HW321

Q1)

a) $V = \alpha/x$ we want $F(x)$

we know $\frac{\partial v}{\partial t} = a$ but we want $\frac{\partial v}{\partial x} = a$
as a in terms x

so we can use the chain rule

$a = \frac{\partial v}{\partial t} = \frac{\partial v}{\partial x} \cdot \frac{\partial x}{\partial t} = v \frac{\partial v}{\partial x}$

now: $\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\alpha}{x}\right) = -\frac{\alpha}{x^2}$

$\therefore = v\frac{\partial v}{\partial x} = \frac{\alpha}{x} \cdot \left(\frac{-\alpha}{x^2}\right)$ where $F(x) = \frac{-m\alpha^2}{x^3}$

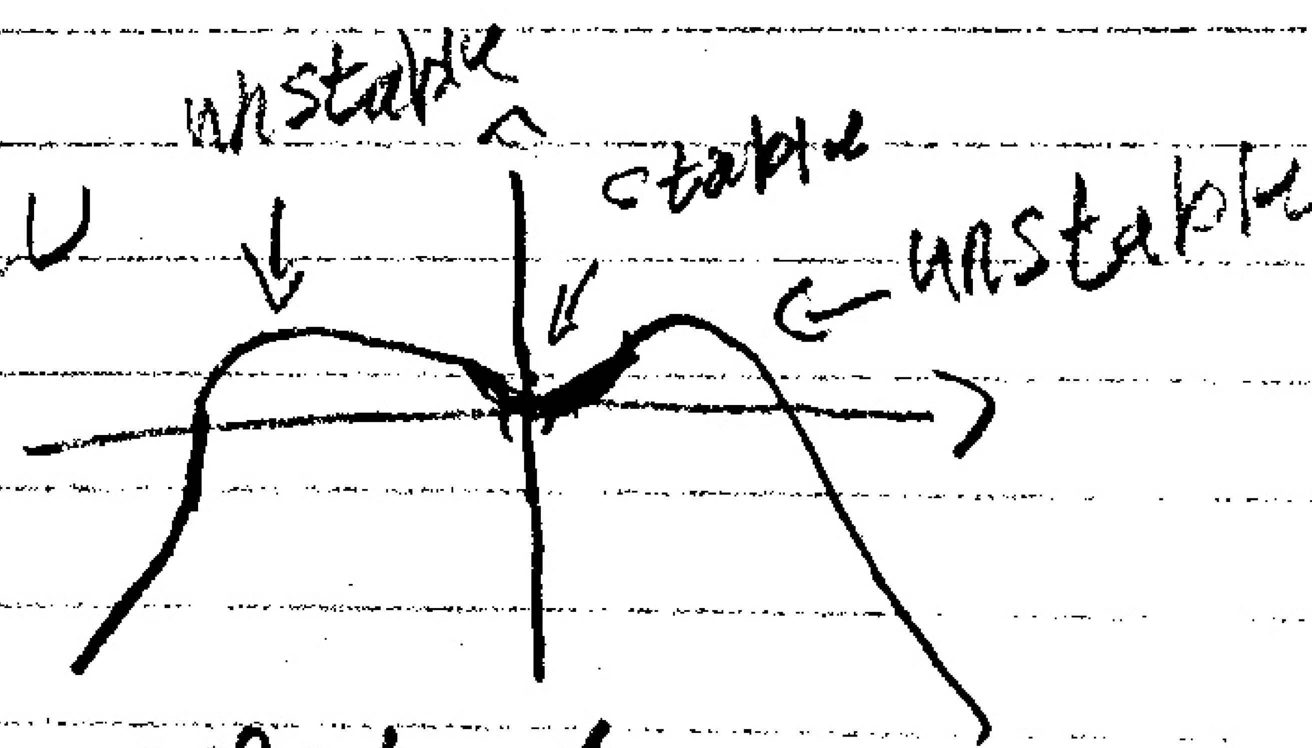
b) $F = -kx + kx^3/\alpha^2$

where $F = \frac{-\partial U}{\partial x}$

so $-kx + kx^3/\alpha^2 = \frac{-\partial U}{\partial x}$, take the negative sign out

