Problem 1.2 - Uncertainty Analysis (Case B)

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Get[ "UCAnalysis.m", Path → {NotebookDirectory[]} ]
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$$\frac{\ell_v}{c_p}\ \frac{\Delta m}{\rho\, V} \quad \mapsto \quad \begin{pmatrix} \ell_v & 2.260 \times 10^3 \pm 0.005 \times 10^3 & \text{UniformD} \\ c_p & 4.19 \pm 0.005 & \text{UniformD} \\ \Delta m & -500 \times 10^{-6} \pm 0.5 \times 10^{-6} & \text{UniformD} \\ \rho & 998 \pm 0.5 & \text{UniformD} \\ V & 200 \times 10^{-6} \pm 0.5 \times 10^{-6} & \text{UniformD} \end{pmatrix}$$

Evaluated Functional Relationship

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?AnalysisEnvironment
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$$y = \frac{x_1\, x_3}{x_2\, x_4\, x_5}$$

Variable		Uncertainty Interval	Distribution	$\lvert \partial f/\partial x_i \rvert$
x_1	ℓ_v	$(2.260 \pm 0.005) \times 10^3$	Uniform	5.97854×10^{-4}
x_2	c_p	$(4.190 \pm 0.005) \times 10^0$	Uniform	3.2247×10^{-1}
x_3	Δm	$(-5.000 \pm 0.005) \times 10^{-4}$	Uniform	2.7023×10^3
x_4	ρ	$(9.980 \pm 0.005) \times 10^2$	Uniform	1.35386×10^{-3}
x_5	V	$(2.000 \pm 0.005) \times 10^{-4}$	Uniform	6.75575×10^3

y	-1.35115098932949	
y_{min}	-1.36119707804894	$= y - 0.0100461$
y_{max}	-1.34118172613204	$= y + 0.00996926$
ε_{max}	0.0100075819322175	$= -0.741\ \%$
$y \pm \varepsilon_{max}$	$(-1.35 \pm 0.01) \times 10^0$	$= -1.35(1)$
u_c	0.00289996122289927	$= -0.215\ \%$
$y \pm u_c$	$(-1.351 \pm 0.003) \times 10^0$	$= -1.351(3)$

Absolute Maximum Uncertainty

$$\varepsilon_{max} = \sum_{i=1}^{n} \lvert \partial_{x_i} f[x] \rvert\, \varepsilon_i ; \quad f[x] \pm \varepsilon_{max} \ \text{// ?UCE}$$

$$-1.35115098932949 \pm 0.0100076$$
$$\in [-1.36116;\ -1.34114]$$
$$\simeq (-1.35 \pm 0.01) \times 10^0 = -1.35(1)$$

Combined Standard Uncertainty

$$u_c = \left(\sum_{i=1}^{n} (\partial_{x_i} f[x])^2\, u_i^2 \right)^{1/2} ; \quad f[x] \pm u_c \ \text{// ?UCA}$$

$$-1.35115098932949 \pm 0.00289996$$
$$\in [-1.354051;\ -1.348251]$$
$$\simeq (-1.351 \pm 0.003) \times 10^0 = -1.351(3)$$

Monte Carlo Simulation

```
Block[ { data, trials = 10^6 },
  data = f @@ Table[RandomReal[fDist[i], {trials}], {i, 1, n}];
  Mean[data] ± StandardDeviation[data] ] // QUCA
```

```
-1.3511522605762 ± 0.00290145
 ∈ [-1.354054; -1.348251]
 ≃ (-1.351 ± 0.003) × 10^0 = -1.351(3)
```