

# Chapter 10

## Frequency Response Techniques

**Figure 10.1**

The HP 35670A Dynamic Signal Analyzer obtains frequency response data from a physical system. The displayed data can be used to analyze, design, or determine a mathematical model for the system.



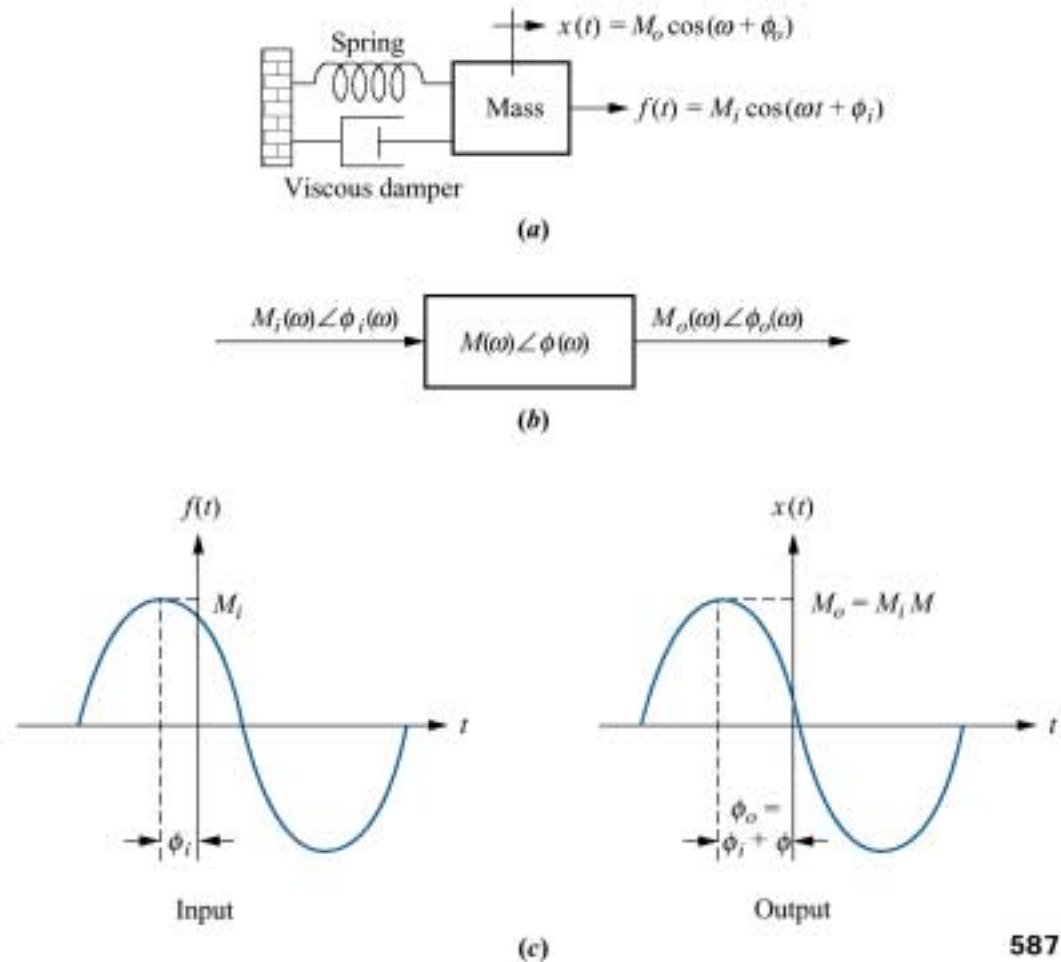
**Figure 10.2**

Sinusoidal frequency response:

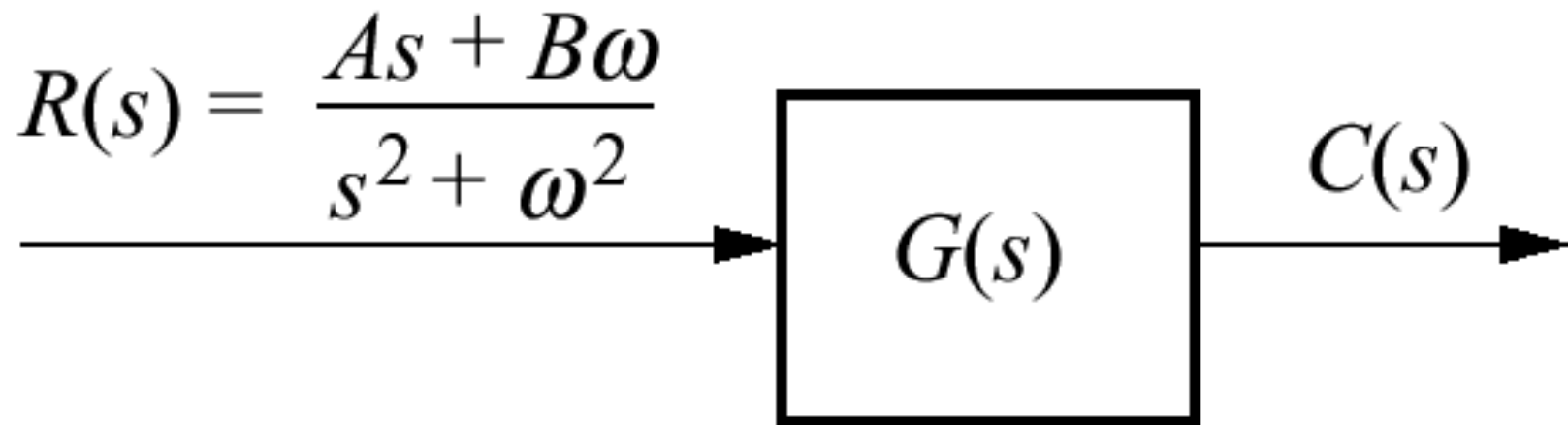
**a.** system;

**b.** transfer function;

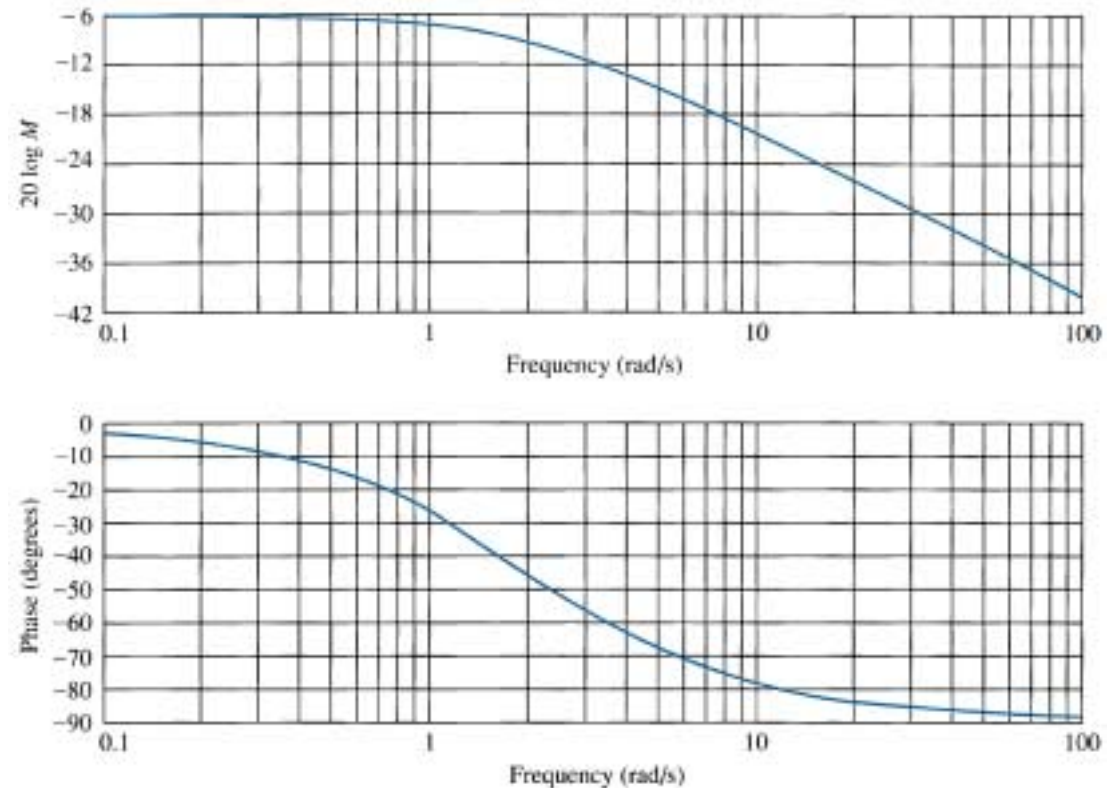
**c.** input and output waveforms



**Figure 10.3**  
System with  
sinusoidal input



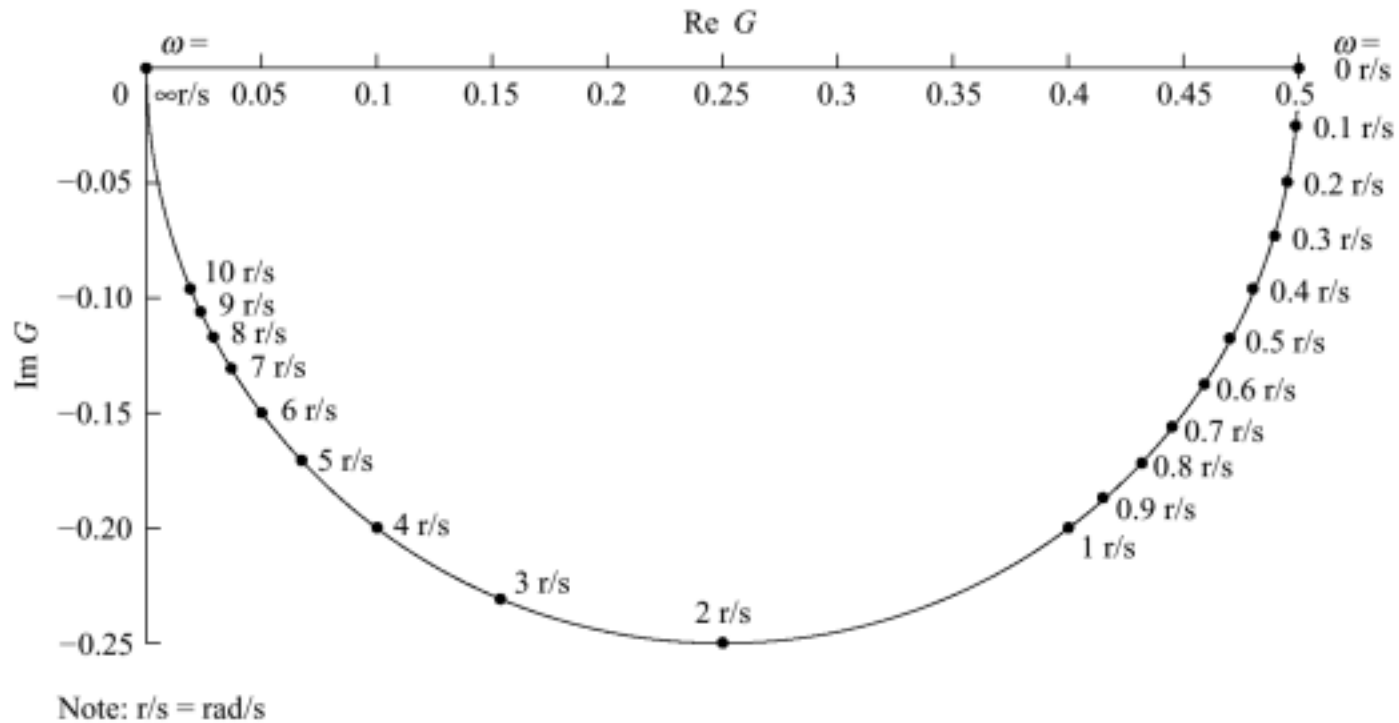
**Figure 10.4**  
Frequency response  
plots for  $G(s) =$   
 $1/(s + 2)$ :  
separate magnitude  
and phase



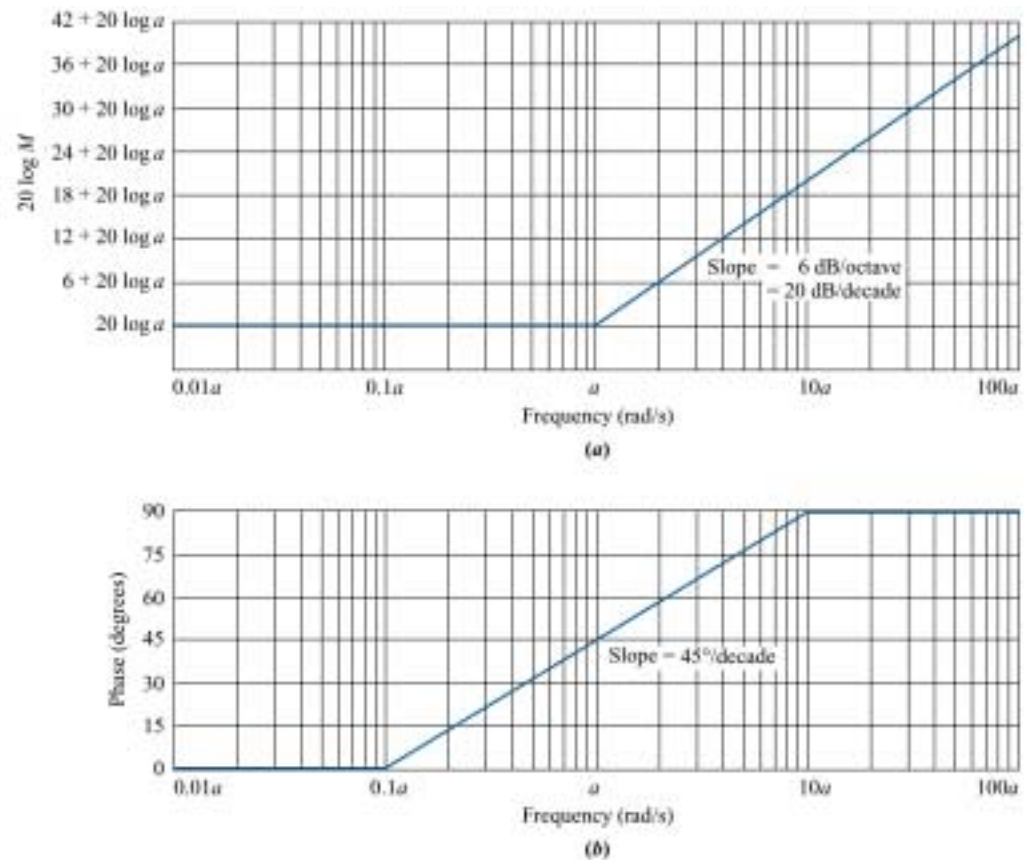
**Figure 10.5**

Frequency response  
plots for  $G(s)$

$= 1/(s + 2)$ : polar plot

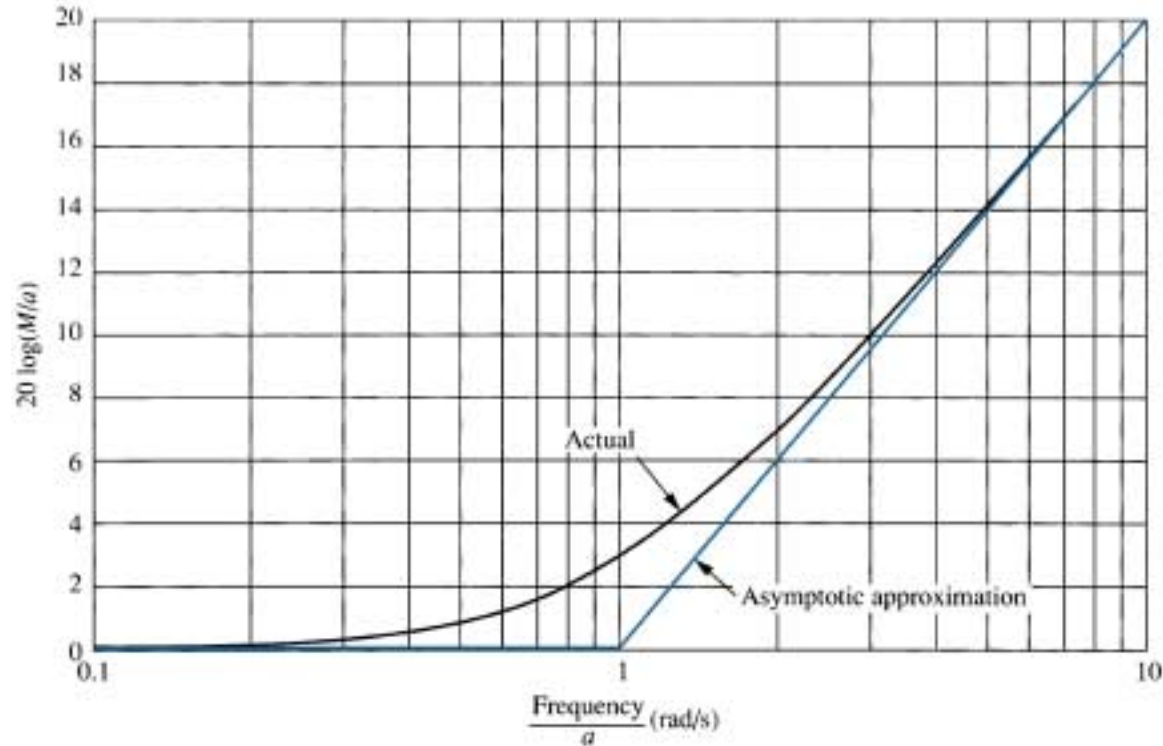


**Figure 10.6**  
 Bode plots of  $(s + a)$ :  
 a. magnitude plot;  
 b. phase plot.



**Figure 10.7**

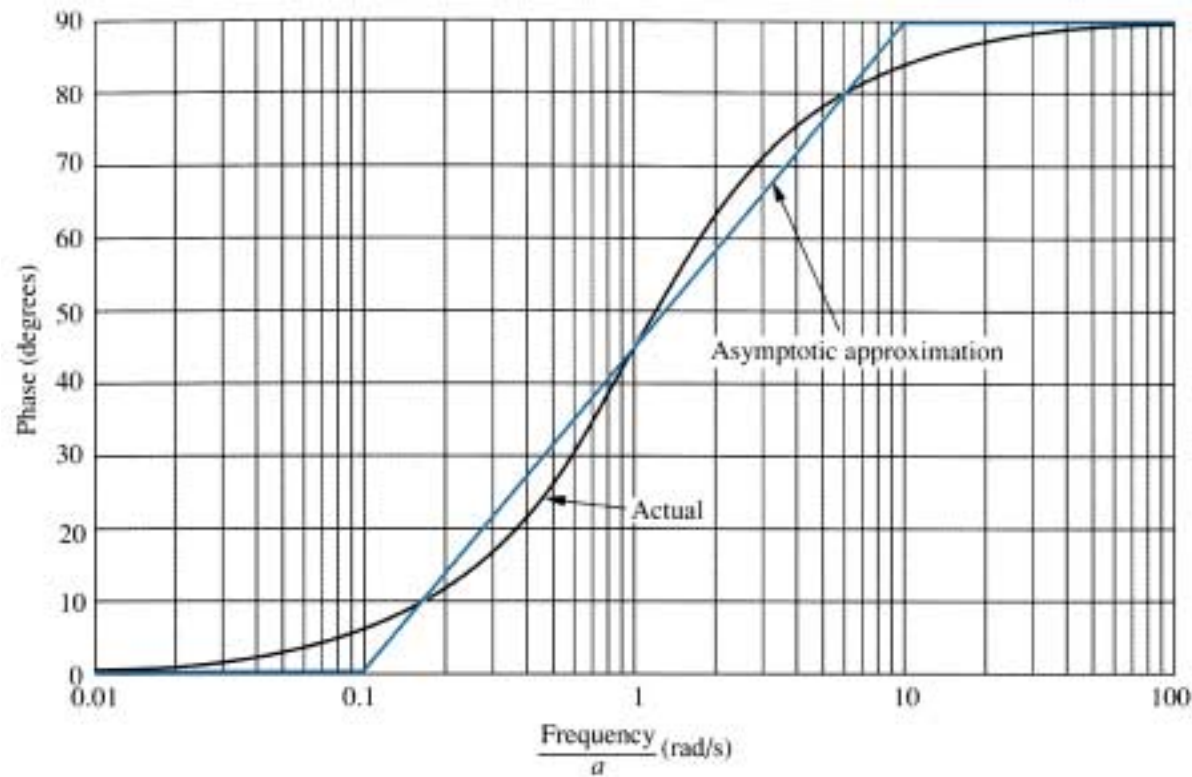
Asymptotic and actual normalized and scaled magnitude response of  $(s + a)$





**Figure 10.8**

Asymptotic and actual normalized and scaled phase response of  $(s + a)$



**Figure 10.9**

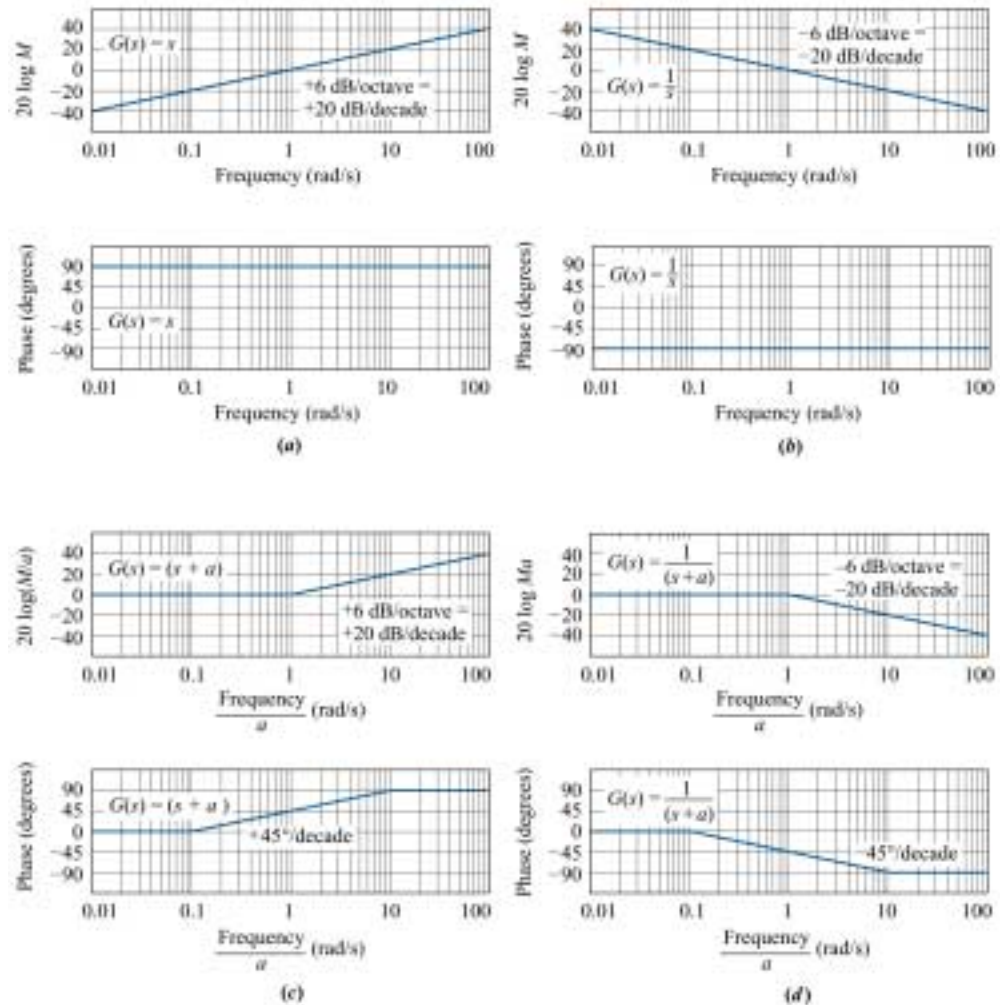
Normalized and scaled  
Bode plots for

a.  $G(s) = s$ ;

