Problem 2.1 - Uncertainty Analysis

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Get[ "UCAnalysis.m", Path → {NotebookDirectory[]} ]
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$$\frac{v_1}{N}\left(\frac{d_1}{d_2}\right)^2 \mapsto \begin{pmatrix} d_1 & 1.80 \pm 0.005 & \text{Uniform}\mathcal{D} \\ d_2 & 1.0 \pm 0.05 & \text{Uniform}\mathcal{D} \\ v_1 & 0.33 \pm 0.005 & \text{Uniform}\mathcal{D} \\ N & 30 \pm 0.5 & \text{Uniform}\mathcal{D} \end{pmatrix}$$

Evaluated Functional Relationship

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?AnalysisEnvironment
```

$$y = \frac{x_1^2\, x_3}{x_2^2\, x_4}$$

Variable		Uncertainty Interval	Distribution	$\lvert \partial f/\partial x_i \rvert$
x_1	d_1	$(1.800 \pm 0.005) \times 10^0$	Uniform	3.96×10^{-2}
x_2	d_2	$(1.00 \pm 0.05) \times 10^0$	Uniform	7.128×10^{-2}
x_3	v_1	$(3.30 \pm 0.05) \times 10^{-1}$	Uniform	1.08×10^{-1}
x_4	N	$(3.00 \pm 0.05) \times 10^1$	Uniform	1.188×10^{-3}

y	0.03564	
y_{min}	0.0311410914092413	$= y - 0.00449891$
y_{max}	0.0409949152542373	$= y + 0.00535492$
ε_{max}	0.004896	$= 13.7\,\%$
$y \pm \varepsilon_{max}$	$(3.6 \pm 0.5) \times 10^{-2}$	$= 3.6(5) \times 10^{-2}$
u_c	0.00211232383880881	$= 5.93\,\%$
$y \pm u_c$	$(3.6 \pm 0.2) \times 10^{-2}$	$= 3.6(2) \times 10^{-2}$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} \lvert \partial_{x_i} f[x] \rvert\, \varepsilon_i ; \quad f[x] \pm \varepsilon_{max} \;\; // \; ?UCE$$

$$0.03564 \pm 0.004896$$
$$\in [0.030744;\ 0.040536]$$
$$\simeq (3.6 \pm 0.5) \times 10^{-2} = 3.6(5) \times 10^{-2}$$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2\, u_i^2 \right)^{1/2} ; \quad f[x] \pm u_c \;\; // \; ?UCA$$

$$0.03564 \pm 0.00211232$$
$$\in [0.033528;\ 0.037752]$$
$$\simeq (3.6 \pm 0.2) \times 10^{-2} = 3.6(2) \times 10^{-2}$$

Monte Carlo Simulation

```
Block[ { data, trials = 10^6 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // QUCA
```

```
0.0357317792753673 ± 0.00212036
 ∈ [0.033611; 0.037852]
 ≃ (3.6 ± 0.2) × 10^-2 = 3.6(2) × 10^-2
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