MATHEMATICS
TEXAS A&M UNIVERSITY

MATH 609

NUMERICAL ANALYSIS

FALL 2022

LAB 06

SUMMARY RESULT

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EXERCISE: ITERATIVE SOLVERS AND LU SOLVERS FOR LEAST SQUARES PROBLEMS

L_1 NORM:

d = 15, tol = 0.01

LU:

[-0.04493857 -0.09654635 0.04411597 0.75271754 1.19925946 0.62242539
1.87890889 2.04617698 3.37441963 3.27439501 -3.88145103 -4.12588748
-3.88077333 5.85075698 -6.19343676]

JACOBI Number of Iteration: 200 – DIVERGED

GAUSS-SEIDEL Number of Iteration: 2

[-6.76407509e-02 -6.99154445e-02 -3.24825138e-03 9.41549659e-01
  1.05416582e+00 9.12642946e-01 2.00169234e+00 2.04083339e+00
  3.08423376e+00 3.04001696e+00 -4.01565621e+00 -4.06460104e+00
 -3.97191380e+00 5.94789955e+00 -6.04145032e+00]

GD Number of Iteration: 2

[-0.2192773 -0.25687584 -0.19629458 0.72997315 0.85695781 0.69010911
1.76880385 1.82238534 2.90084979 2.87563225 -4.17955638 -4.25421399
  -4.15628889 5.74653404 -6.26186204]

d = 15, tol = 0.0001

JACOBI Number of Iteration: 200 – DIVERGED

GAUSS-SEIDEL Number of Iteration: 52

[-0.04475427 -0.09677009 0.042529 0.7494989 1.19567604 0.61924284
1.87662141 2.04544938 3.37468555 3.27615609 -3.87868545 -4.12284991
-3.87854385 5.853618 -6.1923528]

GD Number of Iteration: 4

[[-0.22010878 -0.2577018 -0.19713133 0.72915246 0.85611128 0.68929662
1.76797574 1.82154654 2.89999382 2.87478135 -4.1803969 -4.25503942
-4.15713197 5.74570894 -6.26268486]]

d = 50, tol = 0.01

LU:

[-1.99986027 3.87332345 0.31993312 4.63840414 6.1576226 -1.11828521
1.99047038 4.17825341 6.13822979 0.10875233 3.83569843 2.13762631
0.38557871 0.43888092 -7.14458418 -2.82128437 1.55717569 -0.4698264
-1.39845158 1.39415745 3.72922481 6.61088485 0.13427403 0.40690653
-0.61397708 5.82836862 2.25076054 3.39164693 -2.31384778 0.33131014
4.54431314 3.24762168 -0.79667943 -1.62951752 -1.56788602 -1.33920274
0.88663771 1.22531305 -0.7534489 -0.78001659 -3.47713879 -4.64576357
5.52120913 -0.60730742 4.02141427 0.75643596 1.04478015 1.97800474
3.56789213 3.00613042]

JACOBI Number of Iteration: 200 – DIVERGED

GAUSS-SEIDEL Number of Iteration: 1

[-1.85203963 3.78037815 0.23704546 4.63807023 6.2253363 -1.13614746
2.02281166 4.38707983 5.94109647 0.14410352 3.79860131 2.14016063
0.33703687 0.4684716 -6.87694039 -2.78465492 1.6578261 -0.45225926
-1.49086485 1.59782581 3.76168823 6.71868949 0.09879926 0.29751112
-0.56202222 5.81457123 2.080534 3.26627118 -2.37913187 0.06663743
4.68525869 3.26270959 -0.69357254 -1.65921297 -1.33606463 -1.23913877
0.73797513 1.20910493 -0.62356753 -0.68114025 -3.67144971 -4.53391947
5.43373563 -0.68176834 3.99199805 0.64414041 1.02283512 1.82398973
3.58240708 2.96514379]