$kx - kx^3/\alpha^2 = \frac{\partial U}{\partial x} \Rightarrow \int kx\,\partial x - \int k\frac{x^3}{\alpha^2}\partial x = \partial U$

$\Rightarrow k\int x\,\partial x - \frac{k}{\alpha^2}\int x^3\,\partial x = \partial U$



$\frac{kx^2}{2} + c_1 - \frac{kx^4}{4\alpha^2} + c_2 = U$

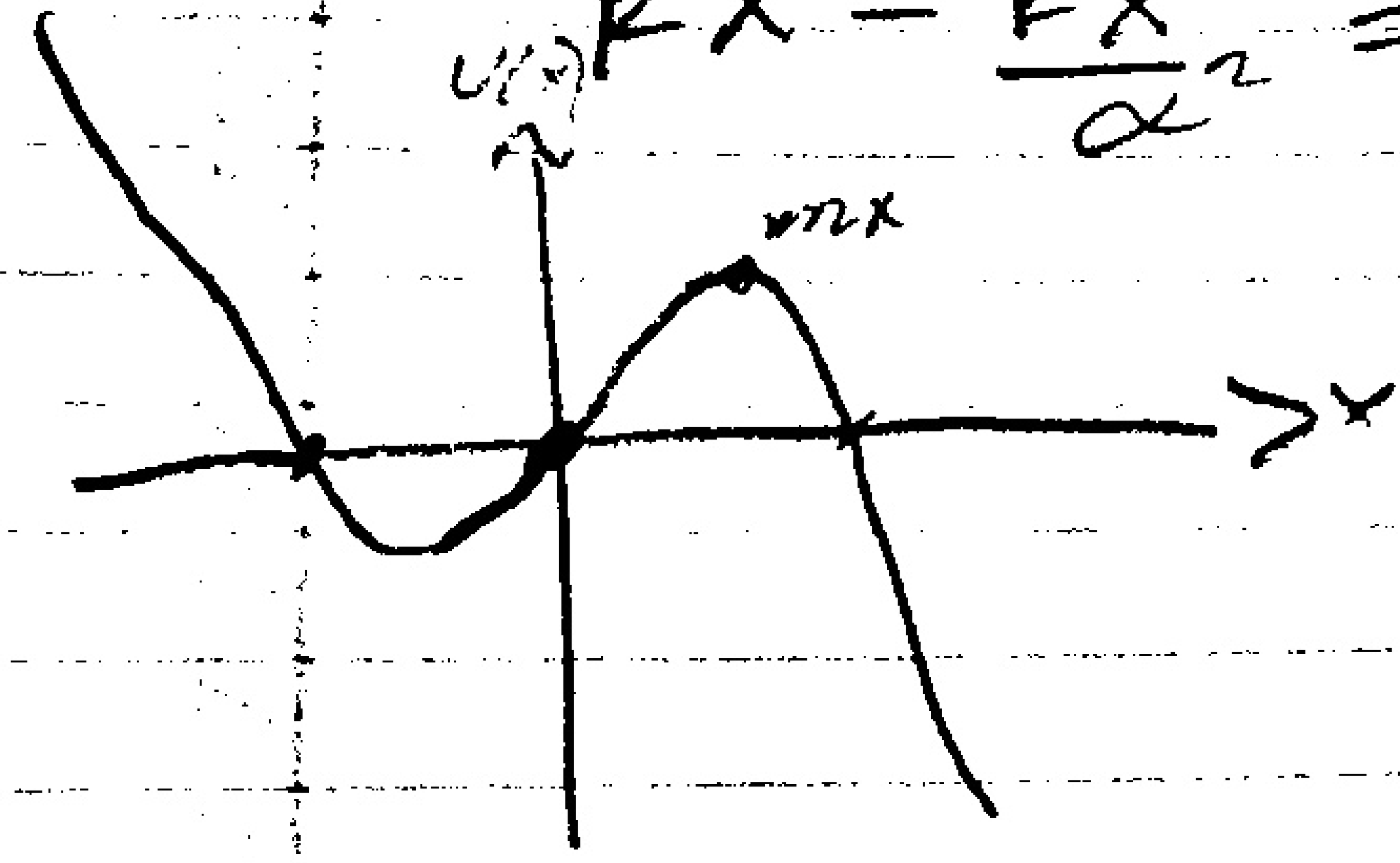
$\frac{kx^2}{2} - \frac{kx^4}{4\alpha^2} + c = U(x)$ where c = constant

