Physics 471 – Fall 2024

Homework #1 – due Wednesday, September 4 at 11:30am

Please show your work and explain your reasoning for each problem. The grader will be looking for this explanation in addition to the "right" answer. Feel free to use notes or texts, and collaborate with each other, but at the end of the day, *all your work should be your own … otherwise what's the point in trying to learn physics.*

Read Appendices A, B, and C in McIntyre, with special emphasis on Appendix B. The first quiz will be a math quiz about complex numbers. Here are some reminders:
Complex numbers can always be written in "standard Cartesian form" as $z=a+bi$, where $a=Re(z)$ is the real part, and $b=Im(z)$ is the imaginary part. In "standard polar form", $z = Ae^{i\theta}$. By convention, θ, the "phase", is real, and A, the "modulus", is positive and real. Euler's theorem says $e^{i\theta} = cos\theta + isin\theta$. The complex conjugate of z is $z^* = a\text{-}bi = Ae^{-i\theta}$. Remember that the phase angle θ of a complex number is always expressed in radians, never in degrees.

1. [3] Consider the complex number z = 2 - 4i.

a) What is the value of $|z|^2 = z^* \cdot z$?

b) Plot z in the complex plane (y-axis = Im, x-axis = Re).

c) Write z in the form $z = Ae^{i\theta}$: what is the value of A? What is the value of the angle θ?

2. [2] Consider these four points on the unit circle in the complex plane: 1, i, -1, -i.

a) Make a single plot showing all four of them in the complex plane and write each of them in the standard polar form $z = Ae^{i\theta}$, with $0 \leq \theta < 2\pi$.

b) Show what you get when you multiply each of them by i. This may help with the next problem.

3. [2] Now consider z $= i\ e^{-i\pi/4}$ (note the "i" out front: this is NOT yet in the "standard polar form")

a) Rewrite z in the form $z = Ae^{i\theta}$ (What is the [real] value of A? What is the value of θ?)

b) Plot z in the complex plane (y-axis = Im, x-axis = Re). *For you to think about: could you describe what "multiply by i" does (in general) to any complex number in the complex plane?*

4. [3] Given matrices $\boldsymbol{A} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1/2 \end{pmatrix}$, and $\boldsymbol{C} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$

a) What is the product **A B = ?** What is the product **A C = ?**

b) What are the eigenvalues of **A**?

c) Can you find the product **C A?** If so, do it. If not, why not?

5. [3] Consider the two complex numbers $z_1 = e^{i\pi/4}$ and $z_2 = e^{i3\pi/4}$.

a) Plot z_1, z_2, z_1^*, and z_2^* in the complex plane and label them.

b) Calculate $z_1 z_2$, z_2/z_1, and $z_2 z_1^*$ using the polar forms given. Simplify your answers.

c) Write z_1 and z_2 in the Cartesian form $a+bi$, then calculate their product $z_1 z_2$ using your results. Check your answer with the result you obtained in part (b).

6. [5] **More practice with complex numbers.**

a) Show that if you have a complex number z, then Re(z) = (z + z*)/2. Now show that Im(z) = (z-z*)/(*what # goes here? You decide!*)

b) There is a useful relation for the product of two complex numbers z_1 and z_2:

It's either $\mathrm{Re}(z_1 z_2) = \mathrm{Re}(z_1)\,\mathrm{Re}(z_2) + \mathrm{Im}(z_1)\,\mathrm{Im}(z_2)$, or else it's

$\mathrm{Re}(z_1 z_2) = \mathrm{Re}(z_1)\,\mathrm{Re}(z_2) - \mathrm{Im}(z_1)\,\mathrm{Im}(z_2)$

Work it out; you decide which sign is correct.

Like above, show that if you have two complex numbers z_1 and z_2, then

$\mathrm{Im}(z_1 z_2)$ = *(+ or -1, you decide!)* $\mathrm{Re}(z_1)\mathrm{Im}(z_2)$ *(+ or -, you decide!)* $\mathrm{Im}(z_1)\mathrm{Re}(z_2)$

c) If you have a complex number z, we define $|z|^2 = z^* z$. Is there any difference between z^2 and $|z|^2$? How about $\mathrm{Re}(z^2)$ and $\mathrm{Re}(|z|^2)$? *Briefly, explain.*

d) Given $z = Ae^{i\theta}$, find expressions for Re(z), Im(z), z*, and |z|, *all* in terms of A and θ.

e) Using Euler's Theorem, find expressions for $cos\theta$ and $sin\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$. You will use these expressions many times in PHY471.