Bayes' Burrito

(a) How many different types of burrito bowls are there

For each type of rice, it can be paired with 4 types of beans; In terms of the topping combination, the customer would be able to choose any subset of the 7 topping options. Therefore the total number of choices for toppings would be: 2^7. Combine it together

$$\begin{aligned}\text{Total} &= 2 \times 4 \times 2^7 \\ &= 8 \times 128 \\ &= 1024\end{aligned}$$

Therefore, there are 1024 different types of burrito bowls possible.

(b) Eshan's burrito bowl

The factorization goes as follows:

$$P_{Eshan}(R, B, T) = P(R) \times P(B) \times P(T \mid R, B)$$

- $P(R)$: need 1 independent value (2 possibilities, knowing 1 would imply the other, since the possibilities must sum up to 1)
- $P(B)$: need 3 independent values (similar reason as above)
- $P(T \mid R, B)$: firstly there are 2 options for R, 4 options for B, and 2^7 options for T, therefore for each combination of R, B, need $(2^7 - 1)$ independent values; lastly calculate the conditional probability for $P(T \mid R, B) = 2 \times 4 \times (2^7 - 1) = 1016$

Putting them tougher, the total number of independent values needed to know is $1 + 3 + 1016 = 1020$. Thus, 1020 values must be known to fully compute $P_{Eshan}(R, B, T)$.

(c) Zafeer's burrito bowl

The factorization goes as follows:

$$P_{Zafeer}(R, B, T) = P(R) \times P(B \mid R) \times P(T)$$

- $P(R)$: need 1 independent value (2 possibilities, knowing 1 would imply the other, since the possibilities must sum up to 1)
- $P(B)$: for each types of rice, have 4 options, but 3 needs calculation (they sum up to 1), therefore the independent values for this conditional probability is $2 \times 3 = 6$
- $P(T \mid R, B)$: need $2^7 - 1 = 127$ independent values (same reason as above)

Putting them tougher, the total number of independent values needed to know is $1 + 6 + 127 = 134$. Thus, 134 values must be known to fully compute $P_{Zafeer}(R, B, T)$.

Bayesian Network

(a) Variables independent of I

Variables A, B, E, D, F are independent of I

- A, B, E because they are not ancestors or descendants of I, and no path from I to them
- D, F because it is not connected to I and does not share common descendants with I

(b) Variables that are conditionally independent of A given K

Given K, A is conditionally independent of J, I, D, F

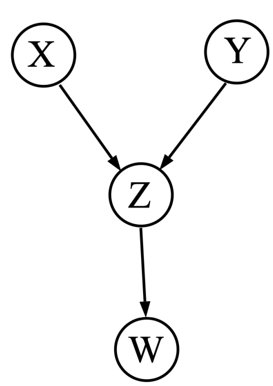
- J because conditioning on K block the path from A to J through K, conditioning on K makes J d-separated from A
- I, D, F because there are no path connecting A to D or F, regardless K is conditioned or not

(c) Variables that are conditionally independent of B given E

Given E, B is conditionally independent of D, I, F

- D because it is a non descendant of B and does not have any influence that has not already being accounted for by E
- I, F because there are no path connecting B to I, F, regardless E is conditioned or not

Drawing a Bayesian Network



Z as the common effect of X, Y, satisfying the condition that X, Y are conditionally independent given Z. When X, Y are both known, Z, W becomes conditionally independent, because X, Y already contains all the information for Z, along with any information that Z will provide for W.

Amazing Race Problem

(a) Calculate all M, N for $P(F = yes \mid N, M)$

First, consider that there are 2 choices for $M : \{train, car\}$ and 3 choices for $N : \{0, 1, 2\}$. Therefore there are 6 possible combinations for making the flight, below it will be calculated separately.

0.1 $M = train, N = 0$

$$\begin{aligned}
P(yes \mid 0, train) = \quad & P(yes \mid early, fast) \cdot P(early \mid train, 0) \cdot P(fast \mid 0) \\
& + P(yes \mid early, slow) \cdot P(early \mid train, 0) \cdot P(slow \mid 0) \\
& + P(yes \mid late, fast) \cdot P(late \mid train, 0) \cdot P(fast \mid 0) \\
& + P(yes \mid late, slow) \cdot P(late \mid train, 0) \cdot P(fast \mid 0) \\
= \quad & (0.95 \cdot 0.9 \cdot 0.9) \quad + \quad (0.7 \cdot 0.9 \cdot [1 - 0.9]) \quad + \quad (0.7 \cdot [1 - 0.9] \cdot 0.9) \\
& + (0.35 \cdot [1 - 0.9] \cdot [1 - 0.9]) \\
= \quad & 0.7695 \quad + \quad 0.063 \quad + \quad 0.063 \quad + \quad 0.0035 \\
= \quad & 0.899
\end{aligned}$$