GD Number of Iteration: 2

[-2.02635801 3.63095045 0.08674888 4.46799213 6.02450298 -1.3026454
1.84276872 4.13367478 5.80406653 -0.0529309 3.63101862 1.95814477
0.1642721 0.28956198 -7.12035959 -2.97968077 1.42691701 -0.64839696
-1.6548726 1.36882904 3.56128521 6.49346431 -0.09072997 0.13995885
-0.75782741 5.6349338 1.9499745 3.12612269 -2.56021209 -0.07166642
4.48709933 3.05394177 -0.89941523 -1.84032753 -1.57866987 -1.4531008
0.58036906 1.04148288 -0.83408662 -0.90385527 -3.819466 -4.73217777
5.29511578 -0.82950376 3.83378966 0.47227129 0.84866512 1.67813543
3.39906761 2.79199382]

d = 50, tol = 0.0001

JACOBI Number of Iteration: 200 – DIVERGED

GAUSS-SEIDEL Number of Iteration: 84

[-1.99627618 3.88025348 0.32388127 4.6400284 6.15925901 -1.11595154

1.98828862 4.17602321 6.1355341 0.10359672 3.83322325 2.12992073
0.3752608 0.43185452 -7.15262089 -2.83142957 1.54916322 -0.48084996
-1.40760391 1.38280675 3.71833355 6.59824148 0.12421659 0.40039409
-0.62153281 5.8221922 2.24690637 3.39094303 -2.31043191 0.33626496
4.55465289 3.25264898 -0.78842191 -1.62269434 -1.55946427 -1.32909335
0.89601625 1.23091692 -0.74783824 -0.77404397 -3.47170309 -4.63733021
5.52908991 -0.60341741 4.02514756 0.75698123 1.04918075 1.98312491
3.5730665 3.01253849]

GD Number of Iteration: 3

[[-2.02642497 3.63088299 0.08668143 4.46792477 6.02443579 -1.30271284
1.8427014 4.13360789 5.80399872 -0.05299815 3.63095117 1.95807742
0.16420465 0.28949463 -7.12042648 -2.97974812 1.42684989 -0.64846431
-1.65494014 1.36876204 3.56121788 6.49339716 -0.09079738 0.13989124
-0.75789466 5.6348664 1.94990679 3.12605509 -2.56027946 -0.07173415
4.48703222 3.05387451 -0.89948245 -1.84039497 -1.57873681 -1.45316803
0.58030139 1.04141543 -0.83415376 -0.90392243 -3.81953371 -4.73224494
5.29504815 -0.82957129 3.8337222 0.47220381 0.84859772 1.67806783
3.39900031 2.7919264]]

L_2 NORM:

d = 15, tol = 0.01

LU :

[-0.02222503 0.22872391 0.08855644 0.78897906 0.80110981 0.87007112
2.17400435 2.23620909 2.98882885 2.95731756 -4.1001633 -4.19894033
-4.0895059 6.00725699 -5.78153049]

JACOBI Number of Iteration: 200 – DIVERGED

GAUSS-SEIDEL Number of Iteration: 1

[-0.01838185 0.01870769 -0.0088272 0.96319898 0.9748403 0.99519606
2.02772544 2.02853519 2.99018319 2.9859254 -4.01610601 -4.02277691
-3.99610586 6.01305593 -5.96881947]

GD Number of Iteration: 2

[-0.0495695 0.02843638 -0.02606381 0.8955759 0.89182832 0.91776336
1.99624146 2.02649542 2.95373219 2.93221268 -4.08375375 -4.11569886
-4.07108461 5.95596385 -5.98444861]

d = 15, tol = 0.0001

JACOBI Number of Iteration: 200 – DIVERGED

GAUSS-SEIDEL Number of Iteration: 38

[-0.01779351 0.23164251 0.08806333 0.78787716 0.80062387 0.8727438
2.17557682 2.23484932 2.98822118 2.95502222 -4.10407071 -4.20199112
-4.09029919 6.00800067 -5.77976537]

