ASEN 5050
SPACEFLIGHT DYNAMICS
Lecture 8: Time Systems

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Announcements

- Homework #4 is posted – due next Thursday.
- No in-class lecture next week – I'll be attending the GRACE Science Team Meeting in Austin, TX.
- Lecture 9 will be pre-recorded tomorrow for you to watch during the week.
- Elena will lead you in doing STK Lab #1:
  - 9/21 Tuesday 11 am (First letter of last name A-K)
  - 9/23 Thursday 11 am (First letter of last name L-Z)

Time systems

- Time is important:
  - Signal travel time of electromagnetic waves
    - Altimetry, GPS, SLR, VLBI
- For positioning
  - Orbit determination
  - One nanosecond (10^{-9} second) is 30 cm of distance
  - Relative motion of celestial bodies

Time Systems

- Question: How do you quantify the passage of time?

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- Year
- Month
- Day
- Second
- Pendulums
- Atoms

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What are some issues with each of these?

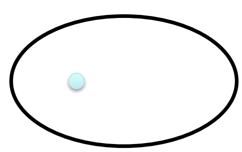
Gravity
Earthquakes
Snooze alarms

Time Systems

- Countless systems exist to measure the passage of time. To varying degrees, each of the following types is important to the mission analyst:
  - **Atomic Time**
    - Unit of duration is defined based on an atomic clock.
  - **Universal Time**
    - Unit of duration is designed to represent a mean solar day as uniformly as possible.
  - **Sidereal Time**
    - Unit of duration is defined based on Earth's rotation relative to distant stars.
  - **Dynamical Time**
    - Unit of duration is defined based on the orbital motion of the Solar System.

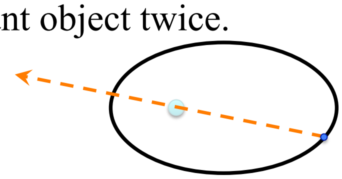
Time Systems: The Year

- The duration of time required to traverse from one perihelion to the next.



(exaggerated)

- The duration of time it takes for the Sun to occult a very distant object twice.



Time Systems: The Year

- Definitions of a Year
  - Julian Year: 365.25 days, where an SI "day" = 86400 SI "seconds".
  - Sidereal Year: 365.256 363 004 mean solar days
    - Duration of time required for Earth to traverse one revolution about the sun, measured via distant star.
  - Tropical Year: 365.242 19 days
    - Duration of time for Sun's ecliptic longitude to advance 360 deg. Shorter on account of Earth's axial precession.
  - Anomalistic Year: 365.259 636 days
    - Perihelion to perihelion.
  - Draconic Year: 365.620 075 883 days
    - One ascending lunar node to the next (two lunar eclipse seasons)
  - Full Moon Cycle, Lunar Year, Vague Year, Heliacal Year, Sothic Year, Gaussian Year, Besselian Year

Time Systems: The Month

- Same variations in definitions exist for the month, but the variations are more significant.

Month	Duration (Earth days)	Duration days	hrs	min	sec
Synodic Month (new Moon to new Moon)	29.53059	29	12	44	03
Anomalistic Month (perigee to perigee)	27.55455	27	13	18	33
Sidereal Month (fixed star to fixed star)	27.32166	27	07	43	12
Tropical Month (equinox to equinox)	27.32158	27	07	43	05
Nodical / Draconic Month (node to node)	27.21222	27	05	05	36

Time Systems: The Day

- Civil day: 86400 SI seconds (+/- 1 for leap second on UTC time system)
- Mean Solar Day: 86400 mean solar seconds
  - Average time it takes for the Sun-Earth line to rotate 360 degrees
  - **True Solar Days** vary by up to 30 seconds, depending on where the Earth is in its orbit.
- Sidereal Day: 86164.1 SI seconds
  - Time it takes the Earth to rotate 360 degrees relative to the (precessing) Vernal Equinox
- Stellar Day: 0.008 seconds longer than the Sidereal Day
  - Time it takes the Earth to rotate 360 degrees relative to distant stars

