Special Results

Within this chapter, SPCRES is presumed to be set. Thus, infinities and non-numeric results are legal – no error message will be thrown if such results happen to occur (cf. the end of previous chapter).

The following monadic functions, if called with ℝ lit (i.e. CPXRES clear), return either ∞, -∞, or NaN under the conditions stated below:

Input x	Operation(s)	Output for ℝ lit [63]
0.	1/x	∞
0 or 0.	ln, lg, lb x	-∞
0 or 0.	Γ(x)	NaN
$\mathrm{Re}(x) < 1$	arcosh	NaN
$\lvert\mathrm{Re}(x)\rvert > 1$	arccos, arcsin	NaN
1.	artanh	∞
$\mathrm{Re}(x) > 1$	artanh	NaN
±90° or equivalents in other *ADM*	tan	NaN

And the following monadic functions operate also on infinities:

Input x	Operation(s)	Output for ℝ lit
-∞	x^3, $\sqrt[3]{x}$	-∞
-∞	arctan	-90.° or equivalents
-∞	e^x, 10^x, 2^x	0.

[63] In this chapter, results were crosschecked against the *WP 34S* wherever possible. Deviations are highlighted. Additionally, *Wolfram Alpha* was used for checking results with finite arguments. Red results are considered wrong although they may concur with the *WP 34S*.

Input x	Operation(s)	Output for ℝ lit
$-\infty \le x < 0$	ln, lg, lb x	NaN
−∞ or ∞	1/x, sinc	0.
−∞ or ∞	x^2	∞
∞	tanh	1.
∞	arctan	90.° or equivalents
∞	ln, e^x, $\sqrt{x}$, lg, 10^x, lb x, x^3, $\sqrt[3]{x}$, sinh, cosh	∞
−∞ or ∞	cos, sin, tan, arcosh, arsinh, artanh	NaN

For dyadic functions, we combined the respective tables:

Input y	x	Op.(s)	Output for ℝ lit
∞	arbitrary $x \neq -\infty$	+	∞ [64]
−∞	arbitrary $x \neq \infty$		−∞ [64]
−∞	∞	+	NaN [64]
∞	arbitrary $x \neq \infty$	−	∞ [65]
−∞	arbitrary $x \neq -\infty$		−∞ [65]
−∞	−∞	−	NaN
∞	∞		
∞	arbitrary $x > 0$	×	∞ [64]
−∞	arbitrary $x < 0$		
∞	arbitrary $x < 0$	×	−∞ [64]
−∞	arbitrary $x > 0$		
0 or 0.	−∞ or ∞	×	NaN [64]

[64] Swapping x and y will return the same result here.

[65] Swapping x and y will return the result times -1.

Input y	x	Op.(s)	Output for ℝ lit
$0 < y \le \infty$	0.	/	∞
$-\infty \le y < 0$	0.	/	−∞
−∞ or ∞	−∞ or ∞	/	NaN
0 or 0.	0.	/, y^x	NaN
−∞ or ∞	0. or 0	y^x	NaN
$-\infty < y < 0$	non-integer x	y^x	NaN (cf. $\tan^{\sin(-.6)}(-.6)$ ∈ ℂ as calculated by *WA* and in *MoHPC*)
−∞	odd $x > 0$	y^x	−∞
−∞	even $x > 0$	y^x	∞
∞	arbitrary $x > 0$	y^x	∞
arbitrary $y \neq 0$	−∞	y^x	0.
	∞	y^x	∞
0.	$0 < x < \infty$	$\log_x y$	−∞

The functions printed on light yellow background in the three tables above will return NaN also with complex results allowed (i.e. CPXRES set). Others will change their output when ℂ is lit. Some particular returns of elementary transient functions operating near ±∞ are listed here: [66]

Input Re(x)	Im(x)	r(x)	φ(x)	Op.	Output for ℂ lit
−∞	—	—		$\sqrt{x}$	∞∡90° = 0.+i×∞
−∞	0	∞	180°	$\sqrt{x}$	NaN+i×∞ (*WP 34S* returns 0.+i×∞)
-10^{999}	0	10^{999}	180°	$\sqrt{x}$	→ ∞∡90° = 0.+i×∞

[66] Following an article of *HP* about the *HP-71*, complex infinities should be treated in polar notation (see http://hparchive.com/Journals/HPJ-1984-07.pdf, p. 27, left column for the reasons).

