C4|

Q01) $\frac{dT}{dt} = -h(T - T_a)$

a) always postive. since the whole law is about measaring the rate of cooling ~~we~~ so, the resalt must be always decreasing

b) seting $T = T_a \Rightarrow \frac{dT}{dt} = -h(T_a - T_a)$

$= -h(0) \Rightarrow \frac{dT}{dt} = 0$

so, if $T = T_a$ it will zero

c) the temp at $t \to \infty$ will eventually reach T_a which is the ambient temp

so at $t \Rightarrow \infty$ $T = T_a$

d) $T(0), t=0$ | $\frac{dT}{dt} = -h(T - T_0)$

$\Rightarrow \frac{dT}{T - T_0} = -h\,dt \Rightarrow \int \frac{dT}{T - T_0} = \int -h\,dt$

$\Rightarrow \int \frac{dT}{T - T_0} = -ht + c$

$= \ln|T - T_0| = -ht + c$

$= |T - T_0| = e^{-ht + c}$ where c is a constant so $A = \pm e^c$

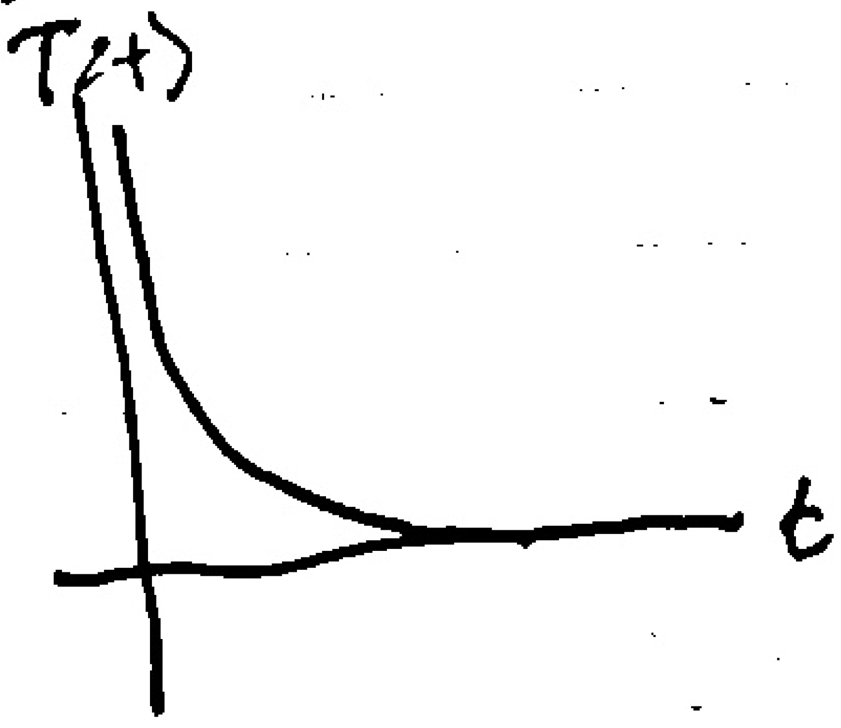
so, $T - T_0 = Ae^{-ht}$ now $T(t) = T_0 + Ae^{-ht}$

