Neutrino Mass Limit

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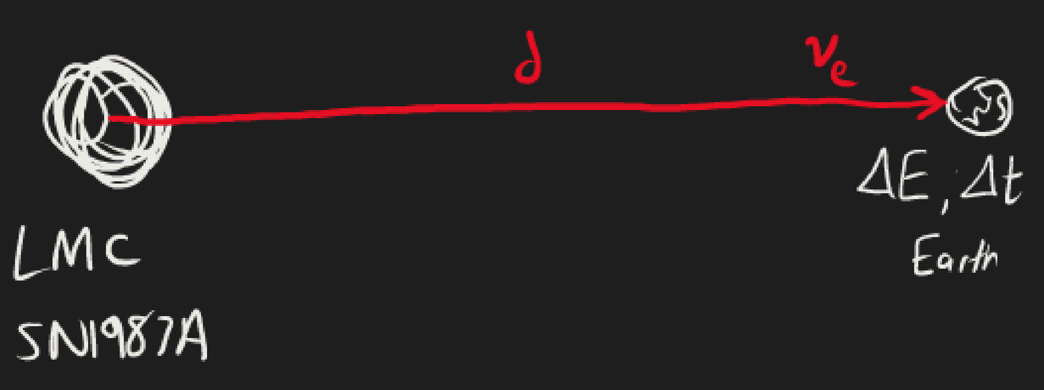

What governs neutrino propagation?

$$d = vt$$

How to relate this to neutrino energy?

$$E = \sqrt{m^2c^4 + p^2c^2}, \quad p = \gamma m v$$

Start with the expression for total energy.

$$E^2 = m^2c^4 + p^2c^2$$

Solve for momentum.

$$\frac{E^2}{c^2} = m^2c^2 + p^2$$

$$p^2 = \frac{E^2}{c^2} - m^2c^2$$

Factor out an $\frac{E^2}{c^2}$

$$p^2 = \left(\frac{E^2}{c^2}\right)\left(1 - \frac{m^2c^4}{E^2}\right)$$

Square-root.

$$p = \frac{E}{c}\sqrt{1 - \frac{m^2c^4}{E^2}}$$

Recognize since $mc^2 \ll E$, $m^2c^4 \ll E^2$

Use the binomial approximation:

$$(1-x)^{1/2} \approx 1 - \frac{x}{2} \quad \text{for } x \ll 1$$

$$\therefore p \approx \frac{E}{c}\left(1 - \frac{m^2c^4}{2E^2}\right)$$

Now use the expression for momentum:

$$p = \gamma m v$$

$$v = \frac{p}{\gamma m} = \frac{E}{m\gamma c}\left(1 - \frac{m^2c^4}{2E^2}\right)$$

Consider the prefactor. Neutrinos are highly relativistic.

$$E = \gamma m c^2$$

$$\frac{E}{mc^2} = \gamma$$

$$\frac{E}{m\gamma c} = \frac{Ec}{m\gamma c^2} = \frac{\gamma c}{\gamma} = c$$

$$\therefore v = c\left(1 - \frac{m^2c^4}{2E^2}\right)$$

Pick some neutrinos with energies E_1 and E_2, velocities v_1 and v_2, arriving at times t_1 and t_2. The time difference is related by

$$\Delta t = d\left(\frac{1}{v_1} - \frac{1}{v_2}\right)$$

$$\Delta t = \frac{d}{c}\left(\frac{1}{1 - \frac{m^2c^4}{2E_1^2}} - \frac{1}{1 - \frac{m^2c^4}{2E_2^2}}\right)$$

Use $\frac{1}{1-x} \approx 1 - x$ for $x \ll 1$

$$\Delta t \approx \frac{d}{c}\left(1 + \frac{m^2c^4}{2E_1^2} - 1 - \frac{m^2c^4}{2E_2^2}\right)$$

$$\Delta t = \frac{d}{2c}\frac{m^2c^4}{E_1^2}\left(1 - \frac{E_1^2}{E_2^2}\right)$$

$$\frac{m^2c^4}{E_1^2} = \left(\frac{2c\Delta t}{d}\right)\left(1 - \frac{E_1^2}{E_2^2}\right)^{-1}$$

$$mc^2 = E_1\left(\frac{2c\Delta t}{d}\right)^{1/2}\left(1 - \frac{E_1^2}{E_2^2}\right)^{-1/2}$$

Use $\Delta t = 13$ s, $E_2 = 40$ MeV, $E_1 = 10$ MeV

$$mc^2 \approx 23 \text{ eV}$$

$$\therefore \boxed{m_{\nu_e}c^2 = 23 \text{ eV}\left(\frac{E_1}{10 \text{ MeV}}\right)\left(\frac{\Delta t}{13 \text{ s}}\right)^{1/2}\left(\frac{50 \text{ kpc}}{d}\right)^{1/2}}$$