so, $U(x) = \frac{kx^2}{2} - \frac{kx^4}{4\alpha^2}$

1) $E = (1/4)k\alpha^2$, $U(x) = \frac{kx^2}{2} - \frac{kx^4}{4\alpha^2}$

$\frac{\partial U}{\partial x} = kx - \frac{kx^3}{\alpha^2}$ solving the cubic equation we find

$kx - \frac{kx^3}{\alpha^2} = 0$ at, $x_1 = 1, x_2 = -1, x_3 = 0$



, so, $E = \frac{1}{4}k\alpha^2$ it has exactly enough energy to reach the top of the potential barier at $x = \pm 1$.

2) a) $K(x) + U(x) = E_{tot}$

$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$\cancel{\frac{1}{2}}mv^2 = \cancel{\frac{1}{2}}kA^2 - \cancel{\frac{1}{2}}kx^2 \Rightarrow \cancel{\frac{1}{2}}mv^2 = k(A^2 - x^2)$

$\Rightarrow v(x) = \sqrt{\frac{k(A^2 - x^2)}{m}}$ where $\sqrt{\frac{k}{m}} = w$

so $v(x) = \pm w\sqrt{(A^2 - x^2)}$

b) We know $V(x) = \sqrt{\frac{k(A^2-x^2)}{m}}$

Where $v(x) = \frac{dx}{dt}$, So, $\frac{dx}{dt} = \sqrt{\frac{k(A^2-x^2)}{m}}$

$dx = \sqrt{\frac{k(A^2-x^2)}{m}}\, dt \Rightarrow \frac{dx}{\sqrt{\frac{k(A^2-x^2)}{m}}} = dt$

So, $\int_0^x \frac{dx}{\sqrt{\frac{k(A^2-x^2)}{m}}} = \int_0^t dt$ ~~we know w=√~~

$\Rightarrow \sqrt{\frac{m}{k}} \int_0^x \frac{dx}{\sqrt{(A^2-x^2)}} = \int_0^t dt$

$\Rightarrow \sqrt{\frac{m}{k}} \int_0^x \frac{dx}{\sqrt{(A^2-x^2)}} = t$ using wolfram alpha

we get: $t = \sqrt{\frac{m}{k}} \sin^{-1}\left(\frac{x}{A}\right)$ or we can solve it by hand

let $x' = 0$ when, $\theta = 0$

$x' = x$ when $\theta = \sin^{-1}\left(\frac{x}{A}\right)$

let $x' = A\sin\theta$, $dx' = A\cos\theta\, d\theta$ where x' is some variable

our integral becomes: $t = \sqrt{\frac{m}{k}} \int_0^{\sin^{-1}\left(\frac{x}{A}\right)} \frac{A\cos\theta}{\sqrt{A^2 - A^2\sin^2\theta}}$

Solving it, $t = \sqrt{\frac{m}{k}} \int_0^{\sin^{-1}\left(\frac{x}{A}\right)} \frac{A\cos\theta}{A\cos\theta}\, d\theta = \sqrt{\frac{m}{k}} \int_0^{\sin^{-1}\left(\frac{x}{A}\right)} d\theta$

So, $t = \sqrt{\frac{m}{k}} \sin^{-1}\left(\frac{x}{A}\right)$, so $\boxed{x(t) = A\sin\left(\sqrt{\frac{k}{m}}\, t\right)}$

for T ...
from ...

