Problem 2.12 - Uncertainty Analysis, Case A1

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Get[ "UCAnalysis.m", Path → {NotebookDirectory[]} ]
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$$v_1 \left(\frac{d_1}{d_2}\right)^2 \mapsto \begin{pmatrix} d_1 & 30 \pm 0.5 & \text{Uniform}\mathcal{D} \\ d_2 & 20 \pm 0.5 & \text{Uniform}\mathcal{D} \\ v_1 & 4.0 \pm 0.05 & \text{Uniform}\mathcal{D} \end{pmatrix}$$

Evaluated Functional Relationship

φAnalysisEnvironment

$$y = \frac{x_1^2\, x_3}{x_2^2}$$

Variable		Uncertainty Interval	Distribution	$|\partial f/\partial x_i|$
x_1	d_1	$(3.00 \pm 0.05) \times 10^1$	Uniform	$6. \times 10^{-1}$
x_2	d_2	$(2.00 \pm 0.05) \times 10^1$	Uniform	$9. \times 10^{-1}$
x_3	v_1	$(4.00 \pm 0.05) \times 10^0$	Uniform	2.25

y	9	
y_{min}	8.1796252230815	$= y - 0.820375$
y_{max}	9.90798816568047	$= y + 0.907988$
ε_{max}	0.8625	$= 9.58\,\%$
$y \pm \varepsilon_{max}$	$(9.0 \pm 0.9) \times 10^0$	$= 9.0(9)$
u_c	0.318933770554327	$= 3.54\,\%$
$y \pm u_c$	$(9.0 \pm 0.3) \times 10^0$	$= 9.0(3)$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} |\partial_{x_i} f[x]|\, \varepsilon_i;\quad f[x] \pm \varepsilon_{max}\ //\ \text{φUCE}$$

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   9 ± 0.8625
 ∈ [8.1375; 9.8625]
 ≃ (9.0 ± 0.9) × 10⁰  =  9.0(9)
```

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2\, u_i^2\right)^{1/2};\quad f[x] \pm u_c\ //\ \text{φUCA}$$

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   9 ± 0.318934
 ∈ [8.6811; 9.3189]
 ≃ (9.0 ± 0.3) × 10⁰  =  9.0(3)
```

Monte Carlo Simulation

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Block[ { data, trials = 10⁶ },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // φUCA
```

$9.00671923680429 \pm 0.319334$

$\in [8.6874; 9.3261]$

$\simeq (9.0 \pm 0.3) \times 10^0 = 9.0(3)$