Lossy Source Compression of multiple Gaussian sources

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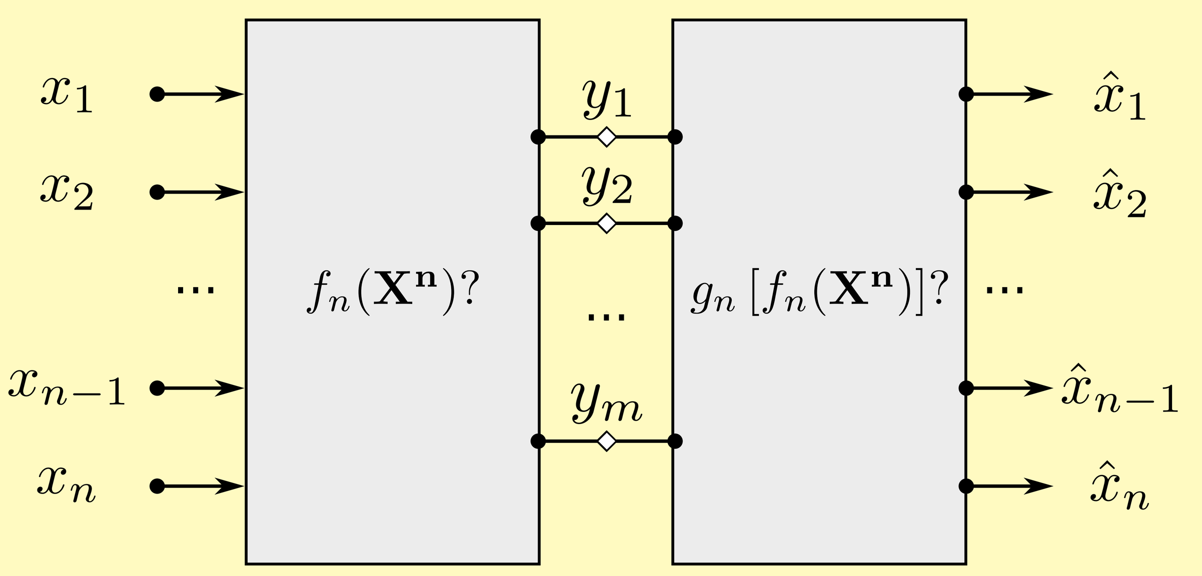
Summary

Rate Distortion Theory ~~is at~~ the foundation of lossy data compression; ~~given some~~ distortion constraint, it ~~gives us a theoretical lower bound corresponding to~~ the minimum rate ~~or minimum amount of entropy that should be communicated over a channel~~. In this poster, we propose to use sparse graphs to encode continuous sources (specifically Gaussian) to their ~~minimum rate for a given distortion~~. First, we show that scalar quantization plus optimal lossy encoding of the binary source does not achieve the rate distortion bound for the continuous source ~~no matter how many bits we use to encode the source with~~. We then ~~show~~ how ~~to use~~ sparse graphs and approximate inference algorithms to directly encode the continuous source into bits.