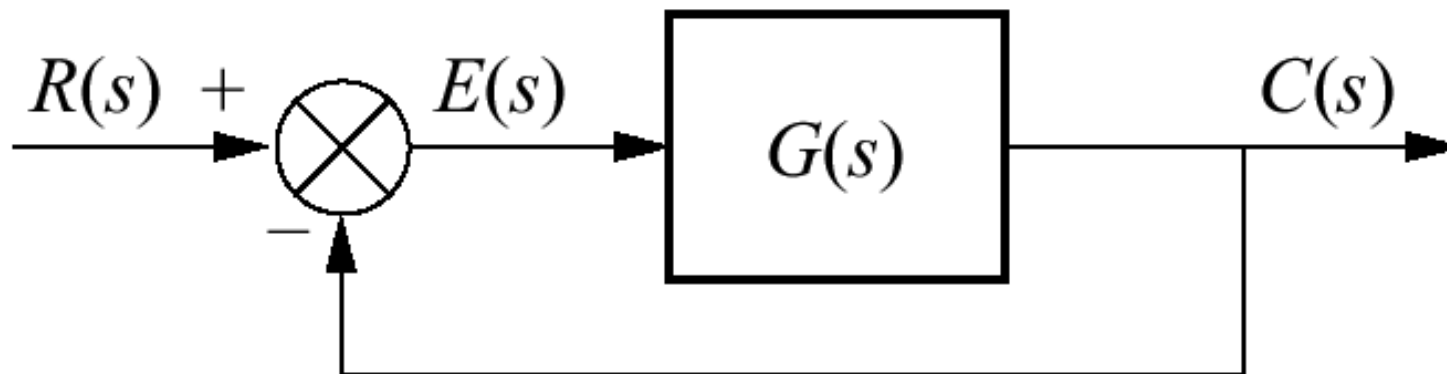
b.  $G(s) = 1/s$ ;

c.  $G(s) = (s + a)$ ;

d.  $G(s) = 1/(s + a)$

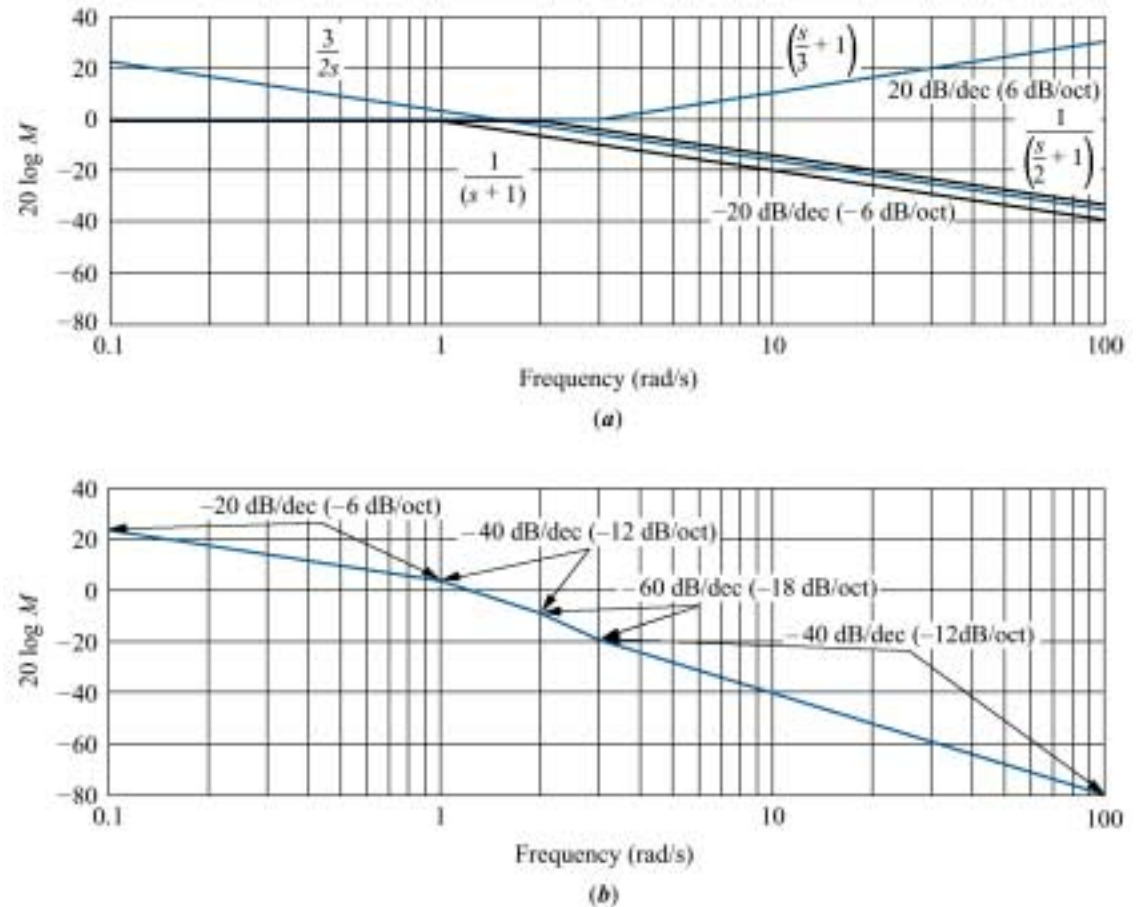


**Figure 10.10** Closed-loop unity feedback system



**Figure 10.11**  
Bode log-magnitude  
plot for Example  
10.2:

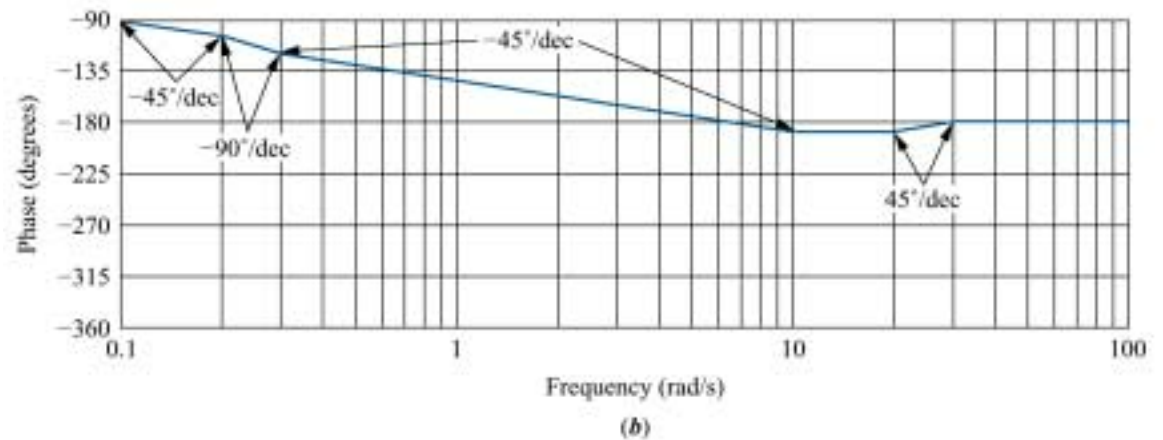
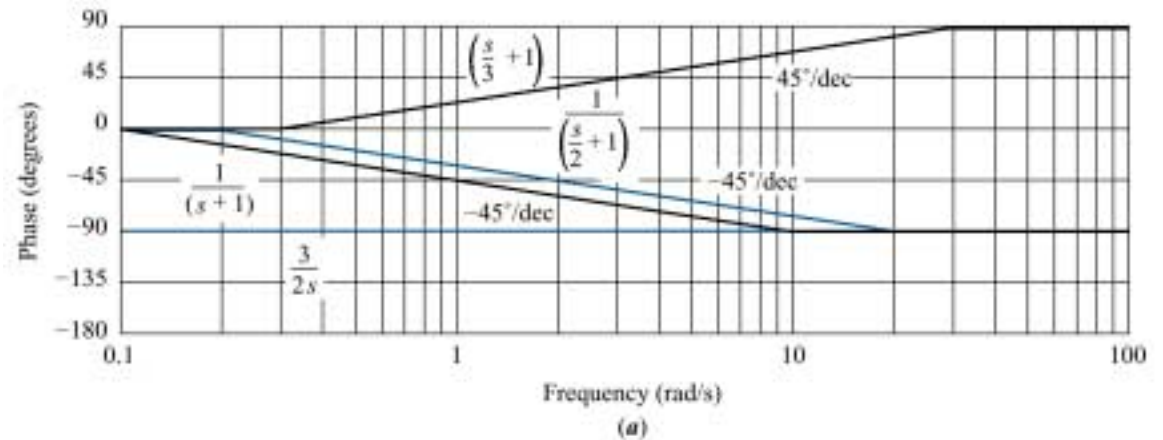
**a.** components;  
**b.** composite



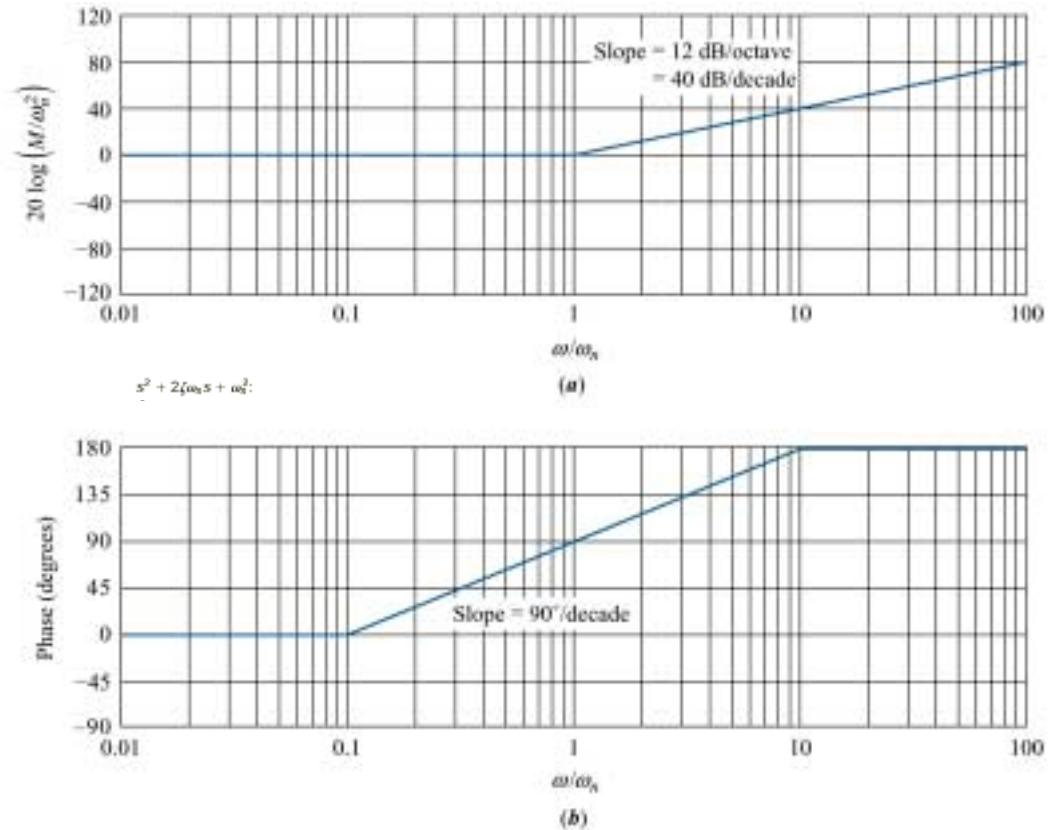
**Figure 10.12**

Bode phase plot  
for Example 10.2:

- a. components;
- b. composite

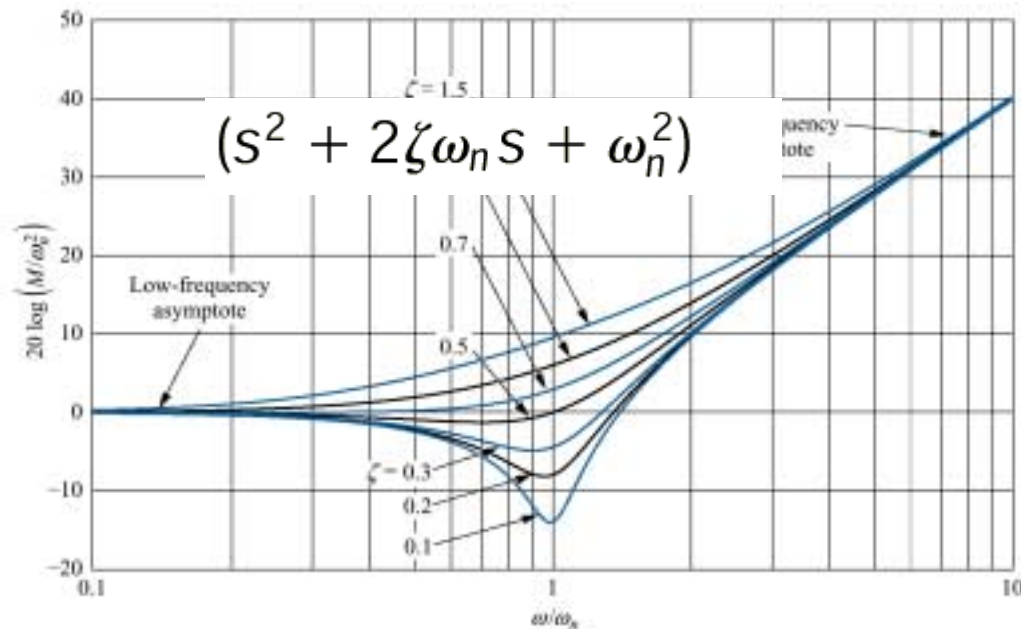


**Figure 10.13**  
Bode asymptotes  
for normalized  
and scaled  $G(s) =$   
 $s^2 + 2\zeta\omega_n s + \omega_n^2$ :  
**a.** magnitude;  
**b.** phase

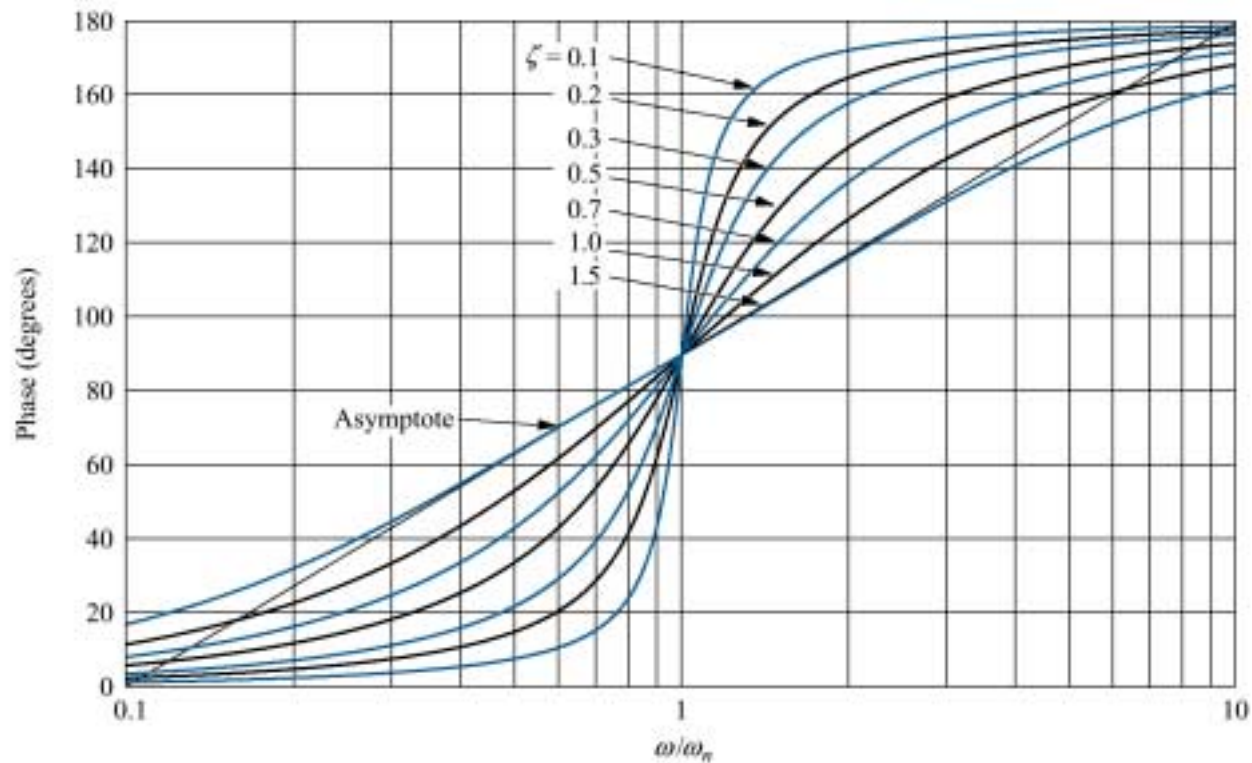


**Figure 10.14**

Normalized and scaled  
log-magnitude response for  
 $(s^2 + 2\zeta\omega_n s + \omega_n^2)$



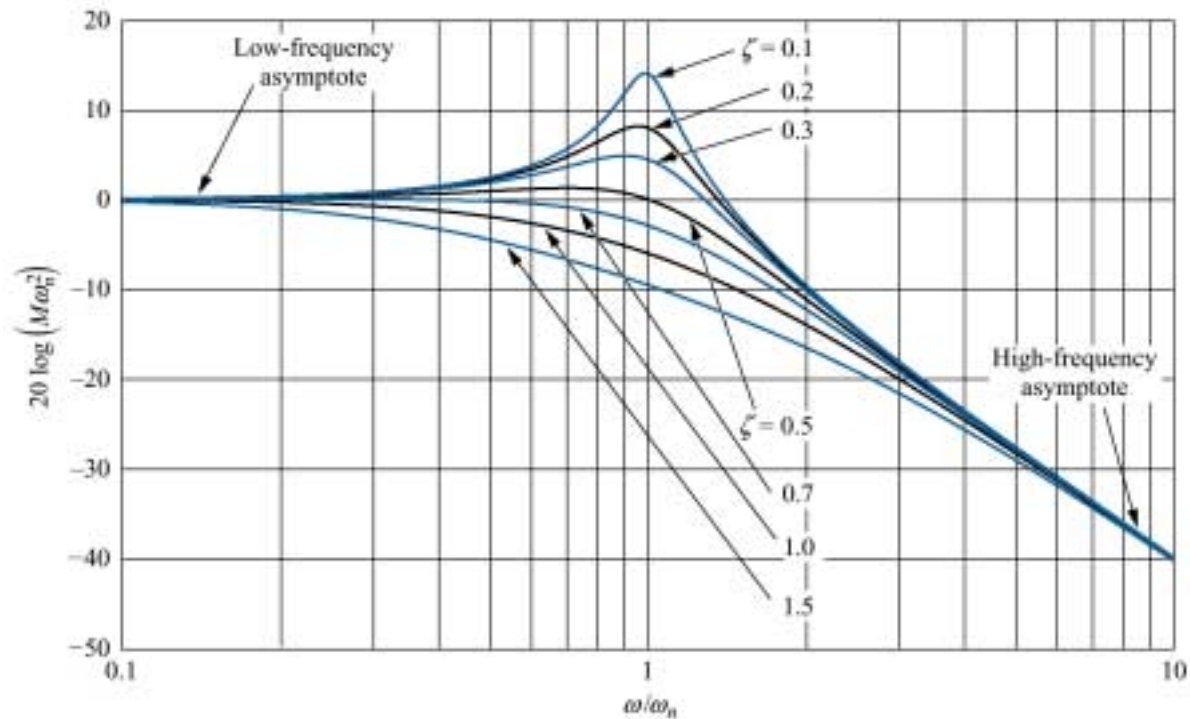
**Figure 10.15**  
Scaled phase response for  
 $(s^2 + 2\zeta\omega_n s + \omega_n^2)$



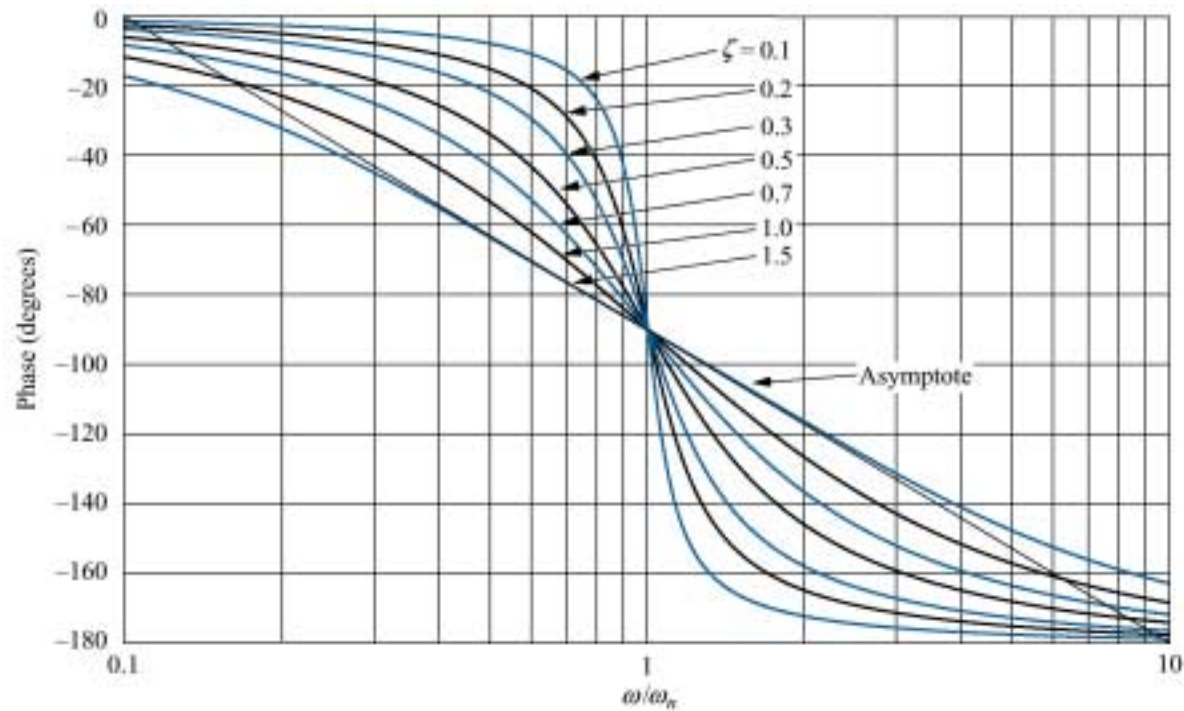


**Figure 10.16**

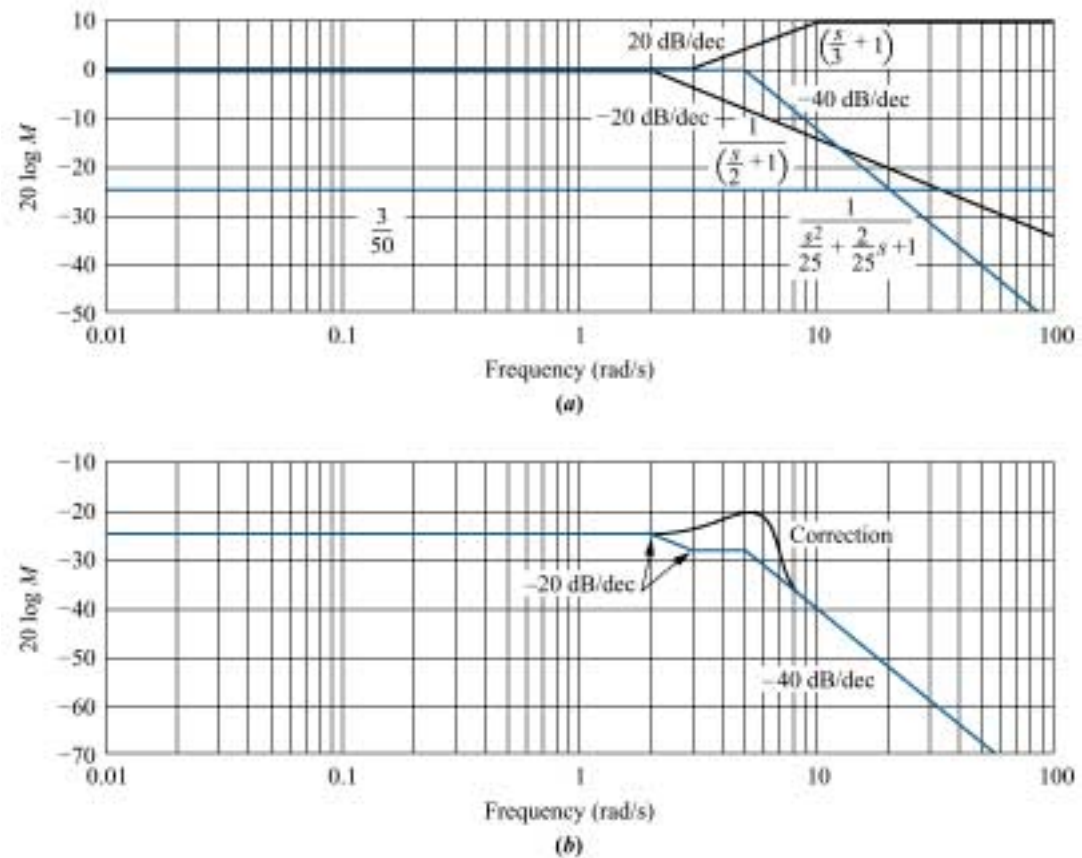
Normalized and scaled log magnitude response for  $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$



**Figure 10.17**  
Scaled phase response for  
 $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$



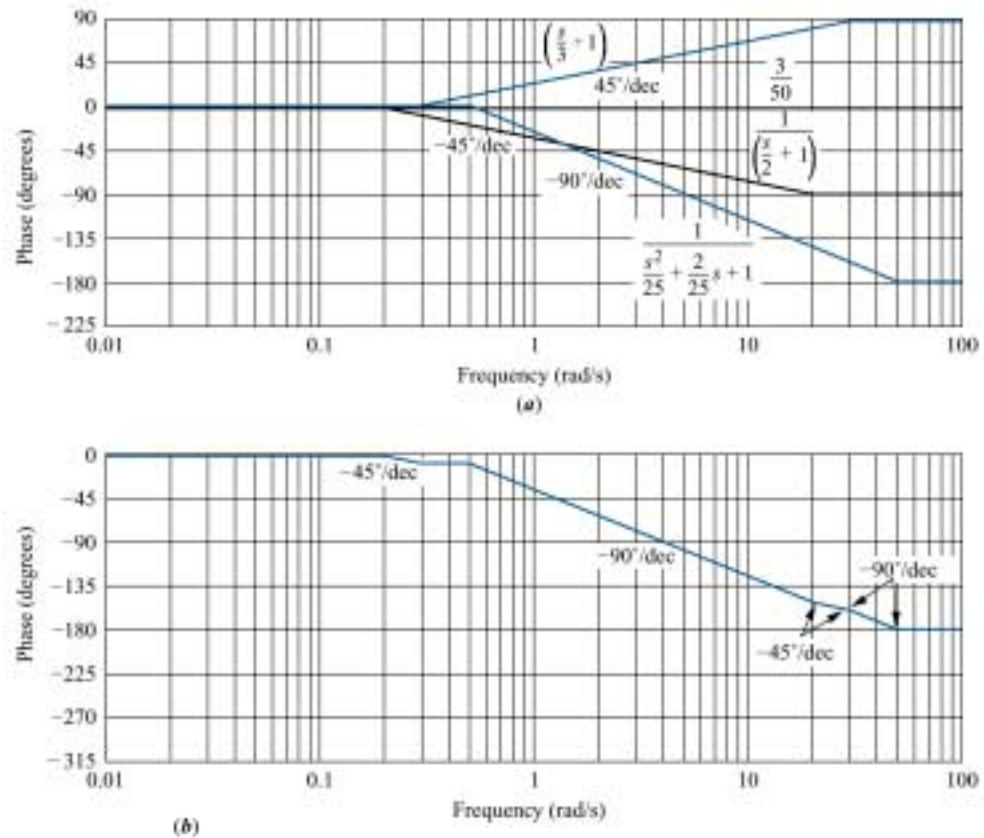
**Figure 10.18**  
 Bode magnitude plot for  $G(s) = (s + 3)/[(s + 2)(s^2 + 2s + 25)]$ :  
**a.** components;  
**b.** composite



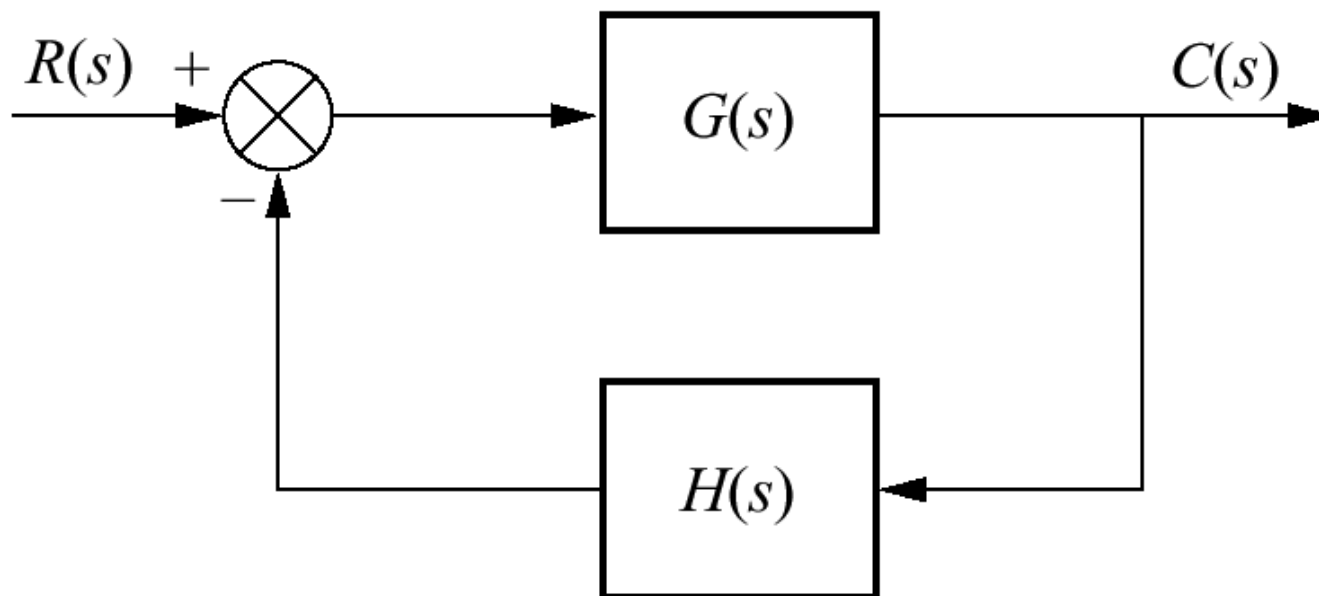
**Figure 10.19**

Bode phase plot for  
 $G(s) = (s + 3)/[(s + 2)(s^2 + 2s + 25)]$ :