Solar versus Sidereal Time

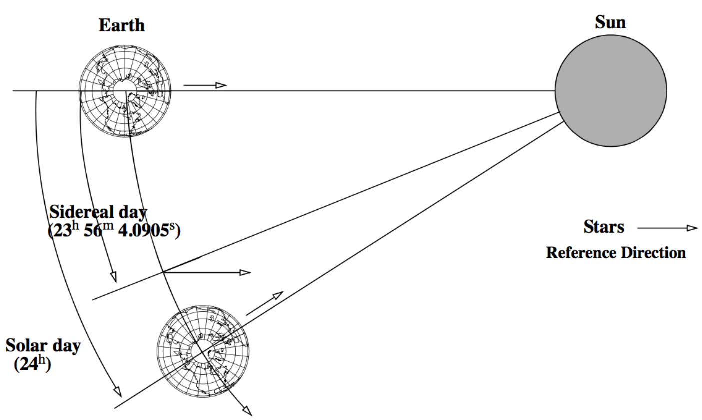
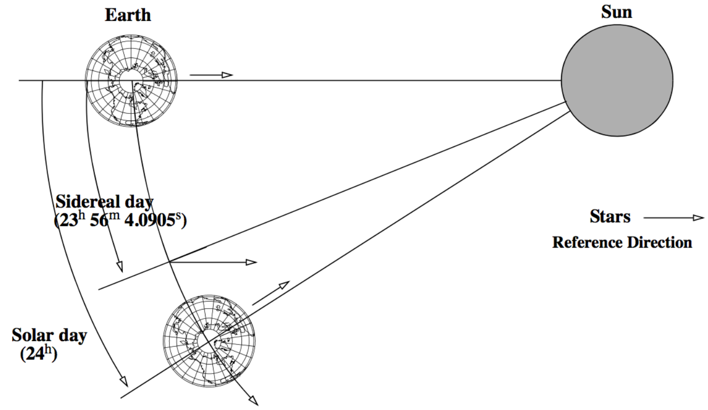


Figure 3-21. Measuring Solar and Sidereal Time. Distance and angular motion are greatly exaggerated for clarity. One solar day is the time required for an observer on the Earth to revolve once *and* observe the Sun at the same location. A sidereal day uses observations of the stars instead of the Sun. Thus, the sidereal day is slightly shorter than the solar day because the stars remain essentially in the same location over time.

Solar versus Sidereal Time



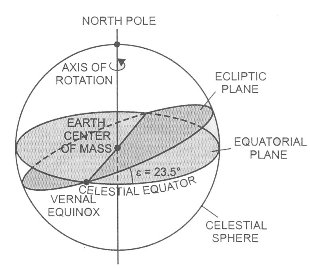
Nonuniform due to changes in Earth's Rotation

- Solar Time – determined from the rotation of the Earth WRT the Sun (position of the Sun WRT the Greenwich meridian).
- Sidereal Time – period of rotation of the Earth WRT the stars (or the hour angle of the Vernal Equinox) – Greenwich Sidereal Time:

$$\theta_{GST} = \theta_{GST0} + \omega_{\oplus} \cdot \Delta UT1$$

Sidereal time

- *Sidereal time* is directly related to the rotation of the Earth.
- *Local Apparent* (or true) *Sidereal Time* (LAST) refers to the observer's local meridian. It is equal to the hour angle of the true vernal equinox.
- The *vernal equinox* is the intersection of the ecliptic and the equator, where the sun passes from the southern to the northern hemisphere.
  - The vernal equinox is affected by precession and nutation and experiences long and short-period variations.



Local and Greenwich Sidereal Time

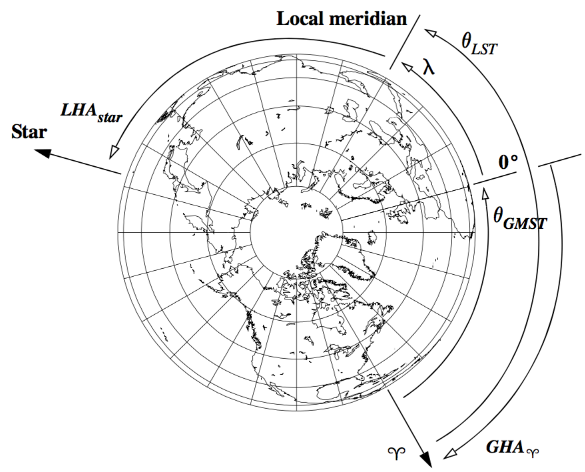


Figure 3-23. Geometry for LST and GMST. Sidereal time is measured positive to the east from the vernal equinox to the location of interest. If we try to locate a star, we can also use hour angles with an appropriate subscript (LHA_{star}). In this example, both sidereal times are positive in a right-handed system, whereas $GHA_{♈}$ is positive and LHA_{star} is negative in a left-handed system.

