{pmatrix}, \quad \boxed{\Psi^{\rm R} \equiv P_{\rm R} \Psi} = \begin{pmatrix} 0 \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}$$

 $\Psi^{\mathsf{c}} \equiv C_0 \Psi^*$, $C_0 \equiv \begin{pmatrix} 0 & arepsilon_{lphaeta} \\ arepsilon^{\dot{lpha}\dot{eta}} & 0 \end{pmatrix}$, $\Psi^{\mathsf{c}} = \begin{pmatrix} \psi_{lpha} \\ ar{\psi}^{\dot{lpha}} \end{pmatrix}$ conjugate: $(D_{\mu}\Psi)^{c} = C_{0}(D_{\mu}\Psi)^{*} = D_{\mu}^{*}\Psi^{c}$

> [†] transpose, * complex conjugate, [†] hermitian conjugate α, β, \dots L-type 2-component spinor indices (dotted = R-type)

The electroweak charges



$$\begin{pmatrix} W_{\mu}^{+} \\ W_{\mu}^{-} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\mathrm{i} \\ \mathrm{i} & 1 \end{pmatrix} \begin{pmatrix} W_{\mu}^{1} \\ W_{\mu}^{2} \end{pmatrix}, \quad \begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix} = \begin{pmatrix} \cos \theta_{\mathrm{W}} & -\sin \theta_{\mathrm{W}} \\ \sin \theta_{\mathrm{W}} & \cos \theta_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} W_{\mu}^{3} \\ B_{\mu} \end{pmatrix}$$

The Weinberg angle: $\tan \theta_{\rm W} \equiv \frac{g'}{g}$, $e = g' \cos \theta_{\rm W} = g \sin \theta_{\rm W}$, $\sin \theta_{\rm W} \equiv \frac{g'}{\sqrt{g^2 + g'^2}}$, $\cos \theta_{\rm W} \equiv \frac{g}{\sqrt{g^2 + g'^2}}$