$\boxed{\text{simp solution } T(t) = T_a + (T_0 - T_a)e^{-ht}}$

where $T(0) = T_0$ so
$T_0 = T_0 + Ae^{-0} = T_0 - T_a = A$

e) $T(t) = T_a + (T_0 - T_a) e^{-kt}$

~~we~~ the general look would be

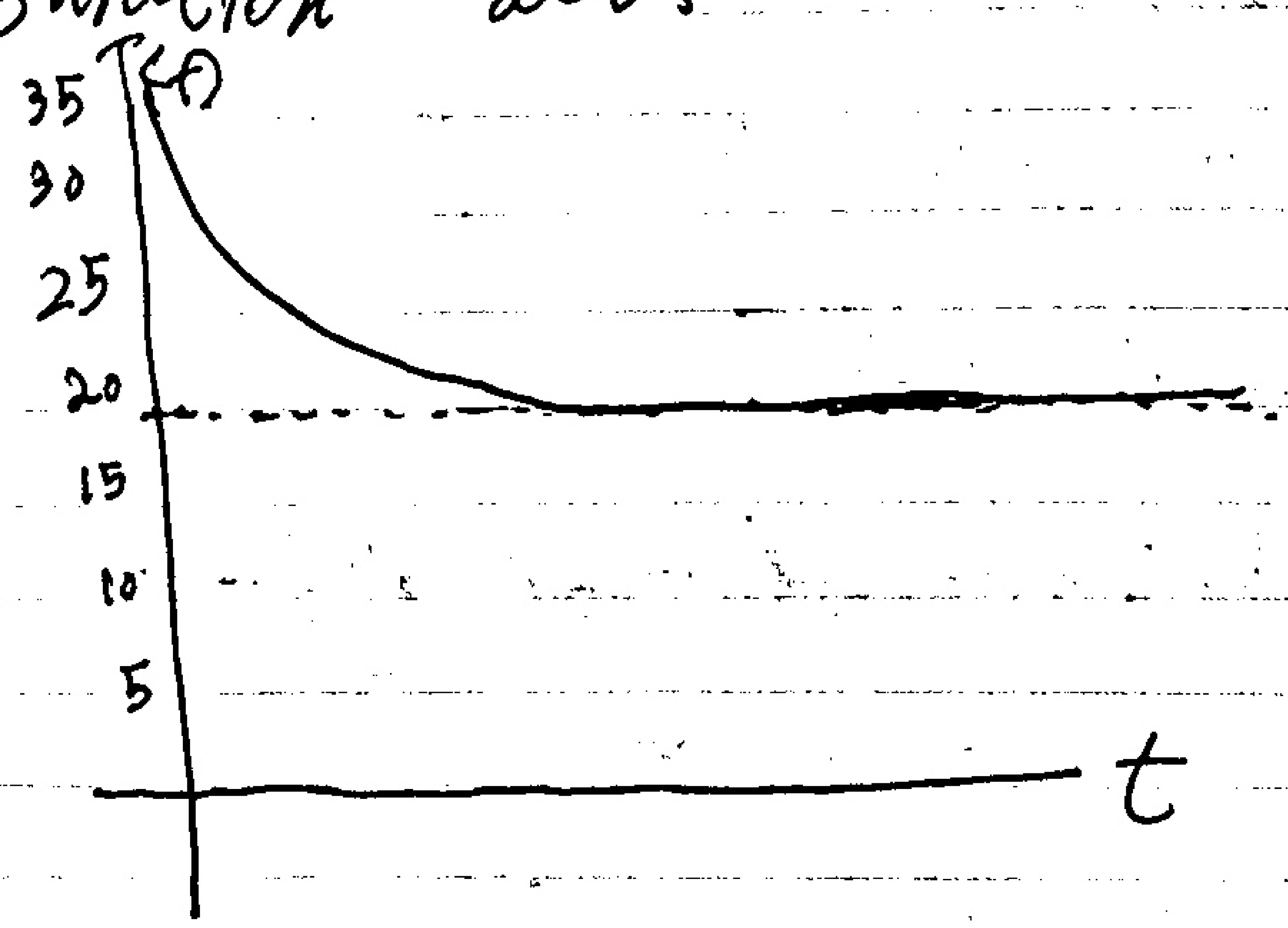


where it decreases exponentially

but we can plug some values to check how our function acts

let:
$k = 0.1$
$T_a = 20$
$T = 200$

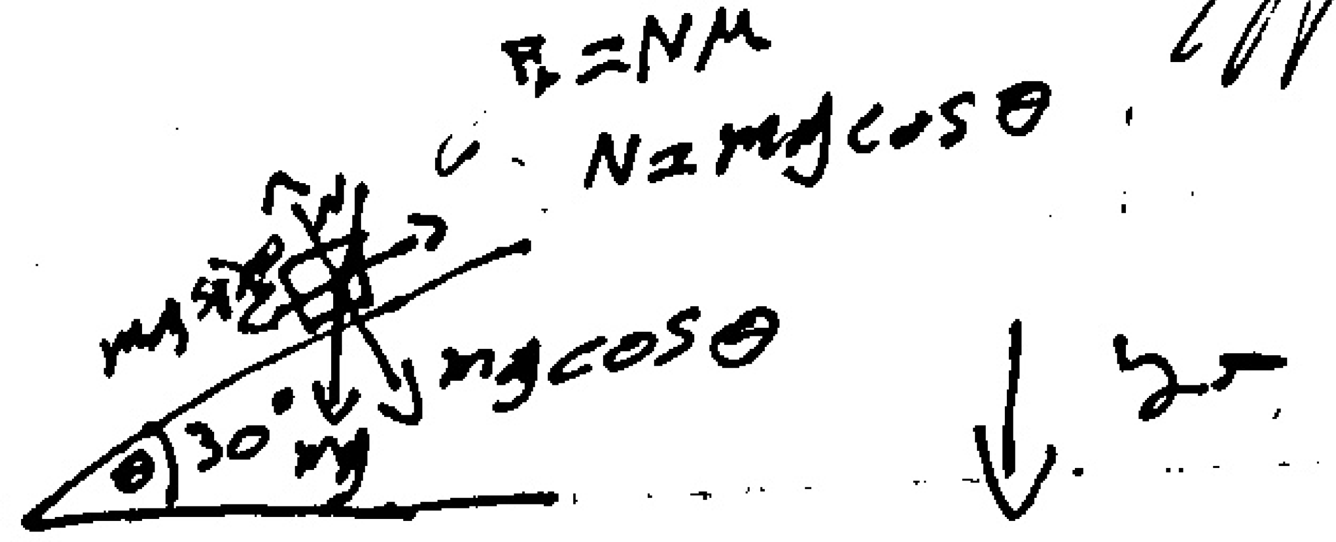
so



~~10 ft/s~~ ~~m/s~~ ~~5 ft/s → 8 m/s~~
~~= 1.524 m/s~~

2)

a)



b) $F_{net} = -mg\sin\theta - \mu mg\cos\theta$

$\cancel{m}a = -\cancel{m}g\sin\theta - \mu \cancel{m}g\cos\theta$

$a = -g\sin\theta - \mu g\cos\theta$

plug values