Time Systems: The Second

- From 1000 AD to 1960 AD, the “second” was defined to be 1/86400 of a mean solar day.
- Now it is defined using atomic transitions – some of the most consistent measurable durations of time available.
  - One SI second = the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cesium 133 atom.
  - The atom should be at rest at 0K.

Astronomical clocks

- We commonly define time in terms of astronomical and geodetic periods:
  - Rotation of the Earth (day)
  - Revolution of the Earth around the sun (year)
  - Orbit of the moon around the Earth (month)
  - Number of bodies in the solar system visible to the naked eye or 1/4 of a lunar cycle (week)
- These astronomical clocks are not consistent
  - The current length of a year is 365.242190 days
  - 1900: 365.242196 days
  - 2100: 365.242184 days

Mechanical clocks

- When mechanical clocks were less accurate than variations in astronomical clocks, astronomical clocks were used to correct mechanical clocks.
  - The pendulum clock was invented in the 17th century
- In 1928 with the invention of the quartz clock.
  - Apparent that the uncertainty in the astronomical day was 10^{-7} due to irregularities in Earth rotation
- Atomic clocks
  - The idea of using hyperfine quantum states of atoms for a clock was first proposed by U.S. physicist Isador Rabi in 1945.

Atomic clocks

- Certain atoms in a the magnetic field can exhibit one of two hyperfine states
  - The spin of the outermost electron of an atom either points in the same direction as the magnetic field of the nucleus, or it points opposite.
  - The laws of quantum physics forbid other orientations.
- Generally, an atom remains in its hyperfine state. But when prodded by electromagnetic radiation at a specific frequency, it will switch to the other state, undergoing the so-called “hyperfine transition”.
- Essentially, an electronic clock selects atoms in one hyperfine state and exposes them to radiation which causes them to switch to the other state. The frequency of the radiation causing the transition becomes the regular beat that the clock counts to register time.
- Hydrogen-masers, cesium, and rubidium standards are usually used
  - Each GPS satellite has 2 Cs and 2 Ru clocks

International Atomic Time (TAI)

- *Atomic Time*, with the unit of duration the Systeme International (SI) second defined in 1967 as the duration of 9,192,631,770 cycles of radiation corresponding to the transition between two hyperfine levels of the ground state of cesium-133 The second was defined in 1967 to correspond to traditional measurement.
- A uniform time-scale of high accuracy is provided by the International Atomic Time (Temps Atomique International, TAI). The origin of TAI was chosen to start on 1 January 1958 0h. It is estimated by a large set (> 200) of atomic clocks.
  - Mostly cesium atomic clocks and a few hydrogen masers at 60 laboratories worldwide.
- Time centers compare clocks, using GPS observations as time links. A weighted mean of local time center results in the TAI.

Solar time and Universal Time

- *Solar time* is used in everyday life
- It is related to the apparent diurnal motion of the sun about the Earth.
- This motion is not uniform, assumes a constant velocity in the motion about the Sun.
- Mean solar time is equal to the hour angle of the mean sun plus 12 hours. If referred to the Greenwich mean astronomical meridian, it is termed *Universal Time* (UT). Its fundamental unit is the mean solar day, the interval between two transits of the sun through the meridian.
- Conversion of UT to GMST is defined by convention, based on the orbital motion of the Earth (about 360°/365 days):

1 mean sidereal day = 1 mean solar day – 3 m 55.90 s = 86164.10 s.

Fundamentals of Time

Universal Time (UT) – Greenwich hour angle of a fictitious Sun uniformly orbiting in the equatorial plane, augmented by 12 hours (eliminates ecliptic motion of the Sun). UT0 is determined from motions of the stars, thus function of sidereal time, and the fixed numerical relationship between universal and sidereal time (3 min 56.555 s). UT1 is UT0 corrected for polar motion. Closely approximates mean diurnal motion of the Sun (Solar Time). UT2 is UT1 corrected for seasonal variations in the Earth's rotation rate.

