Problem 2.15 - Uncertainty Analysis

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Get[ "UCAnalysis.m", Path → {NotebookDirectory[]} ]
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$$\sqrt{2\,\frac{R}{M}\,\frac{T}{p}\,\frac{F}{A}+v_1^2}\ \mapsto\ \begin{pmatrix} v_1 & 100 \pm 0.5 & \text{Uniform}\mathcal{D} \\ R & 8.3144621 \pm 0.0000075 & \text{Normal}\mathcal{D} \\ M & 28.97\times10^{-3} \pm 0.005\times10^{-3} & \text{Uniform}\mathcal{D} \\ T & (273.15-15) \pm 0.5 & \text{Uniform}\mathcal{D} \\ p & 60\times10^{3} \pm 5\times10^{3} & \text{Uniform}\mathcal{D} \\ F & 1000 \pm 100 & \text{Uniform}\mathcal{D} \\ A & 1 & \end{pmatrix}$$

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QUCU[ QfEstimate ± QfStandardUncertainty, v, "m s^-1" ]
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$$v = (111.667615788844 \pm 0.870427)\ \mathrm{m\,s^{-1}}$$
$$\approx (1.117 \pm 0.009)\times10^{2}\ \mathrm{m\,s^{-1}} = 1.117(9)\times10^{2}\ \mathrm{m\,s^{-1}}$$

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QAnalysisEnvironment
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$$y = \sqrt{x_1^2 + \frac{2\,x_2\,x_4\,x_6}{x_3\,x_5\,x_7}}$$

Variable		Uncertainty Interval	Distribution	$\lvert \partial f/\partial x_i \rvert$
x_1	v_1	$(1.000 \pm 0.005)\times10^{2}$	Uniform	8.95515×10^{-1}
x_2	R	$(8.3144621 \pm 0.0000075)\times10^{0}$	Normal	1.32998
x_3	M	$(2.8970 \pm 0.0005)\times10^{-2}$	Uniform	3.81708×10^{2}
x_4	T	$(2.582 \pm 0.005)\times10^{2}$	Uniform	4.28358×10^{-2}
x_5	p	$(6.0 \pm 0.5)\times10^{4}$	Uniform	1.84301×10^{-4}
x_6	F	$(1.0 \pm 0.1)\times10^{3}$	Uniform	1.10581×10^{-2}
x_7	A	$1\times(\text{exact})\ 10^{0}$		1.10581×10^{1}

y	111.667615788844	
y_{min}	109.305223006439	$= y - 2.36239$
y_{max}	114.324530864478	$= y + 2.65692$
ε_{max}	2.49842642729096	$= 2.24\,\%$
$y \pm \varepsilon_{max}$	$(1.12 \pm 0.03)\times10^{2}$	$= 1.12(3)\times10^{2}$
u_c	0.87042725547714	$= 0.779\,\%$
$y \pm u_c$	$(1.117 \pm 0.009)\times10^{2}$	$= 1.117(9)\times10^{2}$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} \lvert \partial_{x_i} f[x] \rvert\, \varepsilon_i ; \quad f[x] \pm \varepsilon_{max} \quad // \text{QUCE}$$

$$111.667615788844 \pm 2.49843$$
$$\in [109.169;\ 114.166]$$
$$\approx (1.12 \pm 0.03)\times10^{2} = 1.12(3)\times10^{2}$$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2 u_i^2\right)^{1/2} \quad ; \quad f[x] \pm u_c \;//\; \text{QUCA}$$

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111.667615788844 ± 0.870427
 ∈ [110.7972; 112.538]
≃ (1.117 ± 0.009) × 10^2 = 1.117(9) × 10^2
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Monte Carlo Simulation

```
Block[ { data, trials = 10^6 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // QUCA
```

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111.690909522576 ± 0.873275
 ∈ [110.8176; 112.5642]
≃ (1.117 ± 0.009) × 10^2 = 1.117(9) × 10^2
```