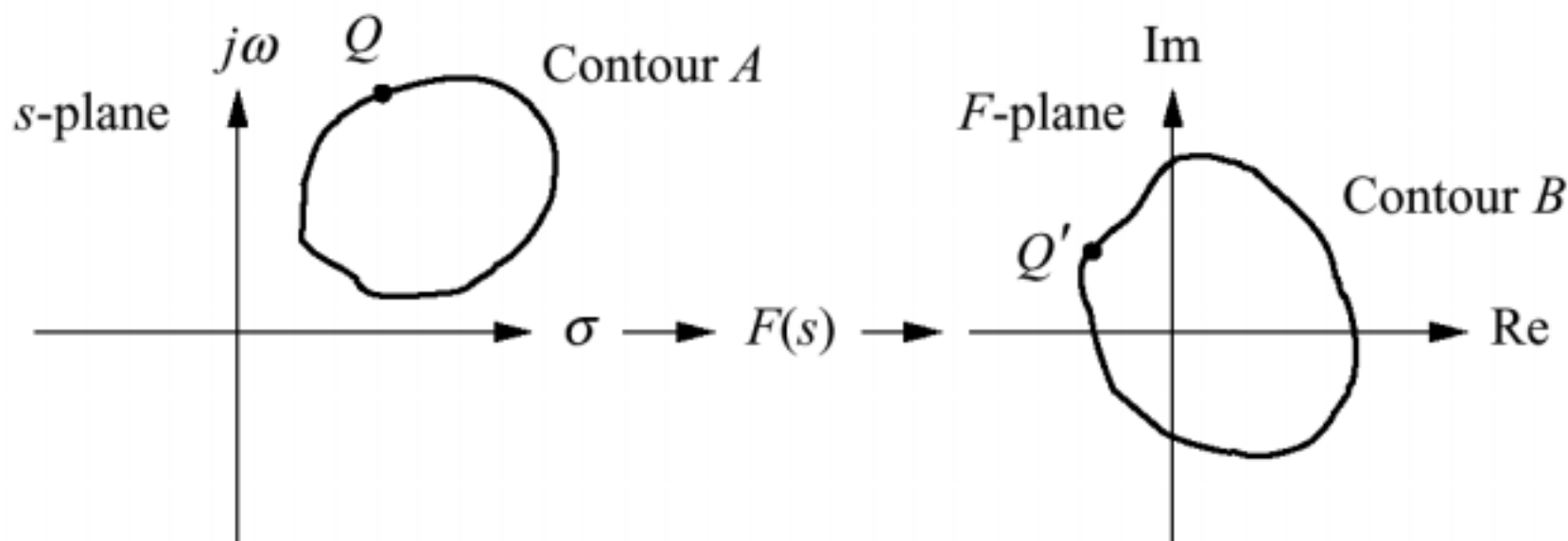
- a. components;
- b. composite



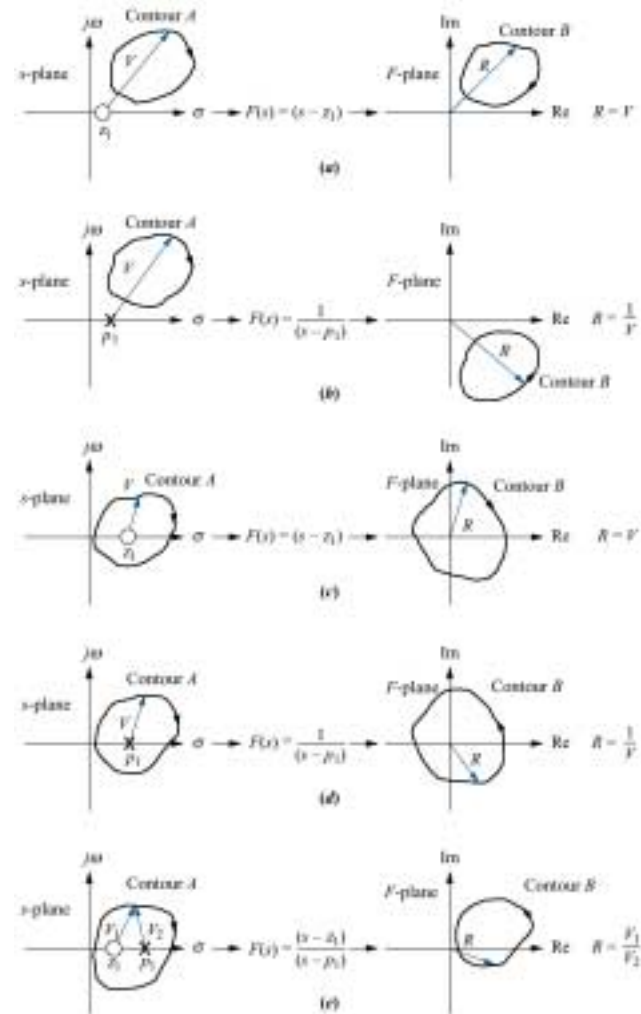
**Figure 10.20**  
Closed-loop control  
system



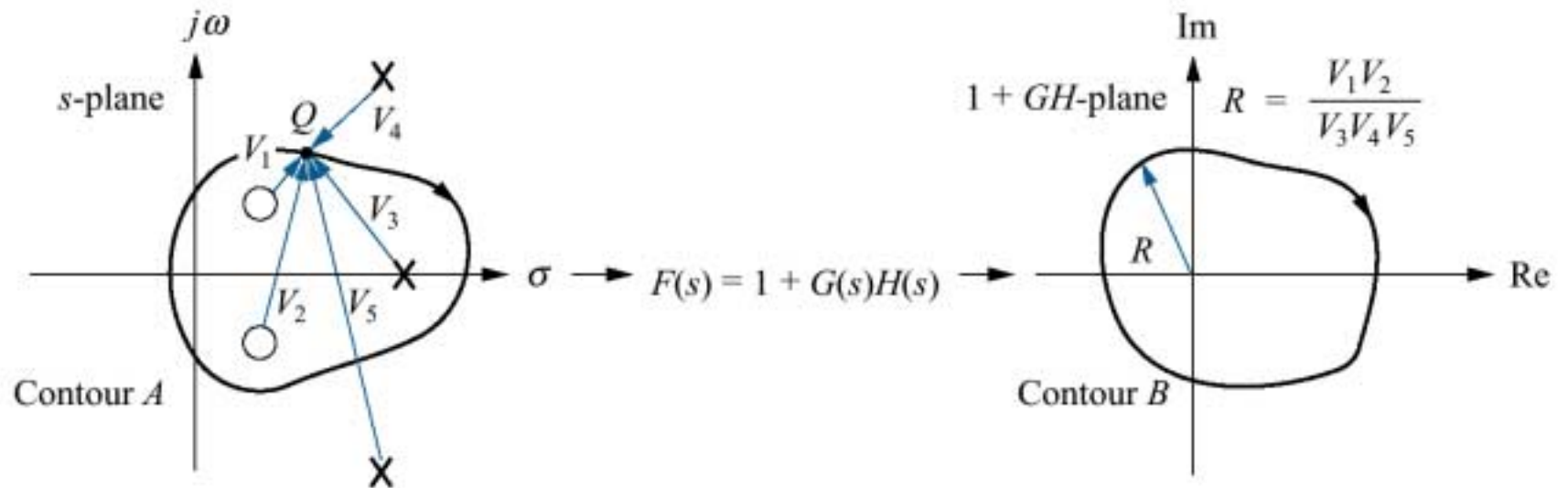
**Figure 10.21**  
Mapping contour  $A$   
through function  $F(s)$   
to contour  $B$



**Figure 10.22**  
Examples of contour mapping



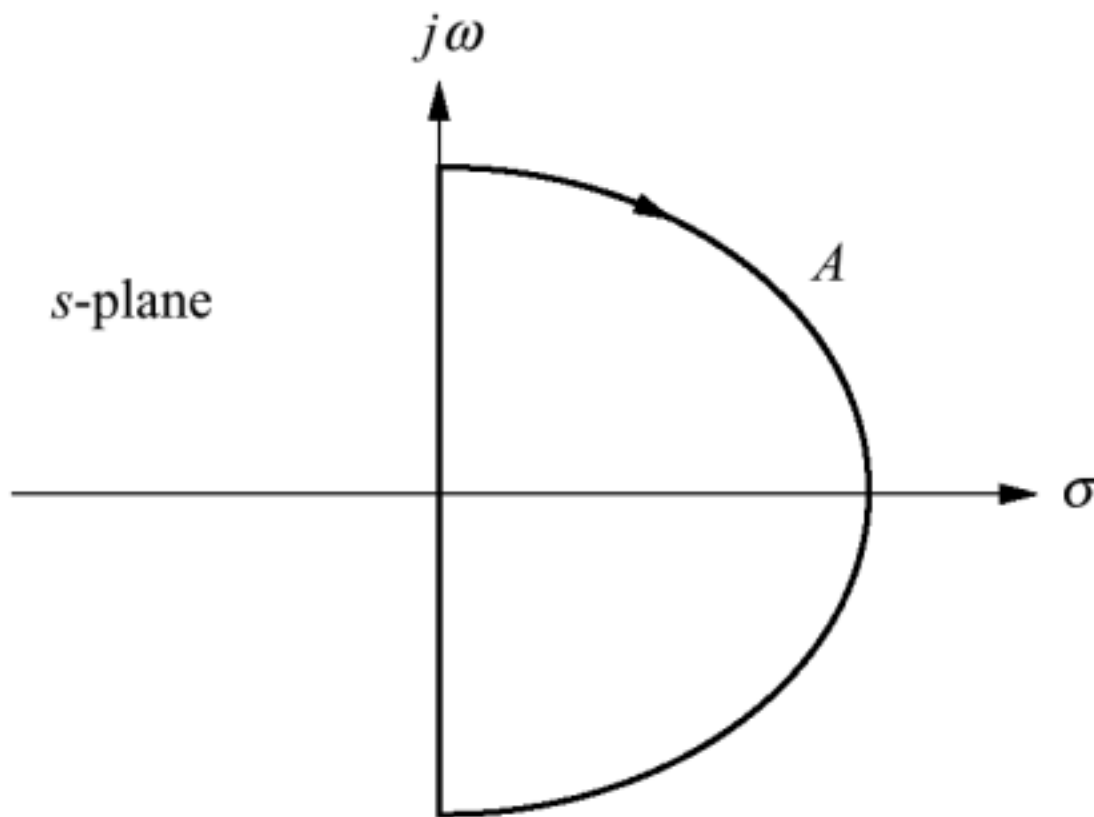
**Figure 10.23**  
Vector representation  
of mapping





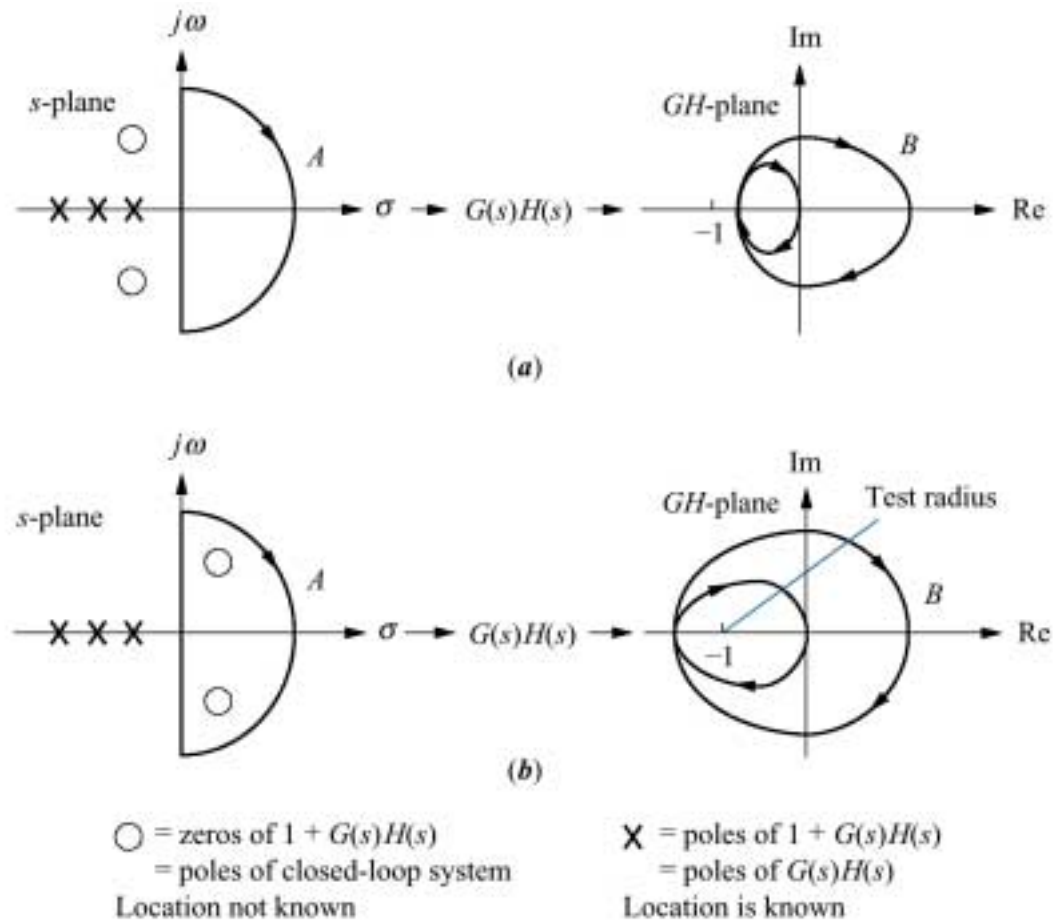
**Figure 10.24**

Contour enclosing  
right half-plane to  
determine stability



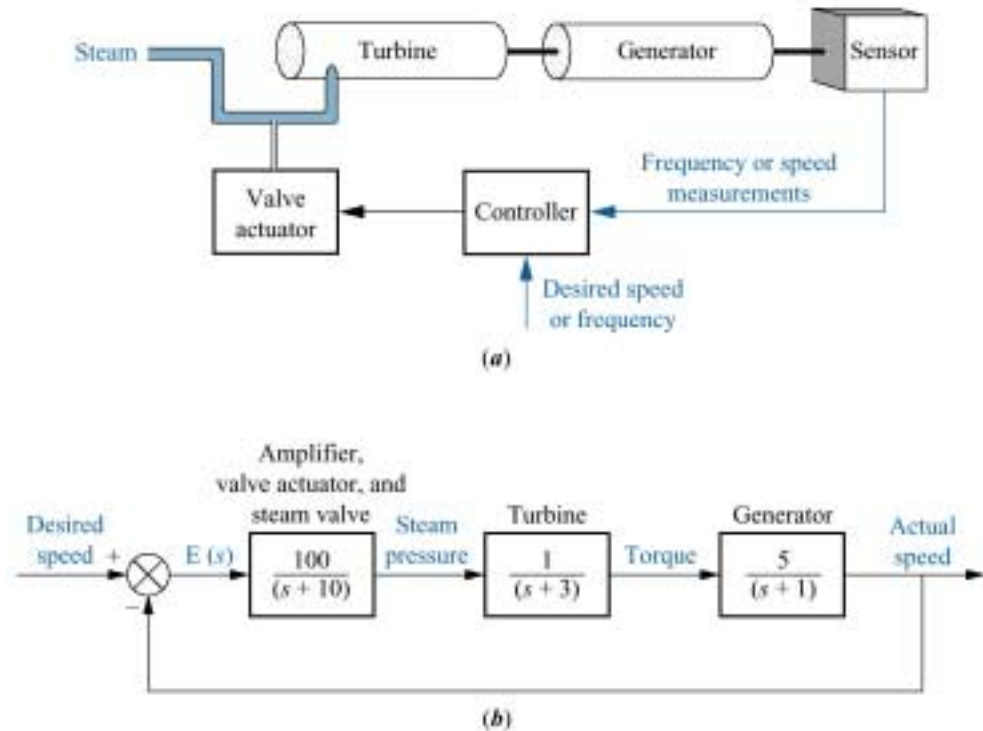
**Figure 10.25**

Mapping examples:  
**a.** contour does not enclose closed-loop poles;  
**b.** contour does enclose closed-loop poles



**Figure 10.26**

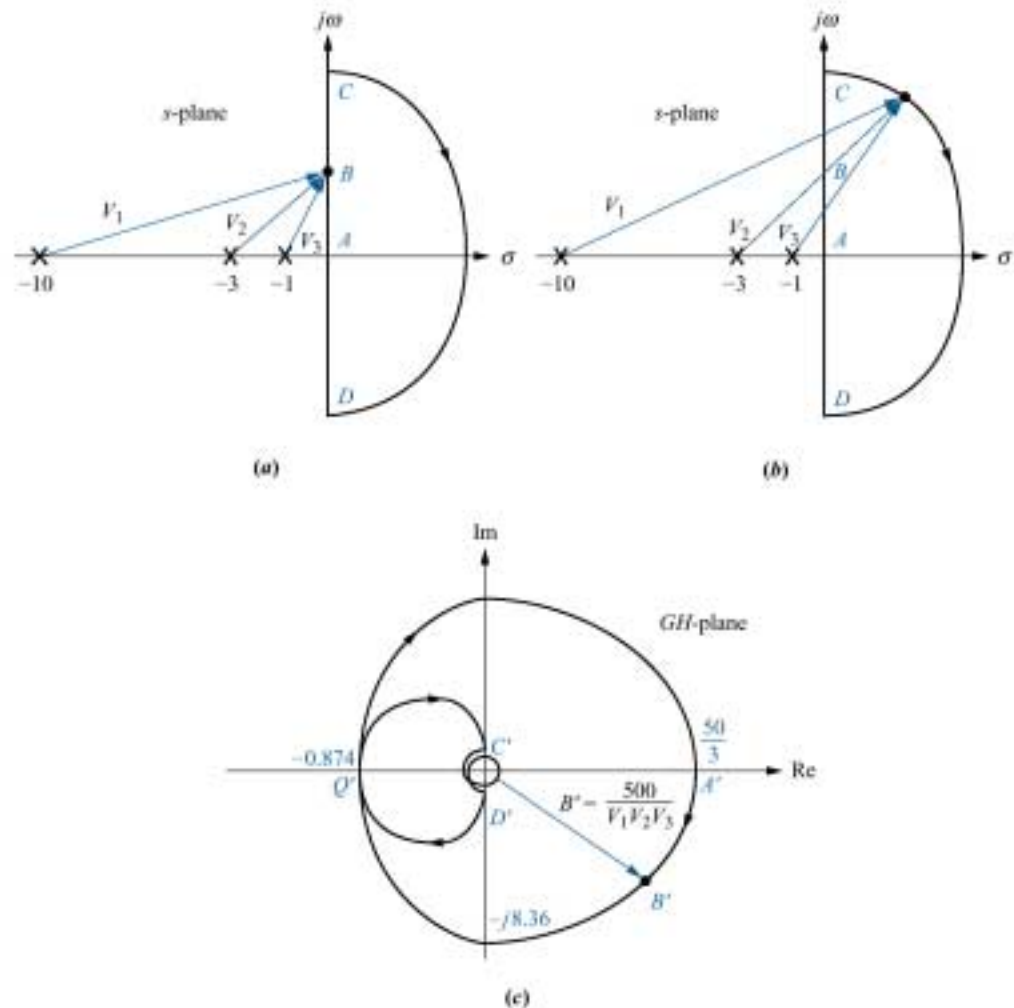
- a.** Turbine and generator;  
**b.** block diagram of speed control system for Example 10.4



**Figure 10.27**

Vector evaluation of the Nyquist diagram for Example 10.4:

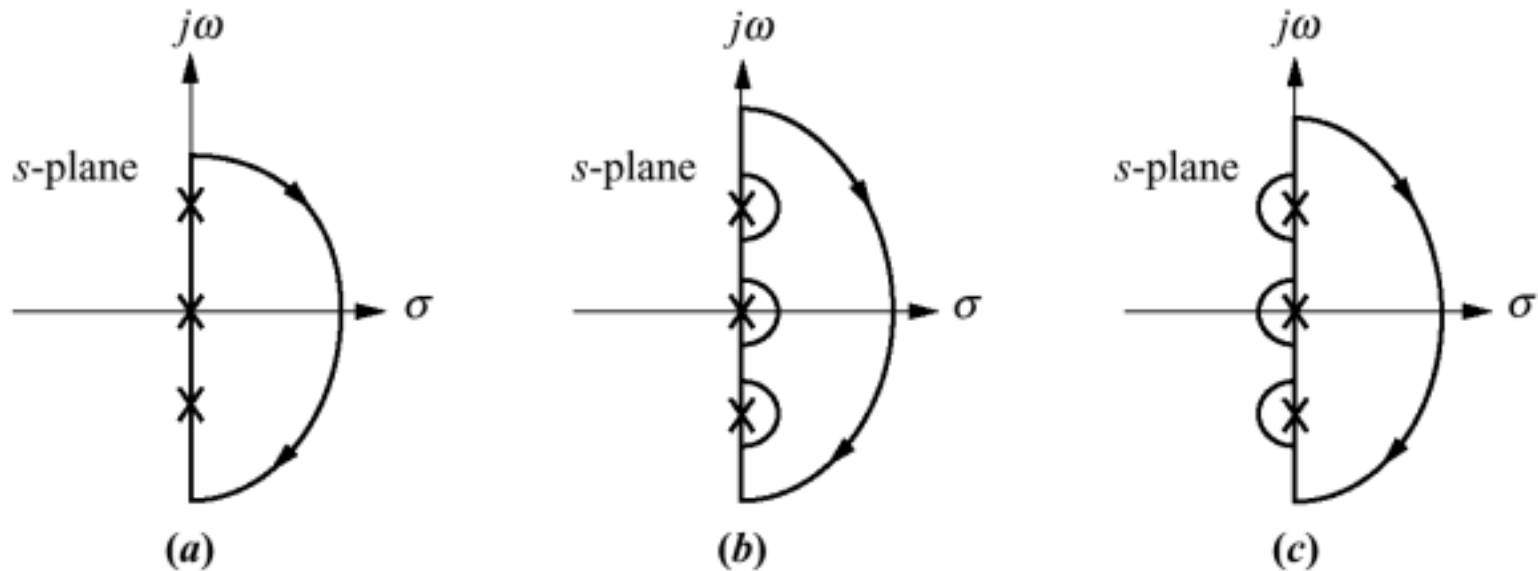
- a.** vectors on contour at low frequency;
- b.** vectors on contour around infinity;
- c.** Nyquist diagram



**Figure 10.28**

Detouring around open-loop poles:

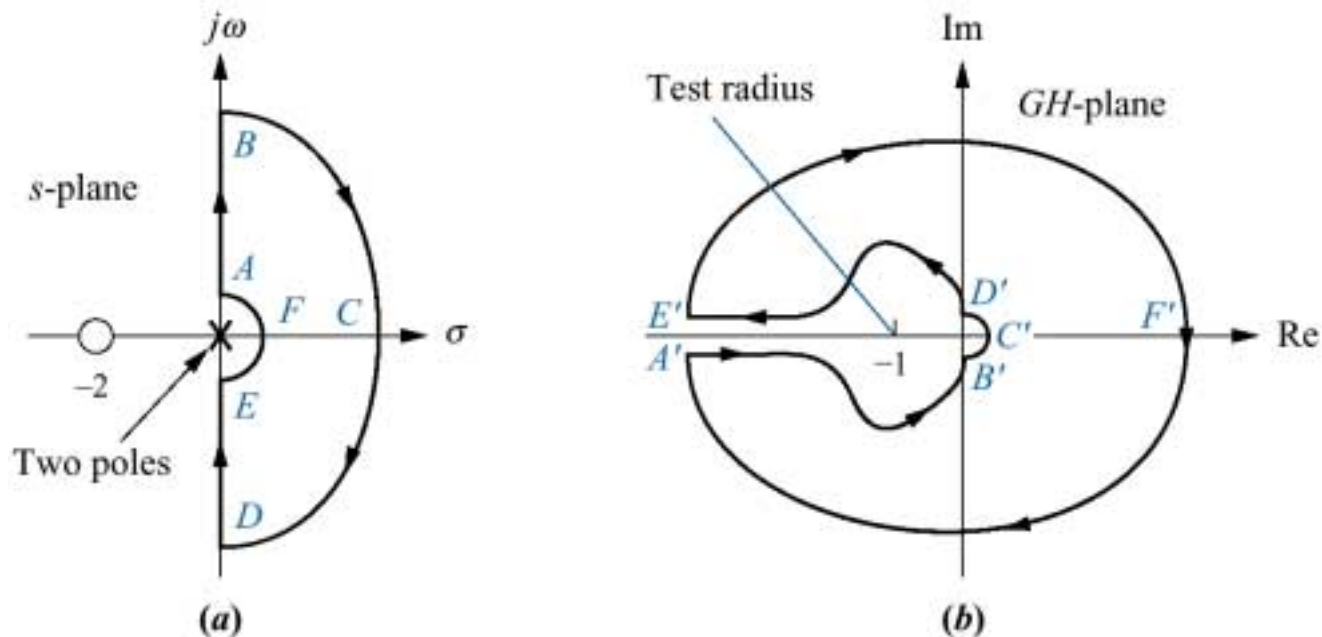
- a. poles on contour;
- b. detour right;
- c. detour left



