Problem 1.1 - Uncertainty Analysis (Case A)

```
Get[ "UCAnalysis.m", Path → {NotebookDirectory[]} ]
```

$$-\frac{c_{Hg}\, m_{Hg}\, (T_{2,Hg} - T_{1,Hg})}{c_{H2O}\, (T_{2,Hg} - T_{1,H2O})} \quad \mapsto \quad \begin{pmatrix} c_{Hg} & 0.14 \pm 0.005 & \text{Uniform}\mathcal{D} \\ c_{H2O} & 4.19 \pm 0.005 & \text{Uniform}\mathcal{D} \\ m_{Hg} & 0.200 \pm 0.0005 & \text{Uniform}\mathcal{D} \\ T_{2,Hg} & (25 + 273.15) \pm 0.5 & \text{Uniform}\mathcal{D} \\ T_{1,Hg} & (80 + 273.15) \pm 0.5 & \text{Uniform}\mathcal{D} \\ T_{1,H2O} & (15 + 273.15) \pm 0.5 & \text{Uniform}\mathcal{D} \end{pmatrix}$$

Evaluated Functional Relationship

```
℘AnalysisEnvironment
```

$$y = -\frac{x_1\, x_3\, (x_4 - x_5)}{x_2\, (x_4 - x_6)}$$

Variable		Uncertainty Interval	Distribution	$\lvert \partial f/\partial x_i \rvert$
x_1	c_{Hg}	$(1.40 \pm 0.05) \times 10^{-1}$	Uniform	2.6253×10^{-1}
x_2	c_{H2O}	$(4.190 \pm 0.005) \times 10^{0}$	Uniform	8.77188×10^{-3}
x_3	m_{Hg}	$(2.000 \pm 0.005) \times 10^{-1}$	Uniform	1.83771×10^{-1}
x_4	$T_{2,Hg}$	$(2.982 \pm 0.005) \times 10^{2}$	Uniform	4.34368×10^{-3}
x_5	$T_{1,Hg}$	$(3.532 \pm 0.005) \times 10^{2}$	Uniform	6.68258×10^{-4}
x_6	$T_{1,H2O}$	$(2.882 \pm 0.005) \times 10^{2}$	Uniform	3.67542×10^{-3}

y	0.0367541766109785	
y_{min}	0.0315170657709394	$= y - 0.00523711$
y_{max}	0.0432247444577194	$= y + 0.00647057$
ε_{max}	0.00579206942316346	$= 15.8\,\%$
$y \pm \varepsilon_{max}$	$(3.7 \pm 0.6) \times 10^{-2}$	$= 3.7(6) \times 10^{-2}$
u_c	0.00182017483569375	$= 4.95\,\%$
$y \pm u_c$	$(3.7 \pm 0.2) \times 10^{-2}$	$= 3.7(2) \times 10^{-2}$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} \lvert \partial_{x_i} f[x] \rvert\, \varepsilon_i; \quad f[x] \pm \varepsilon_{max} \;\; // \; ℘UCE$$

$$0.0367541766109785 \pm 0.00579207$$
$$\in [0.030962;\ 0.042546]$$
$$\simeq (3.7 \pm 0.6) \times 10^{-2} = 3.7(6) \times 10^{-2}$$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2\, u_i^2 \right)^{1/2}; \quad f[x] \pm u_c \;\; // \; ℘UCA$$

$$0.0367541766109785 \pm 0.00182017$$
$$\in [0.034934;\ 0.038574]$$
$$\simeq (3.7 \pm 0.2) \times 10^{-2} = 3.7(2) \times 10^{-2}$$

Monte Carlo Simulation

```
Block[ { data, trials = 10^6 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // QUCA
```

```
0.0368204342273103 ± 0.00182958
 ∈ [0.034991; 0.03865]
 ≃ (3.7 ± 0.2) × 10^-2 = 3.7(2) × 10^-2
```