GD Number of Iteration: 3

[-0.04969424 0.02830199 -0.02619202 0.89545861 0.89171114 0.91764295
1.99611024 2.02636114 2.953607 2.93208968 -4.08387457 -4.11581539
-4.07120658 5.95583822 -5.984582]

d = 50, tol = 0.01

LU:

[2.84824817 -0.5212767 0.99969728 -0.02545016 0.12271017 -0.3221526
-2.89640758 0.63591026 3.26859302 2.53420823 4.06650376 -0.98184886

4.34196489 -3.94984532 4.02155646 5.63693386 -2.95958312 -1.47899187
-1.40169747 -2.48906324 -8.33396397 1.35900873 -0.64152162 8.02116903
2.61347082 2.63558617 -7.39080713 1.28672798 6.49607693 -0.45750691
-1.9518694 0.37068016 0.07976308 0.13704119 0.11170858 -5.77813824
5.58710193 -2.54774407 1.67122007 -3.14201251 3.41195784 6.73656553
1.29450657 0.46677826 4.8841111 1.42824954 -2.54653301 3.90450188
4.47089062 -3.19338061]

JACOBI Number of Iteration: 200 – DIVERGED

GAUSS-SEIDEL Number of Iteration: 1

[2.80883443 -0.37947776 0.96154803 0.12807141 0.16125822 -0.49784447
-2.96916475 0.51693856 3.18002647 2.43318151 4.05102674 -0.73701907
4.58913591 -3.70971495 4.11119346 5.61047797 -2.83515319 -1.58633406
-1.18606197 -2.3778872 -8.36979107 1.3899969 -0.77249459 7.8791714
2.56508285 2.72130327 -7.29769108 1.15191848 6.36948896 -0.42964724
-1.90342631 0.26530731 -0.07650429 0.22990204 0.15238564 -5.90871915
5.45242711 -2.66120344 1.44738038 -3.01058482 3.37775921 6.66109801
1.17456637 0.34927078 4.85293172 1.48445133 -2.26053213 3.81168162
4.67490216 -3.17062226]

GD Number of Iteration: 2

[2.76280756 -0.45842802 0.90523306 0.0325127 0.08482672 -0.52739059
-3.0099309 0.48688955 3.14990438 2.40215077 3.99214412 -0.84168306
4.47715128 -3.82087406 4.02848955 5.55088599 -2.92688866 -1.63024546
-1.28104412 -2.45226149 -8.41954013 1.31518033 -0.80859841 7.85358744
2.52782679 2.63256551 -7.38542383 1.13127674 6.34390378 -0.48002603
-1.96366822 0.22958444 -0.08809847 0.16512287 0.10135585 -5.93308361
5.42292323 -2.69232443 1.44079429 -3.08219005 3.33035259 6.63527001

1.15033094 0.32034837 4.81942278 1.40161137 -2.38271264 3.77738593
4.59266353 -3.24756523]

d = 50, tol = 0.0001

JACOBI Number of Iteration: 200 – DIVERGED

GAUSS-SEIDEL Number of Iteration: 153

[2.85795542 -0.51216235 1.01170432 -0.0144421 0.13496681 -0.30567069
-2.88547955 0.64727451 3.28208422 2.54026927 4.07584865 -0.97492686
4.3501076 -3.94351155 4.02758493 5.64170151 -2.96038356 -1.4788908
-1.40277073 -2.490596 -8.33538492 1.35441065 -0.65090387 8.01008379
2.60713423 2.62801431 -7.40130703 1.27203835 6.48598029 -0.47118869
-1.9655252 0.3639539 0.06887112 0.12834732 0.10080004 -5.78922507
5.57613253 -2.55492144 1.66214372 -3.14742324 3.41181224 6.7328773
1.29737896 0.46770573 4.8885449 1.42754084 -2.5425713 3.91186579
4.47903592 -3.1840655]