$\sqrt{\frac{k}{m}}T = 2$ ...

$\sqrt{\frac{m}{k}} \cdot 2\pi$ ...

3)
a)

OB = O ...
BC = h

now AB ...

CA = ...

ΔCAE, ...

So H ...

H(θ) = ...

b) U = ...

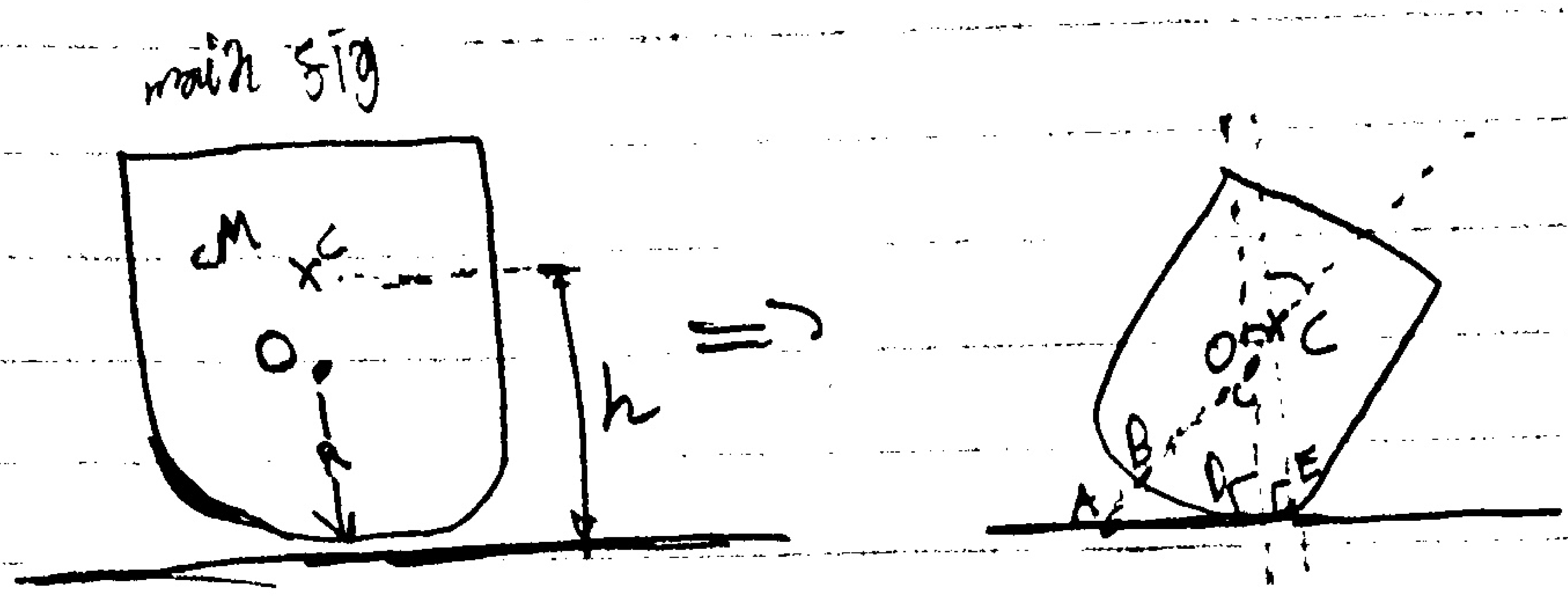
for T note the sine function goes from $0 \to 2\pi$ for a full cycle

$$\sqrt{\frac{k}{m}}\,T = 2\pi$$

$$\boxed{\sqrt{\frac{m}{k}}\cdot 2\pi = T}$$

3)

a)



$OB = OD = R$, from ΔOAD: $OA\cos\theta = OD = R$

$BC = h$ $\qquad OA = \frac{R}{\cos\theta}$

now $AB = OA - OB = \frac{R}{\cos\theta} - R = \boxed{R\left(\frac{1}{\cos\theta} - 1\right)}$

