

Special Relativity Notes

Energy of Newton $\rightarrow ?$

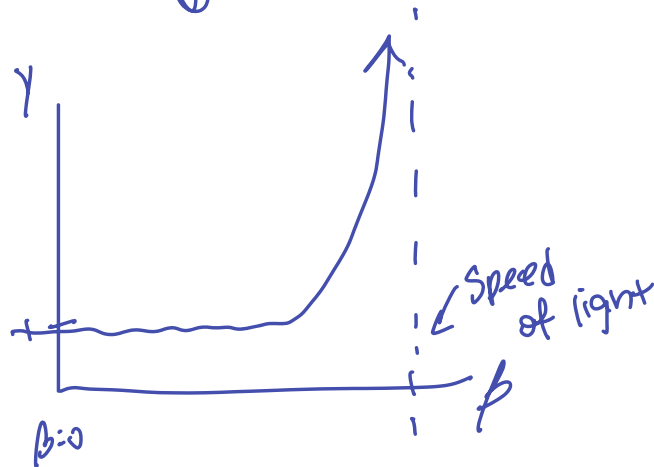
Lorentz Transformations (Motion in x-dir)

$$t' = \gamma(t - \frac{vx}{c^2}) \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \beta = \frac{v}{c}$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$



What to know ab particles

- momentum p
- energy $\rightarrow E_k$
- Force F

Momentum p

Classical: $p = mv$

relativistic: $p = \gamma m v$

Force F

$$p_f - p_i = (F_f - f_i) \Delta t$$

$$F = \frac{dp}{dt}$$

Energy E_k

$$E_k = \frac{1}{2} m v^2$$

$$E_k = \int_0^v F dx = \int_0^v \frac{dp}{dt} dx$$

$$= \int_0^v \frac{d(\gamma m v)}{dt} dx \rightarrow \int_0^v d(\gamma m v) \frac{dx}{dt} \rightarrow \int_0^v \underbrace{v d(\gamma m v)}$$

$$d(\gamma m v) = m \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \cdot dv$$

$$= \int_0^v \gamma m \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} dv = mc^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) = (\gamma - 1) mc^2$$

Total energy E

$$E = E_k + E_0$$

\swarrow
 $= mc^2$

$$E = \gamma mc^2$$

$$E_k = \frac{1}{2} m v^2$$

$$p^2 = (m v)^2 \rightarrow p^2 = m^2 v^2$$

$$\frac{1}{2} \frac{m^2 v^2}{m} = \frac{1}{2} m v^2$$

$$E_k = \frac{p^2}{2m}$$

$$(p^2 + m^2 v^2)^2 \rightarrow p^2 = \gamma^2 m^2 v^2 = \frac{m^2 v^2}{(1 - \frac{v^2}{c^2})}$$

same for v^2 :

$$\gamma = \sqrt{1 + \left(\frac{p}{mc}\right)^2}$$

Plug in for E :

$$E = \sqrt{m^2 c^4 + p^2 c^2}$$

Energy-momentum 4 vector relation