$a = -6.128\,m/s^2$ or $\ddot{x} = -6.128\,m/s^2$

c) ~~$x = x_0 + V_0 t + at^2$~~

$\Rightarrow$ ~~$0 + 1.524\,m/s\,(0.5s) + \frac{(-6.128\,m/s^2)(0.5)^2}{2}$~~

~~$x = -0.004$~~

$\frac{dv}{dt} = -g\sin\theta - \mu g\cos\theta \rightarrow \int_{5}^{v_f} dv = \int_0^t -g\sin\theta - \mu g\cos\theta\, dt$

$[v]_5^{v_f} = -t\,[g\sin\theta + \mu g\cos\theta]$ #

where $\Delta x = -\frac{t^2 \cdot k}{2} + 5$, so it becomes:

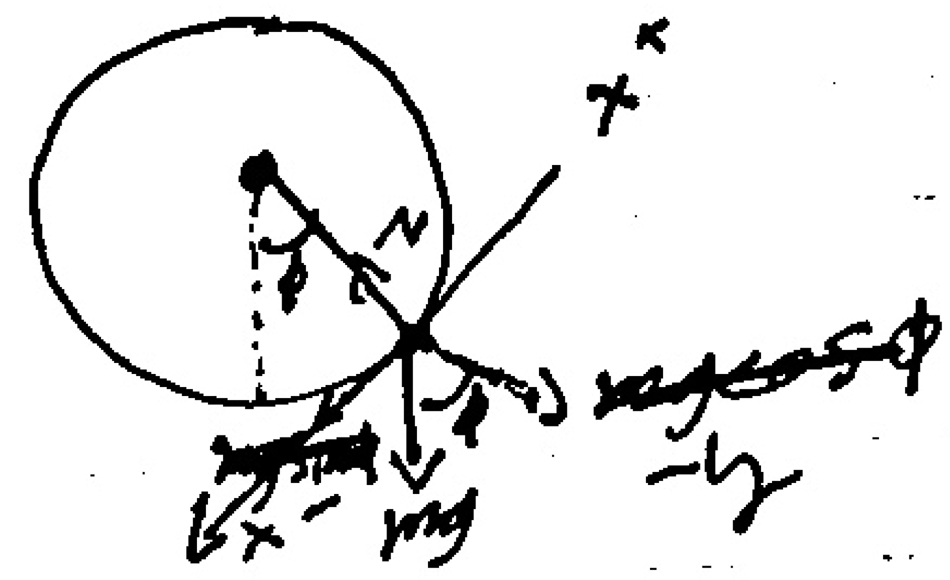
$= -t^2(6.128) + 5t)$

plug values we get

$x = 0.968$

3)

a)



force acting on ϕ direction is $mg\sin\phi$ and in $\hat{\rho}$ direction are $mg\cos\phi$ and $\frac{mv^2}{R}$ and N