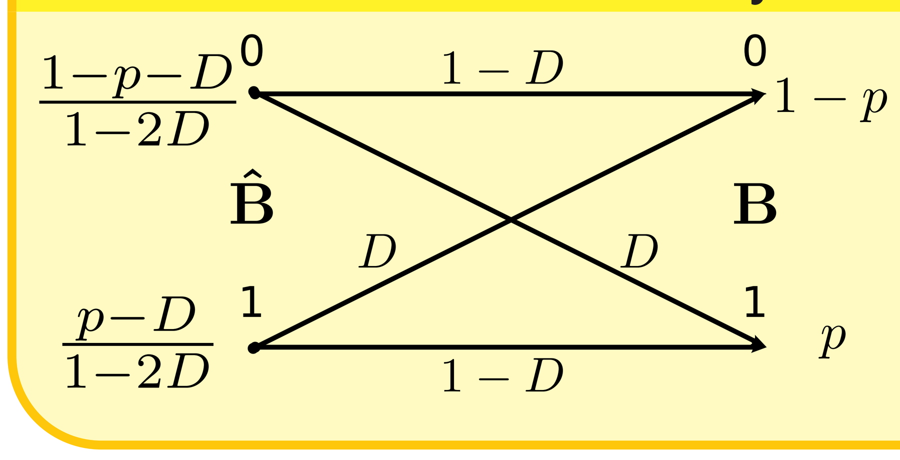
Problem formulation

- **Objective:** Approach the Rate-Distortion (RD) function for Gaussian-distributed sources.
- Given a source distribution $\phi(x)$ and a distortion measure D_{MSE}, find a couple of encoding and decoding functions f_n and g_n that can reach the minimum distorsion D_g for a given rate R_g when $n \to \infty$.



$$D_{MSE} = \frac{1}{n}\sum_{k=1}^{n}(x_k - \hat{x}_k)^2$$

Joint distribution for binary source that reach the RD bound



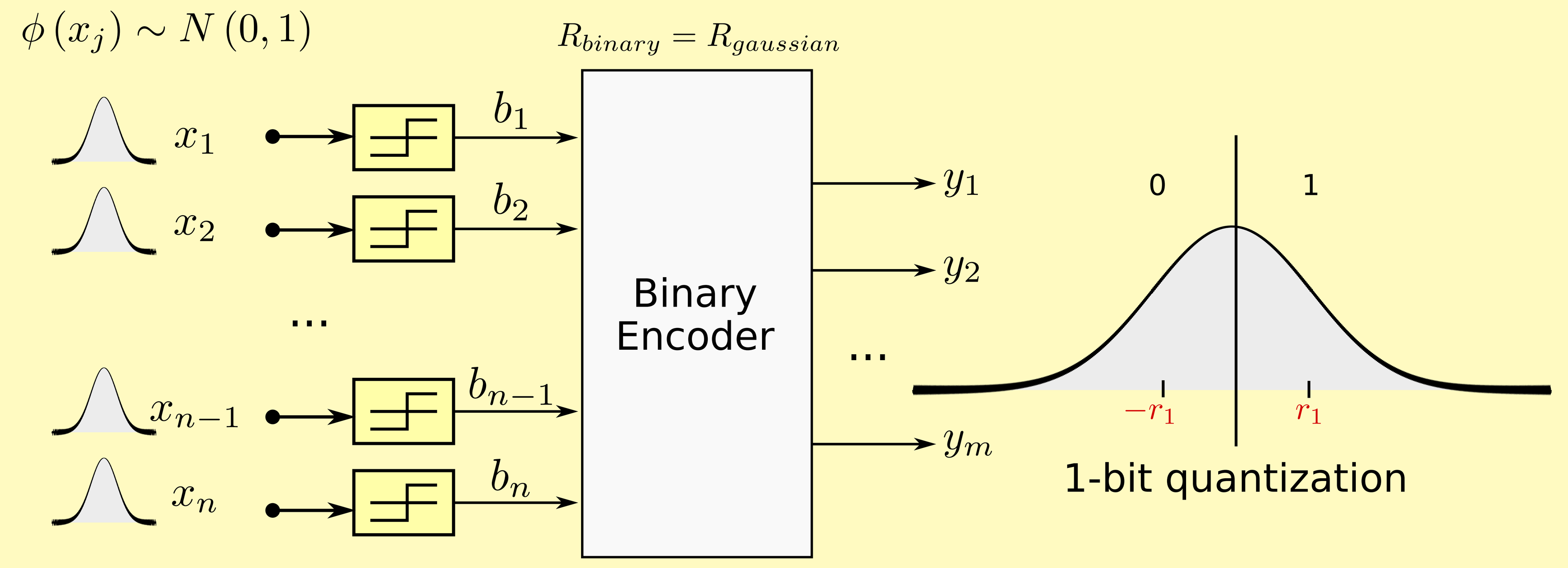
Using Bayes Rule, we have the different error probabilities for each binary symbol:

