Problem 2.8 - Uncertainty Analysis

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$$\Delta p\, V \;\mapsto\; \begin{pmatrix} V & 10 \pm 0.5 & \text{Uniform}\mathcal{D} \\ \Delta p & 0.18 \times 10^6 \pm 0.005 \times 10^6 & \text{Uniform}\mathcal{D} \end{pmatrix}$$

Evaluated Functional Relationship

`φAnalysisEnvironment`

$$y = x_1\, x_2$$

Variable		Uncertainty Interval	Distribution	$|\partial f / \partial x_i|$
x_1	V	$\left(1.00 \pm 0.05\right) \times 10^1$	Uniform	1.8×10^5
x_2	Δp	$\left(1.80 \pm 0.05\right) \times 10^5$	Uniform	$1.\times 10^1$

y	1.8×10^6	
y_{min}	1.6625×10^6	$= y - 137500.$
y_{max}	1.9425×10^6	$= y + 142500.$
ε_{max}	$140\,000$	$= 7.78\,\%$
$y \pm \varepsilon_{max}$	$\left(1.8 \pm 0.2\right) \times 10^6$	$= 1.8\left(2\right) \times 10^6$
u_c	$59\,441.8483337567$	$= 3.3\,\%$
$y \pm u_c$	$\left(1.80 \pm 0.06\right) \times 10^6$	$= 1.80\left(6\right) \times 10^6$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} |\partial_{x_i} f[x]|\; \varepsilon_i; \quad f[x] \pm \varepsilon_{max} \; // \; \text{φUCE}$$

$1.8 \times 10^6 \pm 0.14 \times 10^6$
$\in \left[1.66 \times 10^6;\; 1.94 \times 10^6\right]$
$\simeq \left(1.8 \pm 0.2\right) \times 10^6 \;=\; 1.8\left(2\right) \times 10^6$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2\, u_i^2 \right)^{1/2}; \quad f[x] \pm u_c \; // \; \text{φUCA}$$

$1.8 \times 10^6 \pm 0.0594418 \times 10^6$
$\in \left[1.74056 \times 10^6;\; 1.85944 \times 10^6\right]$
$\simeq \left(1.80 \pm 0.06\right) \times 10^6 \;=\; 1.80\left(6\right) \times 10^6$

Monte Carlo Simulation

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Block[ { data, trials = 10^6 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // φUCA
```

$1.80009595507303 \times 10^6 \pm 0.0594155 \times 10^6$
$\in \left[1.74068 \times 10^6;\; 1.85951 \times 10^6\right]$
$\simeq \left(1.80 \pm 0.06\right) \times 10^6 \;=\; 1.80\left(6\right) \times 10^6$