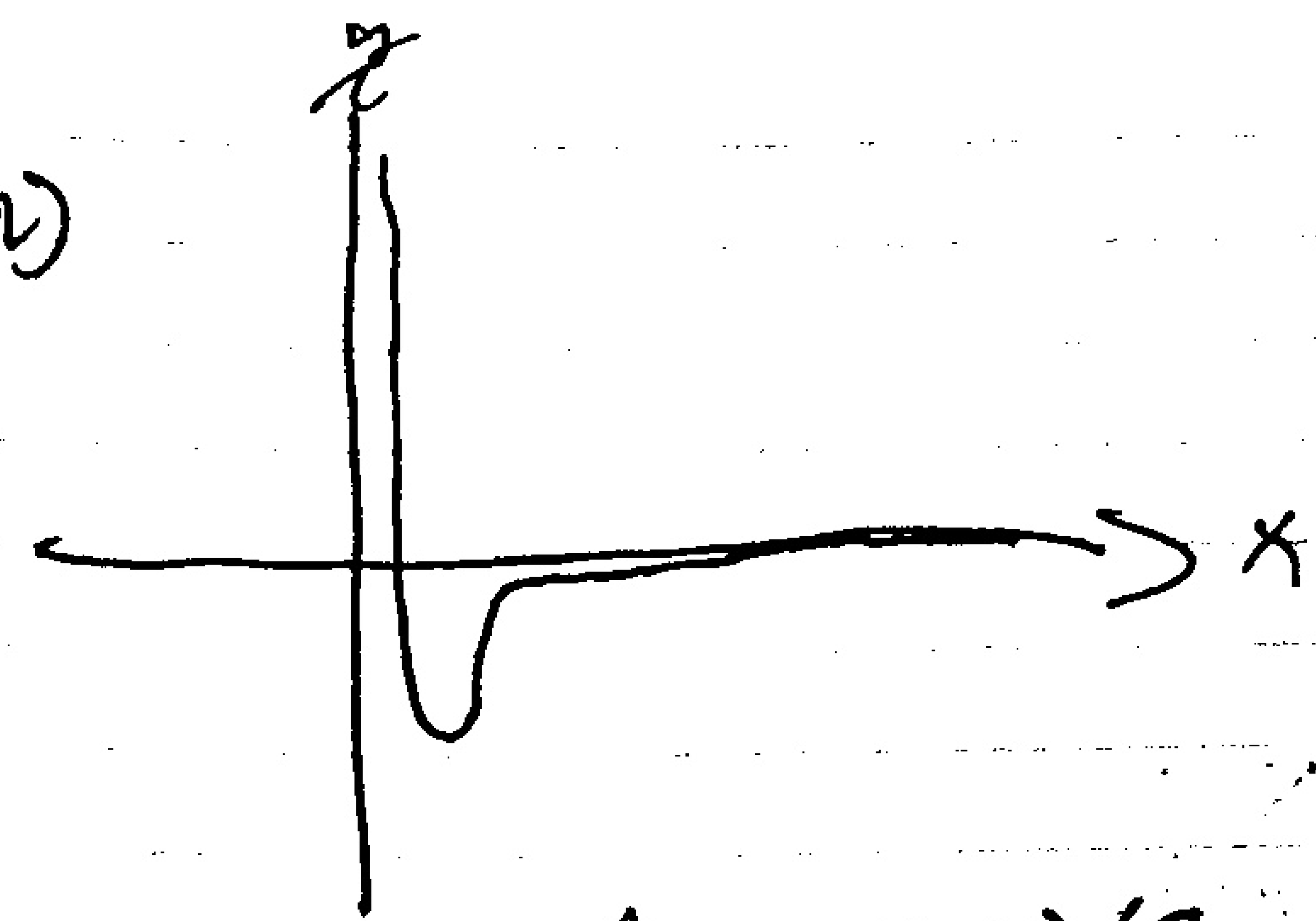
HW321

Q1)

a)



b) $U'(r) = \frac{-2A}{S}(e^{(R-r)/S}-1)e^{(R-r)/S}$

Set $U'(r)=0$

we get $\underbrace{e^{(R-r)/S}}_{J}-1=0$

and this true when $J=1$
and $J=1$ when e^0 where this is true
when $\boxed{R=r}$

