A Game Theoretic Dynamic Pricing for TSRTC

Use Case Report

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Outline

In this work we propose a game-theoritic usecase for the dynamic pricing Telangana State Road Transport Corporation(TSRTC). We have considered TSRTC and private bus service competing with each other to maximise their revenue. For current work, we have considered the usecase to be a two player game i.e., a single TSRTC bus trying to compete with single private bus service. We try to obtain the optimum price for both private and TSRTC services which will be derived by calculating the best reponses of each player and finding the Nash Equilibrium of the game.

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Introduction

Currently, bus services in the state of Telangana are controlled by TSRTC and private bus services. Looking at the high prices being charged by the private bus services, TSRTC have recently announced to adopt dynamic pricing for its buses [1]. This move would result in a game where TSRTC and private bus services should intelligently keep the pricing so as to maximize their revenue. For the usecase, we design a 2 player game with single TSRTC and single private bus service trying to optimize their prices. We assume passengers initially are loyal to only one of the services i.e., they are more inclined to choose TSRTC over private bus or vice versa. Passengers protection level for each of their respective service is defined by loyality factor *i.e.,* customers would choose to stay with their loyal service till the ticket price of other service is above the protection level. As the price of other service falls below the below protection level, customers would switch to other service. Based on this setting we attempt to find the optimum price for both TSRTC and private service.

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Game Description

2.1 Notations

We introduce the following notations

1. Service providers TSRTC and private bus service as A and B respectively.
2. The loyalty of each customer c_i is associated with a random variable $X_{c_i}, (i = 1 \dots n)$ such that X_{c_i} are *i.i.d* and distributed according to P_{X_c} over $\{A, B\}$.
3. k and k' are loyalty factors that represent how loyal customers are to A and B respectively. As long as the inequality $kP_A \leq P_B$ holds, a loyal customer to A won't switch to B. Refer Figure 1 and Figure 2.
4. P_A and P_B denote the price set by A and B respectively.
5. P_{max} is the maximum price limit that A and B can go. We assume that beyond P_{max} customers would prefer other mode of transportation such as trains.
6. r_A and r_B denote the total revenue from a single customer for A and B respectively.
7. $R_A = \sum_1^n r_A$ and $R_B = \sum_1^n r_B$ denote the total revenue from all the customers for A and B respectively.

2.2 Utilities

Before we formulate the revenue expression, we calculate the probabilities of the following events:

Probability of a loyal customer of A staying with A:

$$1 - \frac{kP_A}{P_{max}} \tag{1}$$

Probability of a loyal customer of B switching to A:

$$\frac{k'P_B}{P_{max}} \tag{2}$$

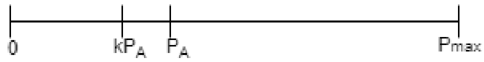

Figure 1: Price range for A

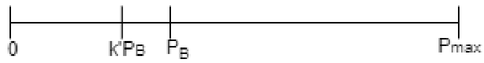

Figure 2: Price range for B

The revenue or payoff function from a single customer for A and B are defined as :

$$\mathrm{r}_A = \left(\mathbb{P}_{X_c}[X_c = A](1 - \tfrac{kP_A}{P_{max}}) + \mathbb{P}_{X_c}[X_c = B](\tfrac{k'P_B}{P_{max}})\right) P_A \tag{3}$$

$$\mathrm{r}_B = \left(\mathbb{P}_{X_c}[X_c = B](1 - \tfrac{k'P_B}{P_{max}}) + \mathbb{P}_{X_c}[X_c = A](\tfrac{kP_A}{P_{max}})\right) P_B \tag{4}$$

The total revenue from all customers for A and B are defined as:

$$\mathrm{R}_A = \textstyle\sum_{i=1}^{n} \left(\mathbb{P}_{X_c}[X_c = A](\tfrac{1-kP_A}{P_{max}}) + \mathbb{P}_{X_c}[X_c = B](\tfrac{k'P_B}{P_{max}})\right) P_A \tag{5}$$

$$\mathrm{R}_B = \textstyle\sum_{i=1}^{n} \left(\mathbb{P}_{X_c}[X_c = B](\tfrac{1-k'P_B}{P_{max}}) + \mathbb{P}_{X_c}[X_c = A](\tfrac{kP_A}{P_{max}})\right) P_B \tag{6}$$

which implies:

$$R_A = n * r_A \tag{7}$$

$$R_B = n * r_B \tag{8}$$

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Existence of Equilibrium

To find the optimum price to maximize revenue, we find the first-order differentiation:

$$\frac{\partial r_A}{\partial P_A} = \frac{\partial}{\partial P_A}\left(\mathbb{P}_{X_c}[X_c = A](1 - \frac{kP_A}{P_{max}}) + \mathbb{P}_{X_c}[X_c = B](\frac{k'P_B}{P_{max}}\right) P_A \tag{9}$$

$$\frac{\partial r_B}{\partial P_B} = \frac{\partial}{\partial P_B}\left(\mathbb{P}_{X_c}[X_c = B](1 - \frac{k'P_B}{P_{max}}) + \mathbb{P}_{X_c}[X_c = A](\frac{kP_A}{P_{max}})\right) P_B \tag{10}$$

