Project One Template

MAT350: Applied Linear Algebra

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Problem 1

Develop a system of linear equations for the network by writing an equation for each router (A, B, C, D, and E). Make sure to write your final answer as A**x**=**b** where A is the 5x5 coefficient matrix, **x** is the 5x1 vector of unknowns, and **b** is a 5x1 vector of constants.

Solution:

While reviewing the connection network to see if current data rates ensure links aren't at risk of reaching capacity, we are reviwing data transmitted from a sender to a receiver over a network of 5 different routers labled A-E. Connections and data rates are labled X1-X5.

Starting with router A, we can see 100mbps are being transmitted from the sender. Looking at the connections being output from router A, we can see an X1, X2, and X2. As an equation this would be: X1 + 2(X2) = 100

(note: reformat to get one sided equation, input will be left side, output will be right side.)

Following this same format for the all of the connections, we'll get:

A) 100 = X1 + X2 + X2

X1 + 2(X2) = 100

B) X1 + X2 = X3 + X5

X1 + X2 - X3 - X5 = 0

C) 50 + X2 = X3 + X5

X2 - X3 - X5 = -50

D) X4 + X5 = X2 + 120

- X2 + X4 + X5 = 120

E) X2 + X3 + X5 = X4

X2 + X3 - X4 + X5 = 0

<u>Coefficient Matrix Ax = b form</u>

(A = 5x5 Coefficient Matrix | x = 5x1 vectors unknown | b = 5x1 vector of constants)

Matrix A = [1 2 0 0 0;

1 1 -1 0 -1;

0 1 -1 0 -1;

0 -1 0 1 1;

0 1 1 -1 1;]

x = [x1; x2; x3; x4; x5]

b = [100; 0; -50; 120; 0]

Problem 2

Use MATLAB to construct the augmented matrix [A **b**] and then perform row reduction using the rref() function. Write out your **reduced matrix and identify the free and basic variables of the system**.

Solution:

```
% Create matrix A
A = [1 2 0 0 0; 1 1 -1 0 -1; 0 1 -1 0 -1; 0 -1 0 1 1; 0 1 1 -1 1]
```

```
A = 5×5
     1     2     0     0     0
     1     1    -1     0    -1
     0     1    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
% Create matrix b
b = [100; 0; -50; 120; 0]
```

```
b = 5×1
   100
     0
   -50
   120
     0
```

```
% Create augmented matrix [A b]
Ab = [A b]
```

```
Ab = 5×6
     1     2     0     0     0   100
     1     1    -1     0    -1     0
     0     1    -1     0    -1   -50
     0    -1     0     1     1   120
     0     1     1    -1     1     0
```

```
% Following 1.6 MatLab:Reduce Matrices
% Perform row reduction using rref() and store as rowreducedAB
[rowreducedAb , pivotvarsAb] = rref(Ab)
```

```
rowreducedAb = 5×6
     1     0     0     0     0    50
     0     1     0     0     0    25
     0     0     1     0     0    30
     0     0     0     1     0   100
     0     0     0     0     1    45
pivotvarsAb = 1×5
```

```
     1     2     3     4     5
```

```
% Basic variables are variables that correspond to pivot columns.
% All remaining variables are free variables.

% Following 1.8 MatLab: Augmented Matrices to find number of free
% variables.
% Size command to find number of variables in system and store in numvars.
[numeqns, numvars] = size(A)
```

```
numeqns = 5
numvars = 5
```

```
% Size command to find number of pivot variables, then store in
% numpivotvars
[numrows, numpivotvars] = size(pivotvarsAb)
```

```
numrows = 1
numpivotvars = 5
```

```
% Use subtraction to find free vars in solution, store in numfreevars
numfreevars = numvars - numpivotvars
```

```
numfreevars = 0
```

```
% There are 5 basic variables and 0 free variables.
```

Problem 3

Use MATLAB to **compute the LU decomposition of A**, i.e., find A = LU. For this decomposition, find the transformed set of equations L**y** = **b**, where **y** = U**x**. Solve the system of equations L**y** = **b** for the unknown vector **y**.

Solution:

```
%Following 2.12.1 MATLAB: LU Decomposition
%Use lu() to find LU decomposition of A
[ L, U] = lu(A)
```

```
L = 5×5
    1.0000         0         0         0         0
    1.0000    1.0000         0         0         0
         0   -1.0000    1.0000         0         0
         0    1.0000   -0.5000    1.0000         0
         0   -1.0000         0   -1.0000    1.0000
U = 5×5
     1     2     0     0     0
     0    -1    -1     0    -1
     0     0    -2     0    -2
     0     0     0     1     1
     0     0     0     0     1
```

```
% LU decomposition used to solve for Ax=b with Ly = b and x = U\y
y = L\b
```

```
y = 5×1
   100
  -100
  -150
   145
    45
```

```
x = U\y
```

```
x = 5×1
    50
    25
    30
   100
    45
```

Problem 4

Use MATLAB to **compute the inverse** of U using the inv() function.

Solution:

```
% Using inv() function to find inverse of U
inv(U)
```

```
ans = 5×5
    1.0000    2.0000   -1.0000         0         0
         0   -1.0000    0.5000         0         0
         0         0   -0.5000         0   -1.0000
         0         0         0    1.0000   -1.0000
         0         0         0         0    1.0000
```

Problem 5

Compute the solution to the original system of equations by transforming **y** into **x**, i.e., compute **x** = inv(U)**y**.

Solution:

```
%Following 2.8.1 MATLAB: Solve Systems of Linear equations Revisited
% Inverse of (U)y = x
x = inv(U)*y
```

```
x = 5×1
    50
    25
    30
   100
    45
```

Problem 6

Check your answer for x_1 using Cramer's Rule. Use MATLAB to compute the required determinants using the det() function.

Solution:

```
% Following 3.16 MATLAB: Cramer's Rule
% Use Cramers rule to check answer for x1
A1 = A
```

```
A1 = 5×5
     1     2     0     0     0
     1     1    -1     0    -1
     0     1    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
% Replace column 1 in A1 with column vector of constants b
A1(:,1)=b
```

```
A1 = 5×5
   100     2     0     0     0
     0     1    -1     0    -1
   -50     1    -1     0    -1
   120    -1     0     1     1
     0     1     1    -1     1
```

```
% Find solution using ratios of determinants.
x = det(A1)/det(A)
```

```
x = 50
```

Problem 7

The Project One Table Template, provided in the Project One Supporting Materials section in Brightspace, shows the recommended throughput capacity of each link in the network. Put your solution for the system of equations in the third column so it can be easily compared to the maximum capacity in the second column. In the fourth column of the table, provide recommendations for how the network should be modified based on your network throughput analysis findings. The modification options can be No Change, Remove Link, or Upgrade Link. In the final column, explain how you arrived at your recommendation.

Solution:

*Fill out the table in the original project document and export your table as an image. Then, use the **Insert** tab in the MATLAB editor to insert your table as an image.*

Network Link	Recommended Capacity (Mbps)	Solution	Recommendation	Explanation
x_1	60	50	No Change	The data rate here is not at risk of reaching capacity.
x_2	50	25	Upgrade Link	The data rate here is less than maximum capacity, can be upgraded.
x_3	100	30	Upgrade Link	The data rate here is less than maximum capacity, can be upgraded.
x_4	100	100	Remove Link	This data rate is at capacity and should be removed to reduce chances of failure.
x_5	50	45	No Change	The data rate here is not at risk of reaching capacity