c) $U(r)=A[(e^{(R-r)/S}-1)^2-1]$

replace: $r=r_e+x=R+x$

$\Rightarrow U(r)=A[(\exp(\frac{R-(R+x)}{S})-1)^2-1]$

$=A[(e^{-x/S}-1)^2-1]$ expand $e^{-x/S}$ around $x=0$ using taylor series

we get $U(r)\approx A[(1-\frac{x}{S}+\frac{x^2}{2S^2}-1)^2-1]$

$=A[(-\frac{x}{S}+\frac{x^2}{2S^2})^2-1]$ $=A[\frac{x^2}{S^2}-\frac{x^3}{S^3}+\frac{x^4}{4S^4}-1]$

we only want the second order, so,

$U(r)\approx A(\frac{x^2}{S^2}-1)$ potential expanded: $U(r)\approx U(R)+\frac{A}{S^2}x^2$

where $U(R)=-A$

2) $k = \frac{2A}{\delta^2}$

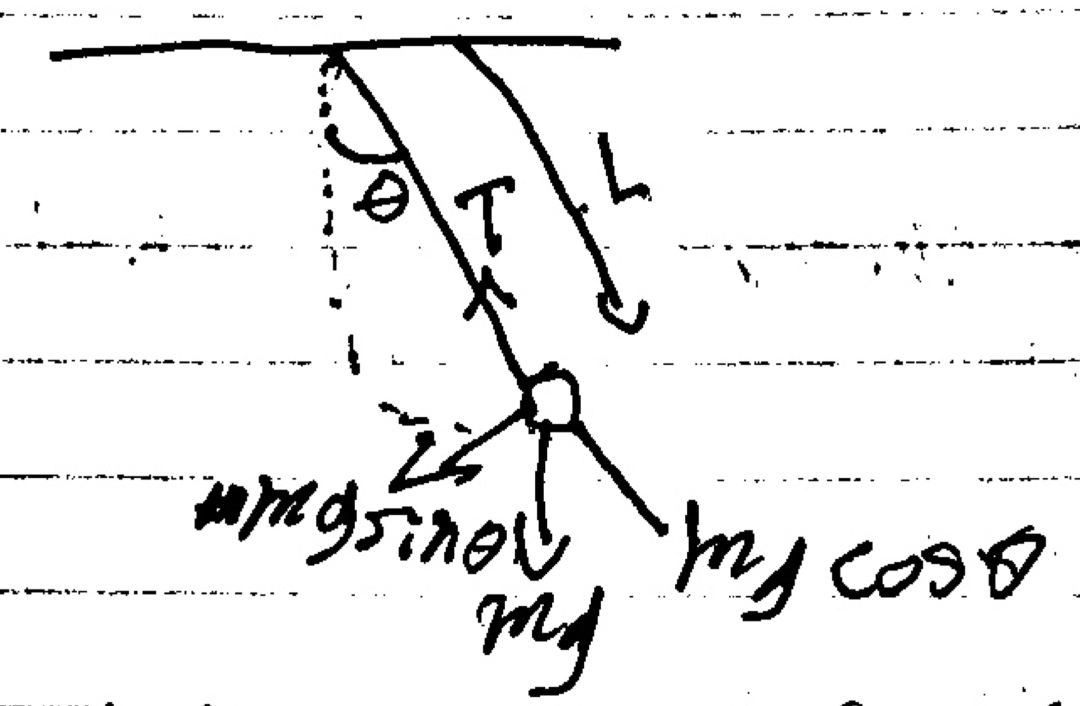
$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{2A}{m\delta^2}}$

$f = \frac{w}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2A}{m\delta^2}}$

---

2)

a)



$\cancel{\tau = I\alpha \Rightarrow -mg\sin\theta\, L = mL^2\frac{\partial^2\theta}{\partial t^2}}$

$T = \frac{1}{2}m(L\dot{\theta})^2 = \frac{1}{2}mL^2\dot{\theta}^2$

$U = -mgL\cos\theta$

$\mathcal{L} = T - U = \frac{1}{2}mL^2\dot{\theta}^2 + mgL\cos\theta$

deriving eom $\frac{d}{dt}\left(\frac{\delta \mathcal{L}}{\partial\dot{\theta}}\right) - \frac{\delta \mathcal{L}}{\delta\theta} = 0$

$\frac{\delta \mathcal{L}}{\delta\dot{\theta}} = mL^2\dot{\theta} \Rightarrow \frac{\partial}{\partial t}(mL^2\dot{\theta}) = mL^2\ddot{\theta}$

$\frac{\delta \mathcal{L}}{\delta\theta} = -mgL\sin\theta \Rightarrow mL^2\ddot{\theta} + mgL\sin\theta = 0$

