

Problem 1.6 - Uncertainty Analysis

```
Get[ "UCAnalysis.m", Path -> {NotebookDirectory[]} ]
```

$$P \tau k / (1000 \times 3600) \rightarrow \begin{pmatrix} P & 60 \pm 5 & \text{Uniform} \\ \tau & 7 \times 24 \times 60 \times 60 & \mathcal{D} \\ k & 0.60 & \end{pmatrix}$$

Evaluated Functional Relationship

```
ΦAnalysisEnvironment
```

$$y = \frac{x_1 x_2 x_3}{3\,600\,000}$$

Variable		Uncertainty Interval	Distribution	$ \partial f / \partial x_i $
x_1	P	$(6.0 \pm 0.5) \times 10^1$	Uniform	1.008×10^{-1}
x_2	τ	$6.048 \times (\text{exact}) 10^5$		$1. \times 10^{-5}$
x_3	k	$6 \times (\text{exact}) 10^{-1}$		1.008×10^1

y	6.048		
y _{min}	5.544	= y - 0.504	
y _{max}	6.552	= y + 0.504	
ε _{max}	0.504	= 8.33 %	
y ± ε _{max}	$(6.0 \pm 0.5) \times 10^0$	= 6.0(5)	
u _c	0.290984535671571	= 4.81 %	
y ± u _c	$(6.0 \pm 0.3) \times 10^0$	= 6.0(3)	

Absolute Maximum Uncertainty

$$\epsilon_{\max} = \sum_{i=1}^n |\partial_{x_i} f[\mathbf{x}]| \epsilon_i; \quad f[\mathbf{x}] \pm \epsilon_{\max} \quad // \quad \Phi UCE$$

$$6.048 \pm 0.504$$

$$\in [5.544; 6.552]$$

$$\approx (6.0 \pm 0.5) \times 10^0 = 6.0(5)$$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^n (\partial_{x_i} f[\mathbf{x}])^2 u_i^2 \right)^{1/2}; \quad f[\mathbf{x}] \pm u_c \quad // \quad \Phi UCA$$

$$6.048 \pm 0.290985$$

$$\in [5.757; 6.339]$$

$$\approx (6.0 \pm 0.3) \times 10^0 = 6.0(3)$$

Monte Carlo Simulation

```
Block[{ { data, trials = 106 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // NUCA
```

6.04837220779298 ± 0.290814

∈ [5.7576; 6.3392]

≈ (6.0 ± 0.3) × 10⁰ = 6.0(3)