Hints for Homework Assignment 1

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1 Exercise 1

In this exercise one has to answer questions concerning the Bayesian network given in Figure 1. This network defines the independencies and dependencies for the global probability function $P(C, D, I, G, S)$. For instance the global probability

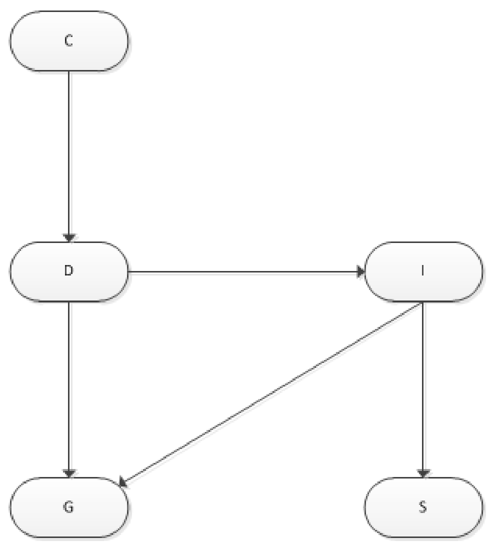

Figure 1: Bayesian network of exercise 1.

$P(C = c_0, D = d_0, I = i_0, G = g_0, S = s_0)$ can be factorized as:

$$P(C = c_0, D = d_0, I = i_0, G = g_0, S = s_0) =$$
$$P(G = g_0|D = d_0, I = i_0)P(S = s_0|I = i_0)P(I = i_0|D = d_0)P(D = d_0|C = c_0)P(C = c_0)$$

Now the right hand side can be computed using the CPDs given in the assignment, the answer is up to you!

1.1 Part a

In order to compute $P(G)$ one has to compute the global probability function as above and marginalize over all variables except G.

$$\begin{aligned}
P(G = g0) &= \sum_{c,d,i,s} P(C = c, D = d, I = i, G = g_0, S = s) \\
&= \sum_{c,d,i,s} P(G = g_0|D = d, I = i)P(S = s|I = i)P(I = i|D = d)P(D = d|C = c)P(C = c) \\
&= \sum_{d,i}[P(G = g_0|D = d, I = i)\sum_{s}[P(S = s|I = i)]P(I = i|D = d)\sum_{c}[P(D = d|C = c)P(C = c)]]
\end{aligned}$$

Observe that in the above formula $\sum_s[P(S = s|I = i)] = 1$ for all i because it is a probability distribution. Moreover

$$\sum_{c}[P(D = d|C = c)P(C = c)]] = (0.52, 0.48)$$

which is a factor multiplication and summation and it defines a probability distribution over $D = (d_0, d_1)$, lets call it $\phi_1(D)$. Hence left to calculate

$$\sum_{d,i}[P(G = g_0|D = d, I = i)P(I = i|D = d)\phi_1(D = d)$$

Now $P(I = i|D = d)\phi_1(D = d)$ is a factor multiplication for which you need to determine the result! Lets call this factor $\phi_2(D, I)$. Then left to compute

$$\sum_{d,i}[P(G = g_0|D = d, I = i)\phi_2(d, i) = ????$$

In a similar way $P(G = g_1)$ and $P(G = g_2)$ can be computed, only the above calculation needs to be adapted, observe that one does not need to recalculate $\phi_2(D, G)$. The resulting probability distribution is ????

1.2 Part b

In order to compute the probability distribution $P(S|C = c_0)$ on can compute the unnormalized distribution $P(S, C = c_0)$ using the same procedure as in Part a but not no marginalization (summing) over C but setting the value equal to c_0. After computing $P(S, C = c_0)$ one just normalizes the distribution to get $P(S|C = c_0)$. Check this!

1.3 Part c

Same procedure as in Part b.

2 Exercise 2

Same procedure as in Exercise 1 but do not forget to include the factor corresponding to (S, I). Does **not** sum up to 1.