$\ddot{\theta} + \frac{g}{L}\sin\theta = 0$

b) $h = L(1-\cos\phi$

$U = mgh$

$U(\phi) = mgL(1-\cos\phi$

c) $U(\phi) \approx mgL(1-(1-\frac{\phi^2}{2}))$

$\approx mgL\frac{\phi^2}{2}$ where $U(\phi) \approx \frac{1}{2}mgL\phi^2$

$k_{eff} = mgL$, $w = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{mgL}{m}} = \sqrt{\frac{g}{L}}$, $f = \frac{w}{2\pi}$

$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

Q3) ~~$f(t)$~~ $x(t) = A\cos(wt+\phi)$

a) $\langle f \rangle = \frac{1}{\tau}\int_0^\tau f(t)\,dt = \frac{1}{\tau}\int_0^\tau A\cos(wt+\phi)\,dt$

$$= \frac{A}{\tau}\int_0^\tau \cos(wt+\phi)\,dt$$

$$\left[\frac{\sin(wt+\phi)}{w}\right]_0^\tau$$

$$= \frac{\sin(w\tau+\phi)-\sin\phi}{w}$$

where $w\tau = 2\pi$

$$\Rightarrow \frac{\sin(\phi)-\sin\phi}{w}$$

$$\boxed{\langle f \rangle = \frac{A}{\tau}\cdot 0 = 0}$$

b) $v(t) = \frac{dx}{dt}(A\cos(wt+\phi))$

$v(t) = -Aw\sin(wt+\phi)$, $\langle v \rangle = \frac{1}{\tau}\int_0^\tau -Aw\sin(wt+\phi)\,dt$

$\Rightarrow$ Solve the integral $\frac{-Aw}{\tau}\int_0^\tau \sin(wt+\phi)\,dt$

$$\Rightarrow \frac{-Aw}{\tau}\left[\frac{-\cos(wt+\phi)}{w}\right]_0^\tau$$

$$-\frac{1}{w}(\cos\phi-\cos\phi) = 0 \Rightarrow \boxed{\frac{-Aw}{\tau}\cdot 0 = 0}$$

c) $T = \frac{1}{2} m v^2$

$= \frac{1}{2} m w^2 A^2 \cos^2(wt+\delta)$

$V = \frac{1}{2} k x^2 = \frac{1}{2} m w^2 A^2 \sin^2(wt+\delta)$

$E = T + V = \frac{1}{2} m w^2 A^2 \cos^2(wt+\delta) + \frac{1}{2} m w^2 A^2 \sin^2(wt+\delta)$

$E = \frac{1}{2} m w^2 A^2 [\cos^2(wt+\delta) + \sin^2(wt+\delta)]$

$E = \frac{1}{2} m w^2 A^2$

$\langle T \rangle = \frac{1}{2} m w^2 A^2 \langle \cos^2(wt+\delta) \rangle$

$= \frac{1}{2} (\frac{1}{2} m w^2 A^2) = \frac{1}{2} E$

$\langle V \rangle = \frac{1}{2} m w^2 A^2 \langle \sin^2(wt+\delta) \rangle$

$= \frac{1}{2} E$ so, $\langle T \rangle = \langle V \rangle = \frac{1}{2} E$

we need to show $\langle \cos^2(wt+\delta) \rangle$ and $\sin^2(wt+\delta) = \frac{1}{2}$

