Thermal Physics : tutorial tips

2020.02.26

- Things to remember / know how to do:

  1. equipartition theorem.
  2. change variables in integrals.
  3. 1^{st}, 2^{nd} moments $\langle v_x \rangle$, $\langle v_x^2 \rangle$.
  4. how to find the probability from a prob. density.
  5. how to choose the domain of the different integrals?

1. Equipartition theorem.

In thermal equilibrium, the energy of the particles is shared in average between the different forms.

For a MONOATOMIC gas in N-dimension:

$$\langle \varepsilon \rangle = \underbrace{d.o.f.}_{\text{degrees of freedom} = N} \cdot \frac{1}{2} K_B T$$

where $\langle \varepsilon \rangle$ is the average energy per particle.

2. Change variables in integrals.

The integrals are mainly of the form:

$$\int_D \alpha \exp[-\underbrace{\beta x}_{\text{new variable}}] \, dx$$

Just practice with the exercises we have done.

3. What are normalisation, expected value (average), variance?

Given a function $f(x)$, we can define its moments:

$$\mu_n = \int_D x^n f(x) \, dx$$

the integral of the function over the entire domain MULTIPLIED by the variable n-th times!

If $f(x)$ is our PDF, we have:

0^{th} moment: NORMALISATION CONDITION

$$\int_D f(x)\,dx = 1$$

1^{st} moment: EXPECTED VALUE (average)

$$\langle x \rangle = \int_D x\, f(x)\,dx$$

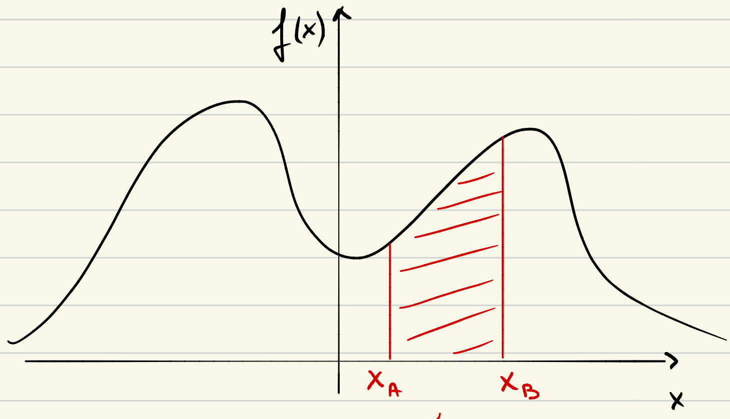
2^{nd} moment: VARIANCE

$$\langle x^2 \rangle = \int_D x^2 f(x)\,dx$$

the RMS is $\sqrt{\langle x^2 \rangle}$

4. **Probability from PDF :**

- $f(x)$ is the Probability Density Function
- To find the PROBABILITY, we INTEGRATE over the domain we are interested in.



$$P(x_A \le x < x_B) = \int_{x_A}^{x_B} f(x)\,dx$$

This is the probability of finding x between x_A and x_B.

5. How to choose the domain of different integrals?

- When we are in 1D, it is likely to be $\int_{-\infty}^{+\infty}$ but in dimensions > 1 it is mostly $\int_0^{+\infty}$. Always <u>CHECK</u>.
- The domain usually change when you change variables.

e.g.

$$\underbrace{P(v_x)\, P(v_y)\, P(v_z)}_{P(v)}\; \underbrace{dv_x\, dv_y\, dv_z}_{4\pi v^2\, dv}$$

Assuming a Gaussian distribution $p(v_i) \propto \exp[-a v_i^2]$, we have

$$p(v) = p(v_x)\, p(v_y)\, p(v_z) \propto e^{-a v_x^2}\, e^{-a v_y^2}\, e^{-a v_z^2} = e^{-a v^2}$$

where $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

v goes from 0 to ∞ because it is a radius therefore a <u>MODULUS</u>

thus,

$$\int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{-\infty}^{+\infty} dv_z\; P(v_x)\, P(v_y)\, P(v_z)$$

becomes

$$\int_0^{+\infty} dv\; 4\pi v^2\, P(v)$$

(we already integrated over $d\theta$ and $d\phi$)