Phy 321

Q1) a) A conservtive force is dependent on 4 main ideas.

1) path independence: where work done by the force on an object moving between two points is indepedent of the path taken. it only depends on the final and intial position

2) potential energy: ~~the~~ force is equal to to the negative difference of the potential energy.

3) The sum of the forces must equal zero

b) $W = \Delta K = W = K_f - K_i$

$W = -\Delta U = W = -(U_f - U_i)$

so; $-U_f + U_i = K_f - K_i$

$K_f + U_f = K_i + U_i \Rightarrow E_f = E_i$

c) $P = \sum_{i=1}^{N} P_i = \sum_{i=1}^{N} m_i v_i$

newton 2nd law:

$$\frac{dP_i}{dt} = F_i^{net} \quad \text{where} \quad F_i^{net} = \sum_{j=1, j\neq i}^{N} F_{ji}$$

$$\frac{dP}{dt} = \sum_{i=1}^{N} \frac{dP_i}{dt} = \sum_{i=1}^{N} F_i^{net} \Rightarrow \frac{dP}{dt} = \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} F_{ji} \quad — (1)$$

where newton 3rd law say:

$$F_{ji} + F_{ij} = 0$$

so $(1) = 0$ thus: $\frac{dP}{dt} = 0$ which implies that:

$$P(t) = P_0$$

therefore the total linear momentum is conserved

from the lecture notes.

2)

a) we know L: angular moment is defined as:

- angular moment: $\vec{L} = \vec{r} \times \vec{P} = m(\vec{r} \times \vec{V})$

- taking the derative:

$$\frac{d\vec{L}_{sys}}{dt} = \frac{d}{dt}(m(\vec{r} \times \vec{V})) = m\underbrace{\frac{d\vec{r}}{dt} \times \vec{V}}_{0} + m\vec{r} \times \frac{d\vec{V}}{dt}$$

$$\frac{d\vec{L}_{sys}}{dt} = \vec{r} \times m\frac{d\vec{V}}{dt} = \boxed{\tau = \vec{r} \times \vec{F}}$$

b) $L_i = r_i \times p_i$, so angular is: $L = \sum_{i=1}^{N} l_i = \sum_{i=1}^{N} r_i \times p_i$

taking the time dervitive:

$\tau_i = r_i \times F_i^{net}$ ← the net ~~tour~~ tourqe of the whole system.

$$\boxed{\Rightarrow \tau = \sum_{i=1}^{N} \tau_i = \sum_{i=1}^{N} \vec{r}_i \times \vec{F}_{ext,i} + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} r_i \times F_{ji}}$$

- internal forces pair cancellation.

$$r_i \times F_{ji} + r_j \times F_{ij} = 0, \quad \sum_{i=1}^{N} \sum_{j=1, j\neq i} r_i \times F_{ji} = 0$$

so,

$$\boxed{\tau = \sum_{i=1}^{N} r_i \times F_{ext,i}, \quad \tau = \frac{dL}{dt}}$$

c) Two conditions need to satissed.

1) $F_{ij} = -F_{ji}$

2) $F_{ij} \propto (r_i - r_j)$

- This works cuz Newton 3rd law ensure that that the internal forces cancels out.

- centeral forces: making the internal forces act along a line connecting the interacting objects. that means:

$$\boxed{(r_i - r_j) \times F_{ij} = 0}$$

1 3) $V(x,y,z) = A\exp\left(-\frac{x^2+z^2}{2a^2}\right)$

from the book:

	x-comp	y comp	z comp
∇	$\delta/\delta x$	$\delta/\delta y$	$\delta/\delta z$
F	F_x	F_y	F_z

$\nabla\times F$ $\quad \delta/\delta_y F_z - \delta/\delta_z F_y \;\Big|\; \delta/\delta_z F_x - \delta/\delta_x F_z \;\Big|\; \delta/\delta_x F_y - \delta/\delta_y F_x$

just in case we need it.