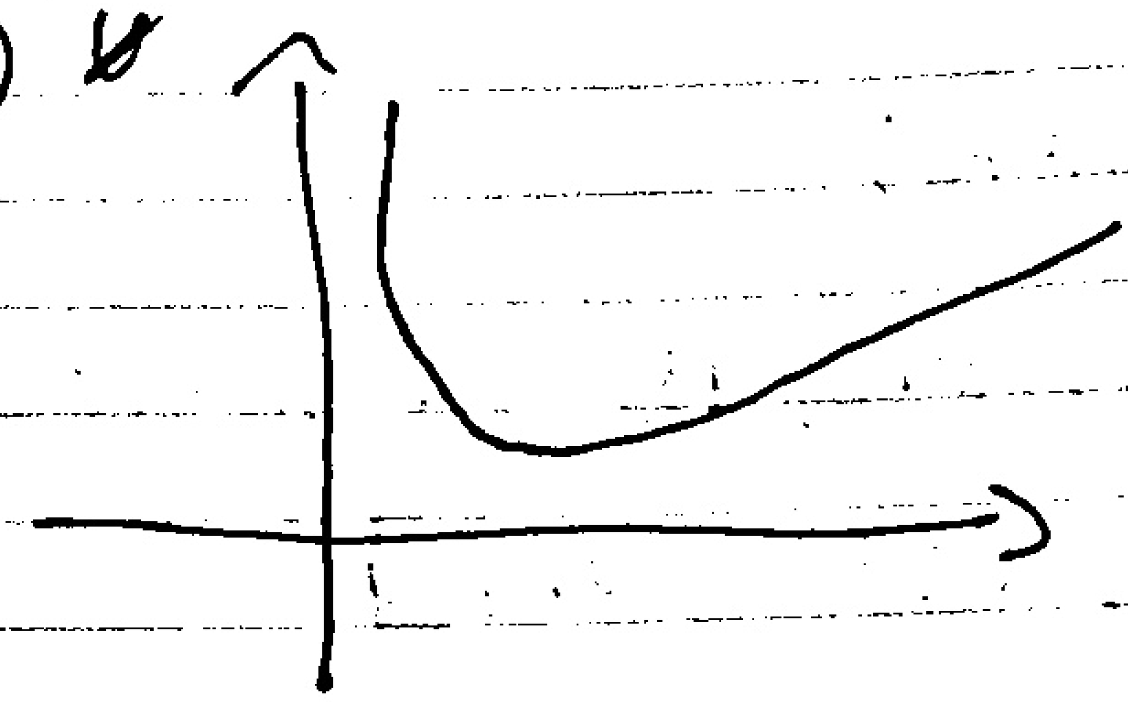
$\frac{1}{\tau} \int_0^\tau \frac{1+2\cos(wt+\delta)}{2} dt$ using wolframe alpha

$= \boxed{\frac{1}{2}}$

$\frac{1}{\tau} \int_0^\tau \sin^2(wt+\delta) dt = \boxed{\frac{1}{2}}$ using wolframe alpha

4) $U(r) = U_0\left(\frac{r}{R} + \lambda^2 \frac{R}{r}\right)$

a) U



b) we take the derivative of $U(r)$

we find $U_0\left(\frac{1}{R} - \frac{\lambda^2 R}{r_e^2}\right) = 0$
we can easily solve it to find that
The equilibrium position r_e is whe $r_e = \lambda R$

c) $U(r_e + x) = U_0\left(\frac{r_e + x}{R} + \frac{\lambda^2 R}{r_e + x}\right)$

using Taylor expansion.

$$U(r_e + x) \simeq U(r_e) + \frac{\partial U}{\partial r}\Big|_{r=r_e} x + \frac{1}{2}\frac{\partial^2 U}{\partial r^2}\Big|_{r=r_e} x^2$$

sub $U(r_e)$, $r_e = \lambda R$

$$U(r_e) = U_0\left(\frac{\lambda R}{R} + \frac{\lambda^2 R}{\lambda R}\right) = U_0(\lambda + \lambda) = 2U_0\lambda$$

find $\frac{\partial^2 U}{\partial r^2}$ at $r_e = r$

$$\frac{\partial^2 U}{\partial r^2} = \frac{2U_0}{\lambda R^2}, \text{ where } k = \frac{\partial^2 U}{\partial r^2}\Big|_{r=r_e} \boxed{= \frac{2U_0}{\lambda R^2}}$$

$$W = \sqrt{\frac{k}{m}} = \boxed{\sqrt{\frac{2U_0}{\lambda R^2 m}}}$$

5) a)