0.2 $M = train, N = 1$

$$\begin{aligned}
P(yes \mid 1, train) = \quad & P(yes \mid early, fast) \cdot P(early \mid train, 1) \cdot P(fast \mid 1) \\
& + P(yes \mid early, slow) \cdot P(early \mid train, 1) \cdot P(slow \mid 1) \\
& + P(yes \mid late, fast) \cdot P(late \mid train, 1) \cdot P(fast \mid 1) \\
& + P(yes \mid late, slow) \cdot P(late \mid train, 1) \cdot P(fast \mid 1) \\
= \quad & (0.95 \cdot 0.85 \cdot 0.8) \quad + \quad (0.7 \cdot 0.85 \cdot [1 - 0.8]) \quad + \quad (0.7 \cdot [1 - 0.85] \cdot 0.8) \\
& + (0.35 \cdot [1 - 0.85] \cdot [1 - 0.8]) \\
= \quad & 0.646 \quad + \quad 0.119 \quad + \quad 0.084 \quad + \quad 0.0105 \\
= \quad & 0.8595
\end{aligned}$$

0.3 $M = train, N = 2$

$$\begin{aligned}
P(yes \mid 2, train) = \quad & P(yes \mid early, fast) \cdot P(early \mid train, 2) \cdot P(fast \mid 2) \\
& + P(yes \mid early, slow) \cdot P(early \mid train, 2) \cdot P(slow \mid 2) \\
& + P(yes \mid late, fast) \cdot P(late \mid train, 2) \cdot P(fast \mid 2) \\
& + P(yes \mid late, slow) \cdot P(late \mid train, 2) \cdot P(fast \mid 2) \\
= \quad & (0.95 \cdot 0.6 \cdot 0.7) \quad + \quad (0.7 \cdot 0.6 \cdot [1 - 0.7]) \quad + \quad (0.7 \cdot [1 - 0.6] \cdot 0.7) \\
& + (0.35 \cdot [1 - 0.6] \cdot [1 - 0.7]) \\
= \quad & 0.399 \quad + \quad 0.126 \quad + \quad 0.196 \quad + \quad 0.042 \\
= \quad & 0.763
\end{aligned}$$

0.4 $M = car, N = 0$

$$\begin{aligned}
P(yes \mid 0, car) = \quad & P(yes \mid early, fast) \cdot P(early \mid car, 0) \cdot P(fast \mid 0) \\
& + P(yes \mid early, slow) \cdot P(early \mid car, 0) \cdot P(slow \mid 0) \\
& + P(yes \mid late, fast) \cdot P(late \mid car, 0) \cdot P(fast \mid 0) \\
& + P(yes \mid late, slow) \cdot P(late \mid car, 0) \cdot P(fast \mid 0) \\
= \quad & (0.95 \cdot 0.75 \cdot 0.9) \quad + \quad (0.7 \cdot 0.75 \cdot [1 - 0.9]) \quad + \quad (0.7 \cdot [1 - 0.75] \cdot 0.9) \\
& + (0.35 \cdot [1 - 0.75] \cdot [1 - 0.9]) \\
= \quad & 0.6375 \quad + \quad 0.0525 \quad + \quad 0.1575 \quad + \quad 0.00875 \\
= \quad & 0.85625
\end{aligned}$$

0.5 $M = car, N = 1$

$$\begin{aligned}
P(yes \mid 1, car) = \quad & P(yes \mid early, fast) \cdot P(early \mid car, 1) \cdot P(fast \mid 1) \\
& + P(yes \mid early, slow) \cdot P(early \mid car, 1) \cdot P(slow \mid 1) \\
& + P(yes \mid late, fast) \cdot P(late \mid car, 1) \cdot P(fast \mid 1) \\
& + P(yes \mid late, slow) \cdot P(late \mid car, 1) \cdot P(fast \mid 1) \\
= \quad & (0.95 \cdot 0.75 \cdot 0.8) \quad + \quad (0.7 \cdot 0.75 \cdot [1 - 0.8]) \quad + \quad (0.7 \cdot [1 - 0.75] \cdot 0.8) \\
& + (0.35 \cdot [1 - 0.75] \cdot [1 - 0.8]) \\
= \quad & 0.57 \quad + \quad 0.105 \quad + \quad 0.14 \quad + \quad 0.0175 \\
= \quad & 0.8325
\end{aligned}$$

0.6 $M = car, N = 2$

$$\begin{aligned}
P(yes \mid 2, car) = \quad & P(yes \mid early, fast) \cdot P(early \mid car, 2) \cdot P(fast \mid 2) \\
& + P(yes \mid early, slow) \cdot P(early \mid car, 2) \cdot P(slow \mid 2) \\
& + P(yes \mid late, fast) \cdot P(late \mid car, 2) \cdot P(fast \mid 2) \\
& + P(yes \mid late, slow) \cdot P(late \mid car, 2) \cdot P(fast \mid 2) \\
= \quad & (0.95 \cdot 0.75 \cdot 0.7) \quad + \quad (0.7 \cdot 0.75 \cdot [1 - 0.7]) \quad + \quad (0.7 \cdot [1 - 0.75] \cdot 0.7) \\
& + (0.35 \cdot [1 - 0.75] \cdot [1 - 0.7]) \\
= \quad & 0.49875 \quad + \quad 0.1575 \quad + \quad 0.1225 \quad + \quad 0.02625 \\
= \quad & 0.805
\end{aligned}$$

(b) Maximize chance to making the flight

Based on all 6 possibilities calculated above, it seems that choosing to go to the airport by **train** and bring **0 bags** will produce the maximized change of making the flight: 89.9%.