$CA = AB + BC = R\left(\frac{1}{\cos\theta} - 1\right) + h$

ΔCAE, $CA\cos\theta = CE \Rightarrow CE = \left(R\left(\frac{1}{\cos\theta} - 1\right) + h\right)\cos\theta$

so $H(\theta)$ the height is: $CE = H(\theta) = h\cos\theta + R(1-\cos\theta)$

$\boxed{H(\theta) = h\cos\theta + R(1-\cos\theta)}$, so $\boxed{U = mgH(\theta)}$ — D

b) $\boxed{U = mg[h\cos\theta + R(1-\cos\theta)]}$

b) continued, simplifying $V(\theta)$

we get $mg[R+(h-R)\cos\theta)$

at $\theta=0$ $V(0)=mgh$;

at $\theta=\pi$ $V(\pi)=mg(2R-h)$

so it oscillate between the two above

c) $\frac{\partial V}{\partial\theta}=mg[-h\sin\theta+R\sin\theta]$

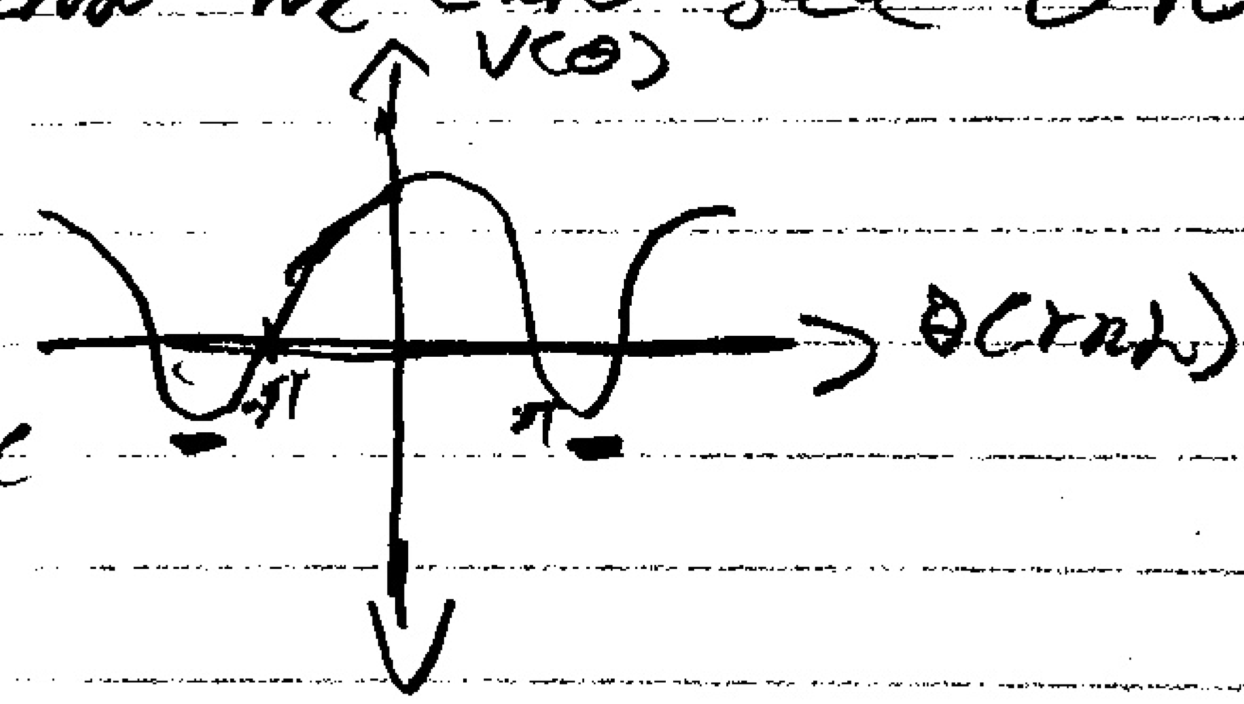
at $\theta=0$ $V'(\theta)=0$ so it stable at 0