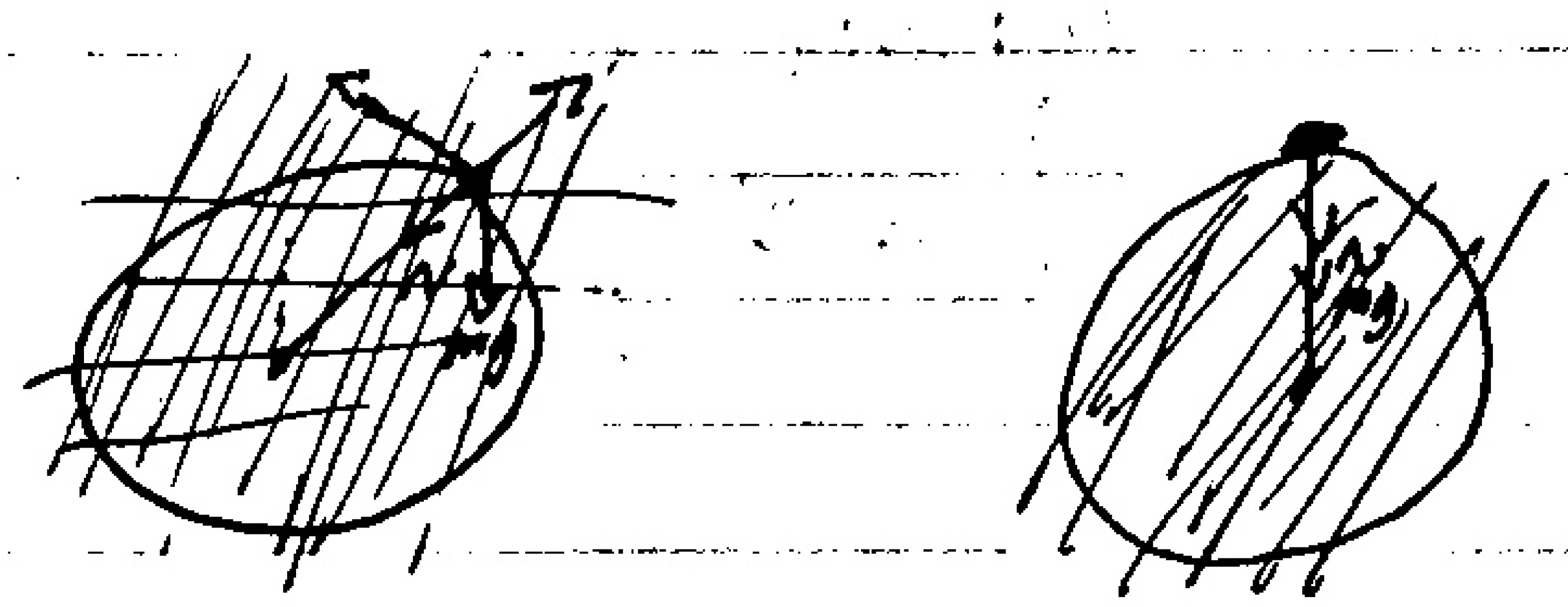
b) $F_{net,\rho} = mg\cos\phi + \frac{mv^2}{R} - N$ since ρ constant $a_\rho = 0$

