

Placing Limits on the Mass of Electron-Neutrinos from SN1987A

Polaris Mentorship Course, Spring 2024

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Several hours before the electromagnetic signal from SN1987A reached the Earth, various ground-based neutrino observatories detected a burst of neutrinos. The Kamiokande II, IMB, and Baksan detectors together measured 25 neutrinos.

The population of neutrinos that were detected ranged in energy $\Delta E = E_2 - E_1$ and were detected over a time range $\Delta t = t_2 - t_1$. Assuming you know the distance to SN1987A, D , place an upper limit on the mass of the electron-neutrino in the following form:

$$m_{\nu_e} = a \text{ eV} \left(\frac{E_1}{b \text{ MeV}} \right)^x \left(\frac{\Delta t}{c \text{ s}} \right)^y \left(\frac{D}{d \text{ kpc}} \right)^z$$

You may use the following assumptions:

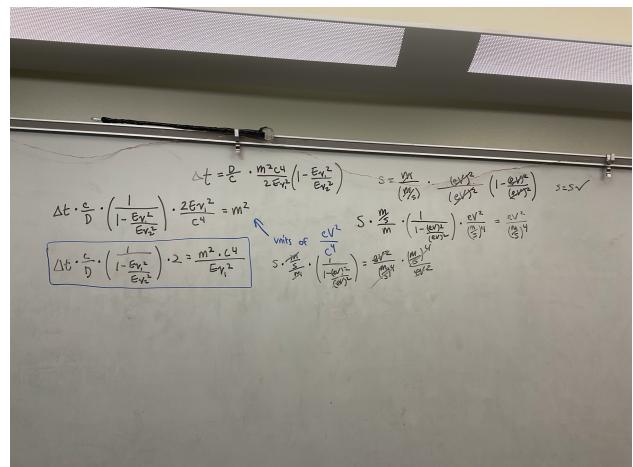
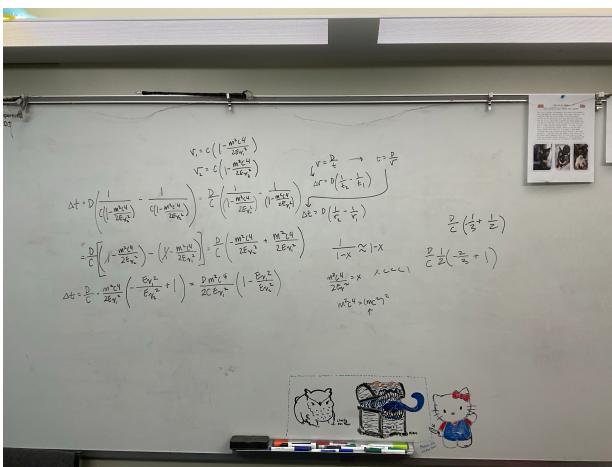
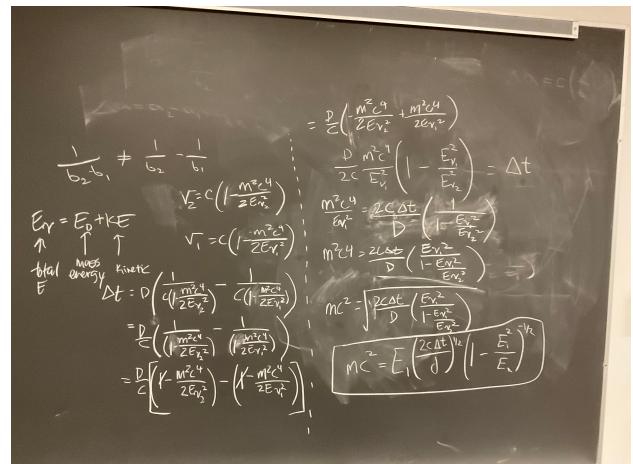
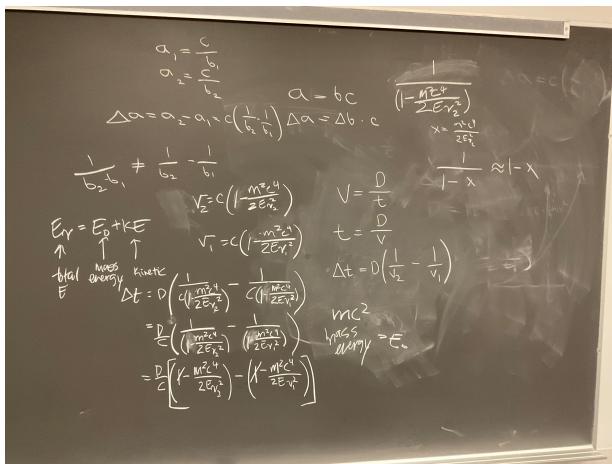
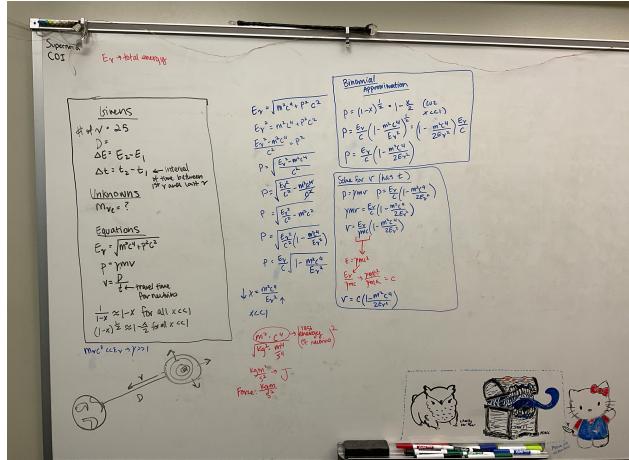
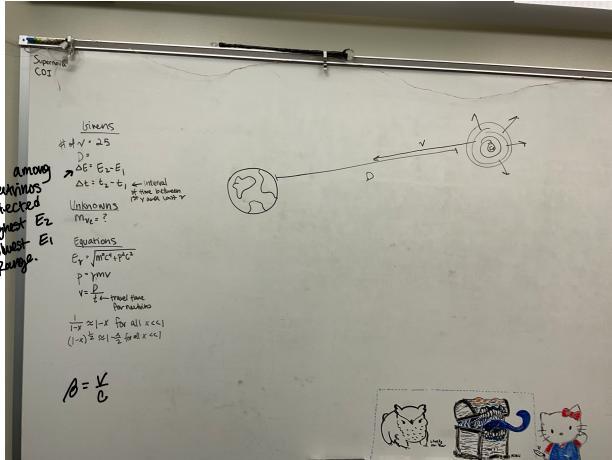
- All neutrinos were emitted at the same time
- The region of neutrino production is negligible compared to the distance d
- Neutrinos are highly relativistic ($m_\nu c^2 \ll E_\nu \rightarrow \gamma \gg 1$)
- Ignore neutrino oscillations