$\frac{\partial^2 V}{\partial\theta^2}=mg\cos\theta[-h+R]$, at $\theta=0$

$mg[-h+R]$, now for stable equilibrium $\theta=0$

$\frac{\partial^2 V}{\partial\theta^2}>0$ which true if $R>h$

now it make sense, and we can see that from graph $U(\theta)$



so it actually oscillate, and it stable when it's concave up

a

4)

$\frac{dx}{dt}=\frac{d}{d}$

$\Rightarrow |v| =$

$=\frac{ds}{dt}$

thus

c) $F_{\parallel}=$

$a_{\parallel}=$

so,

d) $F_{\parallel}=$

thus

two above

e at 0

um θ=0

that

ocrah)

a) and b)

1)

$$|V| = \sqrt{\left(\frac{\partial x}{\partial t}\right)^2 + \left(\frac{\partial y}{\partial t}\right)^2}$$

$$\frac{\partial x}{dt} = \frac{\partial x}{\partial s}\frac{\partial s}{\partial t}, \quad \frac{\partial y}{\partial t} = \frac{\partial y}{\partial t}\frac{\partial s}{dt}$$

$$\Rightarrow |V| = \sqrt{\left(\frac{\partial x}{\partial t}\frac{\partial s}{\partial t}\right)^2 + \left(\frac{\partial y}{\partial t}\frac{\partial s}{\partial t}\right)^2}$$

$$= \frac{\partial s}{\partial t}\sqrt{\left(\frac{\partial x}{\partial s}\right)^2 + \left(\frac{\partial y}{\partial s}\right)^2}, \quad \text{so, } \partial s = \sqrt{\left(\frac{\partial x}{\partial s}\right)^2 + \left(\frac{\partial y}{\partial s}\right)^2}$$

thus $V = \frac{\partial s}{\partial t}$

c) $F_{\parallel} = m\frac{\partial^2 s}{\partial t^2}$

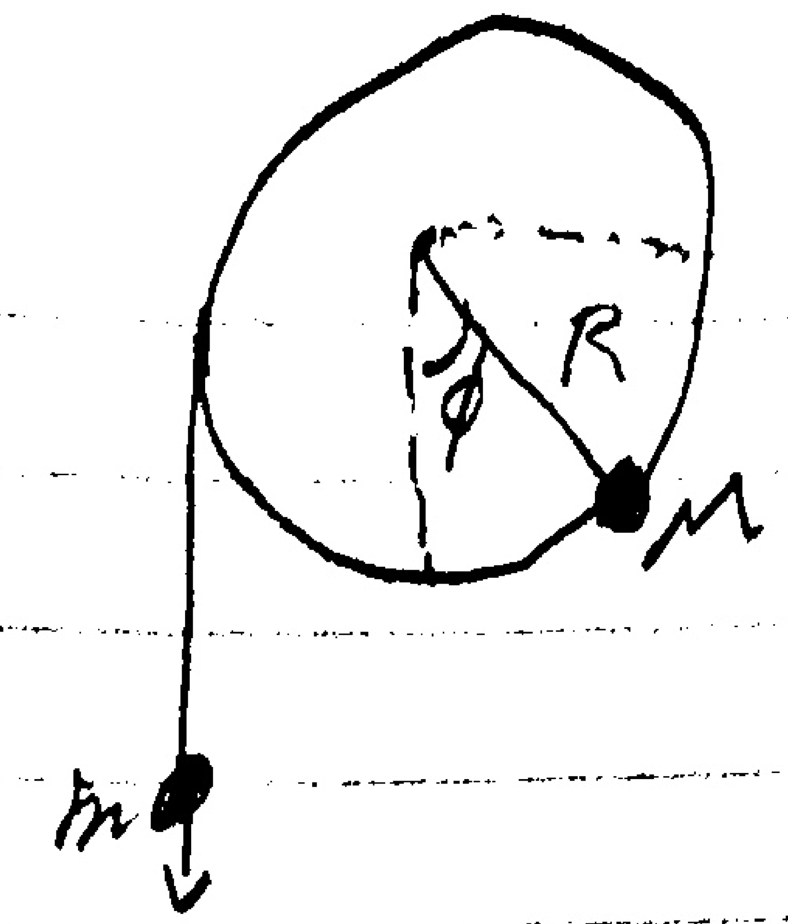
$$a_{\parallel} = \frac{\partial v}{\partial t} = \frac{\partial^2 s}{\partial t^2}, \quad F_{\parallel} = m a_{\parallel}$$