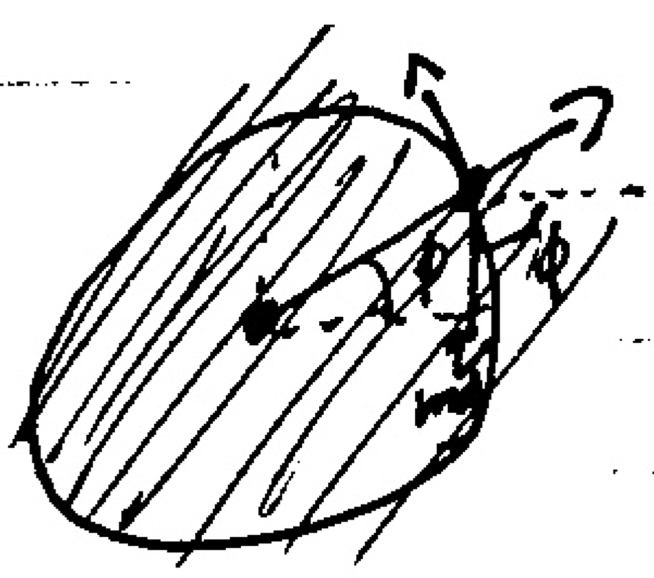
so, $a_\rho m = mg\cos\phi + \frac{mv^2}{R} - N$

$\Rightarrow \boxed{0 = mg\cos\phi + \frac{mv^2}{R}}$

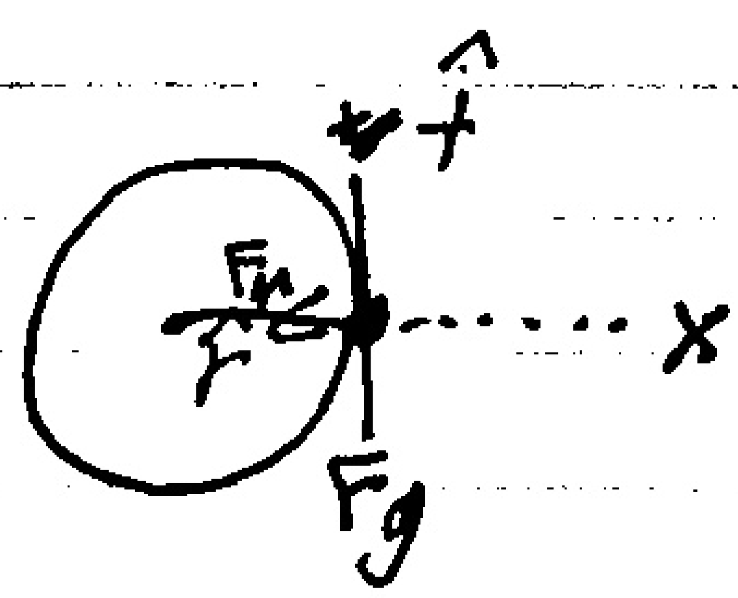
$\boxed{mR\ddot{\phi} = -mg\sin\phi}$ in $\hat{\phi}$ direction

c)





Last one!



~~2)~~ c) continued.

in this excation, there would be nothing to do with the angles with the net equations. one force is going in the x direction ~~They are the~~ gravity is always down, ~~as~~ the components would not be the same are the 90° angle of the forces.

$F_N = N$

d) $F_\rho = m\ddot{\rho} - m\rho\dot{\phi}^2$ ~~m~~ $F_\phi = m\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}$

ρ is constraints $\Rightarrow \rho$, $F_\rho = m(\rho\dot{\phi}^2)$

$$F_N - mg\cos\phi = -m\rho\dot{\phi}^2$$

$$\dot{\phi}^2 = \frac{F_N - mg\cos\phi}{-m\rho}$$

$$F_\phi = mR\ddot{\phi}$$

$$mg\sin\phi = mR\ddot{\phi}$$

$$\ddot{\phi} = \frac{g\sin\phi}{\cancel{R}\rho}$$

4)

a) $v^2 = w^2 + 2gh$

$\Rightarrow v^2 = 0^2 + 2 \cdot 9.81\, m/s^2 \cdot 10$

$= \sqrt{196.2}\, m/s$

$= 14.007\, m/s$

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 10}{9.81}} = 1.427 s$

b) $F_{net} = ma$

$ma = -bv^2$

$m \frac{dv}{dt} = -bv^2 \Rightarrow \boxed{\frac{dv}{v^2} = \frac{-b}{m} dt}$

c) we can take the chain rule

$\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \frac{dv}{dt}$

So, our equation becomes.