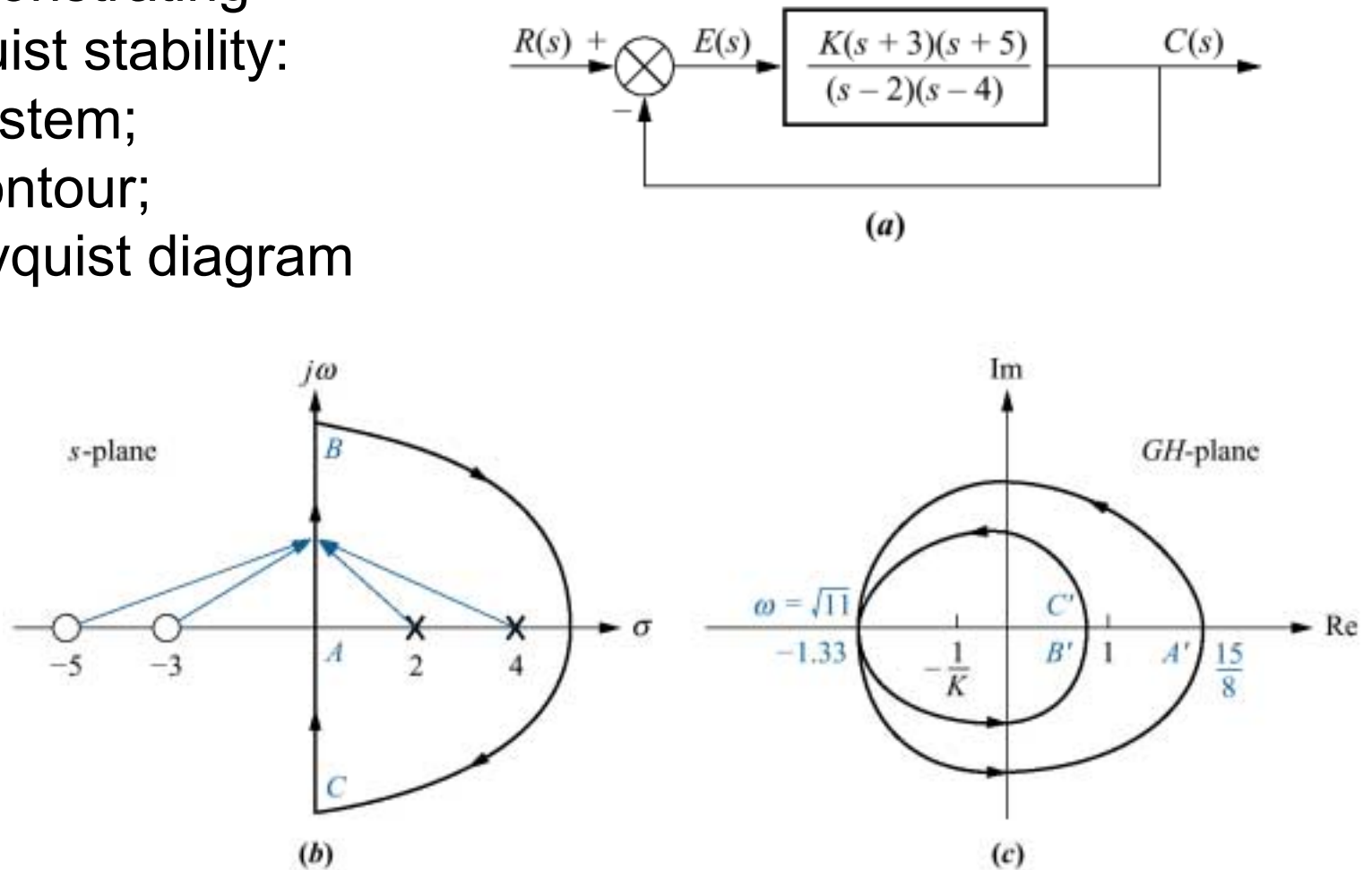
**Figure 10.29**

**a.** Contour for  
Example 10.5;

**b.** Nyquist diagram for  
Example 10.5



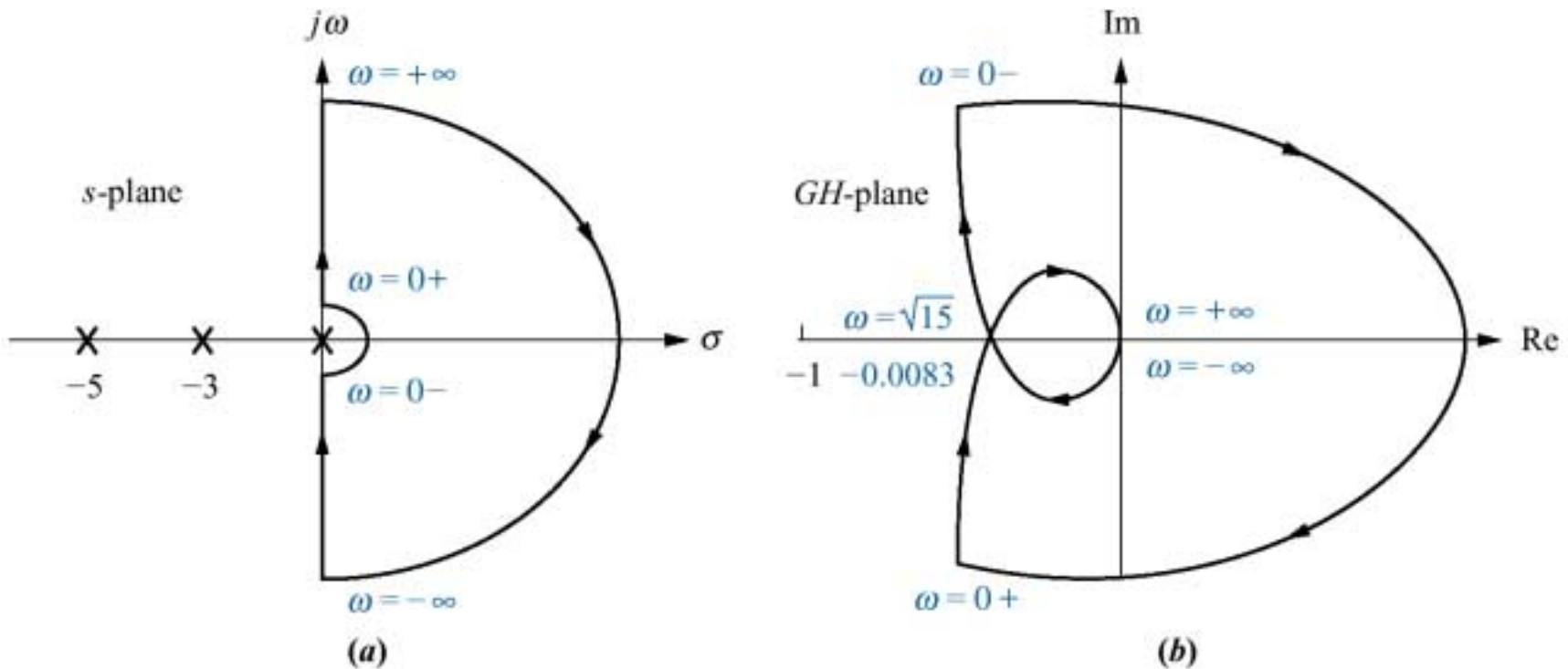
**Figure 10.30**  
 Demonstrating  
 Nyquist stability:  
 a. system;  
 b. contour;  
 c. Nyquist diagram



**Figure 10.31**

a. Contour for  
Example 10.6;

b. Nyquist diagram

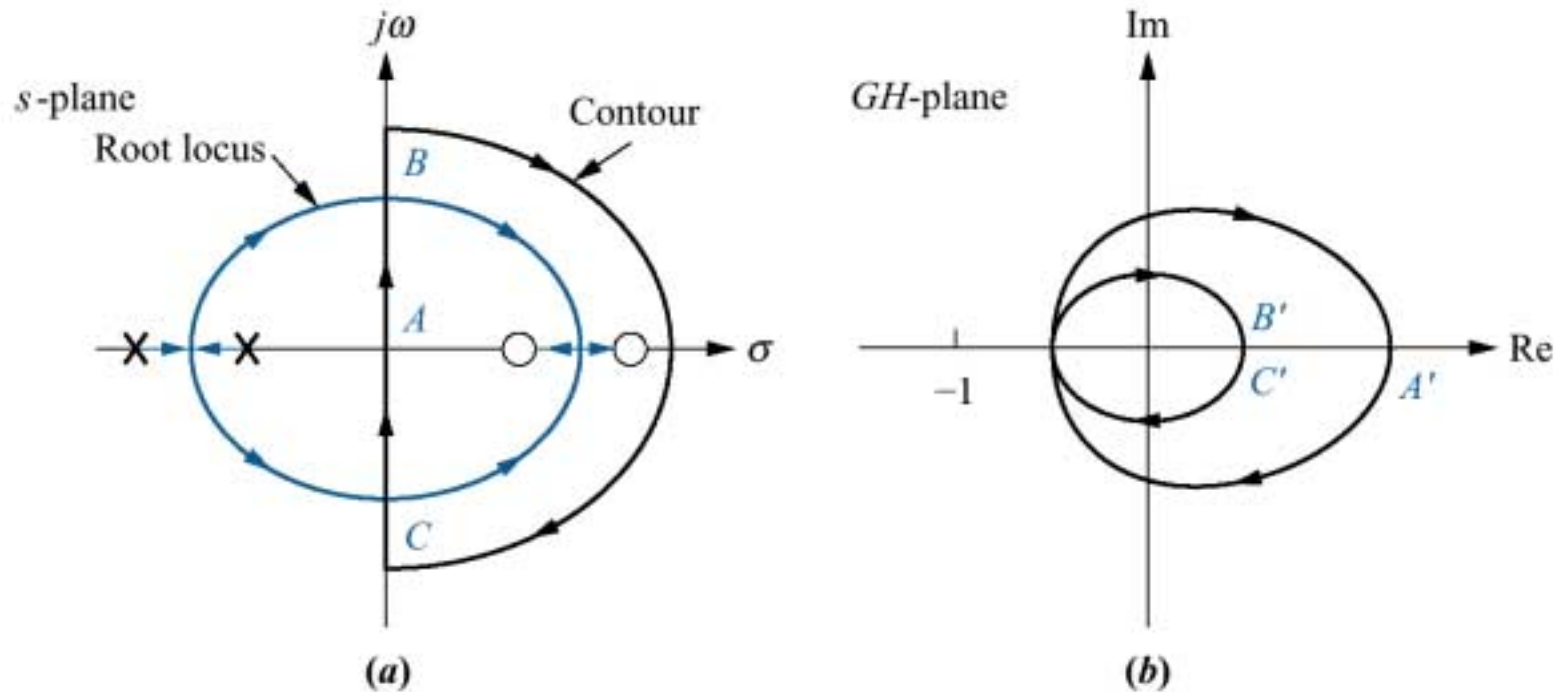




**Figure 10.32**

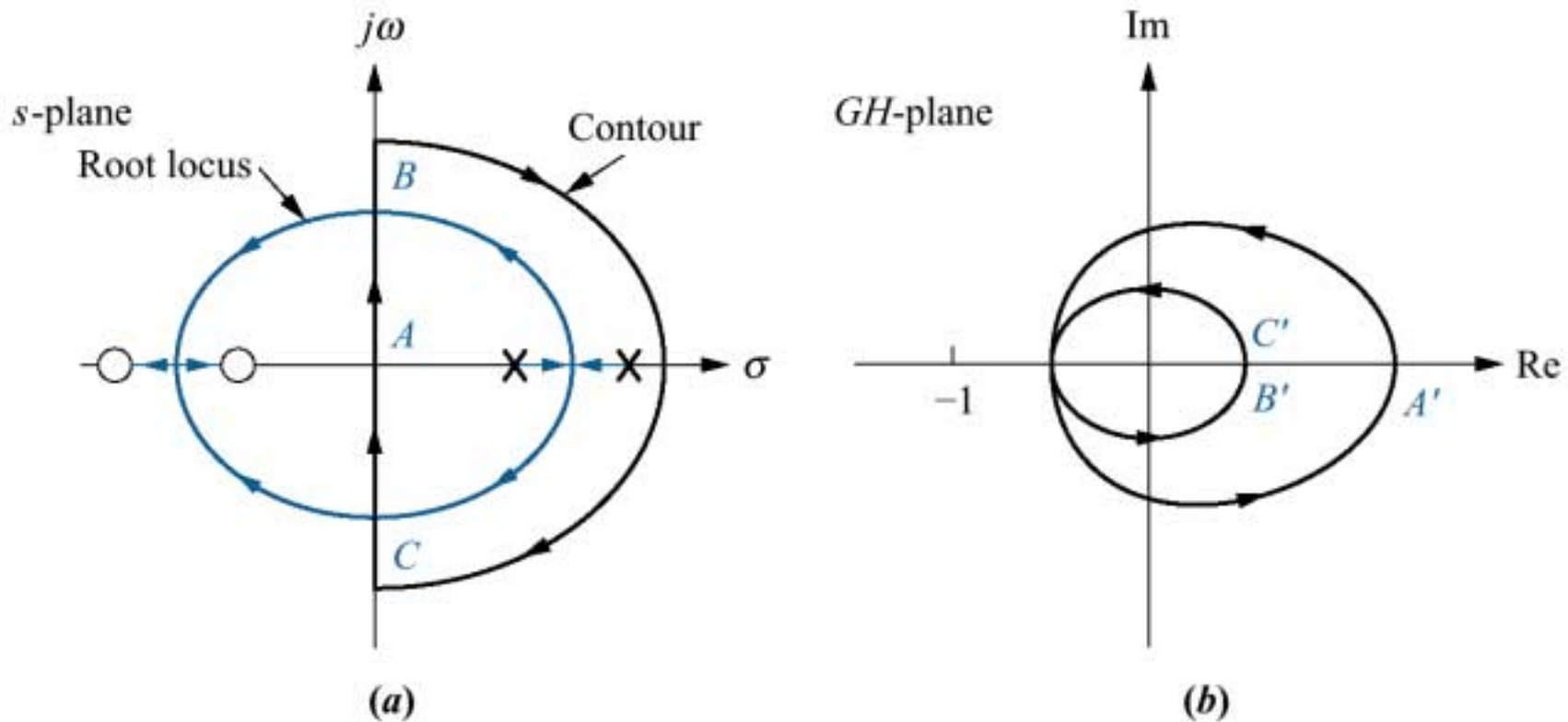
**a.** Contour and root locus of system that is stable for small gain and unstable for large gain;

**b.** Nyquist diagram



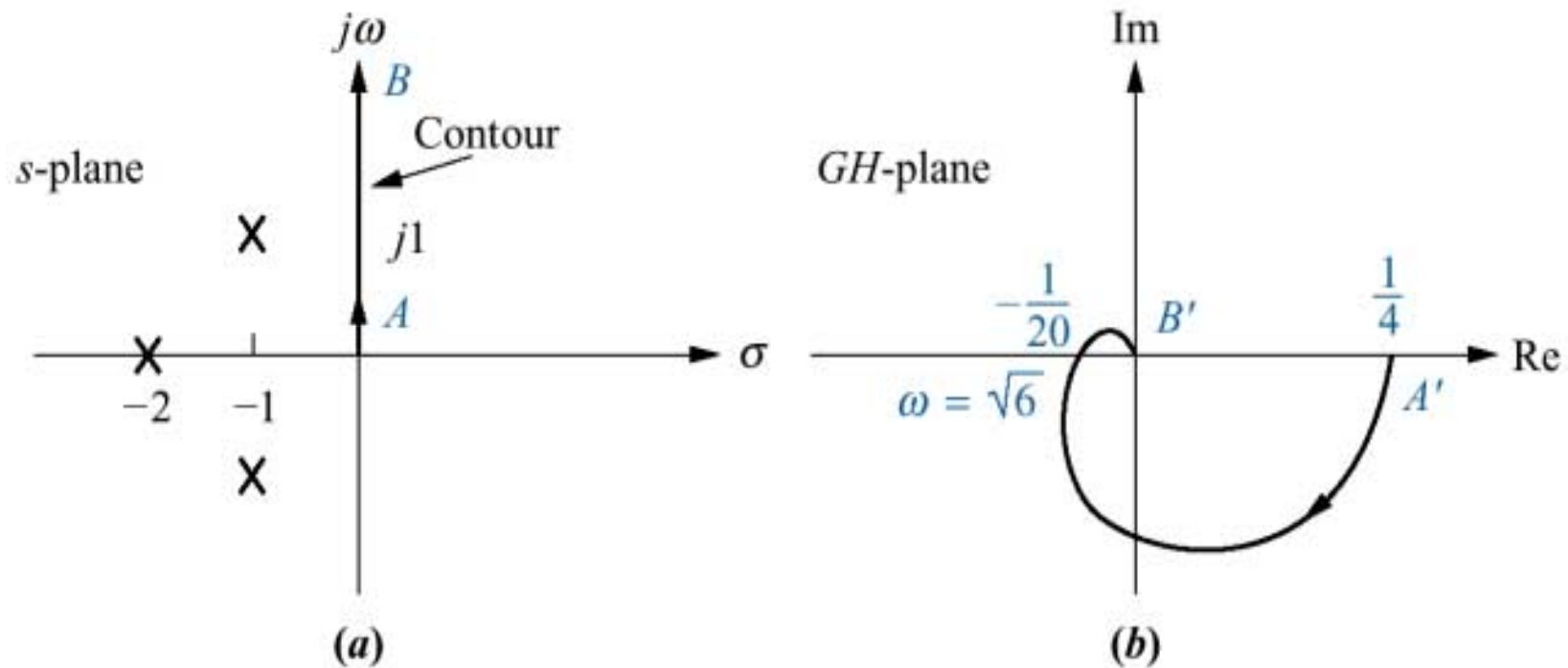
**Figure 10.33**

- a. Contour and root locus of system that is unstable for small gain and stable for large gain;  
b. Nyquist diagram

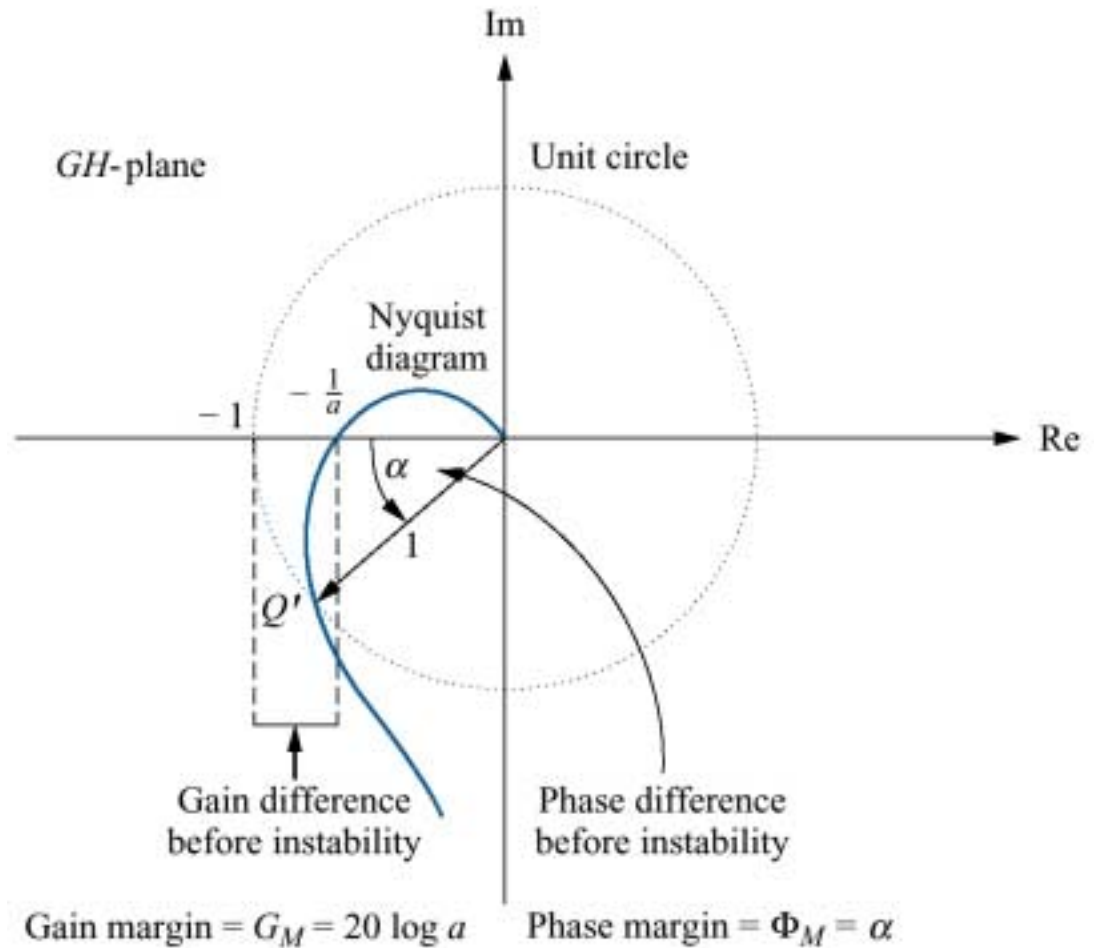


**Figure 10.34**

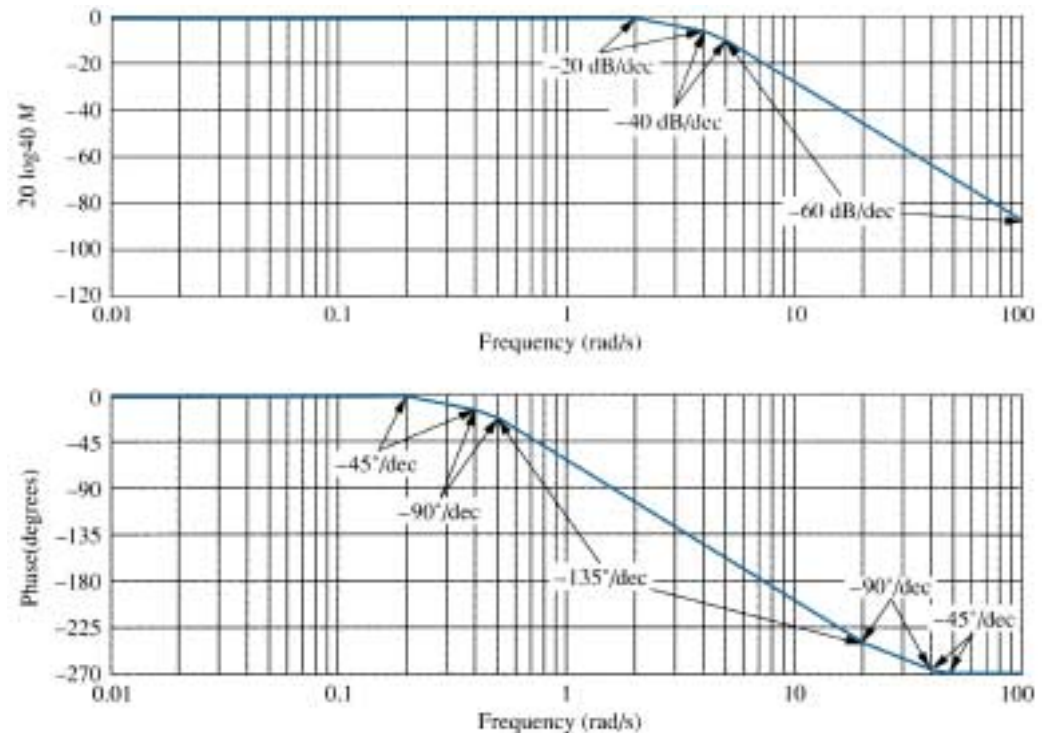
- a. Portion of contour to be mapped for Example 10.7;  
b. Nyquist diagram of mapping of positive imaginary axis



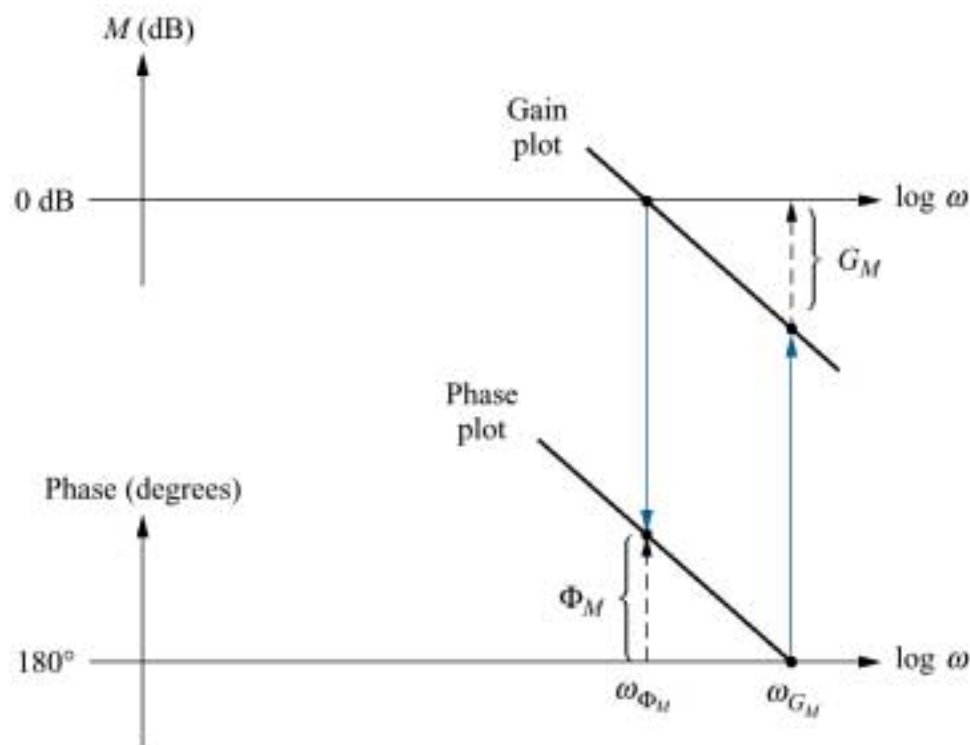
**Figure 10.35**  
Nyquist diagram  
showing gain and  
phase margins



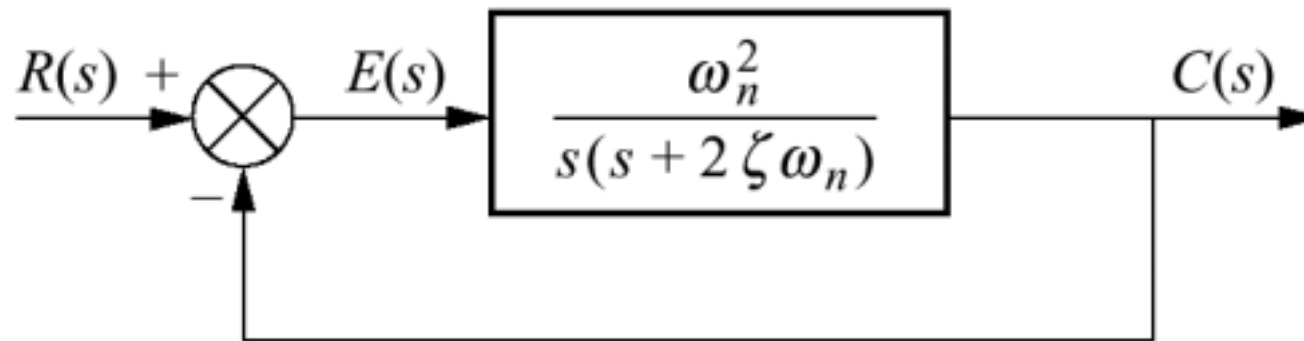
**Figure 10.36**  
Bode log-magnitude  
and phase diagrams  
for the system of  
Example 10.9



