Problem 1.3 - Uncertainty Analysis

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Get[ "UCAnalysis.m", Path → {NotebookDirectory[]} ]
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$$-\frac{m_{water}\, c\, (T_3 - T_{1,water})}{\ell_f + c\, (T_3 - T_{2,ice})} \quad \mapsto \quad \begin{pmatrix} m_{water} & 0.100 \pm 0.0005 & \text{Uniform}\mathcal{D} \\ c & 4.19 \pm 0.005 & \text{Uniform}\mathcal{D} \\ \ell_f & 333 \pm 0.5 & \text{Uniform}\mathcal{D} \\ T_{1,water} & (20.0 + 273.15) \pm 0.05 & \text{Uniform}\mathcal{D} \\ T_{2,ice} & (0 + 273.15) & \\ T_3 & (5.0 + 273.15) \pm 0.05 & \text{Uniform}\mathcal{D} \end{pmatrix}$$

Evaluated Functional Relationship

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φAnalysisEnvironment
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$$y = -\frac{x_1\, x_2\, (-x_4 + x_6)}{x_3 + x_2\, (-x_5 + x_6)}$$

Variable		Uncertainty Interval	Distribution	$\lvert \partial f / \partial x_i \rvert$
x_1	m_{water}	$(1.000 \pm 0.005) \times 10^{-1}$	Uniform	1.77567×10^{-1}
x_2	c	$(4.190 \pm 0.005) \times 10^{0}$	Uniform	3.98705×10^{-3}
x_3	ℓ_f	$(3.330 \pm 0.005) \times 10^{2}$	Uniform	5.01674×10^{-5}
x_4	$T_{1,water}$	$(2.9315 \pm 0.0005) \times 10^{2}$	Uniform	1.18378×10^{-3}
x_5	$T_{2,ice}$	$2.7315 \times (\text{exact})\ 10^{2}$		2.10201×10^{-4}
x_6	T_3	$(2.7815 \pm 0.0005) \times 10^{2}$	Uniform	1.39398×10^{-3}

y	0.0177567453030089	
y_{min} y_{max}	0.0174954104122768 0.0180207995267013	$= y - 0.000261335$ $= y + 0.000264054$
ε_{max} $y \pm \varepsilon_{max}$	0.00026269103590877 1 $(1.78 \pm 0.03) \times 10^{-2}$	$= 1.48\,\%$ $= 1.78(3) \times 10^{-2}$
u_c $y \pm u_c$	0.0000758737820109035 $(1.776 \pm 0.008) \times 10^{-2}$	$= 0.427\,\%$ $= 1.776(8) \times 10^{-2}$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} \lvert \partial_{x_i} f[x] \rvert\, \varepsilon_i; \quad f[x] \pm \varepsilon_{max} \;\; // \; \text{φUCE}$$

$$0.0177567453030089 \pm 0.000262691$$
$$\in [0.0174941;\ 0.0180194]$$
$$\simeq (1.78 \pm 0.03) \times 10^{-2} = 1.78(3) \times 10^{-2}$$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2\, u_i^2 \right)^{1/2}; \quad f[x] \pm u_c \;\; // \; \text{φUCA}$$

$$0.0177567453030089 \pm 0.0000758738$$
$$\in [0.01768087;\ 0.01783262]$$
$$\simeq (1.776 \pm 0.008) \times 10^{-2} = 1.776(8) \times 10^{-2}$$

Monte Carlo Simulation

```
Block[ { data, trials = 10^6 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // QUCA
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0.0177567575851228 ± 0.00007582
 ∈ [0.01768094; 0.01783258]
 ≃ (1.776 ± 0.008) × 10^-2 = 1.776(8) × 10^-2
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