Problem 2.18 - Uncertainty Analysis

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Get["UCAnalysis.m", Path → {NotebookDirectory[]}]
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$$60.\ \frac{1}{2\pi}\sqrt{\frac{6\,m\,g}{\mathcal{N}\,\rho\,(k^2-1)\,b\,(\ell_2^3-\ell_1^3)}} \mapsto \begin{pmatrix} m & 8600 \pm 50 & \text{Uniform}\mathcal{D} \\ g & \text{CODATA[gn]} & \\ \mathcal{N} & 6 & \\ \rho & 1.29 \pm 0.005 & \text{Uniform}\mathcal{D} \\ k & 1.5 \pm 0.05 & \text{Uniform}\mathcal{D} \\ b & 0.40 \pm 0.005 & \text{Uniform}\mathcal{D} \\ \ell_2 & 7.62 \pm 0.005 & \text{Uniform}\mathcal{D} \\ \ell_1 & 1.21 \pm 0.005 & \text{Uniform}\mathcal{D} \end{pmatrix}$$

(Note that standard acceleration of gravity *g* is a defined (exact) quantity.)

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ϘUCResult[f"[min⁻¹]", "min⁻¹", UcPrecision → 2]
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$$f_{[\mathrm{min}^{-1}]} = (164.4901 \pm 5.7393)\ \mathrm{min}^{-1}$$
$$\in [158.7508;\ 170.2294]\ \mathrm{min}^{-1};\ \text{Normal}\mathcal{D}$$
$$\simeq (1.645 \pm 0.058) \times 10^2\ \mathrm{min}^{-1} = 1.645(58) \times 10^2\ \mathrm{min}^{-1}$$

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ϘAnalysisEnvironment
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$$y = 23.3909\sqrt{\frac{x_1\,x_2}{x_3\,x_4\,(-1+x_5^2)\,x_6\,(x_7^3-x_8^3)}}$$

Quantity		Estimate ± Uncertainty	Distribution	$\lvert \partial f/\partial x_i \rvert$
x_1	m	$(8.60 \pm 0.05) \times 10^3$	Uniform	9.56338×10^{-3}
x_2	g	9.80665 (exact)	-	8.38666
x_3	$\mathcal{N}$	6 (exact)	-	1.37075×10^1
x_4	ρ	$(1.290 \pm 0.005) \times 10^0$	Uniform	6.37559×10^1
x_5	k	$(1.50 \pm 0.05) \times 10^0$	Uniform	1.97388×10^2
x_6	b	$(4.00 \pm 0.05) \times 10^{-1}$	Uniform	2.05613×10^2
x_7	ℓ_2	$(7.620 \pm 0.005) \times 10^0$	Uniform	3.25101×10^1
x_8	ℓ_1	$(1.210 \pm 0.005) \times 10^0$	Uniform	8.19746×10^{-1}

y	164.490124079408	
y_{min}	153.4263311	$= y - 11.0638$
y_{max}	177.2452202	$= y + 12.7551$
ε_{max}	11.8610683024866	= 7.21 %
$y \pm \varepsilon_{max}$	$(1.64 \pm 0.12) \times 10^2$	$= 1.64(12) \times 10^2$
u_c	5.739303752976773	= 3.49 %
$y \pm u_c$	$(1.645 \pm 0.058) \times 10^2$	$= 1.645(58) \times 10^2$

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ϘfEstimate ± ϘfMaximumUncertainty // ϘUC
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164.49 ± 11.8611

∈ [152.629; 176.351]

$\simeq (1.64 \pm 0.12) \times 10^2 = 1.64(12) \times 10^2$

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ϘfEstimate ± ϘfStandardUncertainty // ϘUC
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164.4901 ± 5.7393

∈ [158.7508; 170.2294]

$\simeq (1.645 \pm 0.058) \times 10^2 = 1.645(58) \times 10^2$

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φMonteCarlo[10^6] // φUC
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164.7405 ± 5.76712

∈ [158.9734; 170.5076]

≃ $\left(1.647 \pm 0.058\right) \times 10^{2} = 1.647\left(58\right) \times 10^{2}$

Sanity Check

⚲DumpRelationship

$$f_{[\text{min}^{-1}]} == 23.3909 \sqrt{\frac{g\, m}{b\left(-1+k^2\right) \mathcal{N} \rho \left(-\ell_1{}^3+\ell_2{}^3\right)}}$$

⚲DumpQuantities

$f_{[\text{min}^{-1}]} = (164.4901 \pm 5.7393)\ \text{min}^{-1}$
$\in [158.7508;\ 170.2294]\ \text{min}^{-1}$; Normal$\mathcal{D}$
$\simeq (\mathbf{1.645 \pm 0.058}) \times \mathbf{10^2}\ \text{min}^{-1} = \mathbf{1.645(58) \times 10^2}\ \text{min}^{-1}$

$m = 8600 \pm 50$
$\in [8550;\ 8650]$; Uniform$\mathcal{D}$
$\simeq (\mathbf{8.60 \pm 0.05}) \times \mathbf{10^3} = \mathbf{8.60(5) \times 10^3}$

$g = 9.80665\ (\text{exact})\ \frac{\text{m}}{\text{s}^2}$
▷ standard acceleration of gravity; $g_n \left[\frac{\text{m}}{\text{s}^2}\right]$

$\mathcal{N} = 6\ (\text{exact})$

$\rho = 1.29 \pm 0.005$
$\in [1.285;\ 1.295]$; Uniform$\mathcal{D}$
$\simeq (\mathbf{1.290 \pm 0.005}) \times \mathbf{10^0} = \mathbf{1.290(5)}$

$k = 1.5 \pm 0.05$
$\in [1.45;\ 1.55]$; Uniform$\mathcal{D}$
$\simeq (\mathbf{1.50 \pm 0.05}) \times \mathbf{10^0} = \mathbf{1.50(5)}$

$b = 0.4 \pm 0.005$
$\in [0.395;\ 0.405]$; Uniform$\mathcal{D}$
$\simeq (\mathbf{4.00 \pm 0.05}) \times \mathbf{10^{-1}} = \mathbf{4.00(5) \times 10^{-1}}$

$\ell_2 = 7.62 \pm 0.005$
$\in [7.615;\ 7.625]$; Uniform$\mathcal{D}$
$\simeq (\mathbf{7.620 \pm 0.005}) \times \mathbf{10^0} = \mathbf{7.620(5)}$

$\ell_1 = 1.21 \pm 0.005$
$\in [1.205;\ 1.215]$; Uniform$\mathcal{D}$
$\simeq (\mathbf{1.210 \pm 0.005}) \times \mathbf{10^0} = \mathbf{1.210(5)}$