

## Problem 1.4 - Uncertainty Analysis

```
Get[ "UCAnalysis.m", Path -> {NotebookDirectory[]} ]
```

$$\frac{1}{4} \frac{(v/3.6)^2}{c} \mapsto \left( \begin{array}{cc} c & 0.13 \times 10^3 \pm 0.005 \times 10^3 \\ v & 1440 \pm 5 \end{array} \quad \begin{array}{l} \text{Uniform} \mathcal{D} \\ \text{Uniform} \mathcal{D} \end{array} \right)$$

### Evaluated Functional Relationship

```
ΦAnalysisEnvironment
```

$$y = \frac{0.0192901 x_2^2}{x_1}$$

Variable		Uncertainty Interval	Distribution	$ \partial f / \partial x_i $
$x_1$	c	$(1.30 \pm 0.05) \times 10^2$	Uniform	2.36686
$x_2$	v	$(1.440 \pm 0.005) \times 10^3$	Uniform	$4.2735 \times 10^{-1}$

y	307.692307692308
y <sub>min</sub>	294.242255372657 = y - 13.4501
y <sub>max</sub>	322.226080246914 = y + 14.5338
ε <sub>max</sub>	13.9710716633794 = 4.54 %
y ± ε <sub>max</sub>	$(3.1 \pm 0.2) \times 10^2 = 3.1(2) \times 10^2$
u <sub>c</sub>	6.94302595780411 = 2.26 %
y ± u <sub>c</sub>	$(3.08 \pm 0.07) \times 10^2 = 3.08(7) \times 10^2$

### Absolute Maximum Uncertainty

$$\varepsilon_{\max} = \sum_{i=1}^n |\partial_{x_i} f[\mathbf{x}]| \varepsilon_i; \quad f[\mathbf{x}] \pm \varepsilon_{\max} \quad // \quad \Phi UCE$$

$$\begin{aligned} & 307.692307692308 \pm 13.9711 \\ & \in [293.72; 321.66] \\ & \approx (3.1 \pm 0.2) \times 10^2 = 3.1(2) \times 10^2 \end{aligned}$$

### Combined Standard Uncertainty

$$u_c = \left( \sum_{i=1}^n (\partial_{x_i} f[\mathbf{x}])^2 u_i^2 \right)^{1/2}; \quad f[\mathbf{x}] \pm u_c \quad // \quad \Phi UCA$$

$$\begin{aligned} & 307.692307692308 \pm 6.94303 \\ & \in [300.749; 314.635] \\ & \approx (3.08 \pm 0.07) \times 10^2 = 3.08(7) \times 10^2 \end{aligned}$$

## Monte Carlo Simulation

```
Block[{ { data, trials = 106 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // ϕUCA
```

307.852502313854 ± 6.94724  
 ∈ [300.905; 314.8]  
 $\approx (3.08 \pm 0.07) \times 10^2 = 3.08(7) \times 10^2$