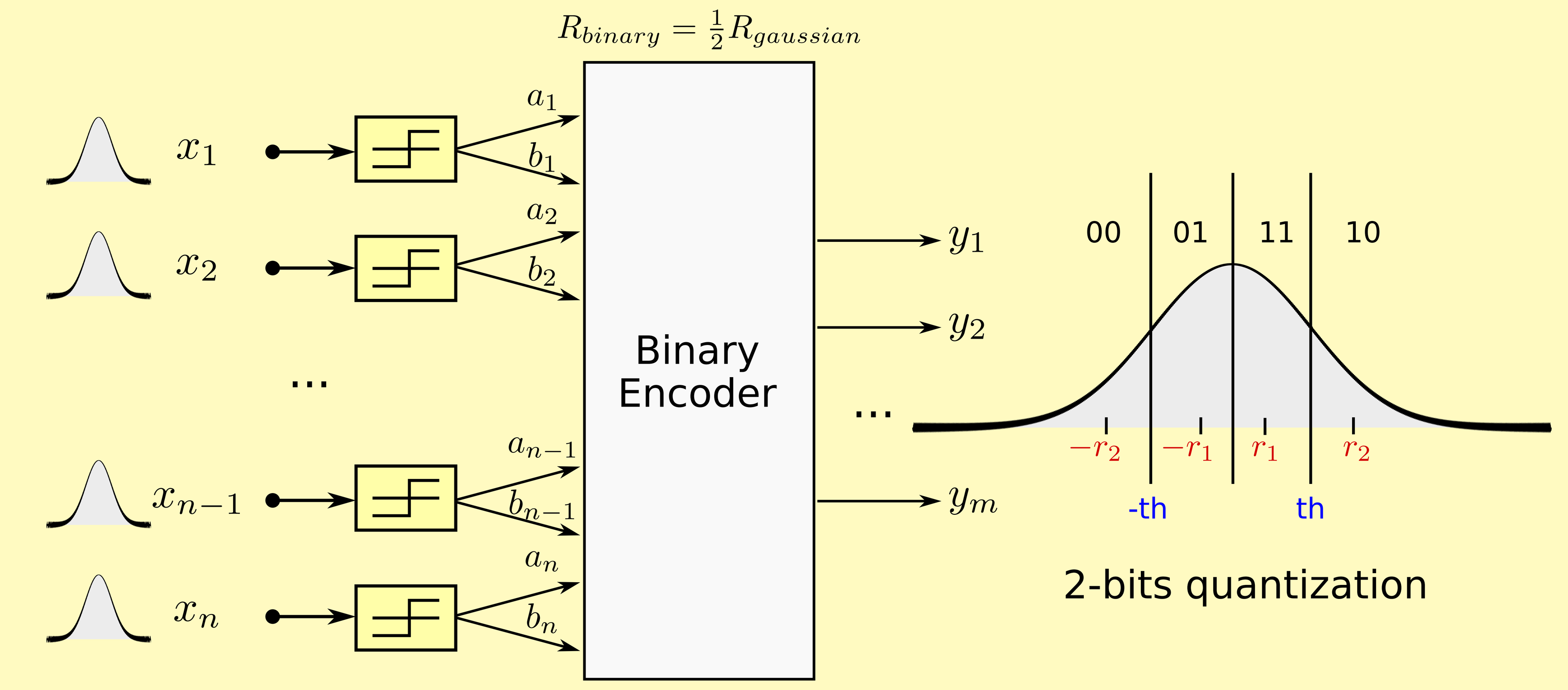
$$P_e^{0\to 1} = \frac{D}{p}\cdot\frac{1-p-D}{1-2D}$$

