ASEN 5050 – Homework #2

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1. **(18 pts (1 pt each))** For an Earth orbiting satellite with a semi-major axis of twice the Earth's radius and an eccentricity of 0.15, complete the following table (T-Tp is the time past periapse passage).

Note 1: Computing the eccentric anomaly requires iteratively solving Kepler's equation.

Note 2: You don't need to show your work. Please show 2 digits past the decimal.

Case	True Anomaly (deg)	Eccentric Anomaly (deg)	Mean Anomaly (deg)	$T - T_p$ (min)
A	0.00	0.00	0.00	0.00
B	30.00	25.95	22.18	14.73
C	-162.76 (=197.24)	200.00	202.94	134.71
D	90.00	81.37	72.88	48.38
E	-106.94 (=253.06)	261.50	270.00	179.23
F	49.88	43.59	37.66	25.00

2. **(14 pts)** The space shuttle orbiter is to deploy a satellite in 65 minutes. If the perigee height of the shuttle's orbit is 321 km, the apogee height is 551 km, and the true anomaly is 330°, what is the orbiter's true anomaly at the moment the satellite is deployed? Show your work.

Find: v_2

Given:

$v_1 = 330^o = 5.76\,rad$ $\quad$ $h_a = 551\,km$

$t_2 = 65\,min = 3900\,sec$ $\quad$ $R_E = 6378.137\,km$

$h_p = 321\,km$ $\quad$ $\mu = 398600.4418\,km^3/s^2$

Analysis:

Determine the perigee and apogee of orbit by including the radius of the Earth, R_E :

$$r_p = R_E + h_p = 6699.137\,km \qquad r_a = R_E + h_a = 6929.137\,km$$

Now, compute the semi-major axis, a , mean motion, n , and eccentricity, e , for this orbit:

$$a = \frac{r_p + r_a}{2} = 6814.137\,km \qquad n = \sqrt{\frac{\mu}{a^3}} = 0.001122\,rad/s \qquad e = \frac{r_a - r_p}{r_a + r_p} = 0.01688$$

With these orbital elements, it is possible to determine the Eccentric Anomaly, E_1 , and Mean Anomaly, M_1 , at current orbit position:

$$\tan\left(\frac{E_1}{2}\right) = \sqrt{\frac{1-e}{1+e}}\tan\left(\frac{v_1}{2}\right) \rightarrow E_1 = 330.48^o$$

$$M_1 = E_1 - e\sin\left(E_1\right) = 330.96^o$$

Because the Mean Anomaly advances at a constant rate, it is now possible to compute the Mean Anomaly at t_2 , labeled here as M_2 :

$$M_2 = M_1 + n\left(t_2 - t_1\right) = 221.76^o$$

Using M_2 , the Newton-Raphson technique allows for the computation of the Eccentric Anomaly at time t_2 , labeled here as E_2 :

$$E_2 = 221.13^o \quad \text{(iteration threshold was set 1e-8)}$$

The Eccentric Anomaly can be converted to the True Anomaly at t_2 , labeled here as v_2 :

$$\tan\left(\frac{v_2}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E_2}{2}\right) \qquad \boxed{\rightarrow v_2 = 220.50^o}$$

3. **(15 pts (5 pts each))** A satellite has been launched into a 798×817 km orbit (perigee height x apogee height), which is very close to the planned orbit of 794×814 km. What is the error in the semi-major axis, eccentricity, and the orbital period?

$\boldsymbol{Find}: \delta_a, \delta_e, \delta_P$

$\boldsymbol{Given}:$

$h_{p1} = 798\,km$ $\quad h_{p2} = 794\,km$

$h_{a1} = 817\,km$ $\quad h_{a2} = 814\,km$

$R_E = 6378.137\,km$ $\quad \mu = 398600.4418\,km^3/s^2$

$\boldsymbol{Analysis}:$

First, convert the perigee and apogee heights to radius of perigee and apogee, respectively:

$r_{p1} = h_{p1} + R_E = 7176.137\,km$ $\quad r_{p2} = h_{p2} + R_E = 7172.137\,km$

$r_{a1} = h_{a1} + R_E = 7195.137\,km$ $\quad r_{a2} = h_{a2} + R_E = 7192.137\,km$

These radii can be used to compute the eccentricity, semi-major axis, and orbital period for the planned and actual orbits:

$$Eccentricity \rightarrow e = \frac{r_a - r_p}{r_a + r_p}$$

$$Semi-Major\ Axis \rightarrow a = \frac{r_a + r_p}{2}$$

$$Period \rightarrow P = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$Error \rightarrow \delta = |Planned - Actual|$$

$$Percent\ Error \rightarrow \left|\frac{Planned - Actual}{Planned}\right|$$

These relations yield the following results for the planned and actual orbits:

	Actual Orbit	**Planned Orbit**	**Error**	**% Error**
Eccentricity (unitless)	1.322e-3	1.392e-3	7.026e-5	5.046
Semi-Major Axis (km)	7185.637	7182.137	3.500	0.049
Period (s)	6061.902	6057.473	4.428	0.073

4. **(28 pts (4 pts each))** Given the following position and velocity vector for a satellite:

$$\bar{r} = \begin{bmatrix} -5650.0 \\ -2650.0 \\ 2850.0 \end{bmatrix} km \qquad \bar{v} = \begin{bmatrix} 2.415 \\ -7.032 \\ -1.796 \end{bmatrix} km/sec$$

Compute the following:

a. The orbit semi-major axis
b. The orbit eccentricity
c. The minimum and maximum altitudes of the satellite above the Earth's (spherical) surface
d. The energy per unit mass of the satellite
e. The angular momentum vector
f. The inclination
g. The flight path angle

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a) Semi-major axis: $\frac{-\mu}{2\,a} = \frac{v^2}{2} - \frac{\mu}{r} \rightarrow a = \frac{-\mu}{v^2 - \frac{2\,\mu}{r}} \rightarrow a = 6908.941\ km$

b) Eccentricity: $e = \sqrt{1 - \frac{h^2}{\mu\ a}} = 0.00742$

c) Minimum altitude: $h_{min} = r_p - R_E = a\ (1 - e) - 6378.137 = 479.563\ km$

Maximum altitude: $h_{max} = r_a - R_E = a\ (1 + e) - 6378.137 = 582.044\ km$

d) Energy per unit mass: $\epsilon = \frac{-\mu}{2\ a} = -28.847\ km^2/s^2$

e) Angular momentum vector: $\bar{h} = \bar{r} \times \dot{\bar{r}} = \begin{bmatrix} 24801 \\ -3265 \\ 46131 \end{bmatrix} km^2/s$

f) Inclination: $\cos(i) = \frac{\hat{K} \cdot \bar{h}}{|\hat{K}|\,|\bar{h}|} = 28.469^o$

g) Flight path angle: $\cos(\varphi_{FPA}) = \sqrt{\frac{a^2\,(1 - e^2)}{r\,(2\ a - r)}} \qquad \sin(\varphi_{FPA}) = \frac{v}{\sqrt{\frac{2\ \mu}{r} - \frac{\mu}{a}}}$

To ensure correct quadrant is used, atan2.m function was used in Matlab to get the final FPA:

$$\tan^{-1}\left(\frac{\sin(\varphi_{FPA})}{\cos(\varphi_{FPA})}\right) = 45.000^o$$

5. **(10 pts (5 pts each))** A satellite is at a distance of 8200 km from the Earth's center of mass. What are the conditions required on the satellite speed and the flight path angle such that the satellite's orbit is circular?

Find : v , φ_{FPA}

Known :

$r = 8200 \ km$

Analysis :

Flight path angle must 0 degrees at every point on the orbit.

Velocity at every point on the orbit must be:

$$\overline{v = \sqrt{\frac{\mu}{r}} \rightarrow v = 6.972 \ km/s}$$

6. **(15 pts (5 pts each))** Let's say that Mercury's orbit about the Sun has a semi-major axis a = 0.387 AU and eccentricity e = 0.205. What is Mercury's orbital period in years? What is the minimum and maximum distance between Mercury and the Sun? What is Mercury's maximum speed? (Use the conversions 1 year = 365.25 days, and 1 day = 86400 seconds).

Find : P , r_a , r_p , v_{max}

Known :

$a = 0.387\ AU = 57894376.077\ km$

$e = 0.205$

$\mu_{sun} = 132712440018\ km^3/s^2$

Analysis :

Mercury's orbital period: $P = 2\pi\sqrt{\dfrac{a^3}{\mu_{sun}}} \rightarrow P = 0.241\ years$

Maximum distance between Mercury and the Sun: $r_a = a(1+e) \rightarrow r_a = 69762723.173\ km$

Minimum distance between Mercury and the Sun: $r_p = a(1-e) \rightarrow r_p = 46026028.981\ km$

The maximum orbital velocity can be computed using the Vis-Viva equation at periapse where the velocity is highest:

$$v_{max} = \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a}} \rightarrow v_{max} = 58.945\ km/s$$