Problem 2.9 - Uncertainty Analysis, Case A

```
Get[ "UCAnalysis.m", Path → {NotebookDirectory[]} ]
```

$$\frac{\rho}{2}\left(v_2^2 - v_1^2\right) \;\mapsto\; \begin{pmatrix} v_2 & 220 \pm 0.5 & \text{UniformD} \\ v_1 & 180 \pm 0.5 & \text{UniformD} \\ \rho & 1.0 \pm 0.05 & \text{UniformD} \end{pmatrix}$$

Evaluated Functional Relationship

```
?AnalysisEnvironment
```

$$y = \frac{1}{2}\left(x_1^2 - x_2^2\right) x_3$$

Variable		Uncertainty Interval	Distribution	$\lvert \partial f/\partial x_i \rvert$
x_1	v_2	$(2.200 \pm 0.005) \times 10^2$	Uniform	2.2×10^2
x_2	v_1	$(1.800 \pm 0.005) \times 10^2$	Uniform	1.8×10^2
x_3	ρ	$(1.00 \pm 0.05) \times 10^0$	Uniform	$8. \times 10^3$

y	8000	
y_{min} y_{max}	7410 8610	$= y - 590.$ $= y + 610.$
ε_{max} $y \pm \varepsilon_{max}$	600 $(8.0 \pm 0.6) \times 10^3$	$= 7.5\,\%$ $= 8.0(6) \times 10^3$
u_c $y \pm u_c$	245.085019262024 $(8.0 \pm 0.3) \times 10^3$	$= 3.06\,\%$ $= 8.0(3) \times 10^3$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} \lvert \partial_{x_i} f[x] \rvert \varepsilon_i ; \quad f[x] \pm \varepsilon_{max} \;\; // \; ?UCE$$

```
8000 ± 600
∈ [7400; 8600]
```

$\simeq (8.0 \pm 0.6) \times 10^3 = 8.0(6) \times 10^3$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2 u_i^2\right)^{1/2} ; \quad f[x] \pm u_c \;\; // \; ?UCA$$

```
8000 ± 245.085
∈ [7754.9; 8245.1]
```

$\simeq (8.0 \pm 0.3) \times 10^3 = 8.0(3) \times 10^3$

Monte Carlo Simulation

```
Block[ { data, trials = 10^6 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // QUCA
```

$$8000.0231617218 \pm 245.375$$
$$\in [7754.6;\ 8245.4]$$
$$\simeq \left(8.0 \pm 0.3\right) \times 10^3 \ = \ 8.0\left(3\right) \times 10^3$$