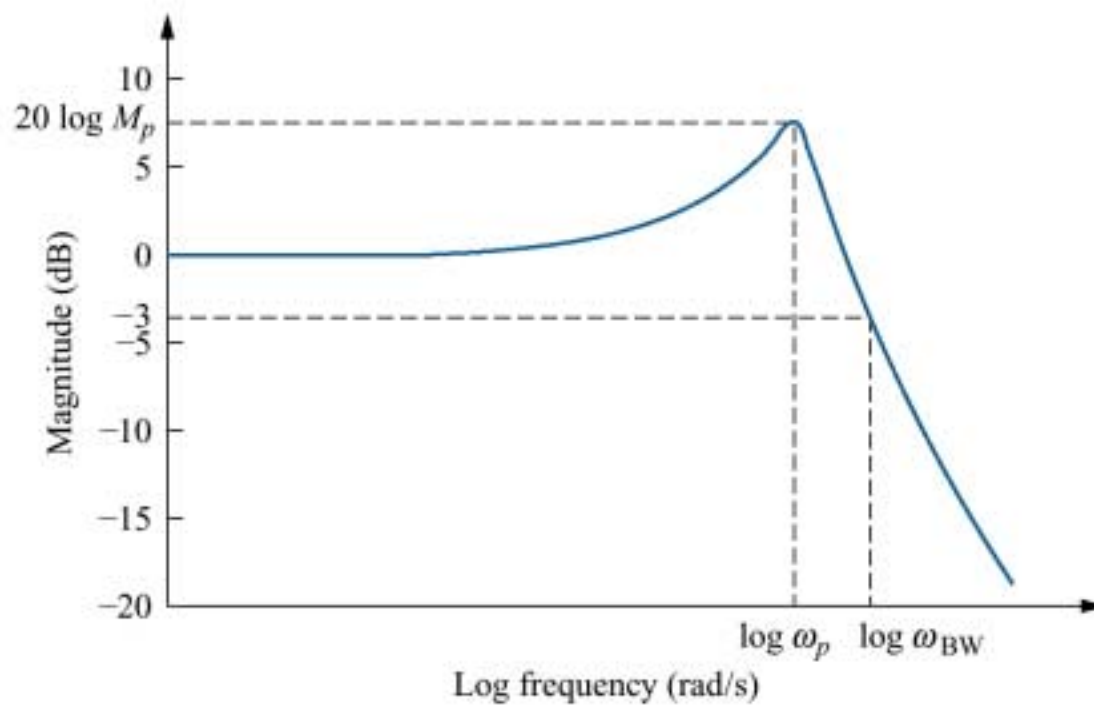
**Figure 10.37**  
Gain and phase  
margins on the Bode  
diagrams



**Figure 10.38**  
Second-order closed-loop  
system



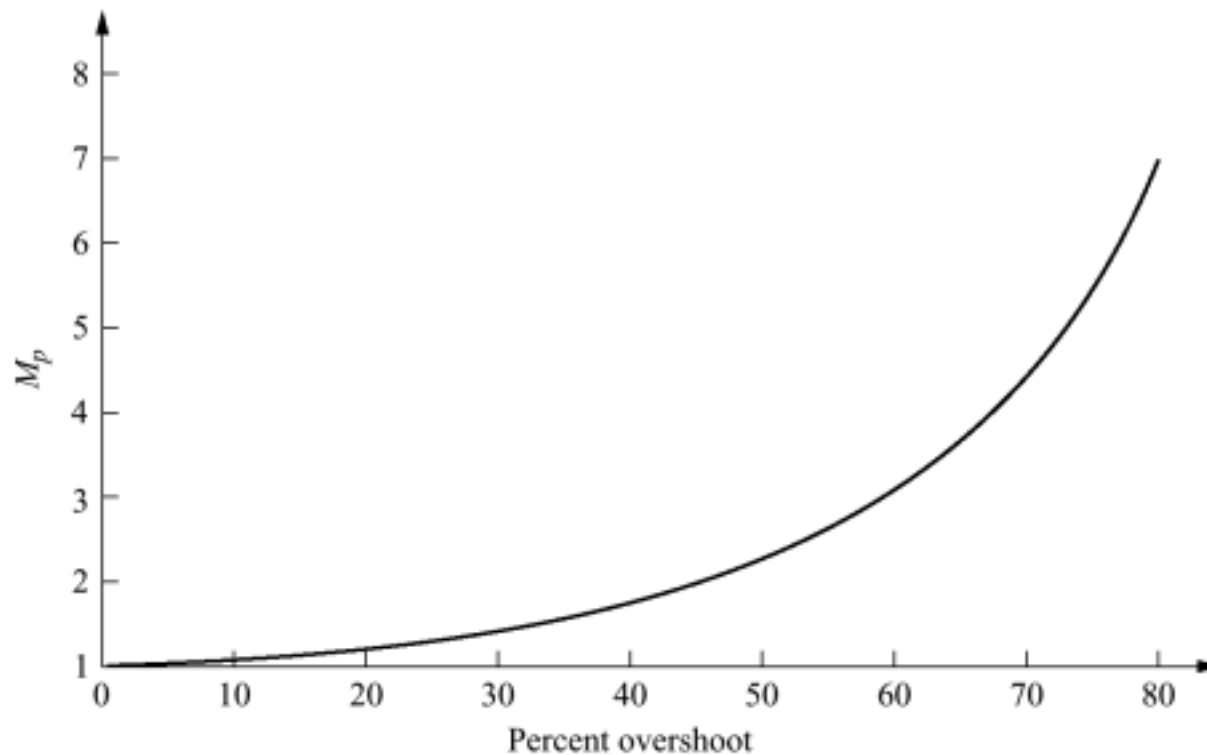
**Figure 10.39**  
Representative log-magnitude plot of  
Eq. (10.51)





**Figure 10.40**

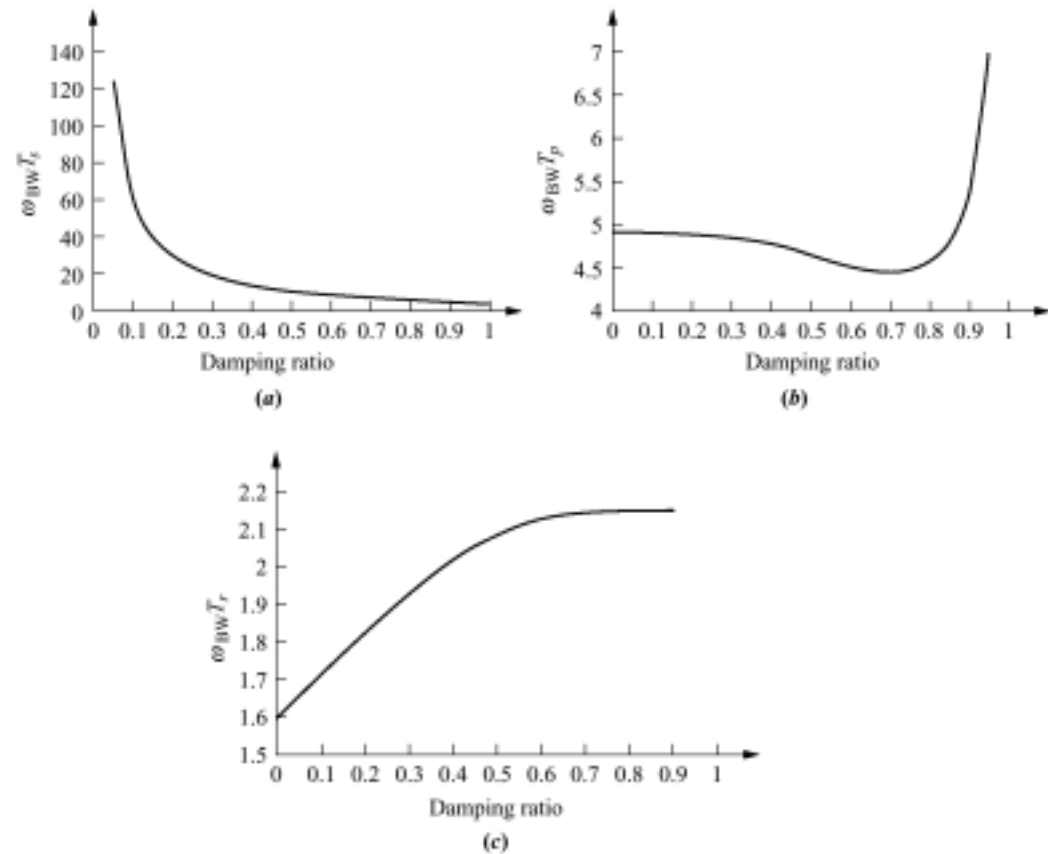
Closed-loop frequency percent overshoot for a two-pole system



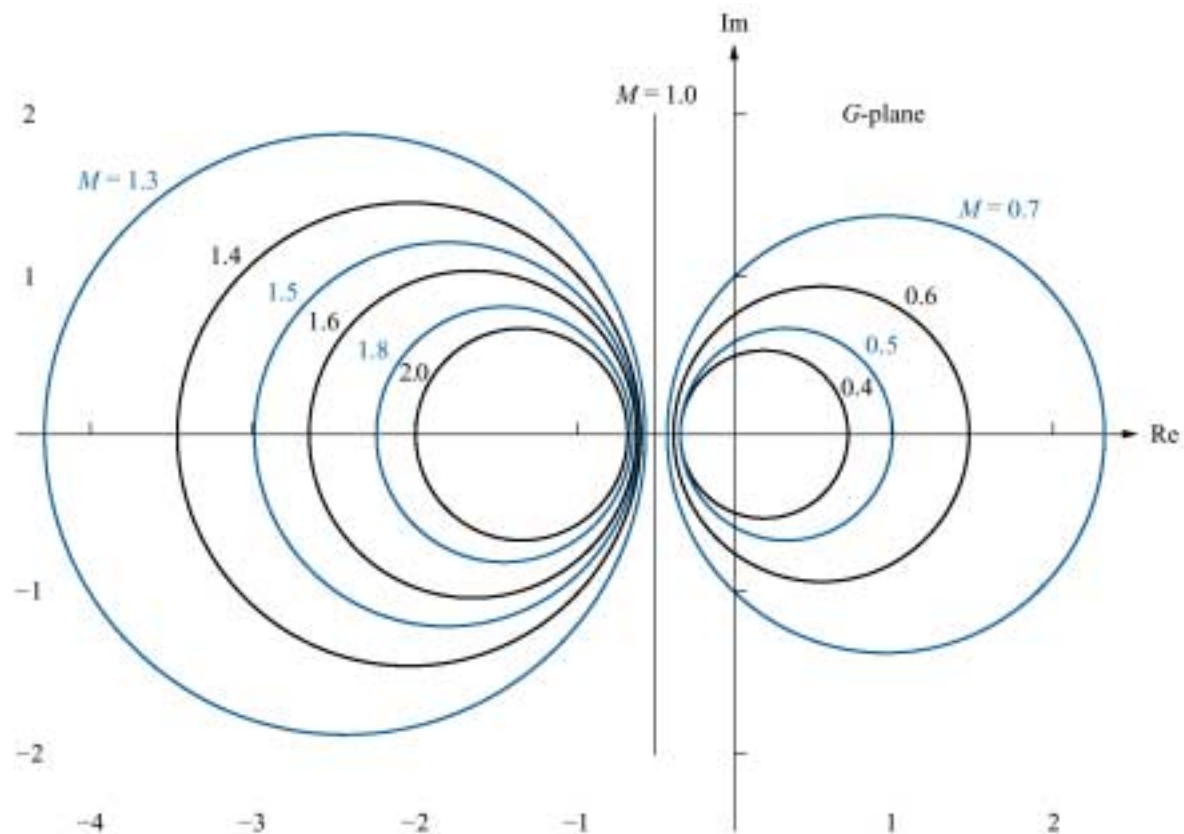
**Figure 10.41**

Normalized bandwidth  
vs. damping ratio for:

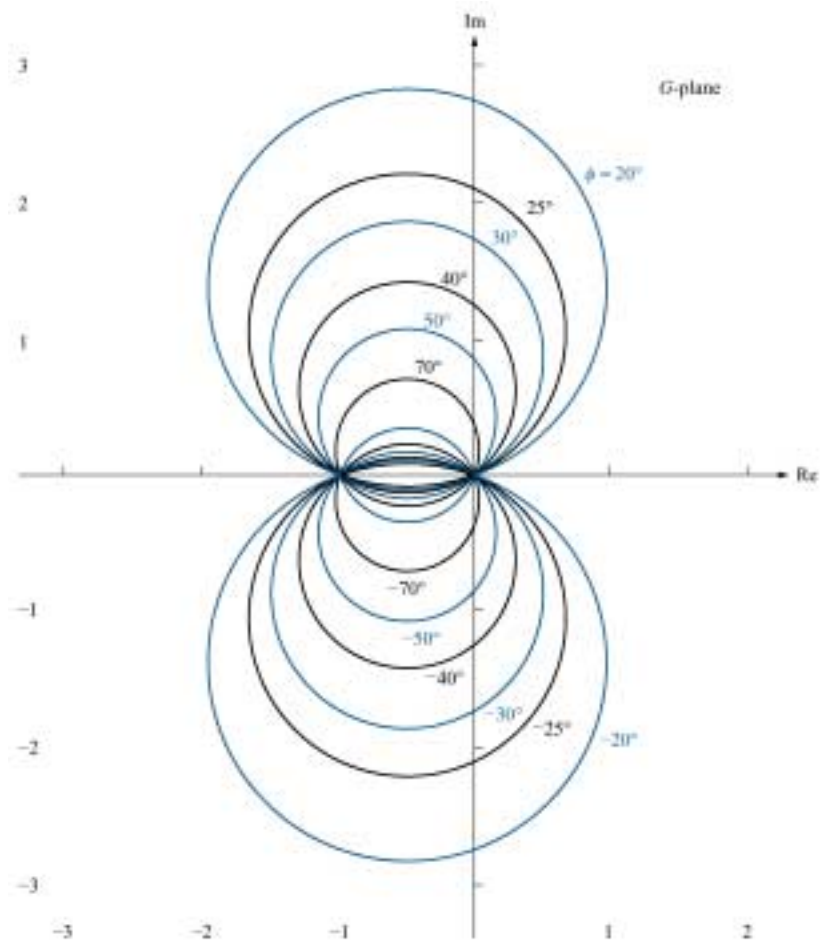
- a.** settling time;
- b.** peak time;
- c.** rise time



**Figure 10.42**  
Constant  $M$   
circles

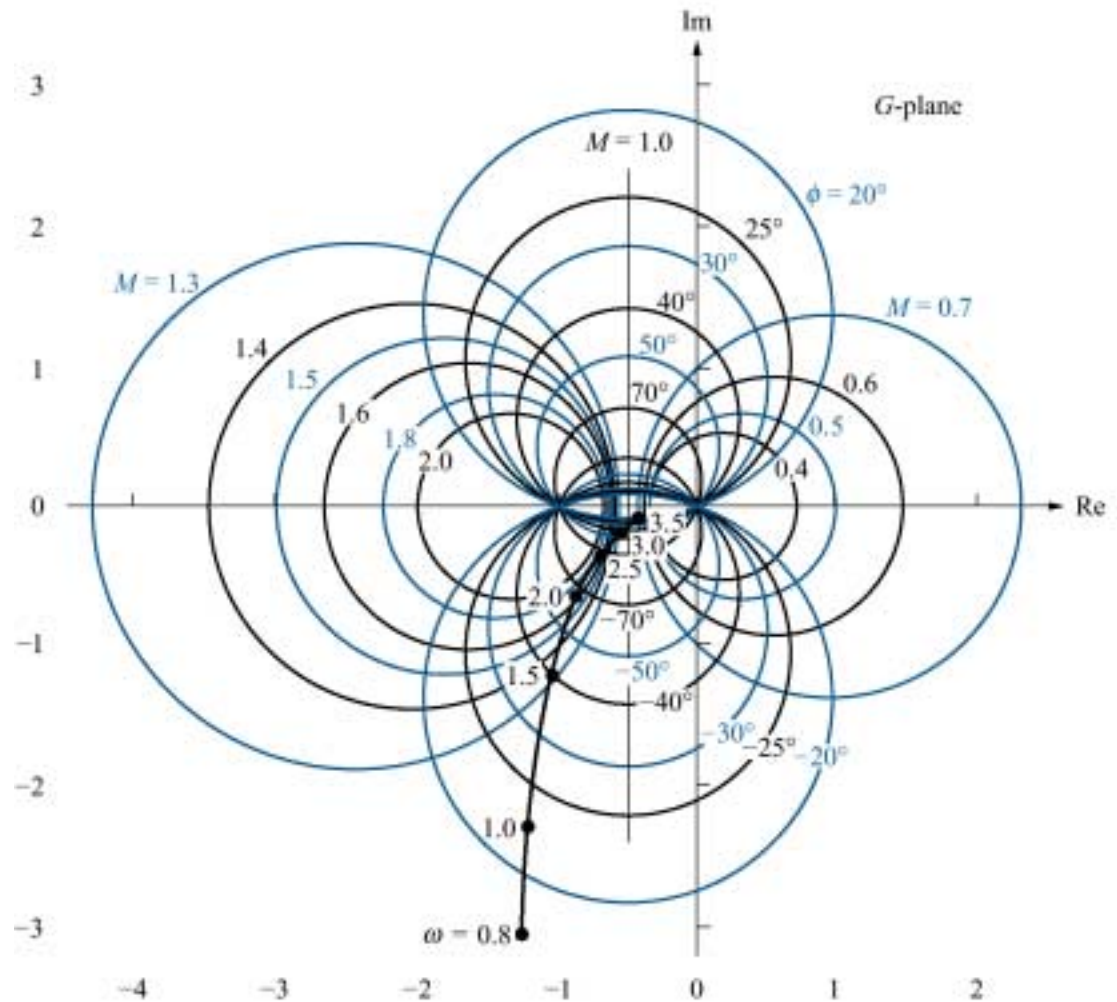


**Figure 10.43**  
Constant  $N$  circles

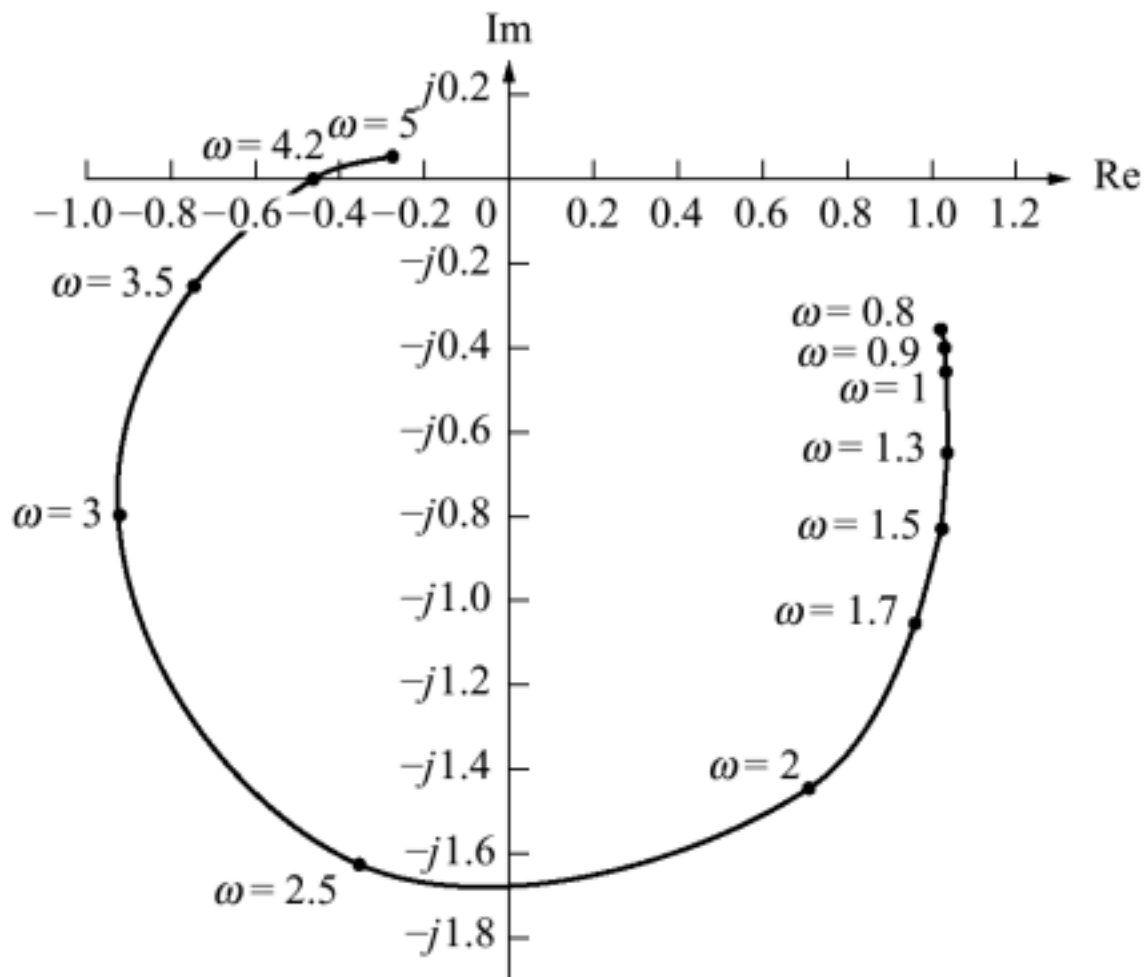


**Figure 10.44**

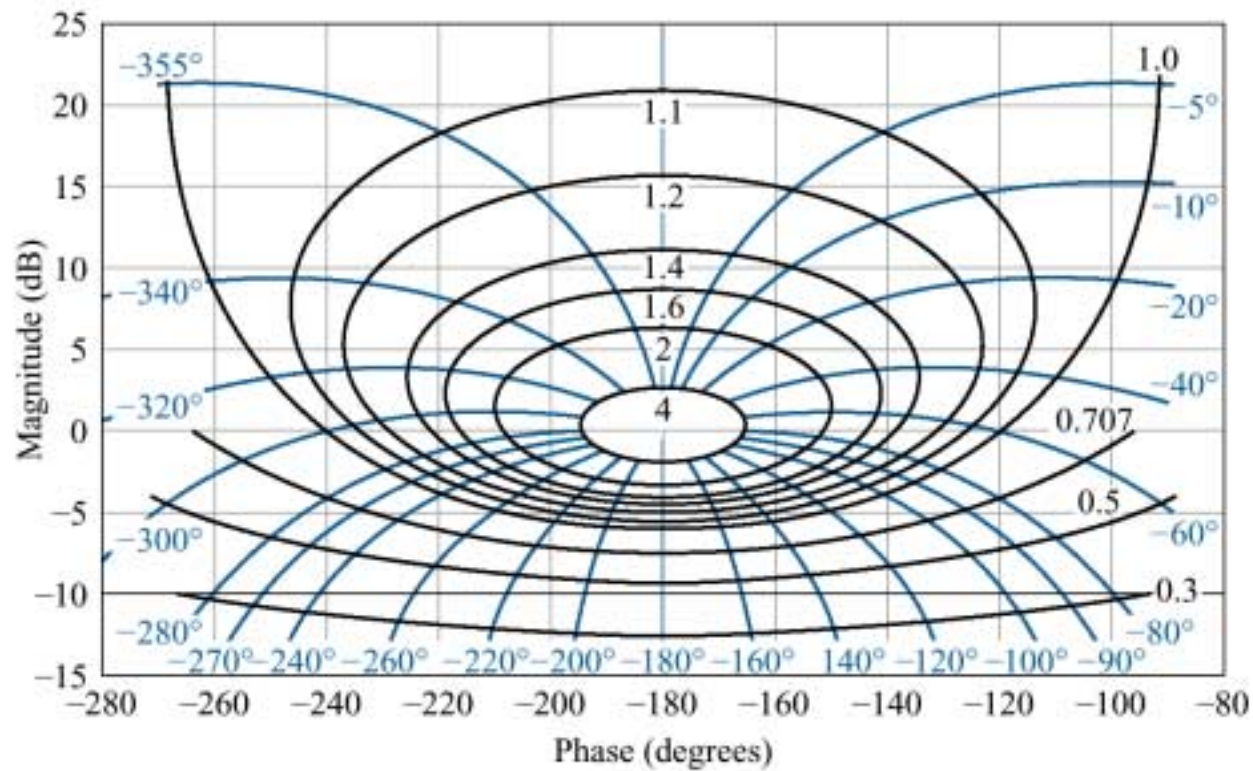
Nyquist diagram for  
Example 10.11 and  
constant  $M$  and  $N$   
circles



