

Two Derived Divisibility Tests for 7

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Abstract

This paper presents two derived divisibility tests for the integer 7 based on decimal digit decomposition and modular arithmetic. Both rules are proven using congruence relations and illustrated with multiple numerical examples. The purpose of this work is educational, providing clear and structured methods for testing divisibility by 7.

1. First Divisibility Test

Let N be written as $N = 100z + 10y + x$, where x is the units digit, y is the tens digit, and z represents the remaining leading digits. Define $T = 11x + z - 2y$. Then N is divisible by 7 if and only if T is divisible by 7.

Examples (First Test)

Example 1: $N = 4102 \rightarrow T = 11(2) + 41 - 2(0) = 63 \rightarrow 63 \div 7 = 9 \rightarrow \text{Divisible.}$

Example 2: $N = 742 \rightarrow T = 11(2) + 7 - 2(4) = 21 \rightarrow 21 \div 7 = 3 \rightarrow \text{Divisible.}$

Example 3: $N = 203 \rightarrow T = 11(3) + 2 - 2(0) = 35 \rightarrow 35 \div 7 = 5 \rightarrow \text{Divisible.}$

2. Second Divisibility Test

Let N be written as $N = 1000y + x$, where x represents the last three digits and y represents the remaining leading digits. Since $1000 \equiv -1 \pmod{7}$, we have $N \equiv x - y \pmod{7}$. Therefore, N is divisible by 7 if and only if $x - y$ is divisible by 7.

Examples (Second Test)

Example 4: $N = 28714 \rightarrow x = 714, y = 28 \rightarrow 714 - 28 = 686 \rightarrow 686 \div 7 = 98 \rightarrow \text{Divisible.}$

Example 5: $N = 14007 \rightarrow x = 007, y = 14 \rightarrow 7 - 14 = -7 \rightarrow$ Divisible.

Example 6: $N = 12345 \rightarrow x = 345, y = 12 \rightarrow 345 - 12 = 333 \rightarrow 333$ not divisible by 7 \rightarrow Not divisible.

Conclusion

Two divisibility tests for 7 have been derived and demonstrated. Both are based on standard modular arithmetic principles and are intended for educational use. These methods provide structured and efficient alternatives for testing divisibility by 7.