Problem 2.12 - Uncertainty Analysis, Case A2

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Get[ "UCAnalysis.m", Path → {NotebookDirectory[]} ]
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$$p_1 + \frac{\rho\, v_1^2}{2}\left(1 - \left(\frac{d_1}{d_2}\right)^4\right) \mapsto \begin{pmatrix} d_1 & 30 \pm 0.5 & \text{UniformD} \\ d_2 & 20 \pm 0.5 & \text{UniformD} \\ v_1 & 4.0 \pm 0.05 & \text{UniformD} \\ \rho & 998 \pm 0.5 & \text{UniformD} \\ p_1 & 0.10 \times 10^5 \pm 0.005 \times 10^5 & \text{UniformD} \end{pmatrix}$$

Evaluated Functional Relationship

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?AnalysisEnvironment
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$$y = \frac{1}{2}\left(1 - \frac{x_1^4}{x_2^4}\right) x_3^2\, x_4 + x_5$$

Variable		Uncertainty Interval	Distribution	$\lvert \partial f/\partial x_i \rvert$
x_1	d_1	$(3.00 \pm 0.05) \times 10^1$	Uniform	5.3892×10^3
x_2	d_2	$(2.00 \pm 0.05) \times 10^1$	Uniform	8.0838×10^3
x_3	v_1	$(4.00 \pm 0.05) \times 10^0$	Uniform	1.62175×10^4
x_4	ρ	$(9.980 \pm 0.005) \times 10^2$	Uniform	3.25×10^1
x_5	p_1	$(1.00 \pm 0.05) \times 10^4$	Uniform	1.

y	$-22\,435$	
y_{min}	$-31\,321.5404485137$	$= y - 8886.54$
y_{max}	$-15\,087.7546840479$	$= y + 7347.25$
ε_{max}	8063.625	$= -35.9\,\%$
$y \pm \varepsilon_{max}$	$(-2.2 \pm 0.8) \times 10^4$	$= -2.2(8) \times 10^4$
u_c	2858.06502714249	$= -12.7\,\%$
$y \pm u_c$	$(-2.2 \pm 0.3) \times 10^4$	$= -2.2(3) \times 10^4$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} \lvert \partial_{x_i} f[x] \rvert\, \varepsilon_i; \quad f[x] \pm \varepsilon_{max} \;\; // \; ?UCE$$

$$-22\,435 \pm 8063.62$$
$$\in [-30\,499;\ -14\,371]$$
$$\simeq (-2.2 \pm 0.8) \times 10^4 = -2.2(8) \times 10^4$$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2\, u_i^2\right)^{1/2}; \quad f[x] \pm u_c \;\; // \; ?UCA$$

$$-22\,435 \pm 2858.07$$
$$\in [-25\,293;\ -19\,577]$$
$$\simeq (-2.2 \pm 0.3) \times 10^4 = -2.2(3) \times 10^4$$

Monte Carlo Simulation

```
Block[ { data, trials = 10^6 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // QUCA
```

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-22 547.8392638872 ± 2865.7
 ∈ [-25 414; -19 682]
≃ (-2.3 ± 0.3) × 10^4 = -2.3(3) × 10^4
```