**Figure 10.45**  
Closed-loop  
frequency response  
for Example 10.11

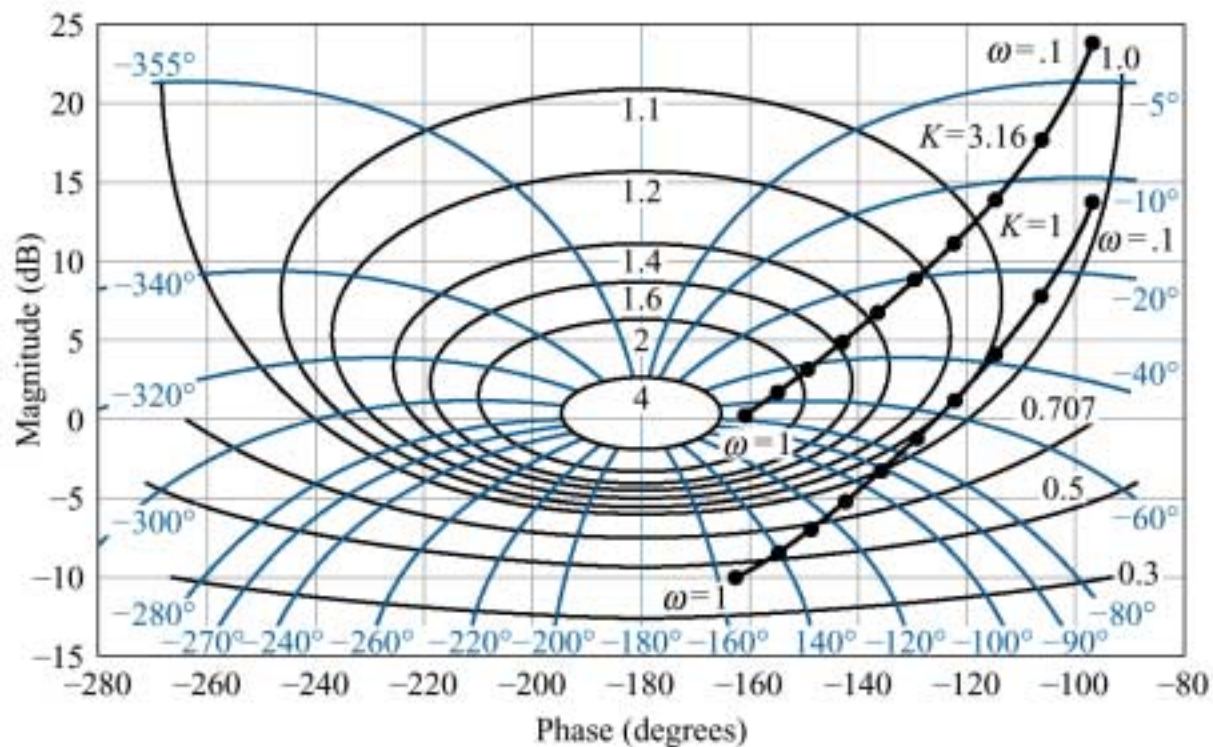


**Figure 10.46**  
Nichols chart



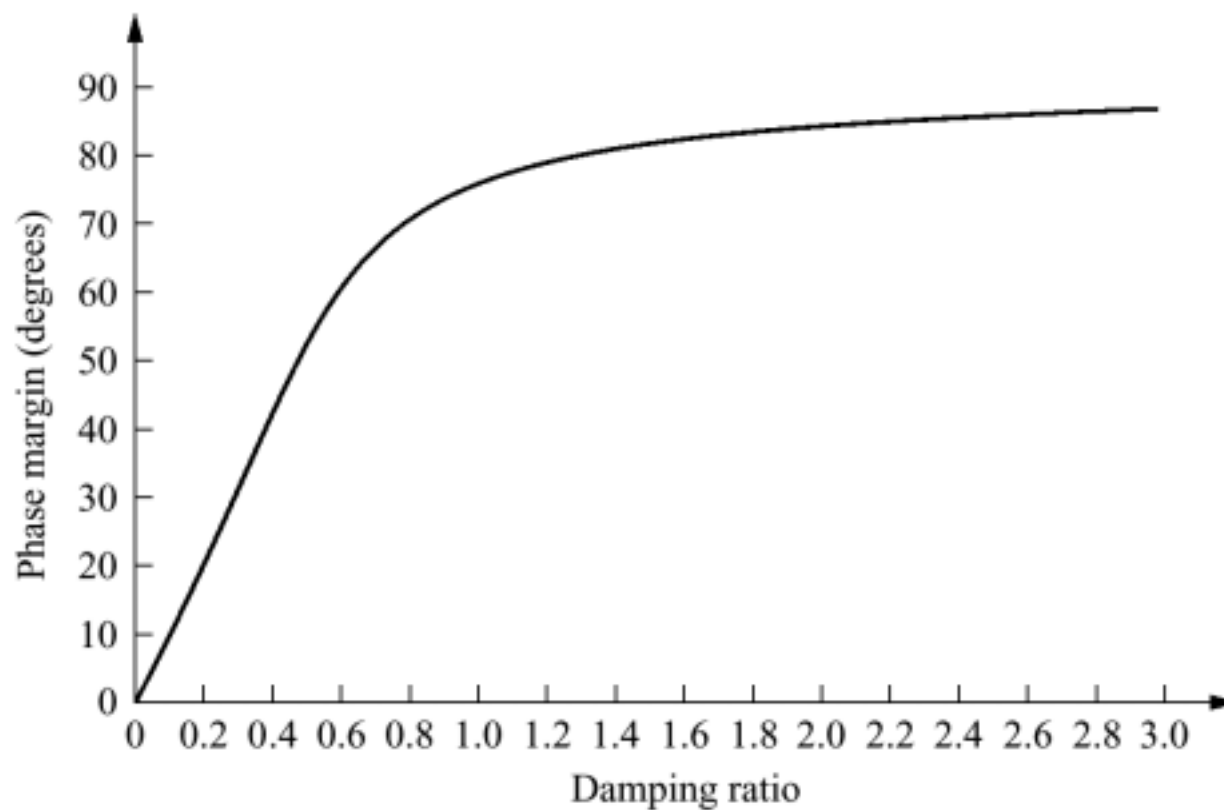
**Figure 10.47**

Nichols chart with frequency response for  $G(s) = K/[s(s + 1)(s + 2)]$  superimposed. Values for  $K = 1$  and  $K = 3.16$  are shown.



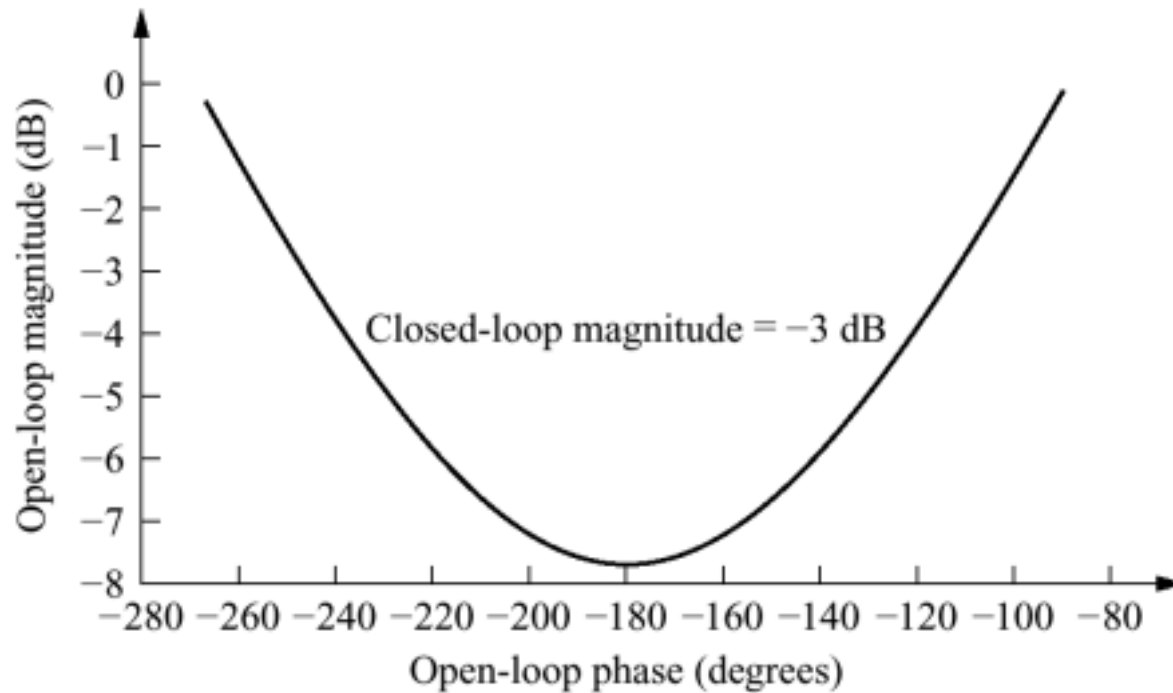


**Figure 10.48**  
Phase margin vs.  
damping ratio



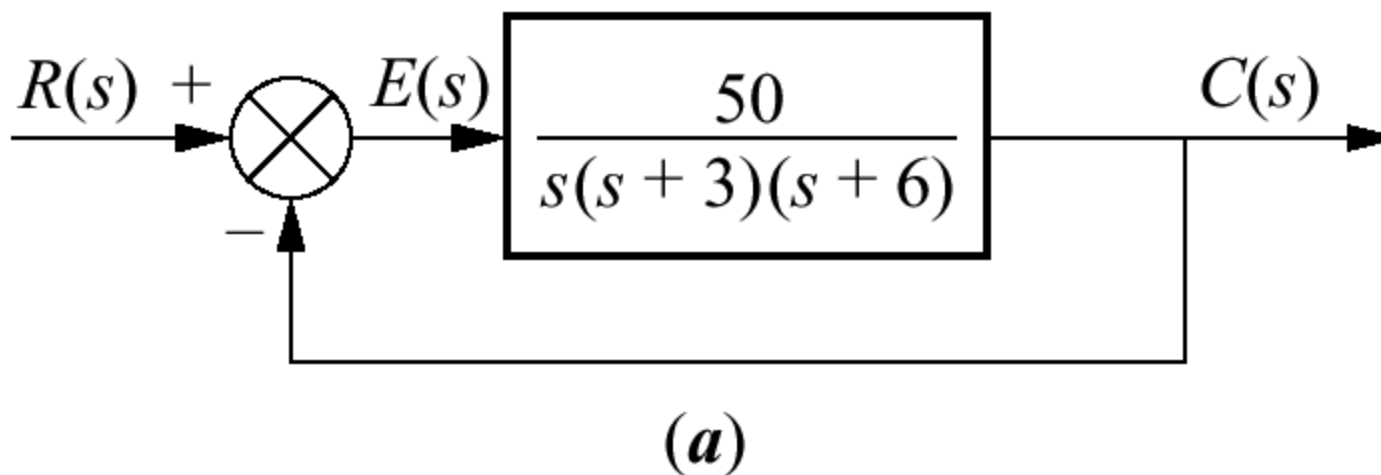
**Figure 10.49**

Open-loop gain vs. open-loop phase angle for  $-3$  dB closed-loop gain



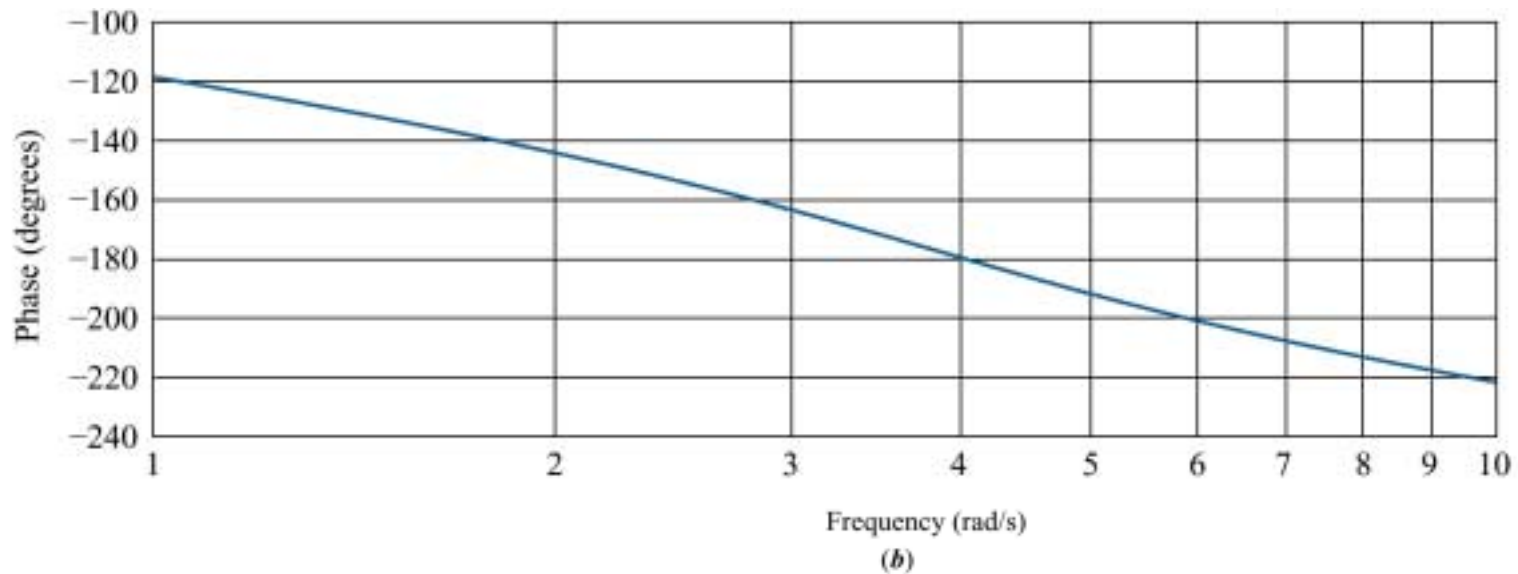
**Figure 10.50**

a. Block diagram  
*(figure continues)*



**Figure 10.50**  
**(continued)**

**b.** Bode diagrams for  
system of Example 10.13



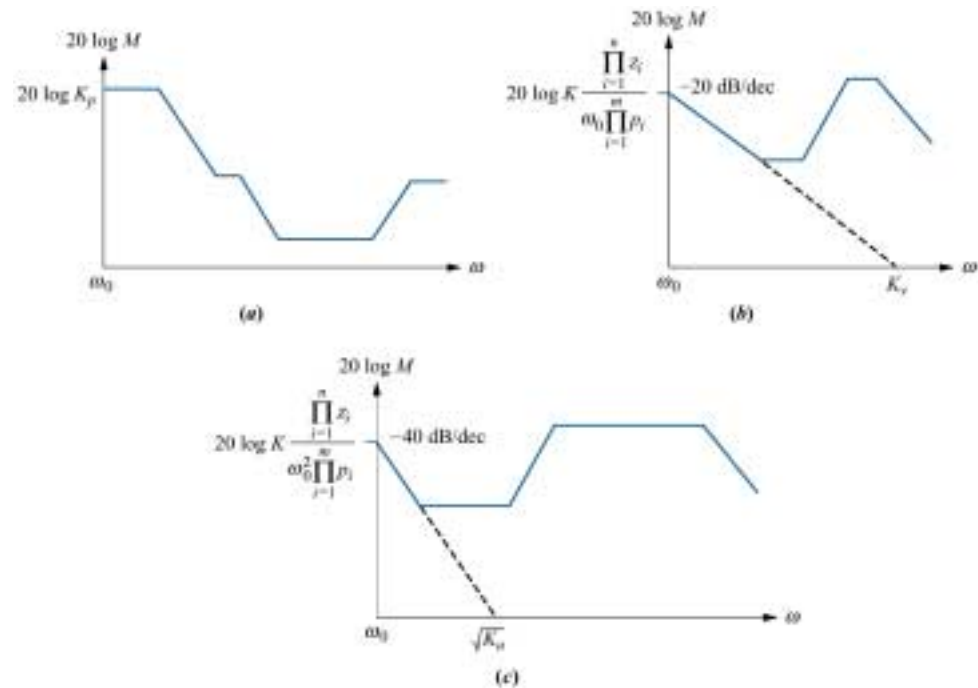
**Figure 10.51**

Typical unnormalized and unscaled Bode log-magnitude plots showing the value of static error constants:

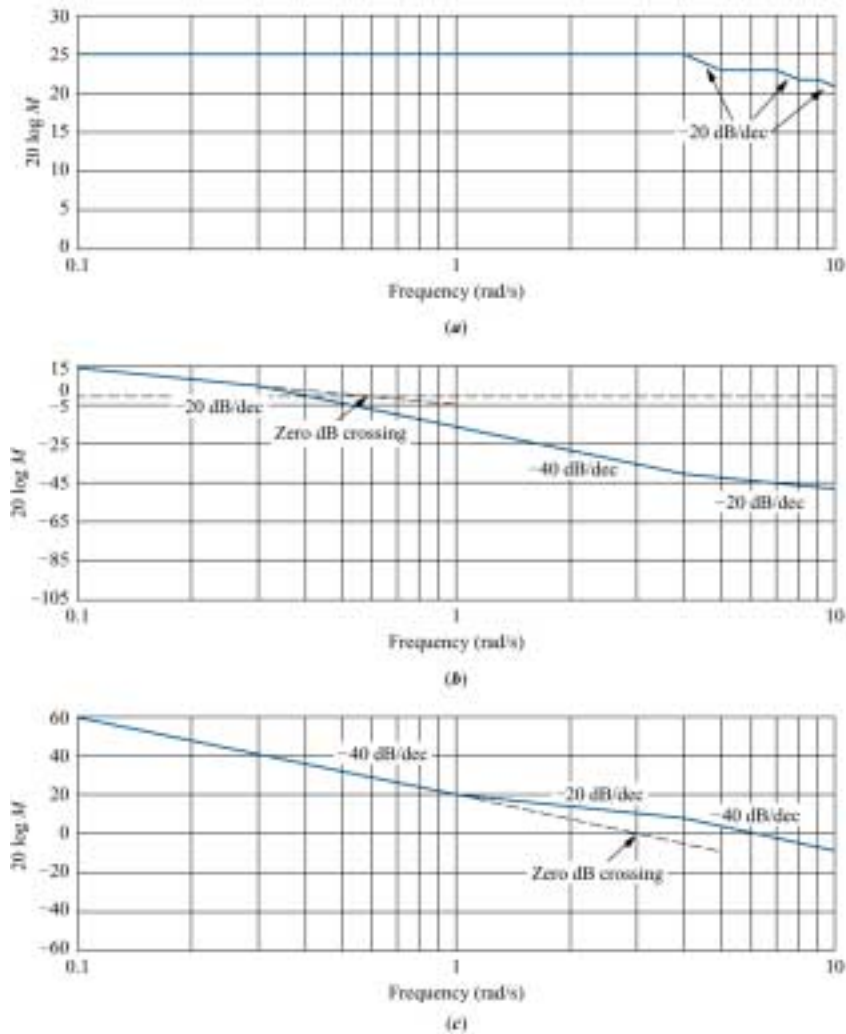
**a.** Type 0;

**b.** Type 1;

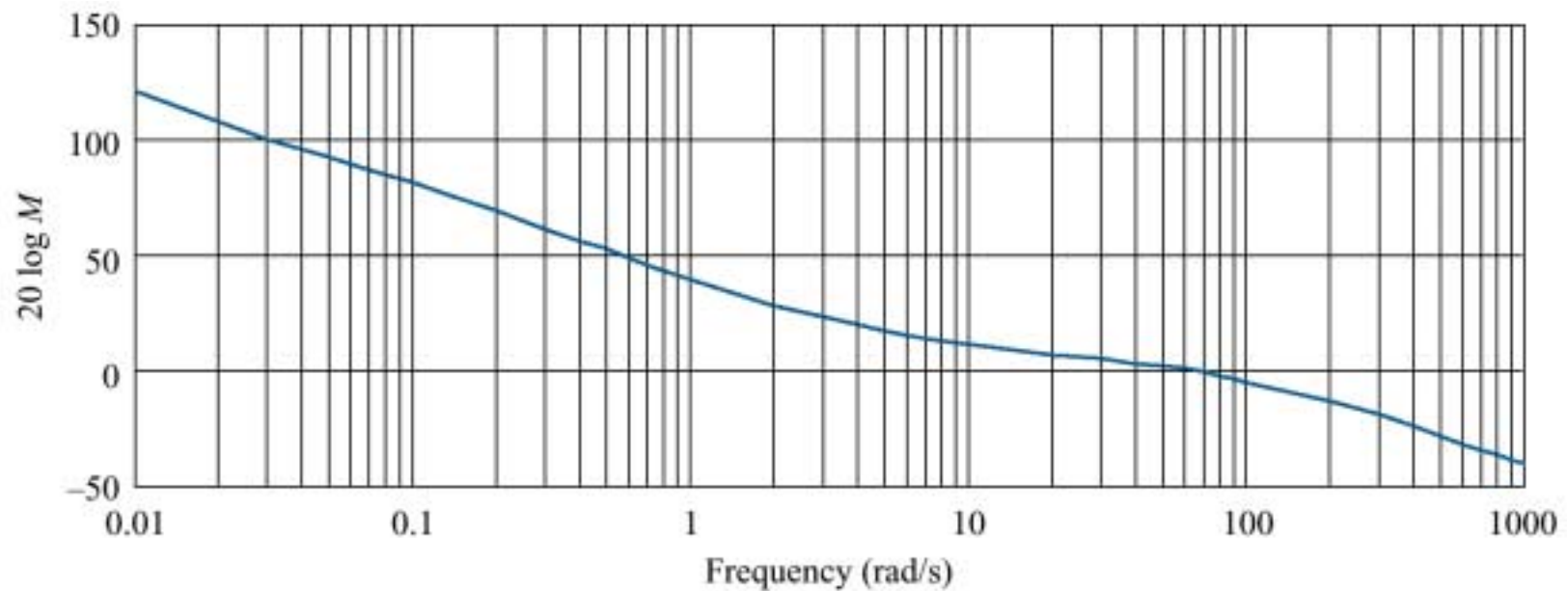
**c.** Type 2



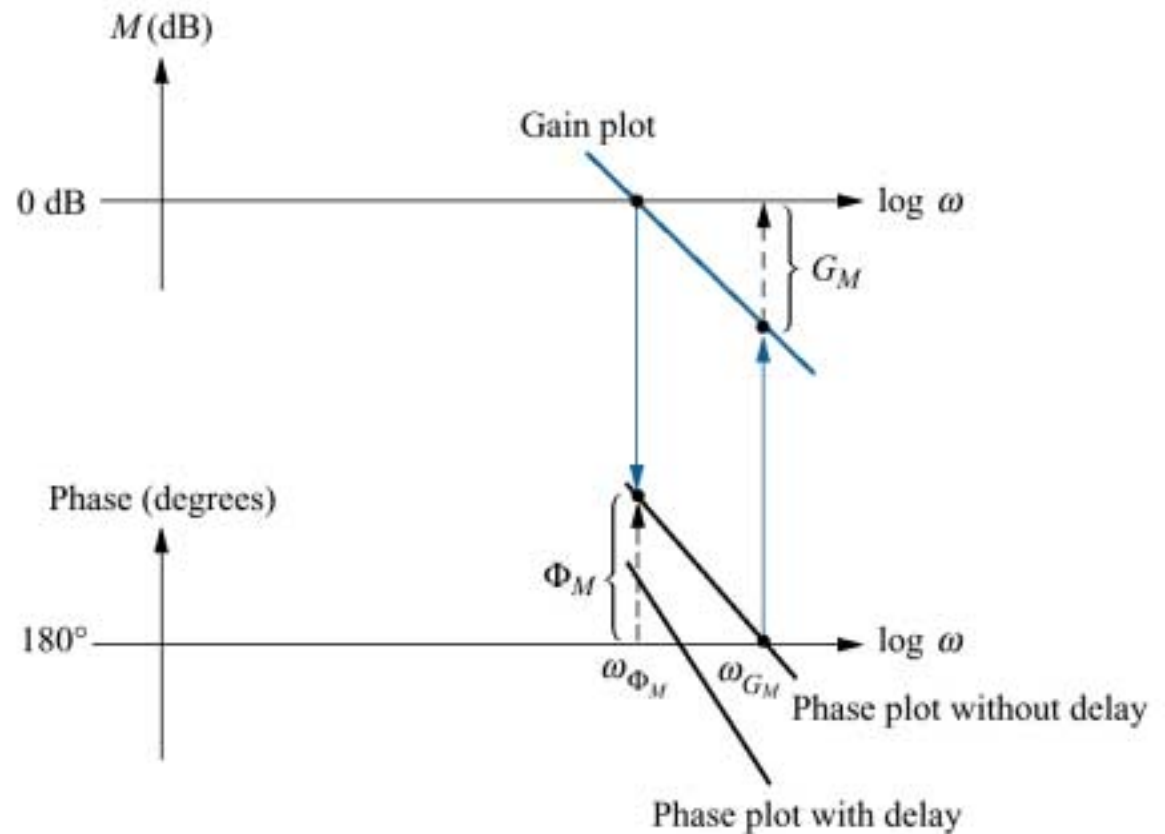
**Figure 10.52**  
Bode log-magnitude  
plots for Example  
10.14



**Figure 10.53**  
Bode log-magnitude  
plot for  
Skill-Assessment  
Exercise 10.10



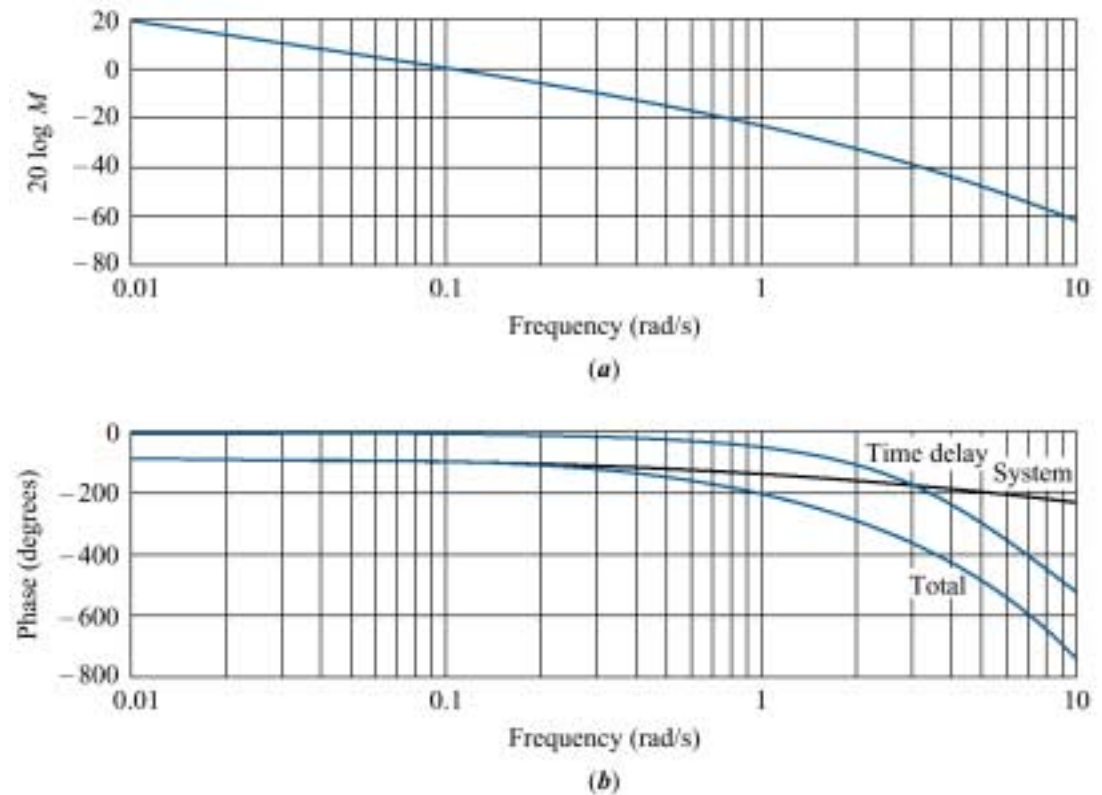
**Figure 10.54**  
Effect of delay  
upon frequency  
response