GD Number of Iteration: 2

[2.76280756 -0.45842802 0.90523306 0.0325127 0.08482672 -0.52739059
-3.0099309 0.48688955 3.14990438 2.40215077 3.99214412 -0.84168306
4.47715128 -3.82087406 4.02848955 5.55088599 -2.92688866 -1.63024546
-1.28104412 -2.45226149 -8.41954013 1.31518033 -0.80859841 7.85358744
2.52782679 2.63256551 -7.38542383 1.13127674 6.34390378 -0.48002603
-1.96366822 0.22958444 -0.08809847 0.16512287 0.10135585 -5.93308361
5.42292323 -2.69232443 1.44079429 -3.08219005 3.33035259 6.63527001
1.15033094 0.32034837 4.81942278 1.40161137 -2.38271264 3.77738593
4.59266353 -3.24756523]

L_∞ NORM:

d = 15, tol = 0.01

LU :

[-0.37209441 -0.04656105 0.20123979 1.09127608 1.0151838   1.08582137
  1.94455694 1.91724271 3.0578017   3.25430324 -3.98731057 -3.74019168
 -4.03529506 5.85578653 -6.22207339]

JACOBI Number of Iteration: 200 – DIVERGED

GAUSS-SEIDEL Number of Iteration: 1

[-0.02993892 0.02126524 0.04735205 1.01118996 0.99456354 1.01093668
  1.99097055 1.98590666 3.01182886 3.02624154 -4.00701587 -3.97844153
 -4.02143451 5.97346105 -6.0301268]

GD Number of Iteration: 3

[5.51058691e-03 6.28905306e-02 1.50341895e-01 1.10943487e+00
  1.07868982e+00 1.11878007e+00 2.06385295e+00 2.03760070e+00
  3.09995490e+00 3.18225814e+00 -3.90888491e+00 -3.83153873e+00
 -3.92499941e+00 5.98067455e+00 -4.45378341e+00]

d = 15, tol = 0.0001

JACOBI Number of Iteration: 200 – DIVERGED

GAUSS-SEIDEL Number of Iteration: 41

[-0.3772694 -0.0539718   0.19749773 1.09037896 1.01626245 1.08795683
  1.94698614 1.92065253 3.0626993   3.26101782 -3.98414407 -3.73815159
 -4.03634819 5.8525055 -6.22544098]

GD Number of Iteration: 4

[5.72240420e-03 6.30927253e-02 1.50540186e-01 1.10963979e+00
  1.07889324e+00 1.11897933e+00 2.06405483e+00 2.03780326e+00
  3.10015170e+00 3.18245865e+00 -3.90868211e+00 -3.83133905e+00
 -3.92479424e+00 5.98088188e+00 -4.45353610e+00]

d = 50, tol = 0.01

LU:

[4.73763863 3.10072339 -0.6065343 -3.17725868 5.73011071 -0.21774834
 -2.52518484 -2.13258315 -1.49430567 2.52740536 -1.32453211 3.02625559
  4.76007434 2.68286524 0.04913592 3.26926182 3.83701014 1.12487598
 -5.96720757 -1.78367365 -1.83042466 1.9402555   3.11508189 8.32166786
 -1.78755974 2.15246212 3.28839825 -0.18683127 -4.85364344 3.95349118
 -0.15755224 -2.06073042 -0.99885258 2.73938315 4.52471011 1.92664367
  3.57827192 2.73888012 5.74839634 -0.03787804 1.43579849 2.37015511
 -0.90279974 3.92239999 -0.7491709   1.2523114 -5.07667045 0.48195819
  2.90655256 1.84329947]