Dynamic Time (Ephemeris Time) – derived from planetary motions in the solar system (deduce time from position of planets and equations of motion). Independent variable in the equations of motion.

- Barycentric Dynamic Time (BDT) is based on planetary motions WRT the solar system barycenter.
- Terrestrial Dynamic Time (TDT) derived from satellite motions around the Earth.

TDT

- *Terrestrial Time* (TT), (or *Terrestrial Dynamical Time*, TDT), with unit of duration 86400 SI seconds on the geoid, is the independent argument of apparent geocentric ephemerides.

$$\text{TDT} = \text{TAI} + 32.184 \text{ seconds}$$

- The epoch of TDT is defined as 1977 January 1, 0h TAI

UTC

- A practical time scale, as needed in navigation for instance, has to provide a uniform unit of time and maintain a close relationship with UT1.
- This led to the introduction of the *Coordinated Universal Time* (UTC).
- Its time interval corresponds to atomic time TAI and its epoch differs by not more than 0.9 sec from UT1. In order to keep the difference

$$|\text{DUT1}| = |\text{UT1} - \text{UTC}| < 0.9\text{sec}$$

  leap seconds are introduced to UTC when necessary.
  - A specification of a UTC clock should always differ from TAI by an integer number of seconds.
- GPS provides easy access to UTC with an accuracy within 100 nanoseconds.
  - GPS navigation data provides the integer offset for TAI.

Leap seconds

- Since the first leap second in 1972, all leap seconds have been positive.
  - Currently the Earth runs slow at roughly 2 milliseconds per day, so a leap second is needed about every five hundred days.
  - There have been 34 leap seconds in the 37 years to January, 2009.
  - This pattern mostly reflects the general slowing trend of the Earth due to tidal braking.
- Sometimes there is a misconception that the regular insertion of leap seconds every few years indicates that the Earth should stop rotating within a few millennia.
- Leap seconds are not a measure of the rate at which the Earth is slowing.
  - The 1 second increments are indications of the accumulated difference in time between the two systems.
  - Usually, you would reset a slow clock to the accurate time
    - We can't alter the Earth's rotation, so we reset the accurate clock

Leap Seconds

Atomic Time – International Atomic Time (TAI) is based on vibrations of the Cesium atom.

$$TDT = TAI + 32.184sec$$

Coordinated Universal Time (UTC) – equal to TAI but augmented by leap seconds to keep it close to UT.

$$TAI = UTC + 1^{s}.0n \quad (n=integer=34 \text{ in early } 2009)$$

GPS Time – needs to be uniform, has a constant offset of 19 secs WRT TAI, and was equal to UTC at the GPS standard epoch of Jan. 6, 1980.

