

## Problem 2.2 - Uncertainty Analysis, Case B

```
Get[ "UCAnalysis.m", Path -> {NotebookDirectory[]} ]
```

$$\frac{(d_1 \text{ in})^2 \pi}{4} v_1 \mapsto \begin{pmatrix} d_1 & 0.5 \pm 0.5 \times 0.05 & \text{Uniform} \\ v_1 & 2.0 \pm 0.05 & \text{Uniform} \\ \text{in} & 25.4 \times 10^{-3} & \end{pmatrix}$$

### Evaluated Functional Relationship

ⓅAnalysisEnvironment

$$y = \frac{1}{4} \pi x_1^2 x_2 x_3^2$$

Variable		Uncertainty Interval	Distribution	$ \partial f / \partial x_i $
$x_1$	$d_1$	$(5.00 \pm 0.25) \times 10^{-1}$	Uniform	$1.01341 \times 10^{-3}$
$x_2$	$v_1$	$(2.00 \pm 0.05) \times 10^0$	Uniform	$1.26677 \times 10^{-4}$
$x_3$	$\text{in}$	$2.54 \times (\text{exact}) 10^{-2}$		$1.99491 \times 10^{-2}$

$y$	0.000253353739548749		
$y_{\min}$	0.000222935456194177	$= y - 0.0000304183$	
$y_{\max}$	0.000286305560298808	$= y + 0.0000329518$	
$\epsilon_{\max}$	0.0000316692174435936	$= 12.5 \%$	
$y \pm \epsilon_{\max}$	$(2.5 \pm 0.3) \times 10^{-4}$	$= 2.5(3) \times 10^{-4}$	
$u_c$	0.0000150775633174256	$= 5.95 \%$	
$y \pm u_c$	$(2.5 \pm 0.2) \times 10^{-4}$	$= 2.5(2) \times 10^{-4}$	

### Absolute Maximum Uncertainty

$$\epsilon_{\max} = \sum_{i=1}^n |\partial_{x_i} f[\mathbf{x}]| \epsilon_i; \quad f[\mathbf{x}] \pm \epsilon_{\max} \quad // \quad \text{ⓅUCE}$$

$$\begin{aligned} & 0.000253353739548749 \pm 0.0000316692 \\ & \in [0.00022168; 0.00028502] \\ & \approx (2.5 \pm 0.3) \times 10^{-4} = 2.5(3) \times 10^{-4} \end{aligned}$$

### Combined Standard Uncertainty

$$u_c = \left( \sum_{i=1}^n (\partial_{x_i} f[\mathbf{x}])^2 u_i^2 \right)^{1/2}; \quad f[\mathbf{x}] \pm u_c \quad // \quad \text{ⓅUCA}$$

$$\begin{aligned} & 0.000253353739548749 \pm 0.0000150776 \\ & \in [0.00023828; 0.00026843] \\ & \approx (2.5 \pm 0.2) \times 10^{-4} = 2.5(2) \times 10^{-4} \end{aligned}$$

## Monte Carlo Simulation

```
Block[{ { data, trials = 106 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // ϕUCA
```

```
0.000253600718358152 ± 0.0000150813
  ∈ [0.00023852; 0.00026868]
  ≈ (2.5 ± 0.2) × 10-4 = 2.5(2) × 10-4
```