**Figure 10.55**

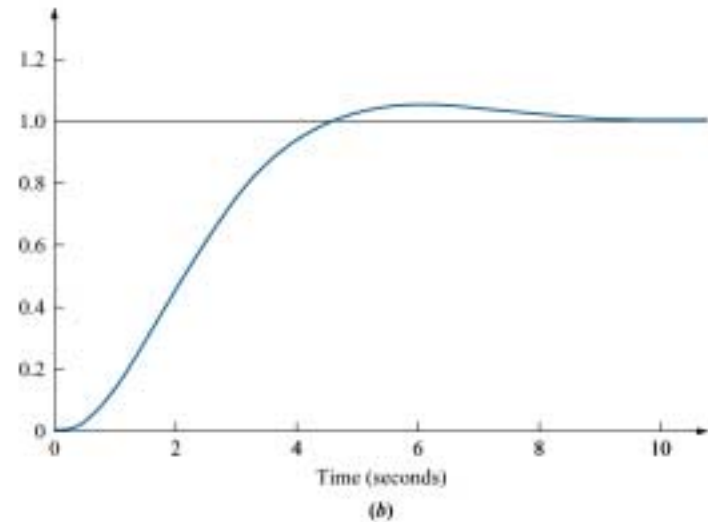
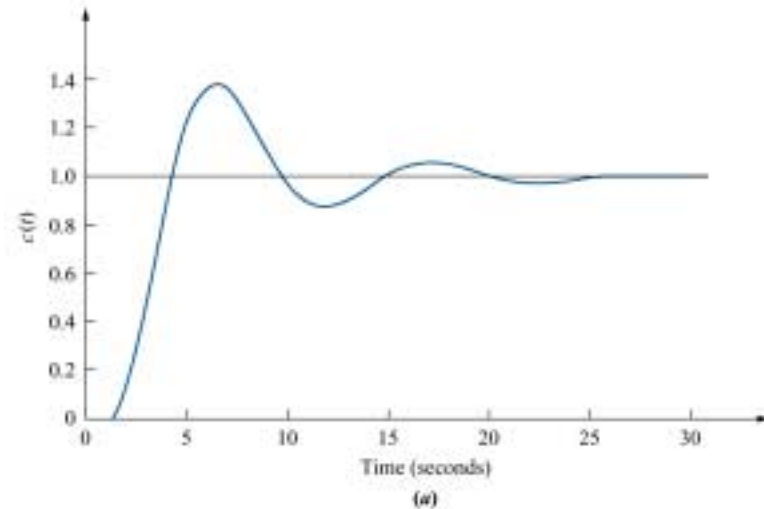
Frequency response plots for  $G(s) = K/[s(s+1)(s+10)]$  with a delay of 1 second and  $K = 1$ :  
**a.** magnitude plot;  
**b.** phase plot



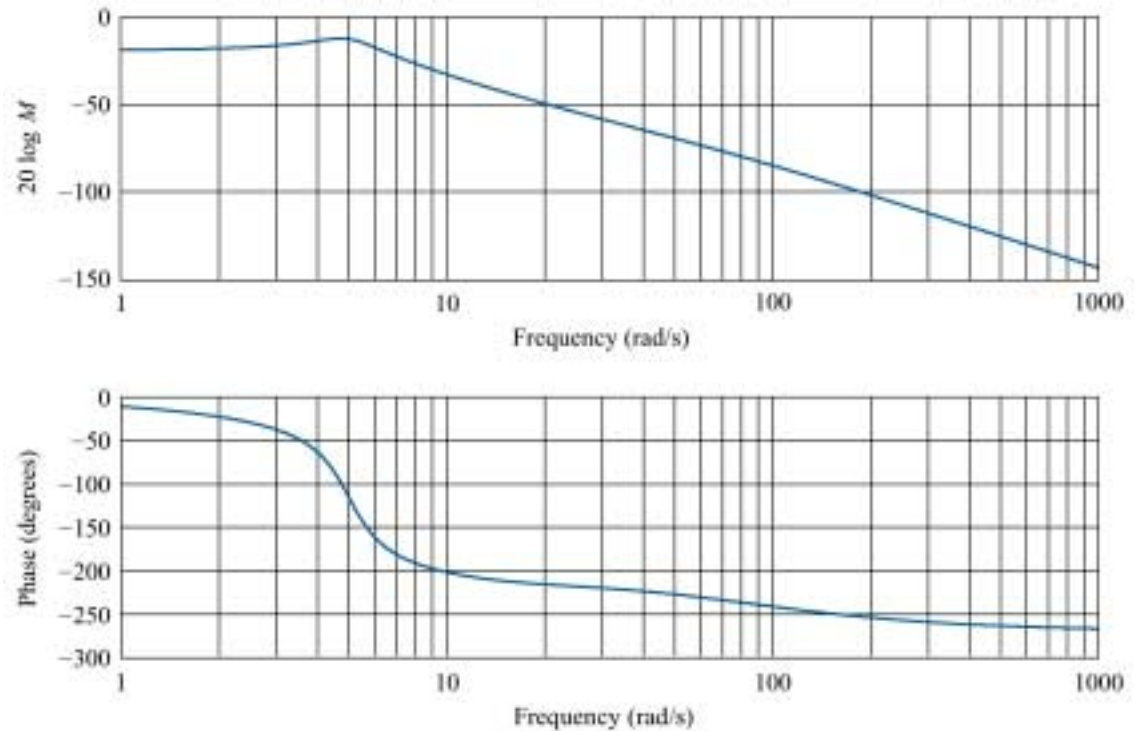
**Figure 10.56**

Step response for closed-loop system with  $G(s) = 5/[s(s + 1)(s + 10)]$ :

- a. with a 1 second delay;
- b. without delay

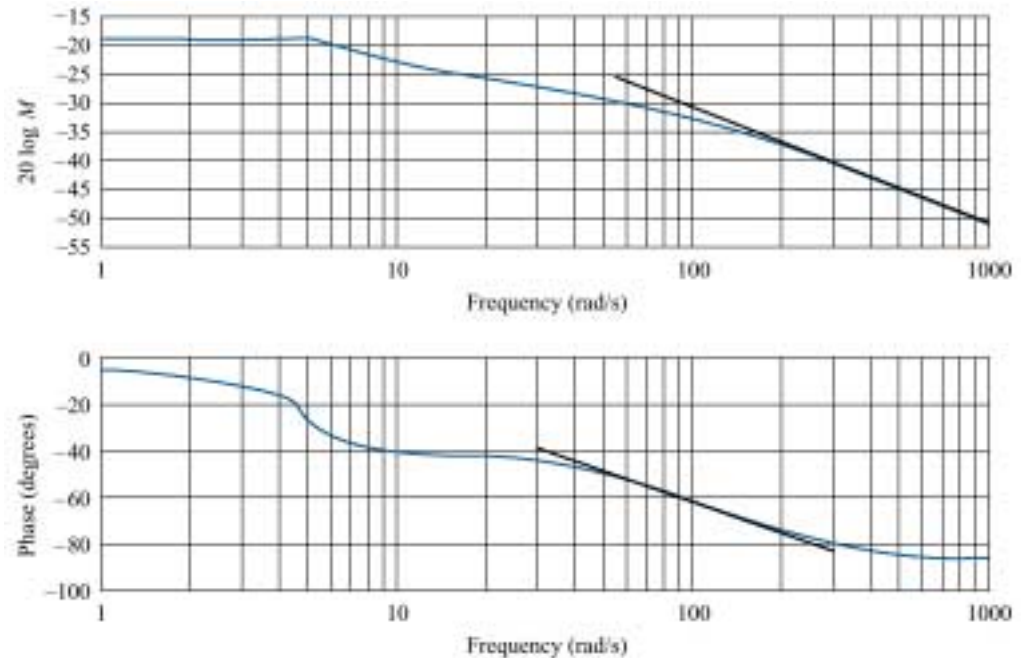


**Figure 10.57**  
Bode plots for  
subsystem with  
undetermined  
transfer function

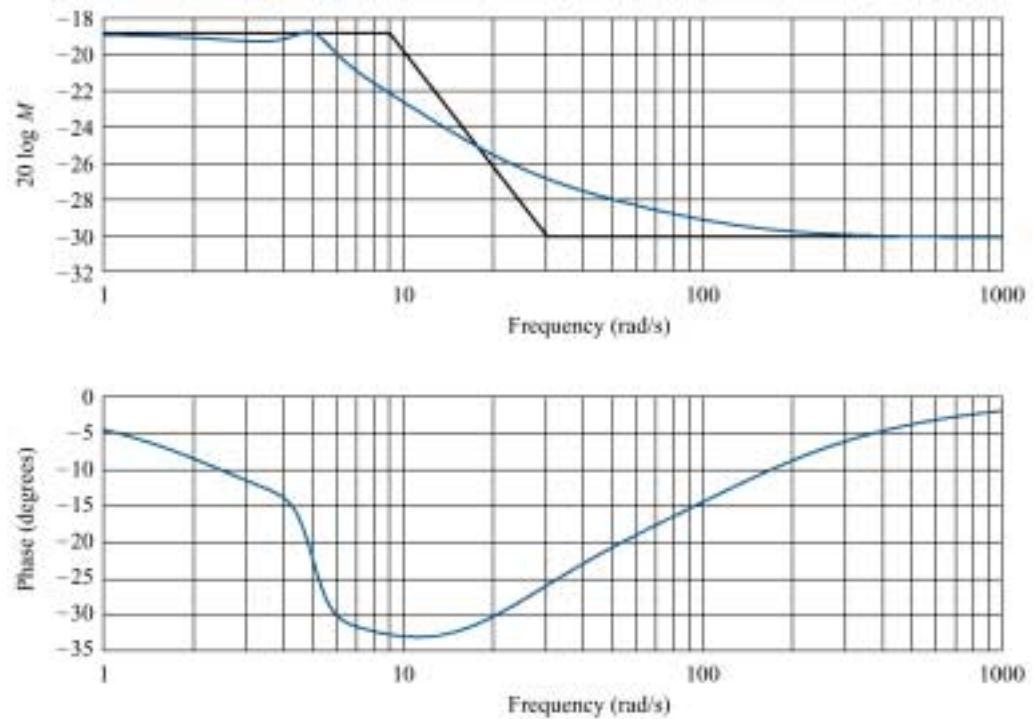


**Figure 10.58**

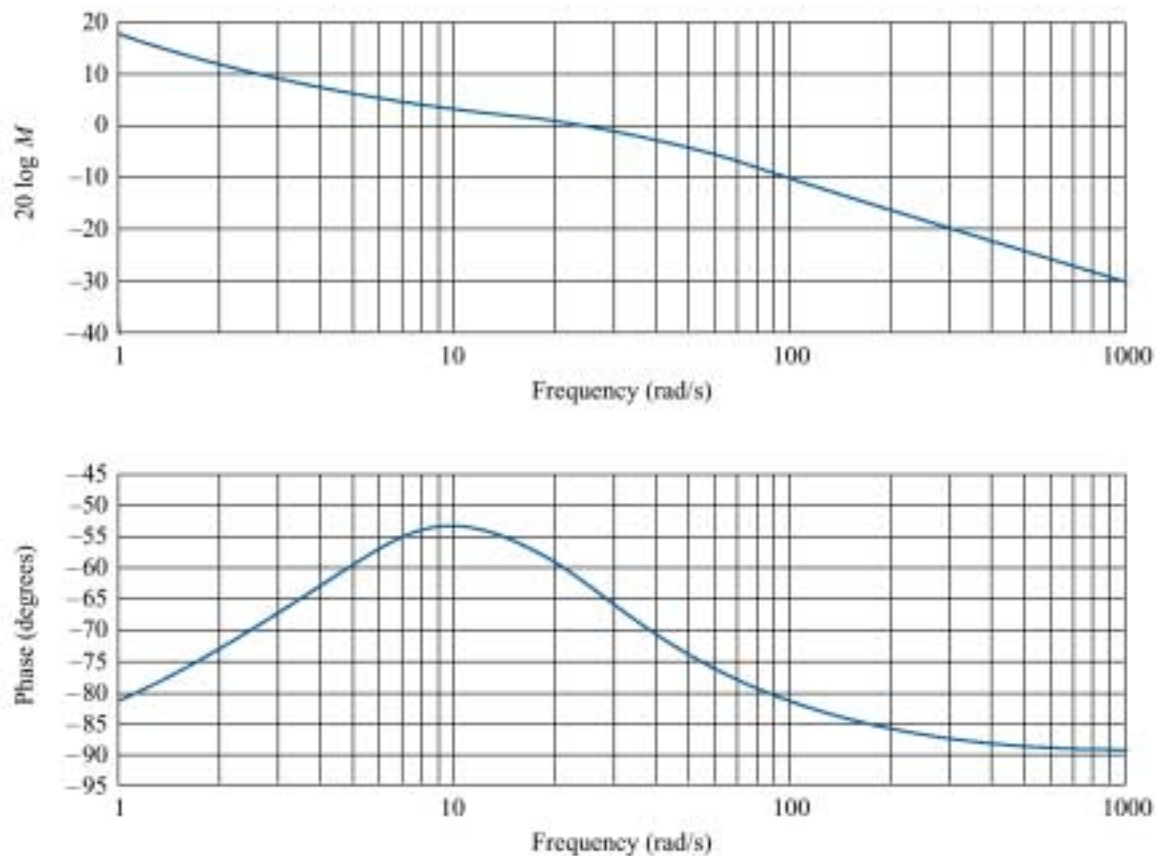
Original Bode plots  
minus response of  
 $G_1(s) =$   
 $25/(s^2 + 2.4s + 25)$



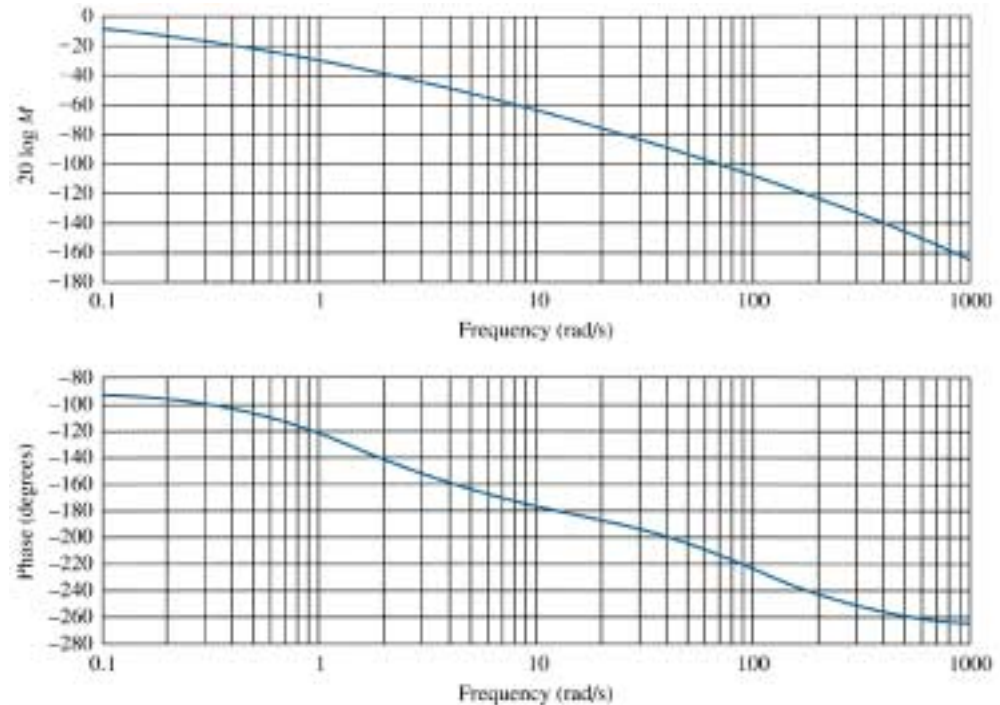
**Figure 10.59**  
Original Bode plot  
minus response  
of  $G_1(s)G_2(s) =$   
 $[25/(s^2 + 2.4s + 25)]$   
 $\cdot [90/(s + 90)]$

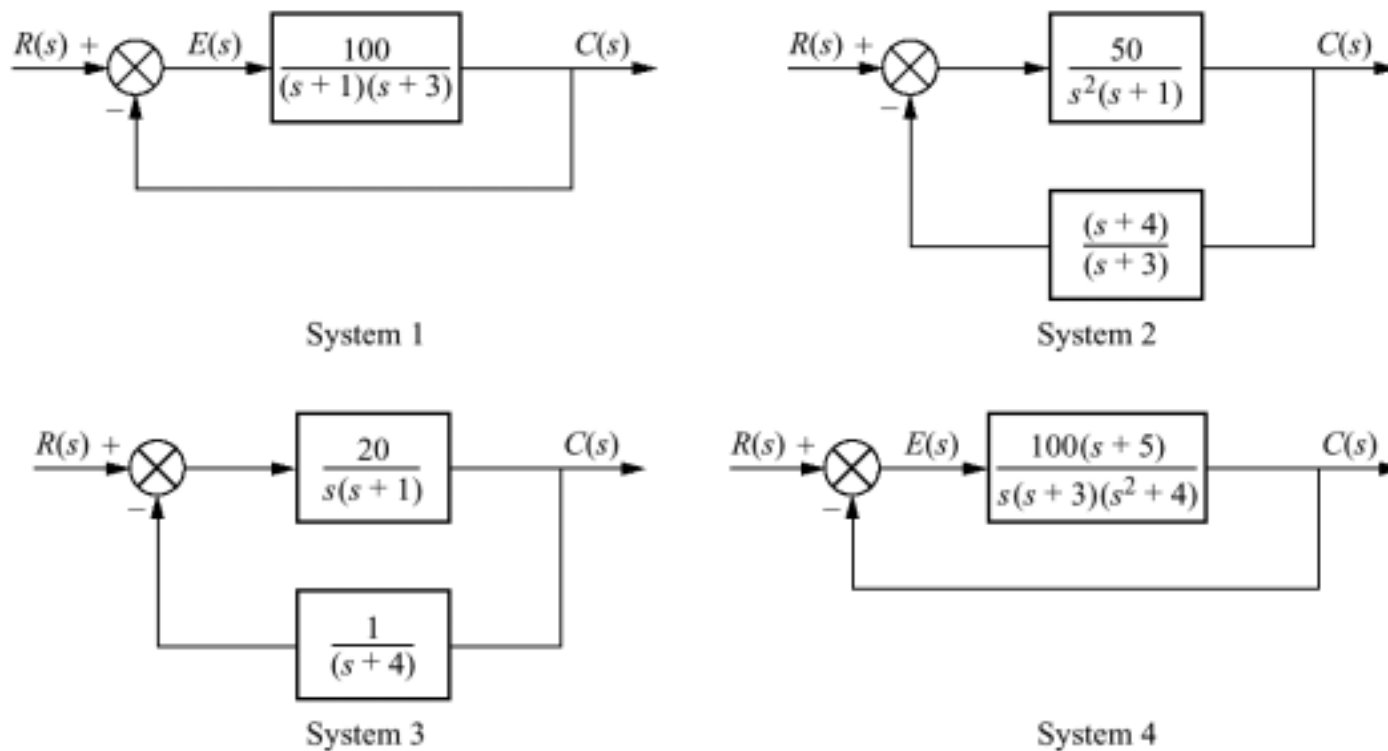


**Figure 10.60**  
Bode plots for  
Skill-Assessment  
Exercise 10.12

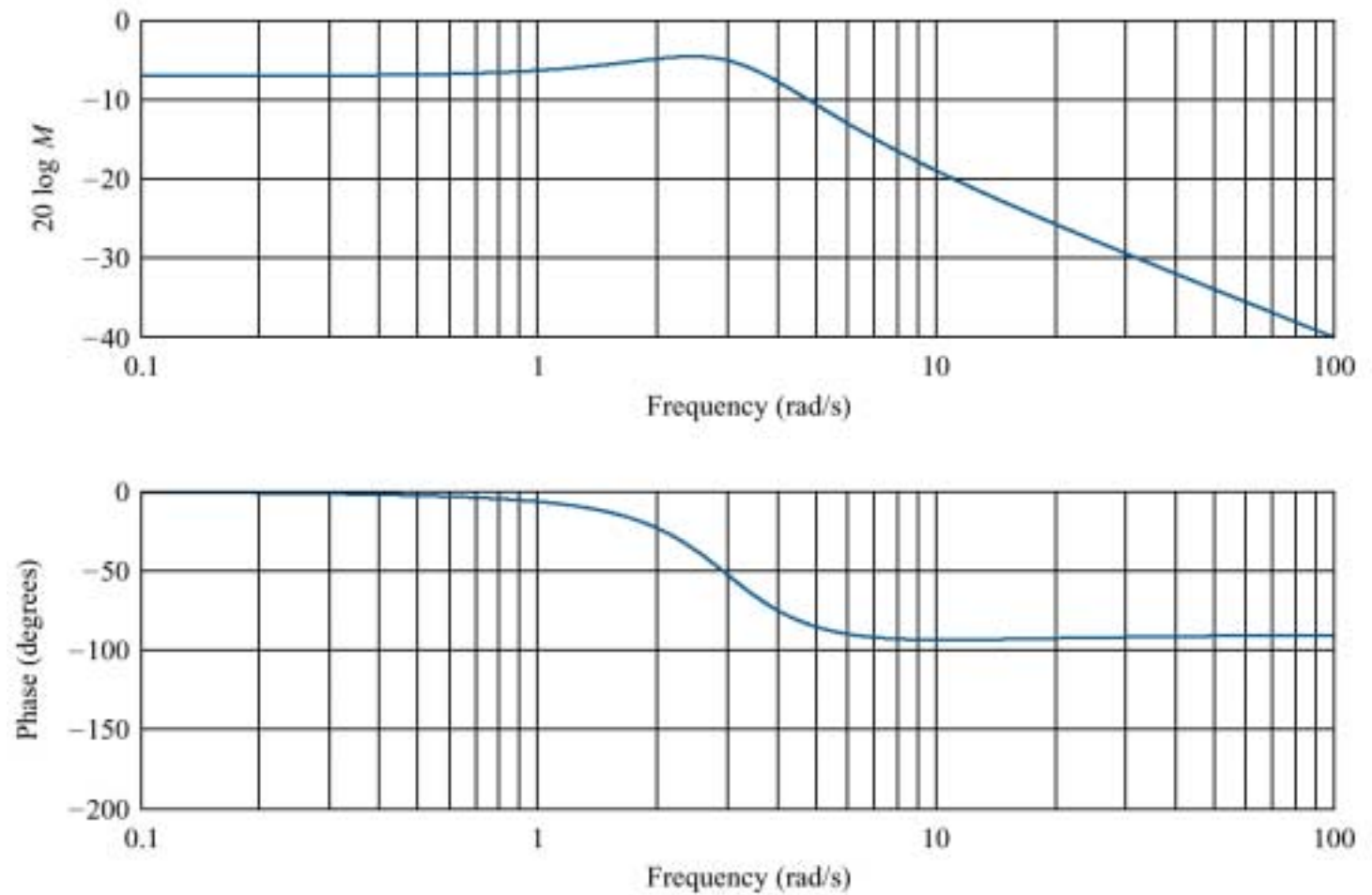


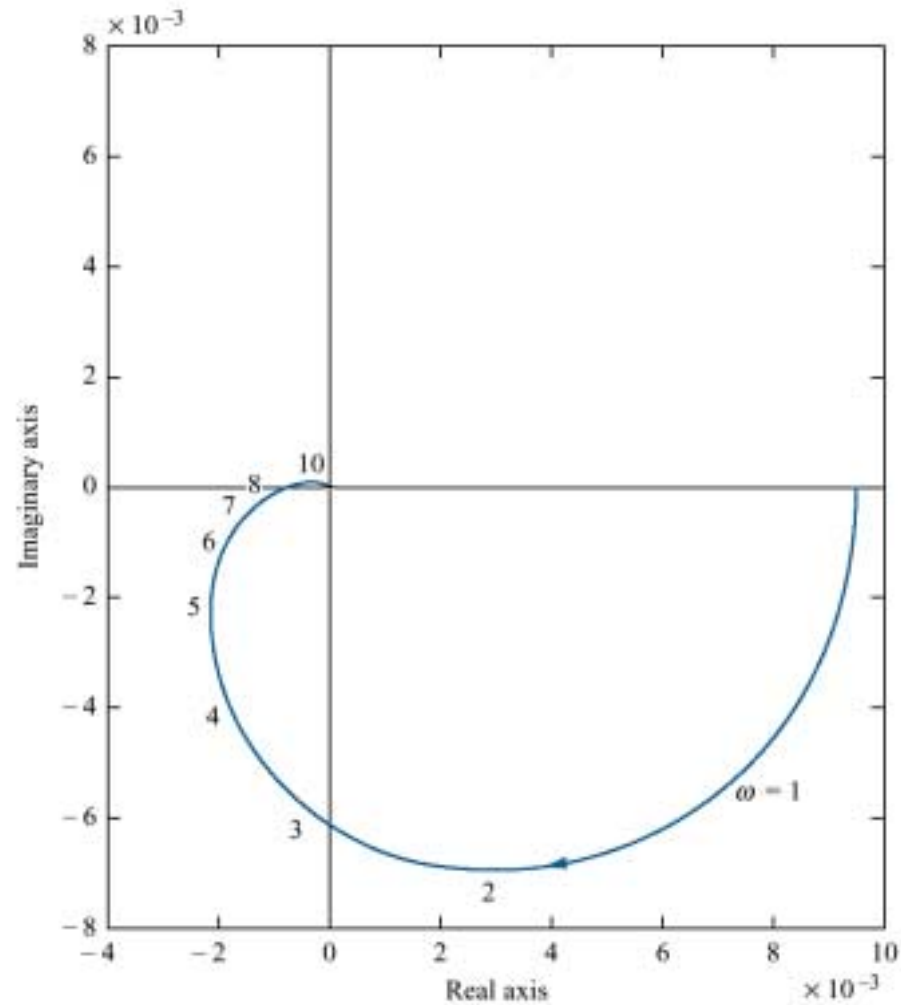
**Figure 10.61**  
Open-loop frequency  
response plots for  
the antenna control  
system ( $K = 1$ )