Leap Seconds Since 1972

1972 JAN	1 =JD 2441317.5	TAI-UTC=	10.0	S + (MJD - 41317.) X 0.0	S
1972 JUL	1 =JD 2441499.5	TAI-UTC=	11.0	S + (MJD - 41317.) X 0.0	S
1973 JAN	1 =JD 2441683.5	TAI-UTC=	12.0	S + (MJD - 41317.) X 0.0	S
1974 JAN	1 =JD 2442048.5	TAI-UTC=	13.0	S + (MJD - 41317.) X 0.0	S
1975 JAN	1 =JD 2442413.5	TAI-UTC=	14.0	S + (MJD - 41317.) X 0.0	S
1976 JAN	1 =JD 2442778.5	TAI-UTC=	15.0	S + (MJD - 41317.) X 0.0	S
1977 JAN	1 =JD 2443144.5	TAI-UTC=	16.0	S + (MJD - 41317.) X 0.0	S
1978 JAN	1 =JD 2443509.5	TAI-UTC=	17.0	S + (MJD - 41317.) X 0.0	S
1979 JAN	1 =JD 2443874.5	TAI-UTC=	18.0	S + (MJD - 41317.) X 0.0	S
1980 JAN	1 =JD 2444239.5	TAI-UTC=	19.0	S + (MJD - 41317.) X 0.0	S
1981 JUL	1 =JD 2444786.5	TAI-UTC=	20.0	S + (MJD - 41317.) X 0.0	S
1982 JUL	1 =JD 2445151.5	TAI-UTC=	21.0	S + (MJD - 41317.) X 0.0	S
1983 JUL	1 =JD 2445516.5	TAI-UTC=	22.0	S + (MJD - 41317.) X 0.0	S
1985 JUL	1 =JD 2446247.5	TAI-UTC=	23.0	S + (MJD - 41317.) X 0.0	S
1988 JAN	1 =JD 2447161.5	TAI-UTC=	24.0	S + (MJD - 41317.) X 0.0	S
1990 JAN	1 =JD 2447892.5	TAI-UTC=	25.0	S + (MJD - 41317.) X 0.0	S
1991 JAN	1 =JD 2448257.5	TAI-UTC=	26.0	S + (MJD - 41317.) X 0.0	S
1992 JUL	1 =JD 2448804.5	TAI-UTC=	27.0	S + (MJD - 41317.) X 0.0	S
1993 JUL	1 =JD 2449169.5	TAI-UTC=	28.0	S + (MJD - 41317.) X 0.0	S
1994 JUL	1 =JD 2449534.5	TAI-UTC=	29.0	S + (MJD - 41317.) X 0.0	S
1996 JAN	1 =JD 2450083.5	TAI-UTC=	30.0	S + (MJD - 41317.) X 0.0	S
1997 JUL	1 =JD 2450630.5	TAI-UTC=	31.0	S + (MJD - 41317.) X 0.0	S
1999 JAN	1 =JD 2451179.5	TAI-UTC=	32.0	S + (MJD - 41317.) X 0.0	S
2006 JAN	1 =JD 2453737.5	TAI-UTC=	33.0	S + (MJD - 41317.) X 0.0	S
2009 JAN	1 =JD 2453737.5	TAI-UTC=	34.0	S + (MJD - 41317.) X 0.0	S
2012 JUL	1 =JD 2453737.5	TAI-UTC=	35.0	S + (MJD - 41317.) X 0.0	S
2015 JUL	1 =JD 2453737.5	TAI-UTC=	36.0	S + (MJD - 41317.) X 0.0	S

Time Systems: Time Scales

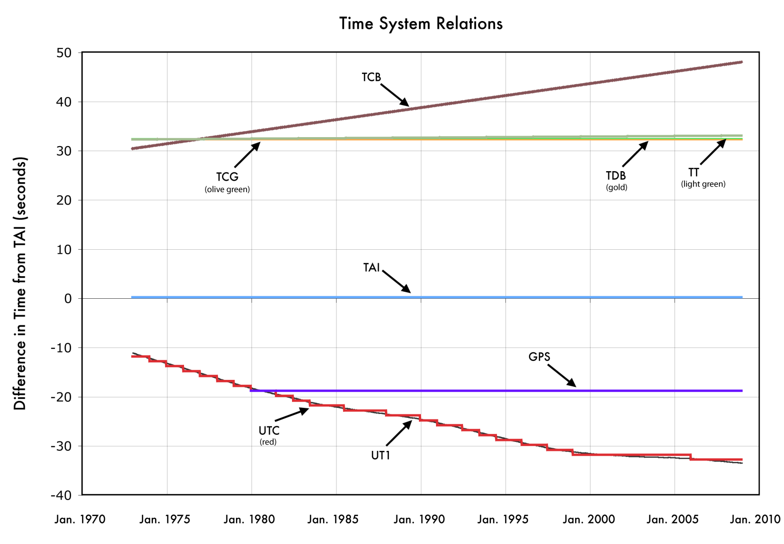
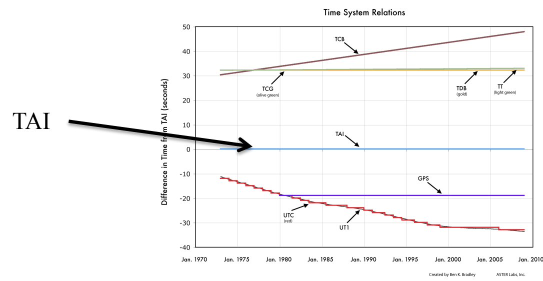