so, $F_{\parallel} = m\frac{\partial^2 s}{\partial t^2}$

d) $F_{\parallel} = m\frac{\partial^2 s}{\partial t^2} = -\frac{\partial U}{\partial s}$

thus, $F_{\parallel} = -\frac{\partial U}{\partial s}$

5)



a)

$U_m = \cancel{mgh}$

$U_M = mgh_M = Mg[R(1-\cos\phi)]$

$U_m = -mgh_m = -mgR\phi$

$\boxed{U(\phi)_{tot} = U_M + U_m} = Mg[R(1-\cos\phi)] - mgR\phi$

b) $\frac{\partial U}{\partial \theta} = MgR\sin\phi - mgR = 0$

$= M\sin\phi = m$ where $\frac{m}{M}$ is $-1 \leq \frac{m}{M} \leq 1$

$\sin\phi = \frac{m}{M}$

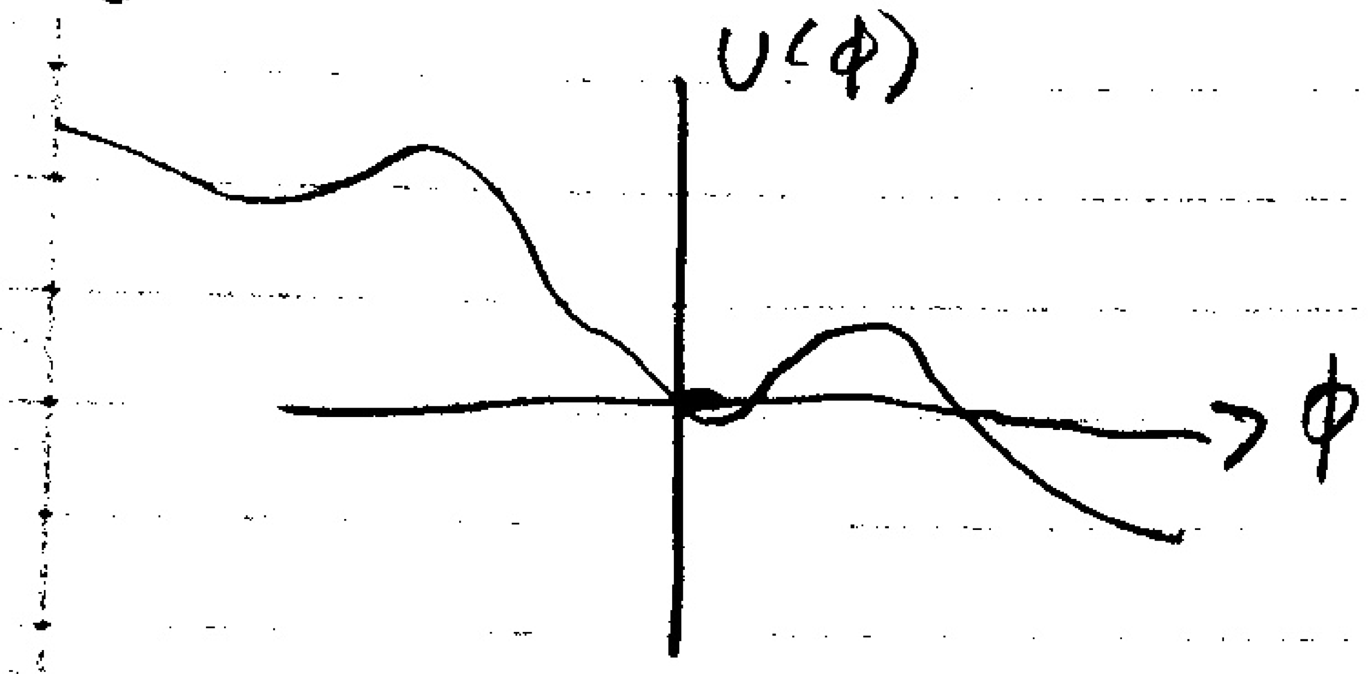
if $\frac{m}{M}$ holds between -1 and 1 then there exist a critical point