$mv \frac{dv}{dx} = -bv^2$

$\Rightarrow \frac{dv}{v} = -\frac{b}{m} dx \quad \int \frac{dv}{v} = \int -\frac{b}{m} dx$

$\Rightarrow \ln|v| = \frac{-b}{m} x + C \Rightarrow |v| = e^{-bx/m} e^{C}$: eat e^C be constant as v_0

$\Rightarrow v(x) = v_0 e^{-\frac{bx}{m}}$ at $v = v_0,\ x = 0$

since; $\boxed{v(x) = v_0 e^{-\frac{bx}{m}}}$ $v_0 e^0 = \boxed{v_0}$

d) our solution from c) becomes

$$\frac{dx}{dt} = v_0 e^{-\frac{bx}{m}}$$

$$\Rightarrow \int \frac{dx}{e^{-\frac{bx}{m}}} = \int v_0 \, dt$$

$$\int e^{bx/m} \, dx = \int v_0 \, dt$$

using wolfram alpha

we get $\boxed{x(t) = \frac{m}{b} \ln\left(1 + \frac{b}{m} v_0 t\right)}$

Q 5)

a) $F = qv \times B$, $B = (0,0,B)$

the force is puerly magnetic so,

$m\frac{\partial v}{\partial t} = qv \times B$, $v(t) = (v_x, v_y, v_z)$

taking crossproduct: $v \times B = (v_y B_z, -v_x B_x, 0)$

the EOM Becomes:

$m\frac{\partial v_x}{\partial t} = q v_y B_z$

$m\frac{\partial v_y}{\partial t} = -q v_x B_z$

$m\frac{\partial v_z}{\partial t} = 0$, so, $v_z(t) = 0$

Solve v_x: $\frac{\partial v_x}{\partial t} = \frac{qB_z v_y}{m}$

v_y: $\frac{\partial v_y}{\partial t} = -\frac{qB_z v_x}{m}$

So, $\frac{\partial^2 v_x}{\partial t^2} = \frac{qB_z}{m}\frac{\partial v_y}{\partial t}$ use $\frac{\partial v_y}{\partial t} = -\frac{qB_z v_x}{m}$

$\Rightarrow \frac{\partial^2 v_x}{\partial t^2} = -\left(\frac{qB_z}{m}\right)^2 v_x$ ← this simple ~~HO~~ HO equation

So, $v_x(t) = A\cos\left(\frac{qB_z t}{m}\right) + B\sin\left(\frac{qB_z t}{m}\right)$, Sm int

So, $F_x = ma_x = q B_z V_y \Rightarrow W = \frac{qB_z}{m}$ $\boxed{V_x' = wV_y}$

$F_y = ma_y = -qB_z V_x$ as we found

$\hookrightarrow \boxed{V_y' = -wV_x}$

$F_z = 0,\ V_z = 0$

$\Delta Z = 0$

$V_x' = wV_y \Rightarrow \int_{V_{0x}}^{V_{Sx}} dV_x = wy(x) \Rightarrow V_{Sx} = wy(x) + V_{0x}$

$V_y' = y'' = -w(wy(t) + V_{0x}) = -w^2 y(t) - wV_0 x$

$y(t) = A\cos(wt) + B\sin(wt) - \frac{wV_0 x t^2}{2}$

for x

$x(t) = A\cos(wt) + B\sin(wt) + \frac{wV_{0y} t^2}{2}$

$y(t) = A\cos(wt) + B\sin(wt) - \frac{wV_{0x} t^2}{2}$

$y(0) = A = 0$

$y'(0) = \cancel{-A\sin(wt) + Bw\cos(wt) - \frac{wV_0 t}{4}}$

$B = \frac{V_0 \sin\theta}{w}$

$y(t)' = \frac{V_0}{w}\sin\theta \sin(wt) - \frac{wV_{0x} t^2}{2}$

$V_0 y = -wx(t) + V_{y0}$

$x'' = -w^2 x + wV_{y0},\ x(t) = A\cos(wt) + B\sin(wt) + \frac{wV_{0x} t^2}{2}$

$B = \frac{-V_0 \cos\theta}{w}$

$$\boxed{x(t) = \frac{-V_0}{w}\cos\theta \sin(wt) + \frac{wV_{y0}}{2} t^2}$$

c)