JACOBI Number of Iteration: 200 – DIVERGED

GAUSS-SEIDEL Number of Iteration: 1

[4.90414970e+00 3.06702247e+00 -7.95523609e-01 -3.30856214e+00
  5.56855394e+00 -1.33040729e-01 -2.52165686e+00 -2.16292405e+00
 -1.40079344e+00 2.53669404e+00 -1.42244562e+00 2.82990376e+00
  4.78376352e+00 2.72892915e+00 1.92953426e-01 3.51064666e+00
  3.77081799e+00 1.11157038e+00 -5.93672437e+00 -1.71557046e+00
 -1.84889930e+00 2.01090365e+00 2.96735996e+00 8.17099786e+00

-2.11470404e+00 2.24822635e+00 3.44461696e+00 -1.55785502e-01
-4.85424876e+00 4.00773331e+00 -1.88817726e-01 -2.19449417e+00
-9.61764887e-01 2.73435195e+00 4.64789045e+00 1.90101867e+00
 3.65522159e+00 2.91442948e+00 6.01496006e+00 -1.50954247e-03
 1.19811007e+00 2.35170205e+00 -9.69355169e-01 3.81635014e+00
-7.14252507e-01 1.11799755e+00 -5.14130740e+00 6.13152364e-01
 2.97575849e+00 1.93096189e+00]

GD Number of Iteration: 2

[4.90825565 3.11065686 -0.70699224 -3.22741629 5.65598617 -0.08451467
 -2.4674943 -2.10835911 -1.33858579 2.58379832 -1.34315043 2.91679169
  4.84452104 2.77016149 0.19976012 3.50613444 3.8293021   1.16500396
 -5.89580302 -1.66224213 -1.80216964 2.05075085 3.04649925 8.27684893
 -1.98551339 2.29640485 3.47914977 -0.08901948 -4.80070839 4.04717145
 -0.12041721 -2.11196704 -0.90378   2.80712485 4.6742139   1.94720288
  3.70583685 2.92198649 5.9947804   0.02176222 1.29268748 2.41630953
 -0.9033144   3.90122655 -0.67417808 1.20213087 -5.06246383 0.64531923
  3.00071218 1.95968546]

d = 50, tol = 0.0001

JACOBI Number of Iteration: 200

GAUSS-SEIDEL Number of Iteration: 50

[4.74223108 3.10875606 -0.60211584 -3.17196013 5.73497119 -0.21152775
 -2.52473042 -2.13253907 -1.49605271 2.52682685 -1.32302815 3.02712933
  4.75494955 2.67577765 0.04099062 3.258589   3.82950816 1.11556286
 -5.97568928 -1.7891422 -1.83438283 1.93703711 3.11620897 8.32189462

-1.78598223 2.15616527 3.28789772 -0.18600687 -4.85062296 3.95300944

-0.15903389 -2.06371216 -1.00096475 2.73586071 4.52014703 1.92188468

3.57473935 2.73908267 5.74582329 -0.03924204 1.43432488 2.37083475

-0.89695136 3.92588117 -0.73921968 1.25898119 -5.0680164   0.48782827

2.91277918 1.84654475]

GD Number of Iteration: 3

[[4.90826549 3.11066655 -0.70698269 -3.22740665 5.65599579 -0.08450489

-2.46748457 -2.10834938 -1.33857605 2.58380807 -1.34314079 2.91680129

4.84453079 2.77017128 0.19977001 3.50614436 3.82931175 1.16501366

-5.89579325 -1.66223238 -1.80215991 2.05076062 3.04650886 8.2768585

-1.98550392 2.29641468 3.47915962 -0.08900978 -4.80069865 4.04718123

-0.12040753 -2.11195743 -0.90377024 2.80713455 4.67422374 1.94721261

3.7058466   2.92199638 5.99479036 0.02177198 1.29269698 2.41631924

-0.90330472 3.90123618 -0.67416828 1.20214049 -5.06245416 0.64532907

3.00072199 1.95969526]]

RESULTS:

- In any case, the Jacobi method diverged.
- Although the difference is not clear enough when the tolerance is small, it is clearly seen that the GD method is more effective than the Gauss-Seidel Method as the tolerance gets smaller and the matrix gets larger.