$$P_e^{1\to 0} = \frac{D}{1-p}\cdot\frac{p-D}{1-2D}$$

Approach 1: Direct quantization + optimal binary compression

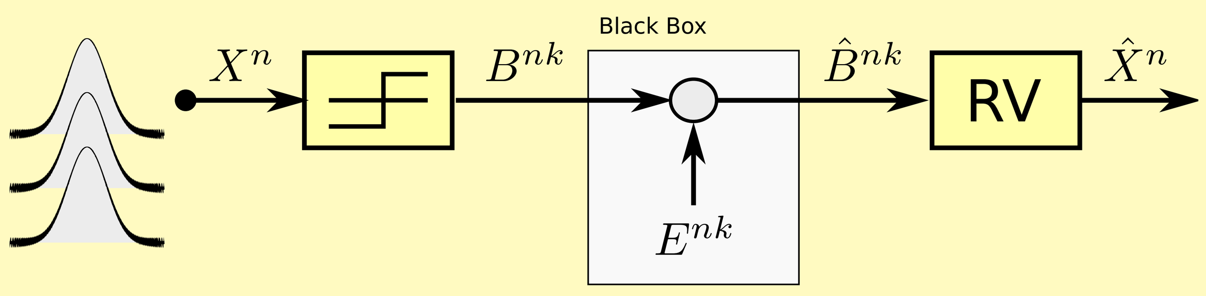
The first idea is to consider 1-bit and 2-bits scalar quantization of the sources together with optimal compression of the binary sources. We need to find the optimal values for the cut th and reconstruction values r_1 and r_2.





Analytical Expression for the RD curve

Let us assume that binary compression can be done optimally. We can then consider the binary processing as a "black box"that returns the encoding bits corrupted by some error probabilities $P_e^{0\to 1}$ and $P_e^{1\to 0}$.



Let us compute the RD function for the 1-bit quantization squeme. The gaussian distorsion D_g can be written as:

$$\begin{aligned} D_g &= (1-P_e)\int_0^{\infty}(x-r_1)^2\phi(x)\,dx + P_e\int_{-\infty}^{0}(x-r_1)^2\phi(x)\,dx \\ &+ (1-P_e)\int_{-\infty}^{0}(x+r_1)^2\phi(x)\,dx + P_e\int_0^{\infty}(x+r_1)^2\phi(x)\,dx \end{aligned} \quad (1)$$

Solving the integrals, we get:

$$D_g = r^2 - \frac{4}{\sqrt{2\pi}}(1-2P_e)\,r_1 + 1 \quad (2)$$

The minimum distorsion is given by:

$$D_g = 1 - \tfrac{2}{\pi}(1-2P_e)^2 \quad \text{with} \quad r_1 = \tfrac{2}{\sqrt{2\pi}}(1-2P_e) \quad (3)$$

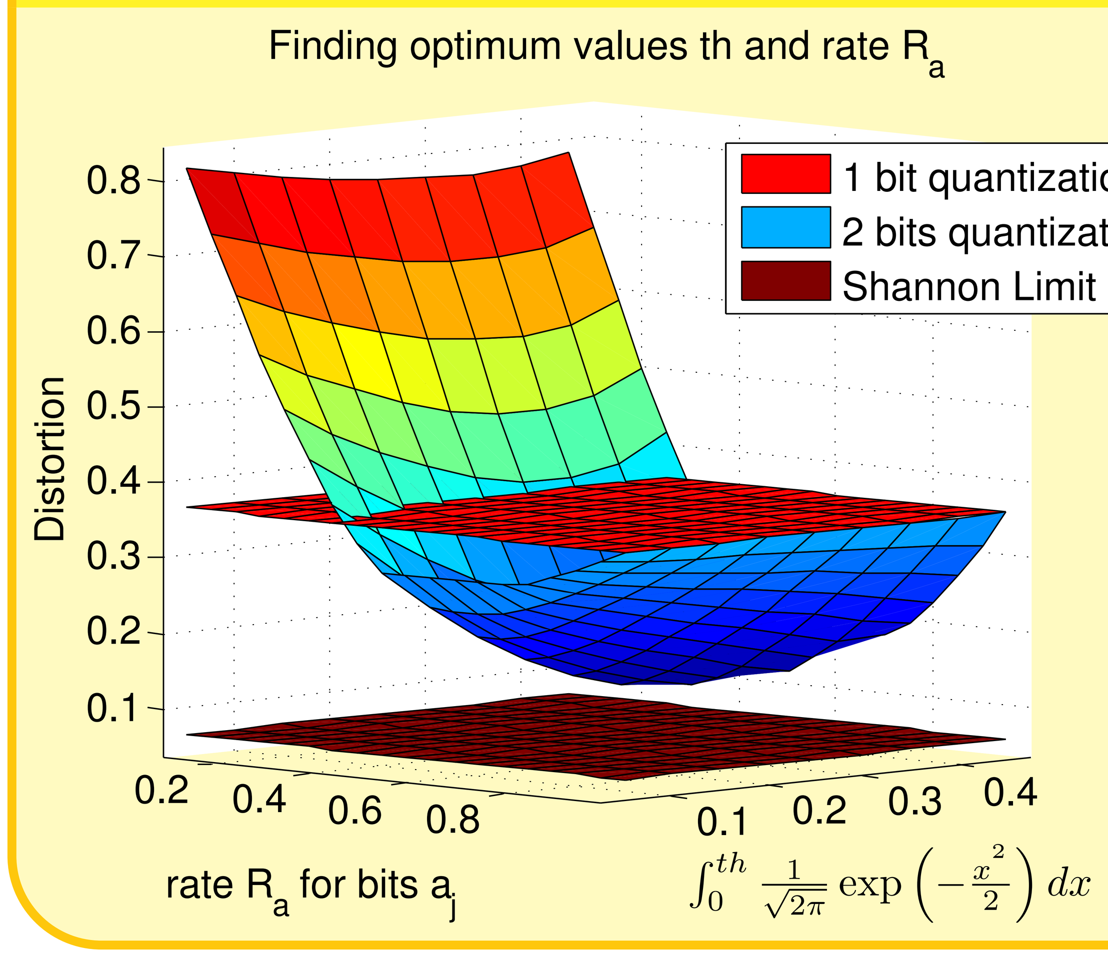
On the other hand, we know the expression for the binary RD function:

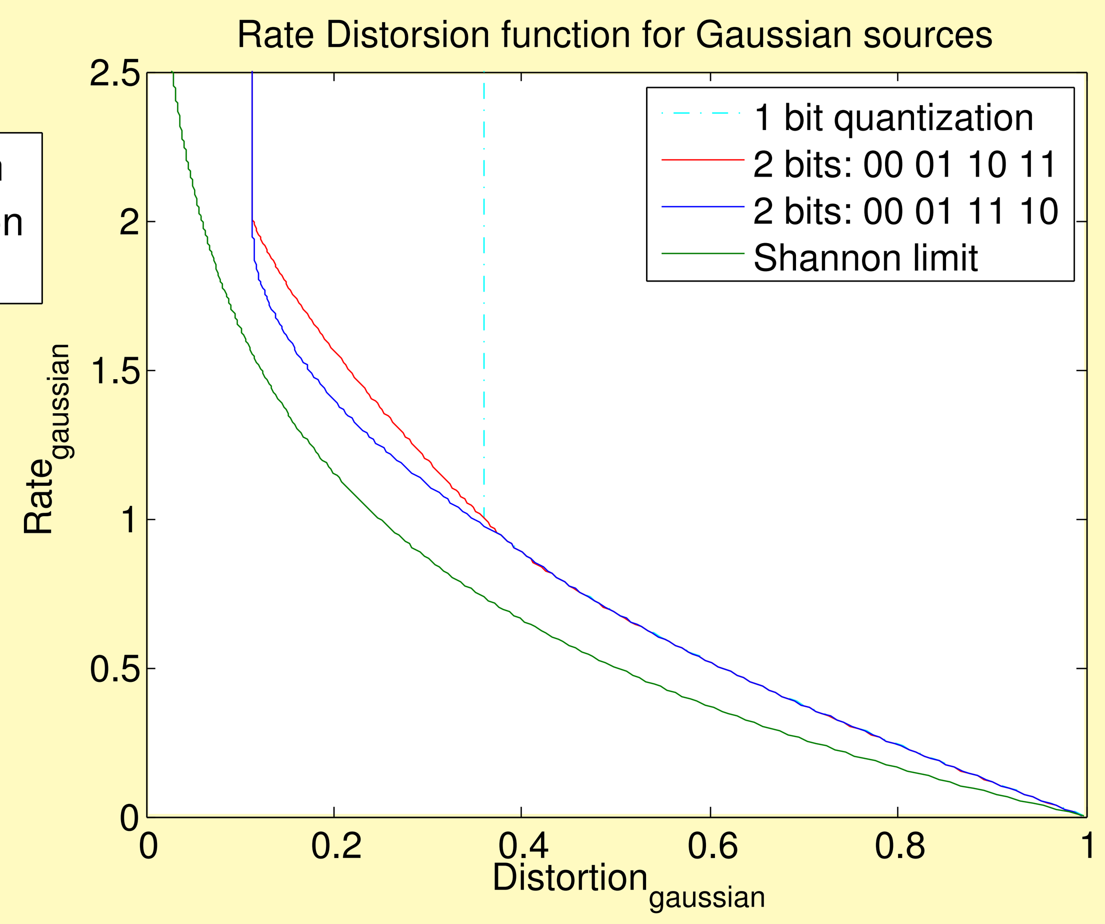
$$R_b(P_e) = \begin{cases} H(p) - H(P_e) & 0 \le P_e \le \min\{p, 1-p\} \\ 0 & P_e > \min\{p, 1-p\} \end{cases} \quad (4)$$

