Problem 1.1 - Uncertainty Analysis (Case B)

```
Get[ "UCAnalysis.m", Path → {NotebookDirectory[]} ]
```

$$\frac{c_{H2O}\, m_{H2O}\, T_{1,H2O} + c_{Hg}\, m_{Hg}\, T_{1,Hg}}{c_{H2O}\, m_{H2O} + c_{Hg}\, m_{Hg}} \mapsto \begin{pmatrix} c_{Hg} & 0.14 \pm 0.005 & \text{UniformD} \\ c_{H2O} & 4.19 \pm 0.005 & \text{UniformD} \\ m_{Hg} & 0.200 \pm 0.0005 & \text{UniformD} \\ m_{H2O} & 0.037 \pm 0.0005 & \text{UniformD} \\ T_{1,Hg} & (273.15 + 80) \pm 0.5 & \text{UniformD} \\ T_{1,H2O} & (273.15 + 15) \pm 0.5 & \text{UniformD} \end{pmatrix}$$

Evaluated Functional Relationship

```
PAnalysisEnvironment
```

$$y = \frac{x_1\, x_3\, x_5 + x_2\, x_4\, x_6}{x_1\, x_3 + x_2\, x_4}$$

Variable		Uncertainty Interval	Distribution	$\lvert \partial f/\partial x_i \rvert$
x_1	c_{Hg}	$(1.40 \pm 0.05) \times 10^{-1}$	Uniform	6.01609×10^{1}
x_2	c_{H2O}	$(4.190 \pm 0.005) \times 10^{0}$	Uniform	2.01015
x_3	m_{Hg}	$(2.000 \pm 0.005) \times 10^{-1}$	Uniform	4.21127×10^{1}
x_4	m_{H2O}	$(3.70 \pm 0.05) \times 10^{-2}$	Uniform	2.27636×10^{2}
x_5	$T_{1,Hg}$	$(3.532 \pm 0.005) \times 10^{2}$	Uniform	1.5298×10^{-1}
x_6	$T_{1,H2O}$	$(2.882 \pm 0.005) \times 10^{2}$	Uniform	8.4702×10^{-1}

y	298.093725072392	
y_{min}	297.151546853375	$= y - 0.942178$
y_{max}	299.043029011412	$= y + 0.949304$
ε_{max}	0.945729701893652	= 0.317 %
$y \pm \varepsilon_{max}$	$(2.981 \pm 0.001) \times 10^{2}$	$= 2.981(1) \times 10^{2}$
u_c	0.310480091278691	= 0.104 %
$y \pm u_c$	$(2.981 \pm 0.003) \times 10^{2}$	$= 2.981(3) \times 10^{2}$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} \lvert \partial_{x_i} f[x] \rvert\, \varepsilon_i ; \quad f[x] \pm \varepsilon_{max} \;\; // \text{PUCE}$$

298.093725072392 ± 0.94573

∈ [297.148; 299.0395]

≃ $(2.981 \pm 0.001) \times 10^{2} = 2.981(1) \times 10^{2}$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2\, u_i^2 \right)^{1/2} ; \quad f[x] \pm u_c \;\; // \text{PUCA}$$

298.093725072392 ± 0.31048

∈ [297.7832; 298.4042]

≃ $(2.981 \pm 0.003) \times 10^{2} = 2.981(3) \times 10^{2}$

Monte Carlo Simulation

```
Block[ { data, trials = 10^6 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // QUCA
```

```
298.0932696678 ± 0.310087
 ∈ [297.7832; 298.4034]
≃ (2.981 ± 0.003) × 10^2 = 2.981(3) × 10^2
```