Input Re(x)	Im(x)	r(x)	φ(x)	Op.	Output for ℂ lit
0.	10^{999}	10^{999}	90°	x^2	→ ∞ ∡ 180° = -∞+i×0.
	∞	∞			-∞+i×NaN
-∞	—	—		$\sqrt[3]{x}$	-∞
-∞	0	∞	180°		∞ ∡ 45° = ∞+i×∞ (*34S*: NaN+i×NaN)
-10^{999}		10^{999}			1.×10^{333} ∡ 60° = 5.×10^{332} + i×8.660 254 037 8×10^{332} $= 5 \times 10^{332}\,(1 + i \times \sqrt{3})$
—		10^{333}	60°	x^3	1.×10^{999} ∡ -180° = -1.×10^{999} + i×0. $\to -\infty + i \times 0$
-∞	—	—		x^3	-∞
-∞	0	∞	180°		NaN+i×NaN
-10^{999}		10^{999}			-1.×10^{2997}+i×0. $\to -\infty + i \times 0$
∞	—	—		ln	∞
∞	0	∞	0°		∞+i×∞ (*WP 34S* returns ∞+i×0.)
10^{999}		10^{999}			$\to \infty + i \times 0$
-10^{999}	0	10^{999}	180°	ln	$\to \infty + i\pi$
-∞		∞			∞+i×∞ (*WP 34S* $= \infty + i\pi$)
-∞	—	—			NaN
∞	∞	∞	45°	ln	∞+i×∞
10^{999}	10^{999}	10^{999}			$\to \infty + i\,{}^{\pi}/_{4}$ (confirmed by *34S* & *WA*)
∞	-∞	∞	-45°	ln	∞-i×∞
10^{999}	-10^{999}	10^{999}			$\to \infty - i\,{}^{\pi}/_{4}$ (confirmed by *34S* & *WA*)
0.	∞	∞	90°	ln	∞+i×∞
	10^{999}	10^{999}			$\to \infty + i\,{}^{\pi}/_{2}$ (confirmed by *34S* & *WA*)

Input Re(x)	Im(x)	r(x)	φ(x)	Op.	Output for ℂ lit
0.	-∞	∞	-90°	ln	∞-i×∞
	-10^{999}	10^{999}			$\to \infty - i\,{}^{\pi}/_{2}$ (confirmed by *34S* & *WA*)
-∞	∞	∞	135°	ln	∞+i×∞
-10^{999}	10^{999}	10^{999}			$\to \infty + i\,{}^{3\pi}/_{4}$ (conf. by *34S* & *WA*)
-∞	-∞	∞	-135°	ln	∞-i×∞
-10^{999}	-10^{999}	10^{999}			$\to \infty - i\,{}^{3\pi}/_{4}$ (conf. by *34S* & *WA*)
0.	0.	0.	0.	ln	NaN+i×NaN
10^{-999}	0.	10^{-999}	0.		$\to -\infty + i \times 0$
0.	—	—			-∞
0.	∞	∞	90°	e^x	NaN+i×NaN
	10^{999}	10^{999}			1.0 ∡ -45° (*WP 34S*: NaN+i×NaN) =0.707 106 781 186... - i×0.707 1... $= \frac{1}{2}(\sqrt{2} - i\sqrt{2})$
0.	-∞	∞	-90°	e^x	NaN+i×NaN
	-10^{999}	10^{999}			1.0 ∡ 45° (*WP 34S*: NaN+i×NaN) =0.707 106 781 186... + i×0.707 1... $= \frac{1}{2}(\sqrt{2} + i\sqrt{2})$
-∞	0	∞	180°	e^x	0.+i×0.
-10^{999}	10^{-999}	10^{999}			0.+i×0.
-∞		∞			NaN+i×NaN
-∞	∞	∞	135°	e^x	NaN+i×NaN
-10^{999}	10^{999}	10^{999}			0.+i×0.
-∞	-∞	∞	-135°	e^x	NaN+i×NaN
-10^{999}	-10^{999}	10^{999}			0.+i×0.