thus, it's $=0$ at $\boxed{\pi/2 + 2\pi n}$ where n is ~~some~~ any integer

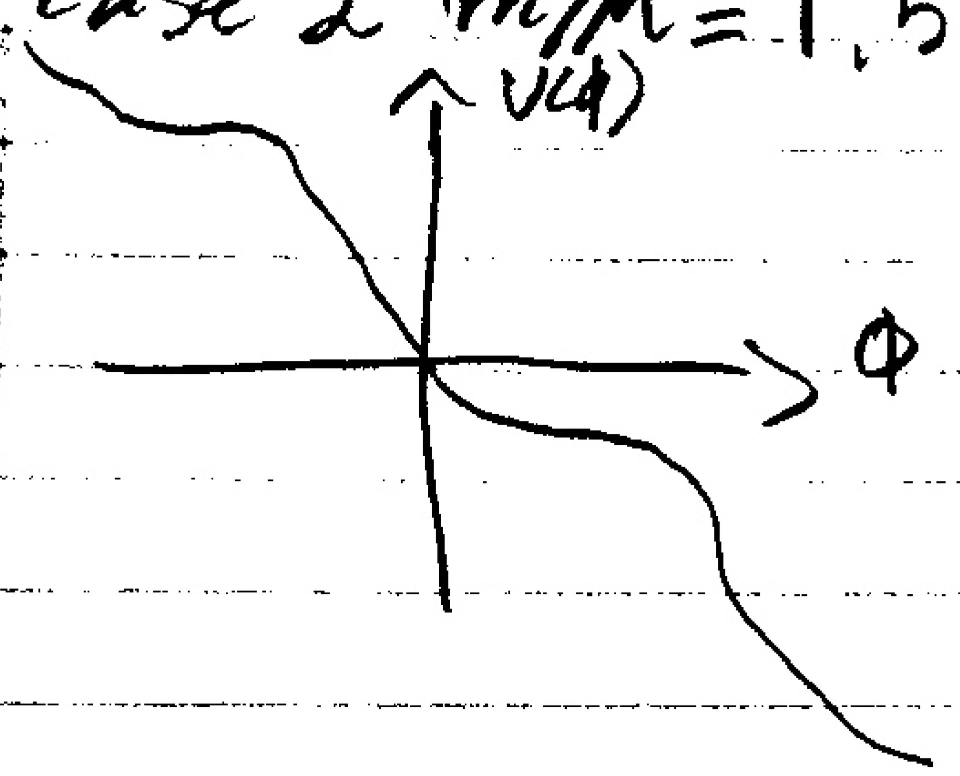
c) $\frac{\partial^2 U}{\partial \phi^2} = MgR\cos\phi$, it's stable when $\frac{\partial^2 U}{\partial \phi^2} > 0$ and unstable when $\frac{\partial^2 U}{\partial \phi^2} < 0$

- this make sense since if $\frac{\partial^2 U}{\partial \phi} > 0$ it make a concave up ($\cup$) and the opposite is $\frac{\partial^2 U}{\partial \phi^2} < 0$ ($\cap$)

d) case 1 $m/M = 0.5$



case 2 $m/M = 1.5$



- increasing m/M changes the potential, removing stable points and leading to more motion

~~e) $U(\phi) = 0$ to oscillate it must be stable equilibrium~~

the system will exhibit ~~ef~~ oscillation if $\frac{m}{M} < 1$ this value serves as a fixed point this can be reached by

solving $\sin\phi = \frac{m}{M}$