Problem 2.9 - Uncertainty Analysis, Case B

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Get[ "UCAnalysis.m", Path → {NotebookDirectory[]} ]
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$$\frac{\rho}{2}\left(v_2^2 - v_1^2\right) D\, L \;\mapsto\; \begin{pmatrix} v_2 & 220 \pm 0.5 & \text{Uniform}\mathcal{D} \\ v_1 & 180 \pm 0.5 & \text{Uniform}\mathcal{D} \\ \rho & 1.0 \pm 0.05 & \text{Uniform}\mathcal{D} \\ D & 3 \pm 0.5 & \text{Uniform}\mathcal{D} \\ L & 8 \pm 0.5 & \text{Uniform}\mathcal{D} \end{pmatrix}$$

Evaluated Functional Relationship

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?AnalysisEnvironment
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$$y = \frac{1}{2}\left(x_1^2 - x_2^2\right) x_3\, x_4\, x_5$$

Variable		Uncertainty Interval	Distribution	$\lvert \partial f/\partial x_i \rvert$
x_1	v_2	$(2.200 \pm 0.005) \times 10^2$	Uniform	5.28×10^3
x_2	v_1	$(1.800 \pm 0.005) \times 10^2$	Uniform	4.32×10^3
x_3	ρ	$(1.00 \pm 0.05) \times 10^0$	Uniform	1.92×10^5
x_4	D	$(3.0 \pm 0.5) \times 10^0$	Uniform	6.4×10^4
x_5	L	$(8.0 \pm 0.5) \times 10^0$	Uniform	2.4×10^4

y	192 000	
y_{min} y_{max}	138 937.5 256 147.5	$= y - 53062.5$ $= y + 64147.5$
ε_{max} $y \pm \varepsilon_{max}$	58 400 $(1.9 \pm 0.6) \times 10^5$	$= 30.4\,\%$ $= 1.9(6) \times 10^5$
u_c $y \pm u_c$	20 589.60255404 $(1.9 \pm 0.2) \times 10^5$	$= 10.7\,\%$ $= 1.9(2) \times 10^5$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} \lvert \partial_{x_i} f[x] \rvert\, \varepsilon_i ; \quad f[x] \pm \varepsilon_{max} \;\; // \; ?UCE$$

192 000 ± 58 400

∈ [133 600; 250 400]

≃ $(1.9 \pm 0.6) \times 10^5 = 1.9(6) \times 10^5$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2\, u_i^2\right)^{1/2} ; \quad f[x] \pm u_c \;\; // \; ?UCA$$

192 000 ± 20 589.6

∈ [171 410; 212 590]

≃ $(1.9 \pm 0.2) \times 10^5 = 1.9(2) \times 10^5$

Monte Carlo Simulation

```
Block[ { data, trials = 10^6 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // QUCA
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192 037.424209331 ± 20 608.6
 ∈ [171 430; 212 650]
 ≃ (1.9 ± 0.2) × 10^5 = 1.9(2) × 10^5
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