Problem 2.12 - Uncertainty Analysis, Case B

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Get[ "UCAnalysis.m", Path → {NotebookDirectory[]} ]
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$$\frac{\rho\, V\, v_1^2}{2}\left(\left(\frac{d_1}{d_2}\right)^4 - 1\right) \mapsto \begin{pmatrix} d_1 & 30 \pm 0.5 & \text{Uniform}\mathcal{D} \\ d_2 & 20 \pm 0.5 & \text{Uniform}\mathcal{D} \\ v_1 & 4.0 \pm 0.05 & \text{Uniform}\mathcal{D} \\ \rho & 998 \pm 0.5 & \text{Uniform}\mathcal{D} \\ V & 1.00 \pm 0.005 & \text{Uniform}\mathcal{D} \end{pmatrix}$$

Evaluated Functional Relationship

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φAnalysisEnvironment
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$$y = \frac{1}{2}\left(-1 + \frac{x_1^4}{x_2^4}\right) x_3^2\, x_4\, x_5$$

Variable		Uncertainty Interval	Distribution	$\lvert \partial f/\partial x_i \rvert$
x_1	d_1	$(3.00 \pm 0.05) \times 10^1$	Uniform	5.3892×10^3
x_2	d_2	$(2.00 \pm 0.05) \times 10^1$	Uniform	8.0838×10^3
x_3	v_1	$(4.00 \pm 0.05) \times 10^0$	Uniform	1.62175×10^4
x_4	ρ	$(9.980 \pm 0.005) \times 10^2$	Uniform	3.25×10^1
x_5	V	$(1.000 \pm 0.005) \times 10^0$	Uniform	3.2435×10^4

y	32 435	
y_{min} y_{max}	25 459.8159106277 41 025.6481507563	$= y - 6975.18$ $= y + 8590.65$
ε_{max} $y \pm \varepsilon_{max}$	7725.8 $(3.2 \pm 0.8) \times 10^4$	$= 23.8\,\%$ $= 3.2(8) \times 10^4$
u_c $y \pm u_c$	2844.99020670546 $(3.2 \pm 0.3) \times 10^4$	$= 8.77\,\%$ $= 3.2(3) \times 10^4$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} \lvert \partial_{x_i} f[x] \rvert\, \varepsilon_i; \quad f[x] \pm \varepsilon_{max} \;\; // \; \text{φUCE}$$

32 435 ± 7725.8

∈ [24 709; 40 161]

≃ $(3.2 \pm 0.8) \times 10^4 = 3.2(8) \times 10^4$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2\, u_i^2\right)^{1/2}; \quad f[x] \pm u_c \;\; // \; \text{φUCA}$$

32 435 ± 2844.99

∈ [29 590; 35 280]

≃ $(3.2 \pm 0.3) \times 10^4 = 3.2(3) \times 10^4$

Monte Carlo Simulation

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Block[ { data, trials = 10^6 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // QUCA
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$$32\,543.2747598863 \pm 2857.69$$
$$\in [29\,686;\ 35\,401]$$
$$\simeq (3.3 \pm 0.3) \times 10^4 = 3.3(3) \times 10^4$$