Problem 1.2 - Uncertainty Analysis (Case A)

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Get["UCAnalysis.m", Path → {NotebookDirectory[]}]
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$$\frac{\ell_v}{c_p}\,\mathrm{Log}\left[1+\frac{\Delta m}{\rho\,V}\right] \mapsto \begin{pmatrix} \ell_v & 2.260\times10^{3}\pm0.005\times10^{3} & \mathrm{Uniform}\mathcal{D} \\ c_p & 4.19\pm0.005 & \mathrm{Uniform}\mathcal{D} \\ \Delta m & -500\times10^{-6}\pm0.5\times10^{-6} & \mathrm{Uniform}\mathcal{D} \\ \rho & 998\pm0.5 & \mathrm{Uniform}\mathcal{D} \\ V & 200\times10^{-6}\pm0.5\times10^{-6} & \mathrm{Uniform}\mathcal{D} \end{pmatrix}$$

Evaluated Functional Relationship

φAnalysisEnvironment

$$y = \frac{\mathrm{Log}\left[1+\frac{x_3}{x_4\,x_5}\right]x_1}{x_2}$$

Variable		Uncertainty Interval	Distribution	$\lvert\partial f/\partial x_i\rvert$
x_1	ℓ_v	$(2.260\pm0.005)\times10^{3}$	Uniform	5.98604×10^{-4}
x_2	c_p	$(4.190\pm0.005)\times10^{0}$	Uniform	3.22875×10^{-1}
x_3	Δm	$(-5.000\pm0.005)\times10^{-4}$	Uniform	2.70909×10^{3}
x_4	ρ	$(9.980\pm0.005)\times10^{2}$	Uniform	1.35726×10^{-3}
x_5	V	$(2.000\pm0.005)\times10^{-4}$	Uniform	6.77272×10^{3}

y	-1.35284614422463	
y_{min} y_{max}	-1.36291174114087 -1.34285918927409	$= y-0.0100656$ $= y+0.00998695$
ε_{max} $y\pm\varepsilon_{max}$	0.0100269311192552 $(-1.35\pm0.01)\times10^{0}$	$= -0.741\,\%$ $= -1.35(1)$
u_c $y\pm u_c$	0.00290557817012747 $(-1.353\pm0.003)\times10^{0}$	$= -0.215\,\%$ $= -1.353(3)$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n}\lvert\partial_{x_i} f[x]\rvert\,\varepsilon_i;\quad f[x]\pm\varepsilon_{max}\ //\ \varphi\mathrm{UCE}$$

$-1.35284614422463 \pm 0.0100269$

$\in [-1.36287;\ -1.34282]$

$\simeq\ (-1.35\pm0.01)\times10^{0}\ =\ -1.35(1)$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n}(\partial_{x_i} f[x])^2\,u_i^2\right)^{1/2};\quad f[x]\pm u_c\ //\ \varphi\mathrm{UCA}$$

$-1.35284614422463 \pm 0.00290558$

$\in [-1.355752;\ -1.349941]$

$\simeq\ (-1.353\pm0.003)\times10^{0}\ =\ -1.353(3)$

Monte Carlo Simulation

```
Block[{data, trials = 10^6},
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data]] // ϘUCA
```

```
-1.35284843192771 ± 0.00290781
 ∈ [-1.355756; -1.349941]
```
$\simeq (-1.353 \pm 0.003) \times 10^0 = -1.353(3)$