Figure 3.24

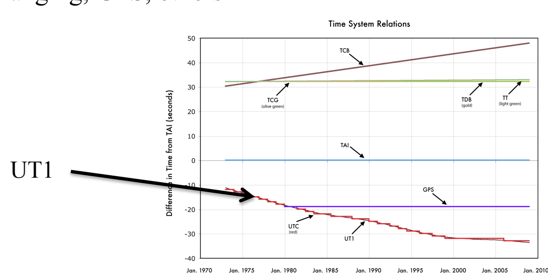
Time Systems: TAI

- TAI = The Temps Atomique International
  - International Atomic Time
- Continuous time scale resulting from the statistical analysis of a large number of atomic clocks operating around the world.
  - Performed by the Bureau International des Poids et Mesures (BIPM)



Time Systems: UT1

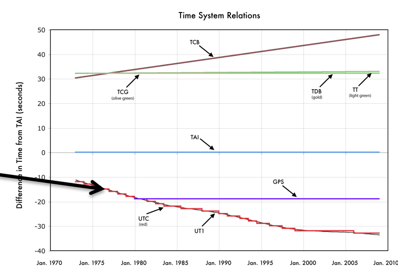
- UT1 = Universal Time
- Represents the daily rotation of the Earth
- Independent of the observing site (its longitude, etc)
- Continuous time scale, but unpredictable
- Computed using a combination of VLBI, quasars, lunar laser ranging, satellite laser ranging, GPS, others



Time Systems: UTC

- UTC = Coordinated Universal Time
- Civil timekeeping, available from radio broadcast signals.
- Equal to TAI in 1958, reset in 1972 such that TAI-UTC=10 sec
- Since 1972, leap seconds keep |UT1-UTC| < 0.9 sec
- In June, 2015, the 26th leap second was added such that TAI-UTC=36 sec

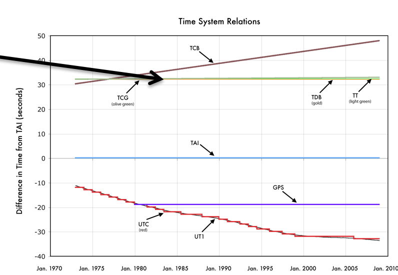
UTC



Time Systems: TT

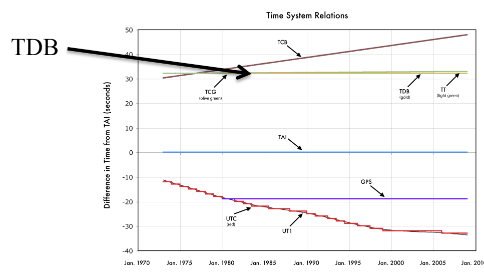
- TT = Terrestrial Time
- Described as the proper time of a clock located on the geoid.
- Actually defined as a coordinate time scale.
- In effect, TT describes the geoid (mean sea level) in terms of a particular level of gravitational time dilation relative to a notional observer located at infinitely high altitude.
  - TT-TAI= ~32.184 sec

TT



Time Systems: TDB

- TDB = Barycentric Dynamical Time
- JPL's "ET" = TDB. Also known as T_{eph}. There are other definitions of "Ephemeris Time" (complicated history)
- Independent variable in the equations of motion governing the motion of bodies in the solar system.
  - TDB-TAI= ~32.184 sec+ relativistic



Time Systems: Summary

- Long story short

- In astrodynamics, when we integrate the equations of motion of a satellite, we're using the time system "TDB" or ~"ET".
- Clocks run at different rates, based on relativity.
- The civil system is not a continuous time system.
- We won't worry about the fine details in this class, but in reality spacecraft navigators do need to worry about the details.
  - Fortunately, most navigators don't; rather, they permit one or two specialists to worry about the details.
  - Whew.

Present time differences

- As of 17 Sept 2015,
  - TAI is ahead of UTC by 36 seconds.
  - TAI is ahead of GPS by 19 seconds.
  - GPS is ahead of UTC by 17 seconds.
    - The Global Positioning System (GPS) epoch is January 6, 1980 and is synchronized to UTC.

