Physics 471 – Fall 2024

Homework #2 – due Wednesday, September 11

Point values for each problem are in square brackets

1. [1] Given 5 consecutive flips of a coin: What is the probability of first getting two heads in a row, followed by three consecutive tails?

2. [2] Sketch a graph of f(x) =$3e^{-(x^2/25)}$ versus. x. (Clearly indicate the x and y scales you choose. "Sketch" means hand-drawn; don't use a computer package to do this for you.)

3. [5] **Practice with Statistics.** Feel free to use a calculator (or spreadsheet) to add/divide/etc, but please <u>don't</u> use built-in statistical functions.

You measure the lengths of 14 objects in a box, and get the following results (in cm):
1; 3; 3; 4; 4; 5; 5; 5; 5; 7; 7; 7; 8; 9

a) What is the probability that an object chosen at random from this box has length 4 cm? (Here and below, assume equal probability for selecting any one object)

b) What's the average length $\langle L\rangle$ (also called the expectation value of length) of the objects? What is the probability that an object chosen at random from the box has length = $\langle L\rangle$?

c) What is the average value of the square of the lengths, $\langle L^2\rangle$?

d) Use your results to find the standard deviation σ of lengths of objects in this box

e) What is the probability that the length of an object chosen randomly from this box is in the range $\langle L\rangle \pm \sigma$? (Given what you know about stand. deviation, does this seem reasonable?)

4. [4] **Orthogonality and inner products.** Consider the following 3 state vectors:

$$|\psi_1\rangle = \frac{2}{\sqrt{5}}\,|+\rangle \;+\; \frac{1}{\sqrt{5}}|-\rangle$$
$$|\psi_2\rangle = \frac{\sqrt{2}}{\sqrt{3}}\,|+\rangle \;+\; \frac{1}{\sqrt{3}}\,i\,|-\rangle$$
$$|\psi_3\rangle = \frac{1}{\sqrt{2}}\,|+\rangle \;+\; \frac{1}{\sqrt{2}}e^{i\pi/4}|-\rangle$$

Use bra-ket notation in your calculations (*not* matrix notation, please)
Use orthogonality and normalization of our basis: $\langle +|+\rangle = \langle -|-\rangle = 1,\ \langle +|-\rangle = \langle -|+\rangle = 0.$

a) [3] For each of the 3 states above, find some vector that is orthogonal to it.
(Use our convention that we keep the coefficient of the $|+\rangle$ basis ket positive and real.)

b) [1] Calculate the inner products $\langle\psi_2|\psi_3\rangle$ and $\langle\psi_3|\psi_2\rangle$.
How are these two results related to one another?

5. [5] **Kets (state vectors).** Consider the following three (candidate) state vectors:

$$|\psi_1\rangle = 3|+\rangle - 4\,|-\rangle$$
$$|\psi_2\rangle = 2|+\rangle + i\,|-\rangle$$
$$|\psi_3\rangle = |+\rangle - 2\,e^{\frac{-i\pi}{4}}\,|-\rangle$$

a) [1] Normalize each of the above states (following our convention that the coefficient of the $|+\rangle$ basis ket is always positive and real.)

b) [1] For *each of these three states*, find the probability that the spin component will be "up" along the Z-direction. Use bra-ket notation in your calculations!

c) [1] For JUST the second state, $|\psi_2\rangle$, find the probability that the spin component will be "up" along the X-direction. Use bra-ket notation in your calculations. (*Hint: you will need McIntyre eq 1.36 for this one. You will need McIntyre 1.60 for the next part!)*

d) [2] For JUST state $|\psi_3\rangle$, find the probability that the spin component will be "up" along the Y-direction. Use bra-ket notation in your calculations. *Be careful, there is some nasty complex-number arithmetic required on this one that is very important, and people frequently make mistakes on it that change the answer significantly!*

6. [2] **The physics of quantum states.**
Using postulate 4, show that the probability to obtain a particular measurement of spin is unaffected by changing a state $|\psi\rangle$ to a state $e^{i\beta}|\psi\rangle$. Briefly discuss the significance of your result, what does it say to you? Also, how is it connected to the convention you imposed in Q3 and 4 above (setting the coefficient of the $|+\rangle$ basis ket to be positive and real?)

7. [1] **Interpreting the coefficients in kets.**
Construct a quantum state for which the probability that it is spin up in the Z-direction is *three times* the probability that it is spin down. Is your answer unique?