Since a customer can be loyal to either A or B, we take X_c to have a Bernoulli distribution, *i.e*, $\mathbb{P}_{X_c}[X_c = A] = q, \mathbb{P}_{X_c}[X_c = B] = 1 - q$. Now,

$$\frac{\partial r_A}{\partial P_A} = \frac{\partial}{\partial P_A}(\left(q(1 - \frac{kP_A}{P_{max}}) + (1-q)(\frac{k'P_B}{P_{max}})\right) P_A) \tag{11}$$

$$\frac{\partial r_B}{\partial P_B} = \frac{\partial}{\partial P_B}\left((1-q)(1 - \frac{k'P_B}{P_{max}}) + q(\frac{kP_A}{P_{max}})\right) P_B \tag{12}$$

$$\frac{\partial r_A}{\partial P_A} = q - q\frac{2kP_A}{P_{max}} + (1-q)(\frac{k'P_B}{P_{max}}) \tag{13}$$

$$\frac{\partial r_B}{\partial P_B} = (1-q) - (1-q)\frac{2k'P_B}{P_{max}} + q(\frac{kP_A}{P_{max}}) \tag{14}$$

Setting $\frac{\partial r_A}{\partial P_A} = 0$ and $\frac{\partial r_B}{\partial P_B} = 0$, in order to maximize r_A and r_B respectively.

On solving these, we get

$$P_A^* = \frac{(1 + \frac{1}{q})}{3k} P_{max} \tag{15}$$

$$P_B^* = \frac{(\frac{2}{q} - 1)}{3k'(\frac{1}{q} - 1)} P_{max} \tag{16}$$

These are the optimum prices for A and B for given k and k'.

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Analysis

The above optima values are the best response of each bus service to other services' action. Further, if their beliefs of other bus services' correspond to their true action, the pair of their maximum prices set should be a pure Nash Equilibrium.

Since both P_A and $P_B \leq P_{max}$. On substituting values from equations 15 and 16 respectively we get,

$$\frac{(1 + \frac{1}{q})}{3k} P_{max} \leq P_{max}$$
$$\frac{(1 + \frac{1}{q})}{3k} \leq 1$$
$$\frac{(1 + \frac{1}{q})}{3} \leq k$$
$$\implies \frac{1}{3k - 1} \leq q$$

Since $0 \leq q \leq 1$, we get $\frac{2}{3} \leq k < 1$

$$\frac{(\frac{2}{q} - 1)}{3k'(\frac{1}{q} - 1)} P_{max} \leq P_{max}$$
$$\frac{(\frac{2}{q} - 1)}{3k'(\frac{1}{q} - 1)} \leq 1$$
$$\frac{1 + \frac{1}{1-q}}{3} \leq k'$$
$$\implies 1 + \frac{1}{1 - 3k'} \leq q$$

Since $0 \leq q \leq 1$, we get $\frac{1}{3} < k < 1$.

We calculated the intersection of the best response linear equations in P_A and P_B for various possible values for k, k', and q. But one of the prices exceeds our model constraint of P_{max}.

Example 1: For $k = 3/4$, k'=2/3, and $q = 4/5$, we get $P_A = P_{max}$ and $P_B = 3P_{max}$.

Example 2: For $k = 3/4$, k'=2/3, and $q = 9/10$, we get $P_A = 76/81P_{max}$ and $P_B = 11/2P_{max}$.

Our modelling of the game for dynamic pricing does have intersection but the values P_A^* and P_B^* are greater than P_{max} which implies that the optimum price offered by bus services exceeds the maximum price that customer can afford to pay for a bus service.

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Conclusion and Future work

We believe we need to consider more parameters to model the game and better the formulation for more accurate revenue function which can have practically valid Nash equilibrium *i.e.,* the optimum price doesn't exceed P_{max} and not less than 0.

We are not able to get a dominant strategy or pure strategy Nash equilibrium probably because our revenue function has taken care of loyalty using loyalty factor k, but it doesn't take into account, the role of P_B while calculating the probability for the customers to switch from the service they are loyal to. But intuitively P_B in essence is determining the disloyalty. So, a better customer choice model for loyalty that takes P_B into account would probably give a quasi-concave function and thus, existence of pure strategy Nash equilibrium.

As a part of future work we wish to work on finding other critical parameters to model the dynamic pricing problem as game and explore the Nash equilibrium under different k and k'.

References

[1] Express News Service. Tsrtc to start dynamic ticket pricing system. https://www.newindianexpress.com/states/telangana/2023/mar/24/tsrtc-to-start-dynamic-ticket-pricing-system-2558946.html. Accessed: 24th March 2023.