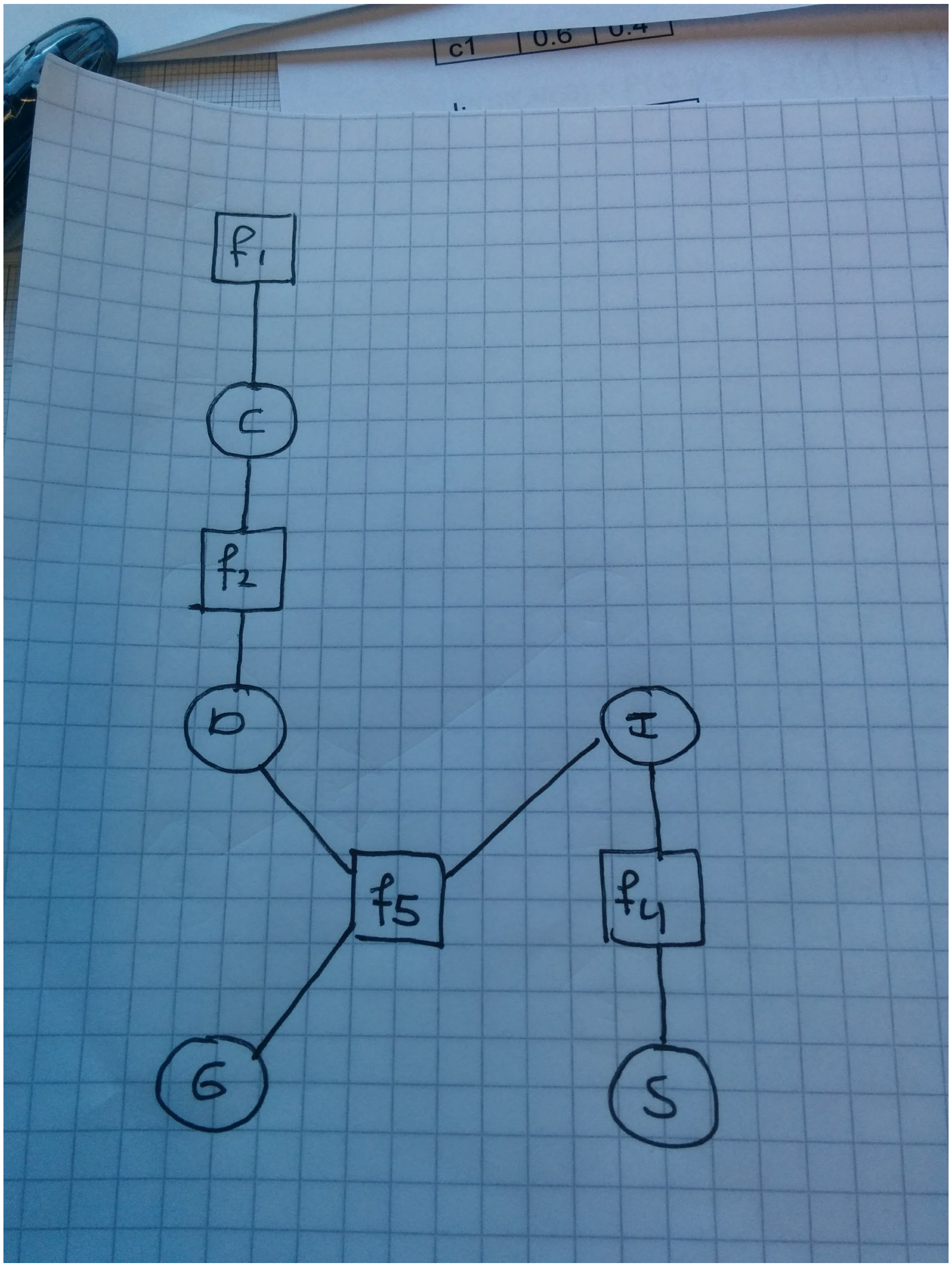

Figure 2: Factor graph for Exercise 3

3 Exercise 3

I will only explain Part a. Other parts are still left as an exercise. First we construct the factor graph. In order to compute $\phi(G)$ the node G is considered the root of the tree and messages are send upwards to G. Moreover we have to marginalize over all variables expect G. G receives a message from f_5 which is the requested marginalized factor. D and I sends a marginalized factors (messages) to f_5. D should receive a message from f_2, etcetera.
Message from f_1 to C is the factor $[3, 10]$, from S to f_4 is $[1, 1]$ (see book). The message sent by C to f_2 is just the factor $[3, 10]$ (variable nodes take the product of

all incoming messages, see pg. 406). Factor node f_2 takes the product of all incoming messages and the factor f_2 and afterwards marginalizes all variables associated with the incoming messages, in this case C, resulting in $[?, ?]$. This message is send to variable node D and D sends the same message to f_5. Factor node f_4 takes the product of $[1, 1]$ (incoming message) and the corresponding factor and marginalizes over S, resulting in $[?, ?]$. This message is send to I, which on its turn sends it to f_5. Now wrap it up yourself.