If we substitute the value of P_e, i.e, equation (3) into equation (4), we get the final expression for the gaussian RD function.

$$R_g = 1 + \frac{1}{2}\left(1-\sqrt{\frac{\pi}{2}(1-D_g)}\right)\log_2\left[\frac{1}{2}\left(1-\sqrt{\frac{\pi}{2}(1-D_g)}\right)\right] + \frac{1}{2}\left(1+\sqrt{\frac{\pi}{2}(1-D_g)}\right)\log_2\left[\frac{1}{2}\left(1+\sqrt{\frac{\pi}{2}(1-D_g)}\right)\right]$$

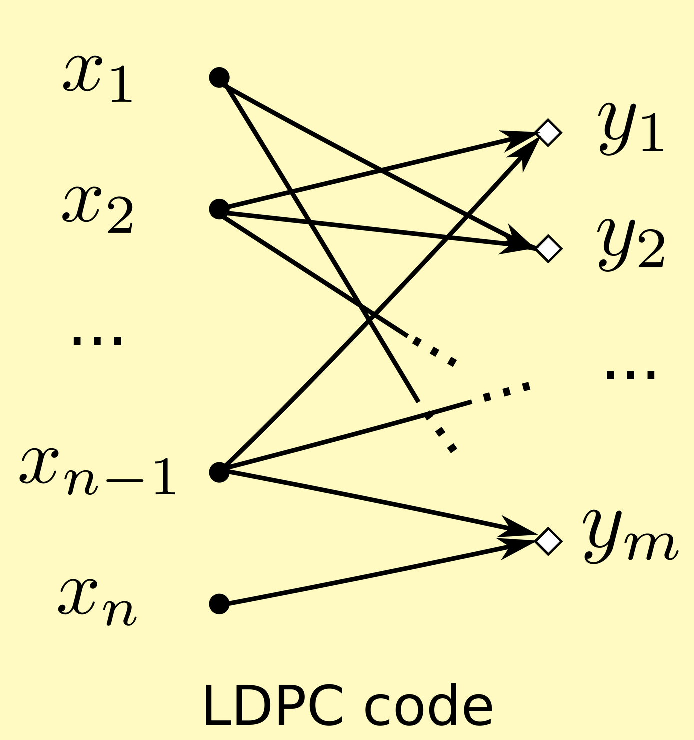
Empirical Validation





Approach 2: Sparse graphs + BP algorithms

Our 2nd approach is to use graphical models such as LDPC or LDGM codes together with message-passing algorithms like ~~BP~~ (Belief Propagation).



Conclusions

- Direct quantization plus optimal lossy encoding of the binary source does not achieve the rate distortion bound
- Sparse graphs together with message-passing algorithms might be the key to approach the RD bound in the case of continuous sources.

Basic References

- Ref 1
- Ref 2
- Ref 3