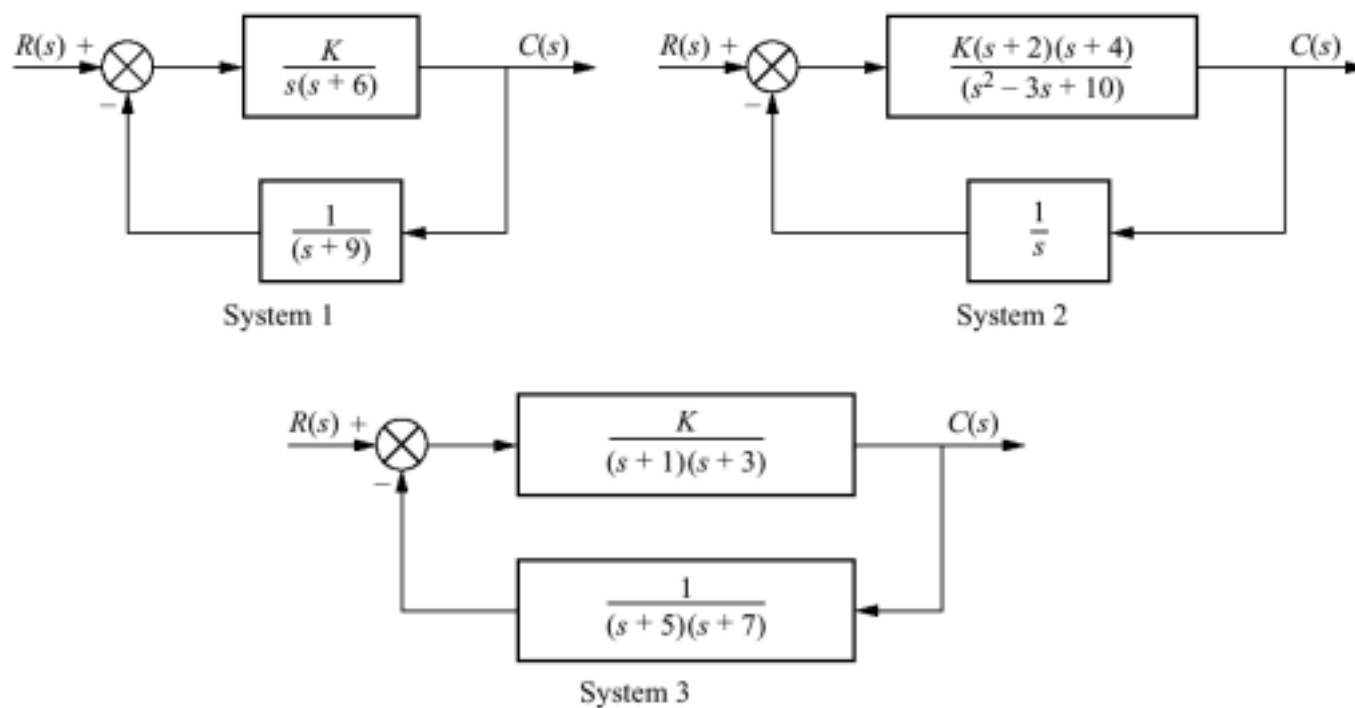


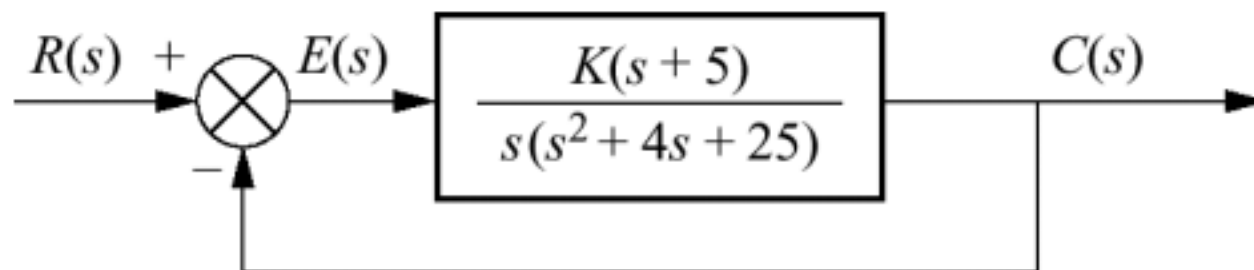
**Figure P10.1**

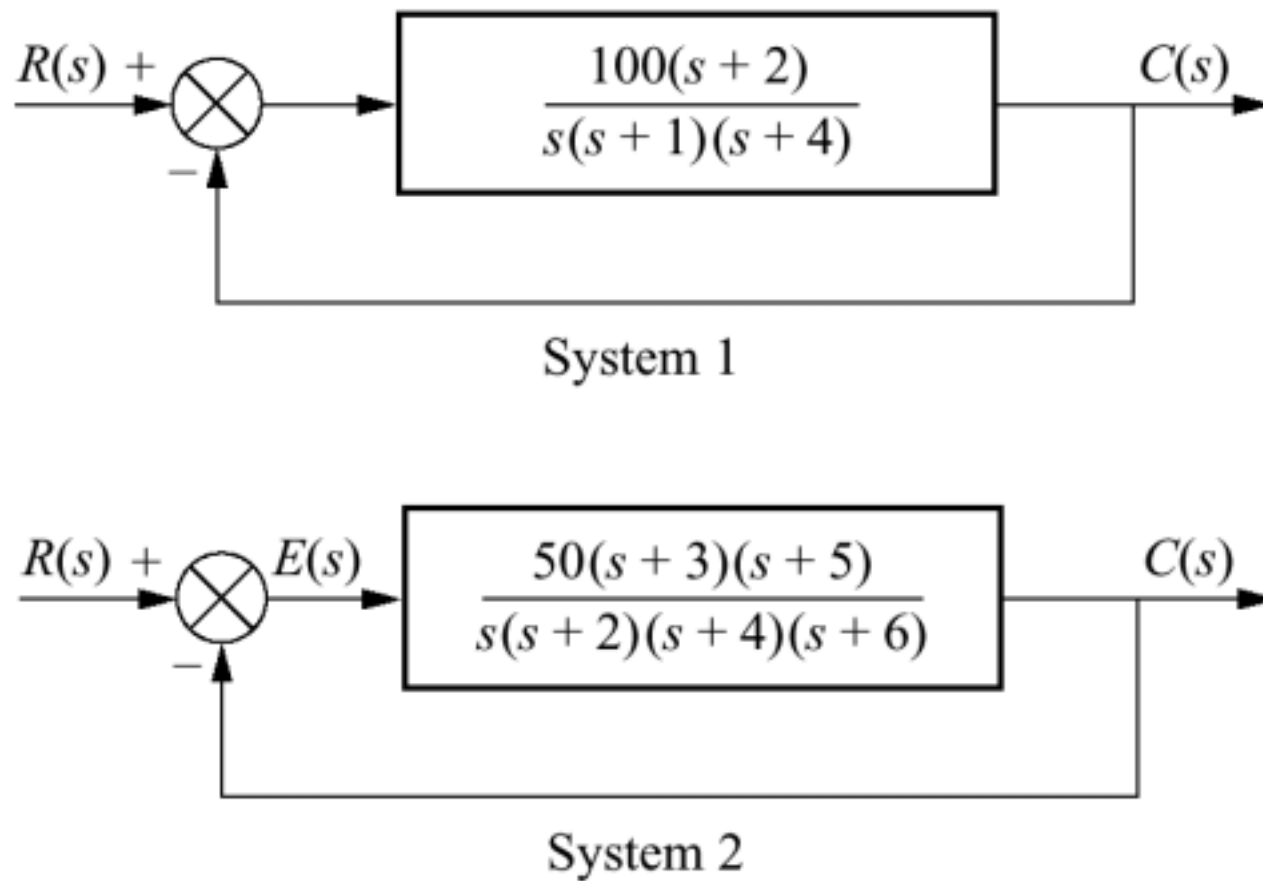


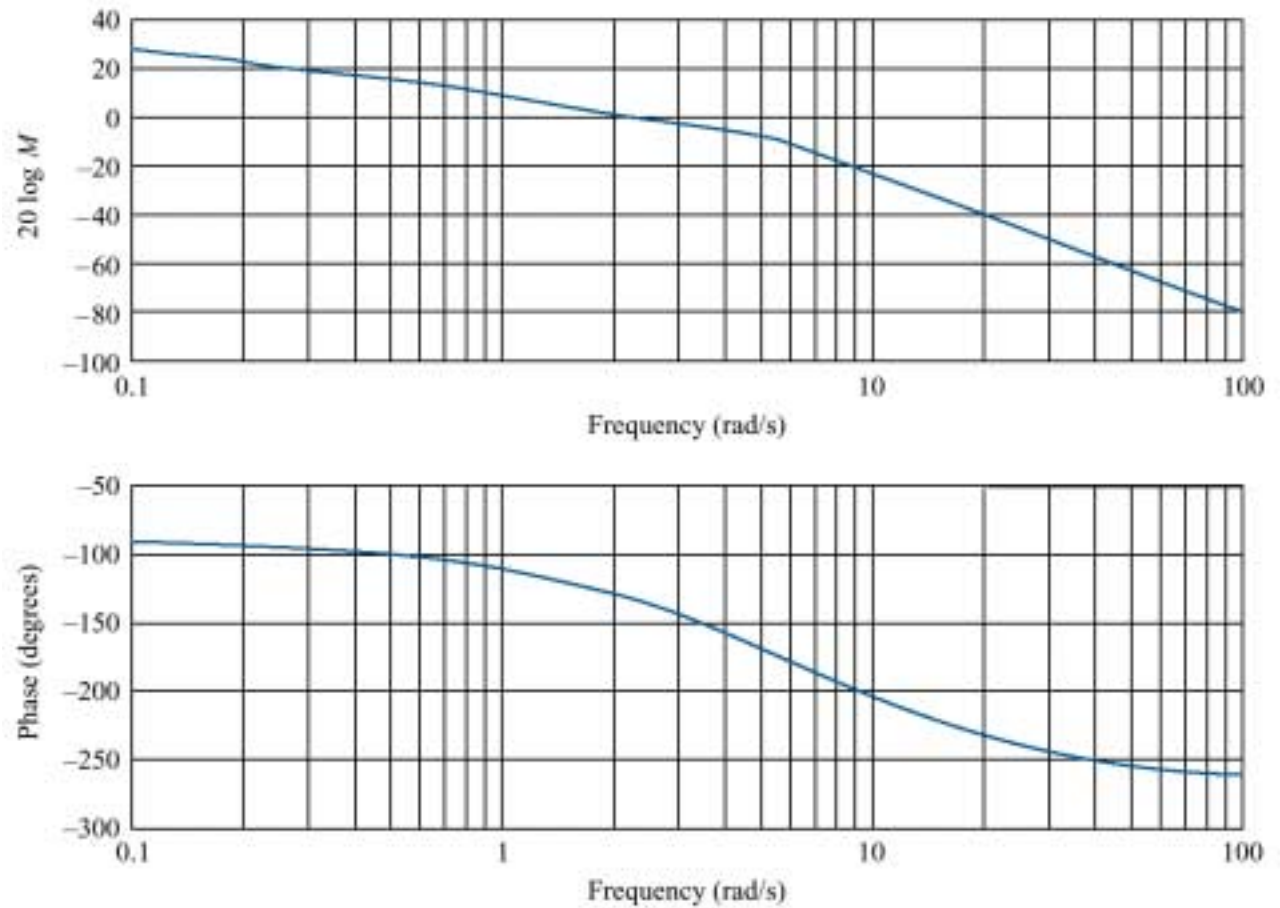
**Figure P10.2**

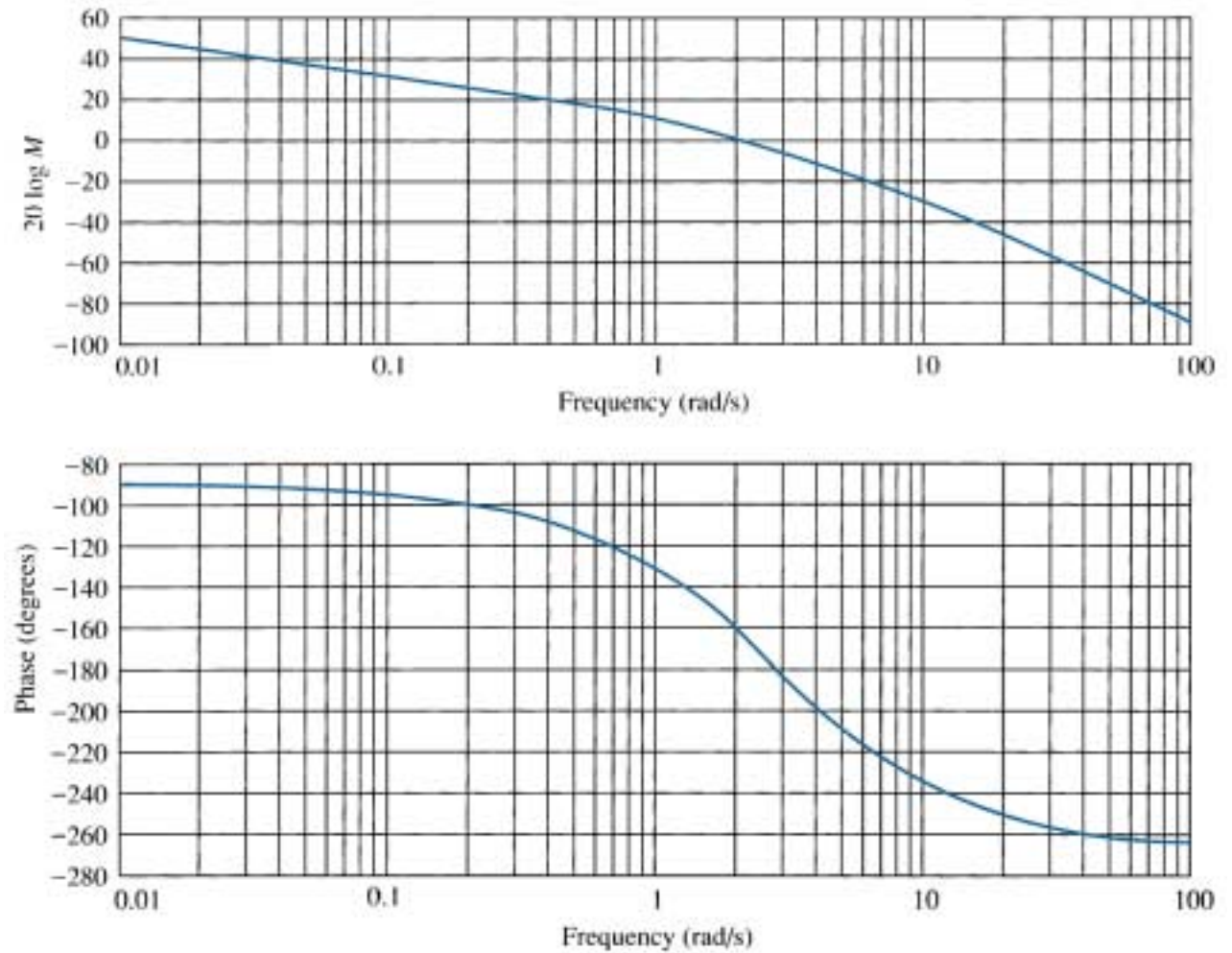
**Figure P10.3**

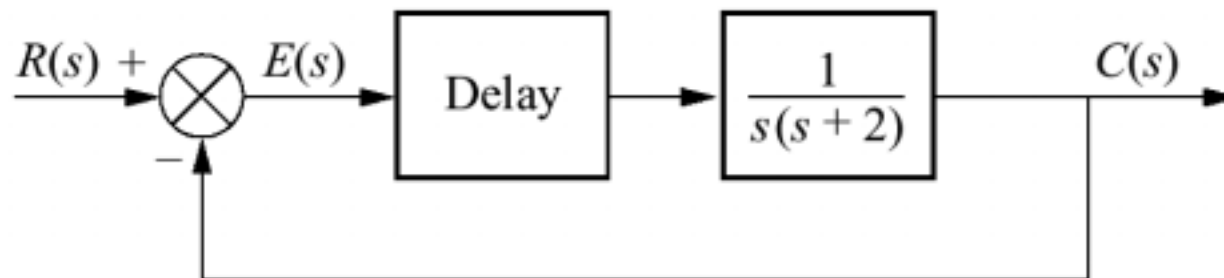
**Figure P10.4**

**Figure P10.5**

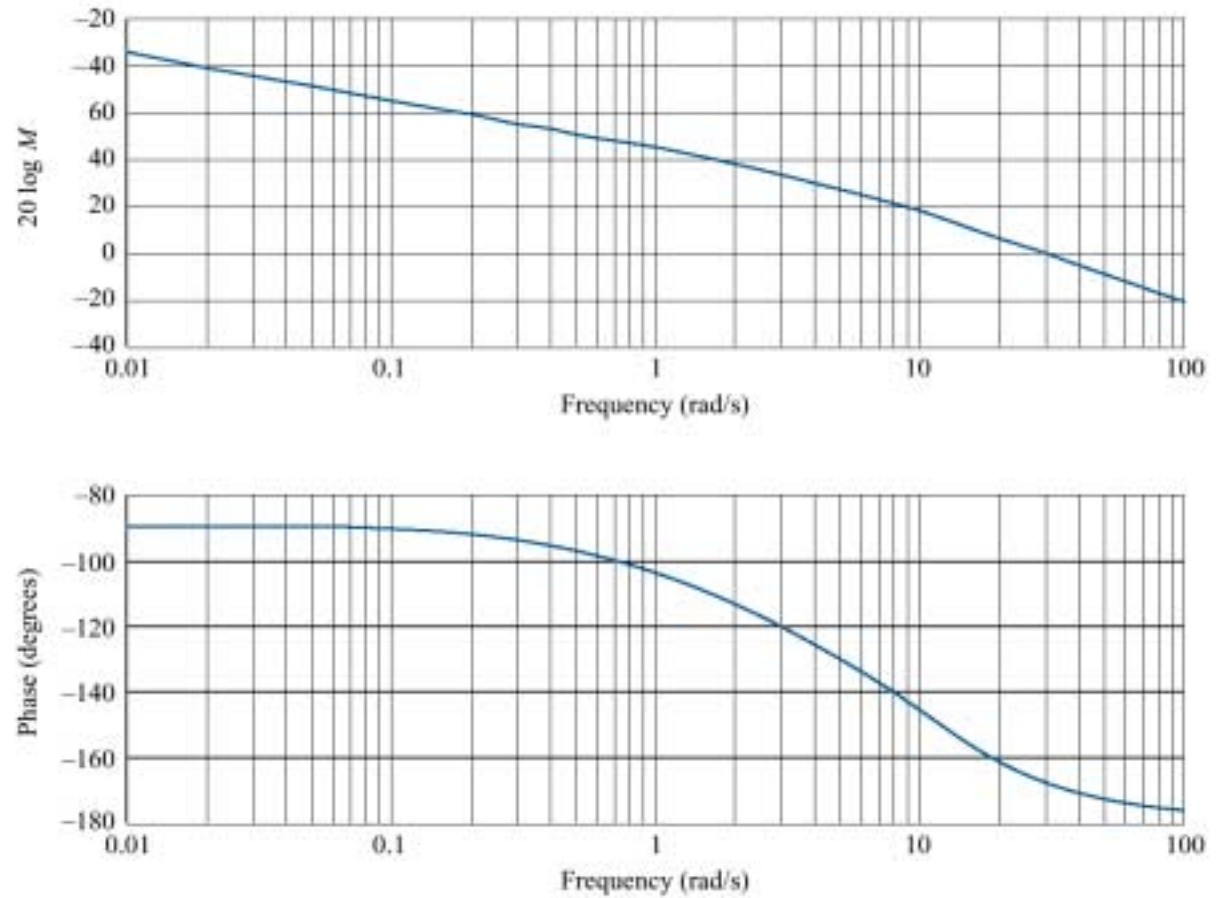
**Figure P10.6**

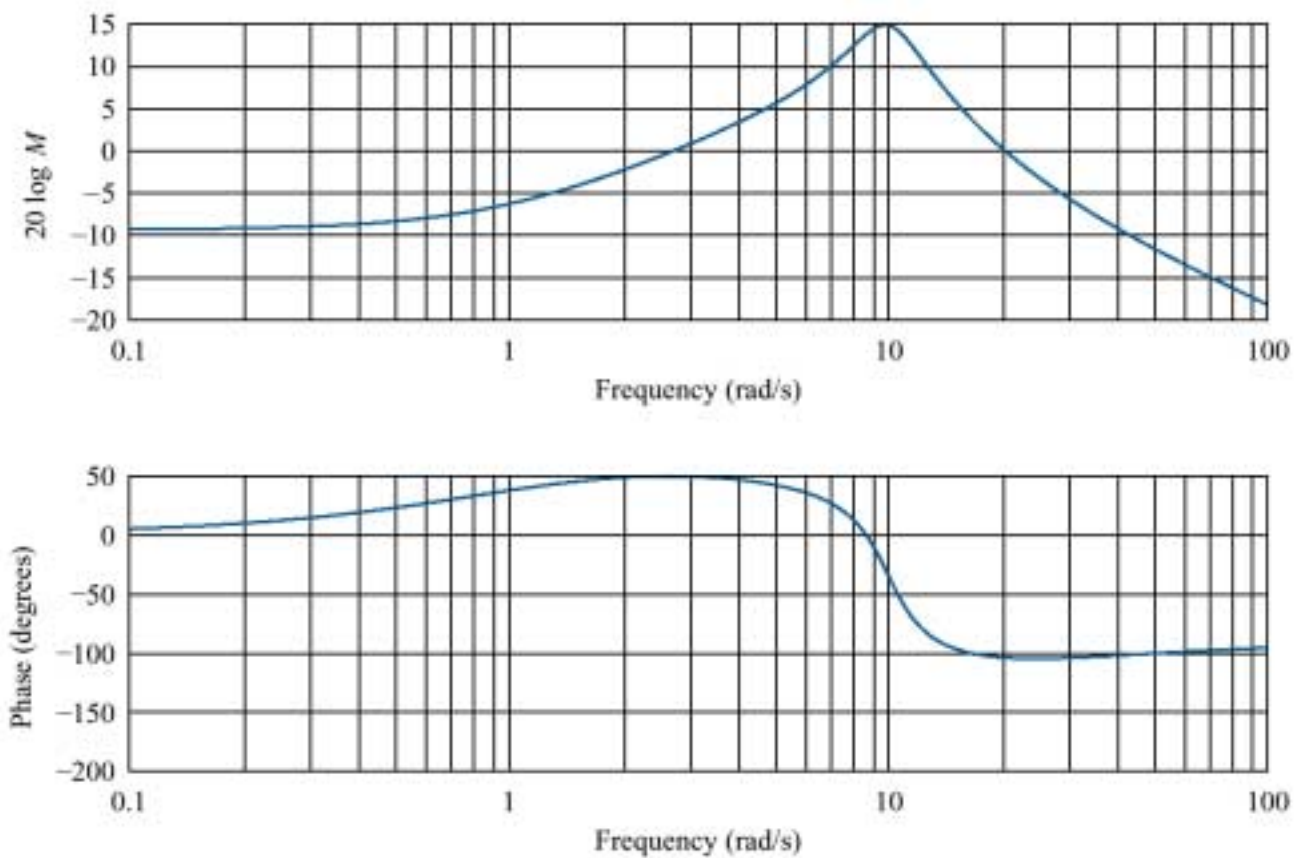
**Figure P10.7**

**Figure P10.8**

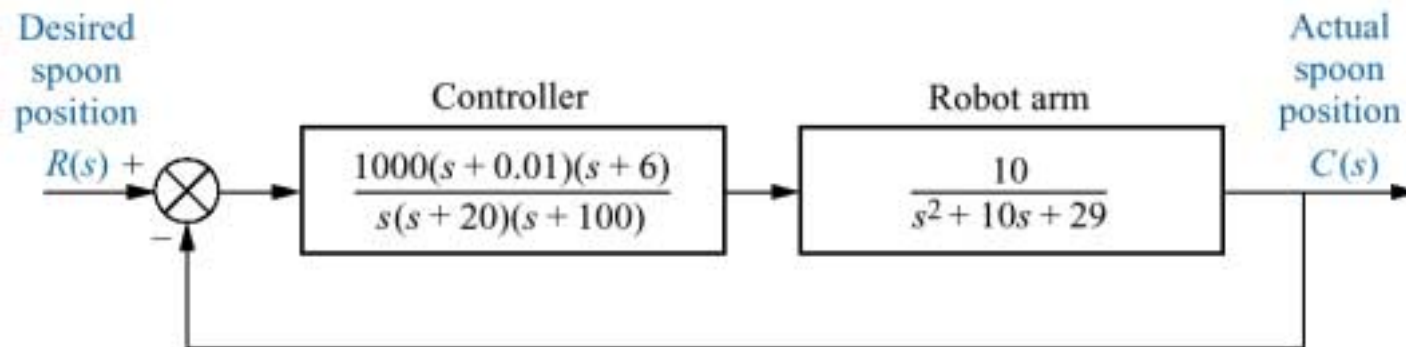
**Figure P10.9**



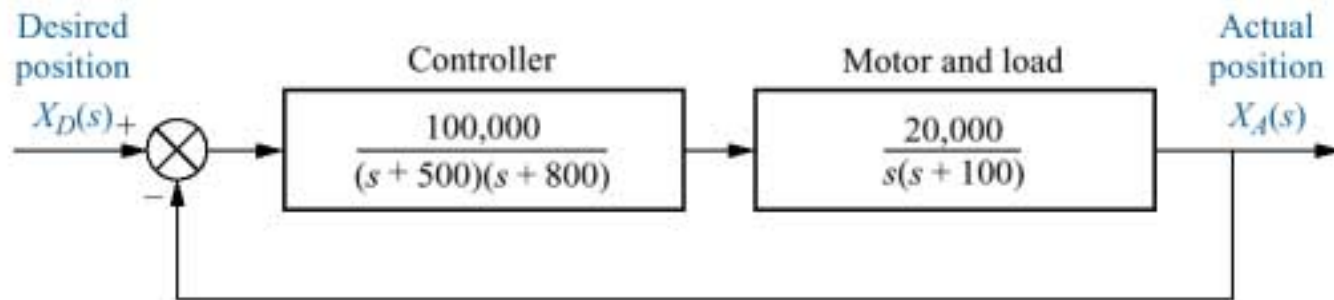
**Figure P10.10**

**Figure P10.11**

**Figure P10.12**  
Soft Arm position  
control system  
block diagram



**Figure P10.13**  
Floppy disk drive  
block diagram



**Figure P10.14**

AdeptOne, a four- or five-axis industrial robot, is used for assembly, packaging, and other manufacturing tasks.

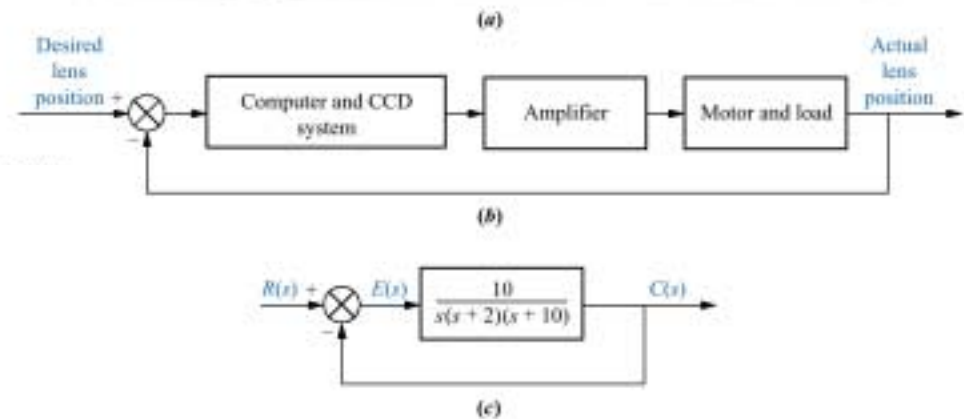


**Figure P10.15**

**a.** A cutaway view of a Nikon 35-mm camera showing parts of the CCD automatic focusing system;

**b.** functional block diagram;

**c.** block diagram



**Figure P10.16**  
Block diagram of a  
ship's roll stabilizing  
system