Some helpful expressions that may help you along the way include

- $\frac{1}{1-x} \approx 1-x$ for $x \ll 1$
- $(1-x)^{1/2} \approx 1-\frac{x}{2}$ for $x \ll 1$
-

$$E = \sqrt{m^2 c^4 + p^2 c^2}$$

All electron
neutrinos →



$$\begin{aligned}
& \Delta t = \frac{c}{D} \cdot \frac{m^2 c^2}{E_{V_1}} \left(1 - \frac{E_{V_2}}{E_{V_1}} \right) \quad S = \frac{m}{(\gamma v)} \cdot \frac{E_{V_1}^2 c^2}{D^2} \left(1 - \frac{E_{V_2}^2}{E_{V_1}^2} \right)^{\frac{1}{2}} \quad \text{so } S \propto \sqrt{S} \\
& \Delta t \cdot \frac{c}{D} \cdot \left(\frac{1}{1 - \frac{E_{V_2}}{E_{V_1}}} \right) \cdot \frac{2 E_{V_2}^2}{c^3} = m^2 \quad \text{with } \alpha = \frac{E_{V_1}}{E_{V_2}} \\
& \Delta t \cdot \frac{c}{D} \cdot \left(\frac{1}{1 - \frac{E_{V_2}}{E_{V_1}}} \right) \cdot 2 \cdot \frac{m^2 \cdot \alpha^2}{c^3} = S \cdot \frac{m}{(\gamma v)} \cdot \left(\frac{1}{1 - \frac{E_{V_2}^2}{E_{V_1}^2}} \right) \cdot \frac{E_{V_1}^4}{(\gamma v)^2} = \frac{E_{V_1}^2}{(\gamma v)^2} \\
& \Delta t \cdot \frac{c}{D} \cdot \left(\frac{1}{1 - \frac{E_{V_2}}{E_{V_1}}} \right) \cdot 2 \cdot \frac{m^2 \cdot \alpha^2}{c^3} = S \cdot \frac{m}{(\gamma v)} \cdot \left(\frac{1}{1 - \frac{E_{V_2}^2}{E_{V_1}^2}} \right) \cdot \frac{E_{V_1}^4}{(\gamma v)^2} \\
& \rightarrow \left(\Delta t \cdot \frac{c}{D} \cdot \left(\frac{1}{1 - \frac{E_{V_2}}{E_{V_1}}} \right) \cdot 2 \right)^{\frac{1}{2}} = \frac{m^2}{E_{V_2}} + \left(2 \cdot \frac{c}{D} \cdot \frac{\alpha^2}{c^3} \cdot \frac{m^2}{E_{V_1}} \right)^{\frac{1}{2}} = \frac{m^2}{E_{V_2}} \\
& \rightarrow \left(\frac{2 E_{V_2}^2}{c^3} \cdot \left(\frac{1}{1 - \frac{E_{V_2}}{E_{V_1}}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} = \frac{m^2}{E_{V_2}} \quad \text{(1) } E_{V_1} = 7.5 \text{ eV} \quad t_c = 203.5 \quad D = 50 \text{ kpc} \\
& \rightarrow \left(\frac{2 E_{V_2}^2}{c^3} \cdot \left(\frac{1}{1 - \frac{E_{V_2}}{E_{V_1}}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} = \frac{m^2}{E_{V_2}} \quad \text{(2) } E_{V_1} = 19.8 \text{ eV} \quad t_c = 191.5 \text{ s} \\
& \rightarrow \left(\frac{2 E_{V_2}^2}{c^3} \cdot \left(\frac{1}{1 - \frac{E_{V_2}}{E_{V_1}}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} = \frac{m^2}{E_{V_2}} \quad \boxed{\left\{ E_{V_1} \left(\frac{2 E_{V_2}^2}{c^3} \right)^{\frac{1}{2}} \left(\frac{1 - E_{V_2}^2}{E_{V_1}^2} \right)^{\frac{1}{2}} = m^2 \right\}}
\end{aligned}$$

$$m_v c^2 = E_{V_1} \left(\frac{2 c \Delta t}{D} \right)^{\frac{1}{2}} \left(1 - \left(\frac{E_{V_1}}{E_{V_2}} \right)^2 \right)^{-\frac{1}{2}}$$

\downarrow
 $\overbrace{\sqrt{\left(1 - \left(\frac{E_{V_1}}{E_{V_2}} \right)^2 \right)}}^{\text{really small}}$
 $\left(\frac{E_{V_1}}{E_{V_2}} \right)^{\frac{1}{2}} \quad \text{really big}$
 neutrinos w/ big ΔE
 small Δt