SK180N Part 1C, Problem 2 - Uncertainty Analysis

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$$\frac{h\,c}{2\,\mathrm{Sin}\left[\theta\,\frac{\pi}{180}\right]\sqrt{K*eV\,(K*eV+2\,m\,c^2)}} \mapsto \begin{pmatrix} \theta & 0.65`2 & \text{"Deg"} \\ K & 0.50`2\times10^{6} & \text{"eV"} \\ h & \mathrm{CODATA}[h] & \\ c & \mathrm{CODATA}[c] & \\ m & \mathrm{CODATA}[me] & \\ eV & \mathrm{CODATA}[evj] & \end{pmatrix}$$

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ϕUCResult[ d, m ]
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$d = (6.264161\times10^{-11} \pm 0.0367563\times10^{-11})$ m

$\in [6.227404\times10^{-11};\ 6.300917\times10^{-11}]$ m; Normal$\mathcal{D}$

$\simeq (6.264 \pm 0.037)\times10^{-11}$ m $= 6.264(37)\times10^{-11}$ m

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ϕAnalysisEnvironment
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$$y = \frac{\mathrm{Csc}\left[\frac{\pi x_1}{180}\right] x_3\, x_4}{2\sqrt{x_2\, x_6\,(2\, x_4^2\, x_5 + x_2\, x_6)}}$$

Quantity		Estimate ± Uncertainty	Distribution	$\lvert \partial f/\partial x_i \rvert$
x_1	θ	$(6.50 \pm 0.05)\times10^{-1}$	Uniform	9.63676×10^{-11}
x_2	K	$(5.00 \pm 0.05)\times10^{5}$	Uniform	8.32204×10^{-17}
x_3	h	$(6.62606957 \pm 0.00000029)\times10^{-34}$	Normal	9.45381×10^{22}
x_4	c	299 792 458 (exact)	-	6.86433×10^{-20}
x_5	m	$(9.10938291 \pm 0.00000040)\times10^{-31}$	Normal	2.30877×10^{19}
x_6	eV	$(1.602176565 \pm 0.000000035)\times10^{-19}$	Normal	2.5971×10^{8}

y	$6.2641607320665320236158937586\times10^{-11}$	
y_{min}	$6.209279946994701\times10^{-11}$	$= y - 5.48808\times10^{-13}$
y_{max}	$6.353786549380712\times10^{-11}$	$= y + 8.96258\times10^{-13}$
ε_{max}	$8.9795332282691344831858254102\times10^{-13}$	$= 1.43\,\%$
$y \pm \varepsilon_{max}$	$(6.264 \pm 0.090)\times10^{-11}$	$= 6.264(90)\times10^{-11}$
u_c	$3.6756331879310589394474372256\times10^{-13}$	$= 0.587\,\%$
$y \pm u_c$	$(6.264 \pm 0.037)\times10^{-11}$	$= 6.264(37)\times10^{-11}$

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ϕMonteCarlo[ 10^6 ] // ϕUC
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$6.26436\times10^{-11} \pm 0.0367251\times10^{-11}$

$\in [6.227634\times10^{-11};\ 6.301085\times10^{-11}]$

$\simeq (6.264 \pm 0.037)\times10^{-11} = 6.264(37)\times10^{-11}$

Sanity Check

ϙDumpRelationship

$$d == \frac{\text{Csc}\left[\frac{\pi \theta}{180}\right] c\, h}{2 \sqrt{\text{eV}\, K \left(\text{eV}\, K + 2 c^2 m\right)}}$$

ϙDumpQuantities

$d = \left(6.264161 \times 10^{-11} \pm 0.0367563 \times 10^{-11}\right)$ m
$\in \left[6.227404 \times 10^{-11};\ 6.300917 \times 10^{-11}\right]$ m; Normal$\mathcal{D}$
$\simeq$ **$\left(6.264 \pm 0.037\right) \times 10^{-11}$** m = **$6.264\left(37\right) \times 10^{-11}$** m

$\theta = (0.65 \pm 0.005)$ Deg
$\in [0.645;\ 0.655]$ Deg; Uniform$\mathcal{D}$
$\simeq$ **$\left(6.50 \pm 0.05\right) \times 10^{-1}$** Deg = **$6.50(5) \times 10^{-1}$** Deg

$K = (500\,000 \pm 5000)$ eV
$\in [495\,000;\ 505\,000]$ eV; Uniform$\mathcal{D}$
$\simeq$ **$\left(5.00 \pm 0.05\right) \times 10^{5}$** eV = **$5.00(5) \times 10^{5}$** eV

$h = \left(6.62606957 \times 10^{-34} \pm 2.9 \times 10^{-41}\right)$ J s
$\in \left[6.62606928 \times 10^{-34};\ 6.62606986 \times 10^{-34}\right]$ J s; Normal$\mathcal{D}$
$\simeq$ **$\left(6.62606957 \pm 0.00000029\right) \times 10^{-34}$** J s = **$6.62606957\left(29\right) \times 10^{-34}$** J s
▷ Planck constant; h [J s]

$c = 299\,792\,458$ (exact) $\frac{m}{s}$
▷ speed of light in vacuum; c, c_0 $\left[\frac{m}{s}\right]$

$m = \left(9.10938291 \times 10^{-31} \pm 4 \times 10^{-38}\right)$ kg
$\in \left[9.10938251 \times 10^{-31};\ 9.10938331 \times 10^{-31}\right]$ kg; Normal$\mathcal{D}$
$\simeq$ **$\left(9.10938291 \pm 0.00000040\right) \times 10^{-31}$** kg = **$9.10938291\left(40\right) \times 10^{-31}$** kg
▷ electron mass; m_e [kg]

$\text{eV} = \left(1.602176565 \times 10^{-19} \pm 3.5 \times 10^{-27}\right)$ J
$\in \left[1.60217653 \times 10^{-19};\ 1.6021766 \times 10^{-19}\right]$ J; Normal$\mathcal{D}$
$\simeq$ **$\left(1.602176565 \pm 0.000000035\right) \times 10^{-19}$** J = **$1.602176565\left(35\right) \times 10^{-19}$** J
▷ electron volt – joule relationship; 1 eV [J]