a) $\nabla V = \left(\frac{\delta V}{\delta x}, \frac{\delta V}{\delta y}, \frac{\delta z}{\delta z}\right)$

so, the force is

$$F = \frac{-Ax}{a^2}\exp\left(\frac{-x^2+z^2}{2a^2}\right)\hat{x} + 0 - \frac{Az}{a^2}\exp\left(\frac{-x^2+z^2}{2a^2}\right)\hat{z}$$

$F = -\nabla V$ so,

$$F = -\left(-\frac{Ax}{a^2}\exp\left(\frac{-x^2+z^2}{2a^2}\right)\hat{x} - \frac{Az}{a^2}\exp\left(\frac{-x^2+z^2}{2a^2}\right)\hat{z}\right)$$

• Satisfying the first condition.

-> second condition curl F=0 or $\nabla\times F = 0$

we already desined our curl F

so

$\frac{\delta F_z}{\delta y} - \frac{\delta F_y}{\delta z} = 0$ since F_z don't depend on y and F_y is 0

$\frac{\delta F_x}{\delta z} - \frac{\delta F_z}{\delta x} = 0$ using wolfram alpha this would give zero now the ~~a~~ east one:

$\frac{\delta F_y}{\delta x} - \frac{\delta F_x}{\delta y} = 0$ it's obvious why.

so $\boxed{\text{curl } E(0,0,0)}$

since the force is derivable from the potential and the curl F. we have satisfied the conditions that the force is conservative

b) For this question and the next we are using the same concept.

• <u>Noether's theorem</u>: if the potential is independent of a particular coordinate, the corresponding component of the momentum along that coordinate is conserved.

Now, $V(x,y,z)$ is dependent on x and z

we can see that by the term $\left(-\frac{x^2+z^2}{2a^2}\right)$

z and x are not invariant under translation of their respected coordinates.

where $\underline{y}$ is invariant under translation.

• <u>conclusion</u>:

P_x is <u>not</u> conserved

P_y is <u>conserved</u>

P_z is <u>not</u> <u>conserved</u>

c) using the same approach rotational symmetries of the potential function

Noether's theorem of potential invariance

$L = (L_x, L_y, L_z)$, $L_x = y p_z - z p_y$

$L_y = z p_x - x p_z$

$L_z = x p_y - y p_x$

- now L_x is not conserved since it rotation would mix y and z
- L_y is ~~not~~ conserved due to it rotational symmetry about it axis
- L_z is not conserved for the same reason L_x is, but it rotation would mix x and y

So,

L_x: not conserved.

L_y: conserved.

L_z: not conserved.

Q 4)

a) $F = m \frac{\partial v}{\partial t}$

$F = F\hat{x}$ So, $F\hat{x} = m\frac{\partial v}{\partial t}$

$\Rightarrow \frac{F}{m}\hat{x}\,\partial t = \partial v$

$\int \frac{F}{m}\hat{x}\,\partial t = \int \partial v$ since $V(0) = 0$

we get $\int_0^t \frac{F}{m}\,\partial t = V(t) = \underline{\underline{\frac{F}{m}t\hat{x}}}$

$P(t) = mV(t) \Rightarrow m\,\frac{F}{m}t\hat{x}$

$\boxed{P(t) = Ft\hat{x}}$

b) $r(t) = \int_0^t V(t)\,\partial t \Rightarrow r(t) = \frac{Ft^2}{2m}\hat{x} + r_0$

Subsituting $\underline{\underline{r_0}}$ ~~(t)~~

final: $\boxed{r(t) = (x_0 + \frac{Ft^2}{2m})\hat{x} + y_0\,\hat{y}}$

c) angular momentum is defined as

$L(t) = r(t) \times p(t)$

we have both taking their cross product

so the result is:

$$\boxed{L(t) = b_0 F t \hat{z}}$$

note*
I used chatgpt to do the cross product.

tourqe is $\boxed{\frac{dL}{dt} = \tau(t)}$

so,

$$\boxed{\tau(t) = b_0 F \hat{z}}$$

• angular momentum is not conserved since

$$\boxed{\tau(t) \neq 0}$$