Christian-Albrechts-Universität zu Kiel

Philosophische Fakultät

Krippendorff‘s Alpha

NLP-Taskforce

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1. Coincidence Matrix

	sample_1	sample_2	sample_3	sample_4	sample_5	sample_6
Annotator 1	1	1	4	3	5	*
Annotator 2	2	2	1	3	5	3

→ remove unpaired labels, create coincidence matrix:

	1	2	3	4	5
1		2		1	
2	2				
3			1		
4	1				
5					1

2. Difference Functions

- *nominal*: agreement =0, disagreement = 1
- *interval*: $(v1 - v2)^2$
- *ordinal*: $\left(\sum_{g=v1}^{g=v2} ng \; - (\frac{nv1+nv2}{2}) \right)^2$

example: weighing of coincidence matrix according to nominal and interval function

	1	2	3	4	5
1	0	1	1	1	1
2	1	0	1	1	1
3	1	1	0	1	1
4	1	1	1	0	1
5	1	1	1	1	0

	1	2	3	4	5
1	0	1	4	9	16
2	1	0	1	4	9
3	4	1	0	1	4
4	9	4	1	0	1
5	16	9	4	1	0

3. Complete Formula

$$\alpha = 1 - \frac{D_o}{D_e}$$

- D_o: sum of all observed disagreements in one triangle (weighed by difference function), example: 2*1 + 1*9 = 11
- D_e: sum of all weighed expected disagreements (normalized):

$$\left(\frac{1}{n-1}\right) \sum_{v1=1,v2=1}^{V} n_{v1}\, n_{v2}\, \partial$$

Coincidence Matrix (see slide 1) with margins

	1	2	3	4	5	Σ fr
1		2		1		3
2	2					2
3			1			2
4	1					1
5					1	2
Σ fr	3	2	2	1	2	n=10

Weight for each value pair

	1	2	3	4	5
1	0	1	4	9	16
2	1	0	1	4	9
3	4	1	0	1	4
4	9	4	1	0	1
5	16	9	4	1	0

example:
do = 11

de = $\frac{1}{9}((3*2*1) + (3*2*4) + (3*1*9) + (3*2*16) + (2*2*1) + (2*1*4) + (2*2*9) + (2*1*1) + (2*2*4) + (1*2*1))$ =

$$\frac{1}{9}(6 + 24 + 27 + 96 + 4 + 8 + 36 + 2 + 16 + 2) = \frac{221}{9} = 24.56$$

$\alpha = 1 - \frac{11}{24.56}$ = 1-0.45 = 0.55

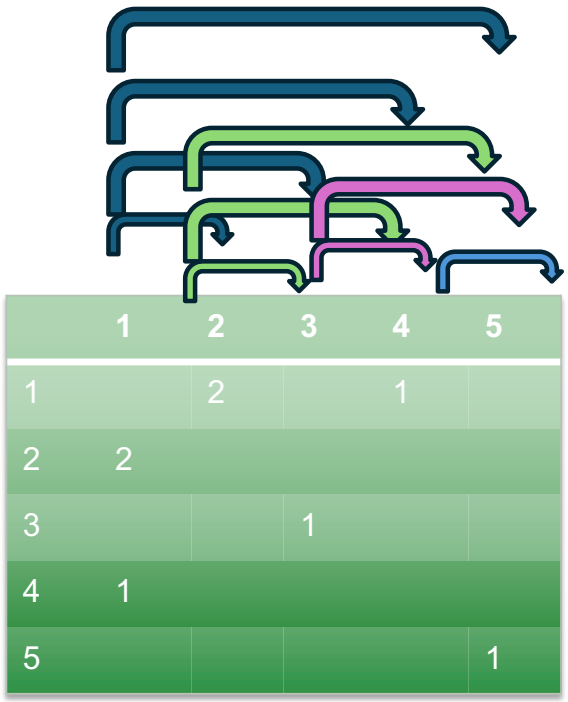
4. Focus: Expected Disagreement

	1	2	3	4	5	Σ fr
1		2		1		3
2	2					2
3			1			2
4	1					1
5					1	2
Σ fr	3	2	2	1	2	n=10

$$D_e = \left(\frac{1}{n-1}\right) \sum_{v1=1,v2=1}^{V} n_{v1}\, n_{v2}\, \partial$$

- n: number of annotations
- nv1, nv2: total number of label 1, total number of label 2
- ∂: difference function

4. Focus: Expected Disagreement



$De = \left(\frac{1}{n-1}\right) \sum_{v1=1,v2=1}^{V} n_{v1}\, n_{v2}\, \partial$

1. Iterate through all possible label pairs: 1,2 / 1, 3 / 1,4 / … / 4,5
2. For e.g. 1,2: calculate number of ways the pair 1,2 can be made 3*2
3. Weigh this product by the difference function (e.g. $(1-2)^2 = 1^2$): 3*2*1
4. Sum up all of these products for all label pairs
5. Normalize: divide sum by number of annotation samples minus one

→ Takeaway: more categories – more products/higher weights – higher expected disagreement; intuition: less labels, higher chance of selecting the same label by chance

5. Summary

$$\alpha = 1 - \frac{D_o}{D_e}$$

1	perfect agreement	Do: low if few disagreements De: high, if many classes and labels are equally distributed
0	agreement no better than chance	
-1	systematic disagreement	