b) cos…

$w_1 t - \delta$…
solvin…

$t_n = $…

$t_{n+1} - t_n$…

We g…

showin…

$2 \cdot \frac{\pi}{w}$…

c) $T_1 = $…

$= \sqrt{w_0^2}$…

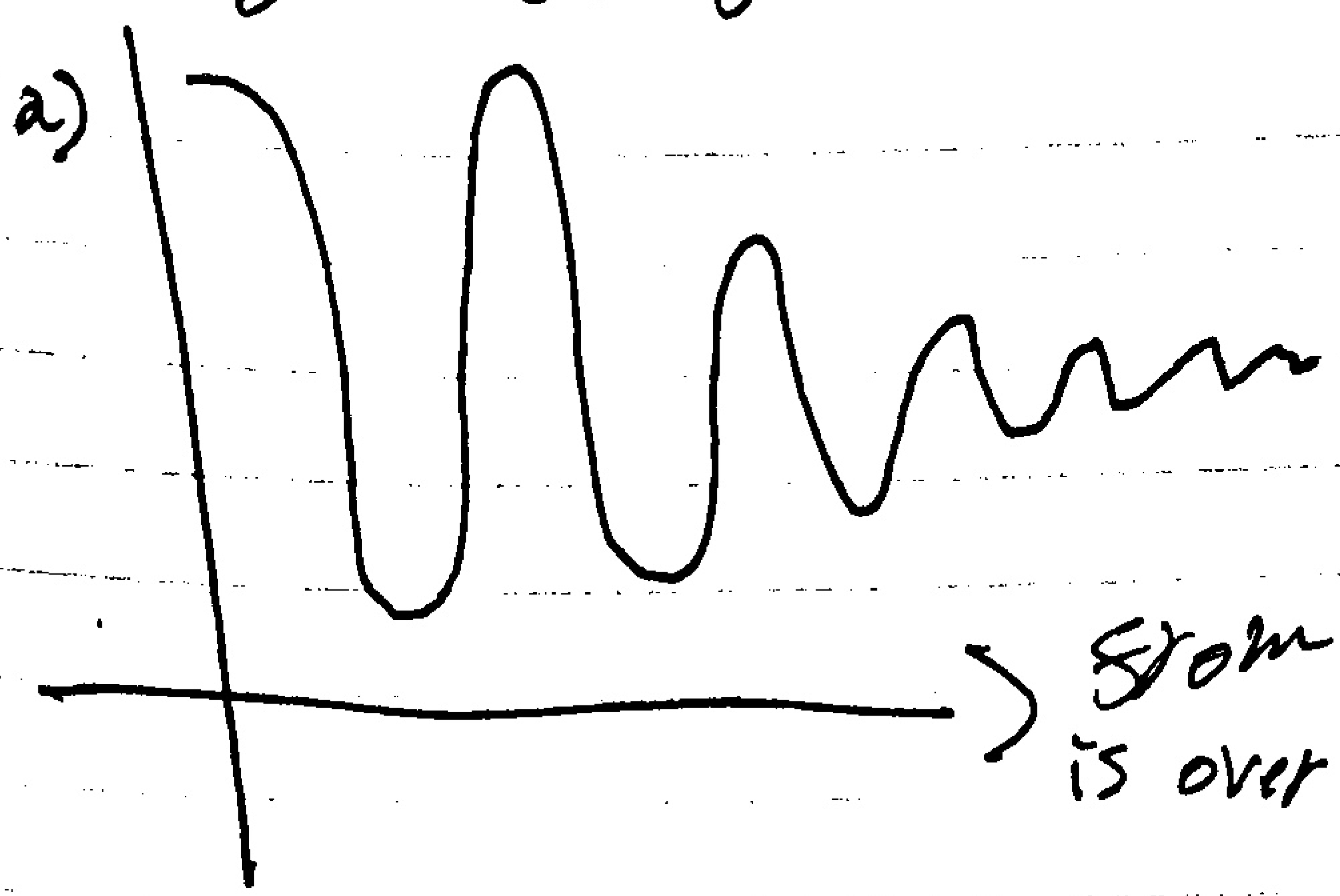
$T_1 = $…

e

· The …
over …

something like that

5) a)



from 1 peak to another is over $\frac{2\pi}{w_1}$

thus: $\tau_1 = \frac{2\pi}{w_1}$

b) $\cos(w_1 t - \delta) = 0$

$w_1 t - \delta = \frac{\pi}{2} + n\pi$ for some n ~~~~ integer

solving for t

$t_n = \frac{\pi}{2w_1} + \frac{n\pi}{w_1} + \frac{\delta}{w_1}$

$t_{n+1} - t_n = \left(\frac{\pi}{2w_1} + \frac{(n+1)\pi}{w_1} + \frac{\delta}{w_1}\right) - \left(\frac{\pi}{2w_1} + \frac{n\pi}{w_1} + \frac{\delta}{w_1}\right)$

we get: $t_{n+1} - t_n = \frac{\pi}{w_1}$

showing $\tau_1 = 2 \cdot (t_{n+1} - t_n)$

$2 \cdot \frac{\pi}{w_1} = \frac{2\pi}{w_1} = \tau_1$

c) $\tau_1 = \frac{2\pi}{w_1}$ plug $w_1 = \sqrt{w_0^2 - \beta^2}$, $\beta = \frac{w_0}{2}$

$= \sqrt{w_0^2 - \frac{w_0^2}{4}} = \frac{\sqrt{3}}{2} w_0$ hence, the period is:

$\tau_1 = \frac{4\pi}{\sqrt{3} w_0}$, the decay factor becomes:

$e^{-\beta\tau_1} = \exp\left(-\frac{w_0}{2} \cdot \frac{4\pi}{\sqrt{3} w_0}\right) = \boxed{e^{-2\pi/\sqrt{3}}}$ — 1

- The amplitude decay with a factor of eq 1 over one period τ_1 when β is half w_0