Julian Dates

- *Julian Day Number* is a count of days elapsed since Greenwich mean noon on 1 January 4713 B.C., Julian calendar. The *Julian Date* is the Julian day number followed by the fraction of the day elapsed since the preceding noon.
  - The Julian Day Number for 7 February 2002 is 2452313
- The *Modified Julian Day*, was introduced by space scientists in the late 1950s. It is defined as

$$\mathrm{MJD} = \mathrm{JD} - 2400000.5$$

Julian Dates

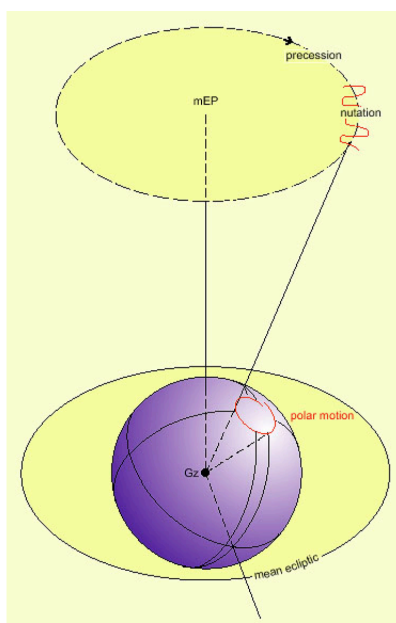
Julian Date (JD) – defines the number of mean solar days since 4713 B.C., January 1, 0.5 (noon).

Modified Julian Date (MJD) – obtained by subtracting 2400000.5 days from JD. Thus, MJD commences at midnight instead of noon.

Civilian Date	*JD*	
1980 Jan 6 midnight	2444244.5	GPS Standard Epoch
2000 Jan 1 noon	2451545.0	J2000 Epoch

Algorithm 14 in book.

Earth Rotation Variations



Earth Orientation Measurements

Length of Day – must be measured, preferably using VLBI.

Polar Motion – must be measured with space geodetic techniques.

Nutation – theory/modeling can generally be used (sometimes apply small corrections from VLBI).

Precession – theory/modeling can generally be used.

Motion of the Coordinate System

For this class, we will ignore precession, nutation, and polar motion.

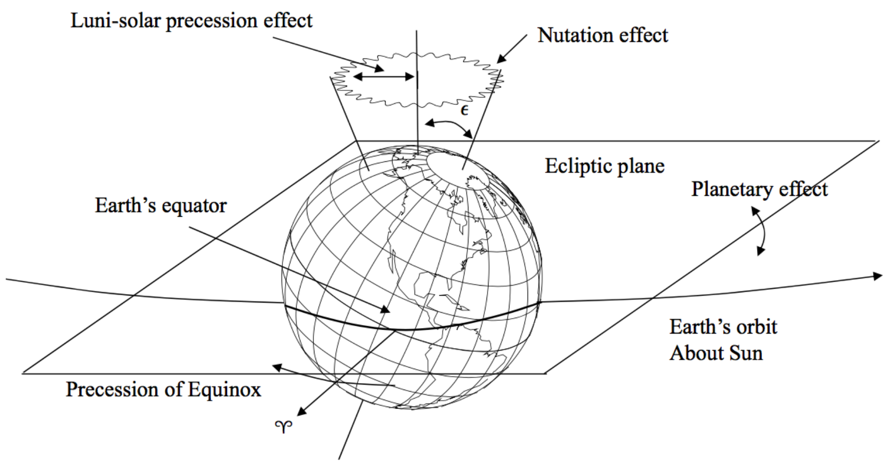
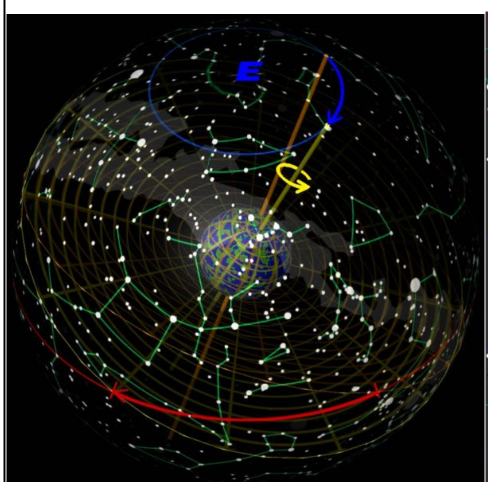


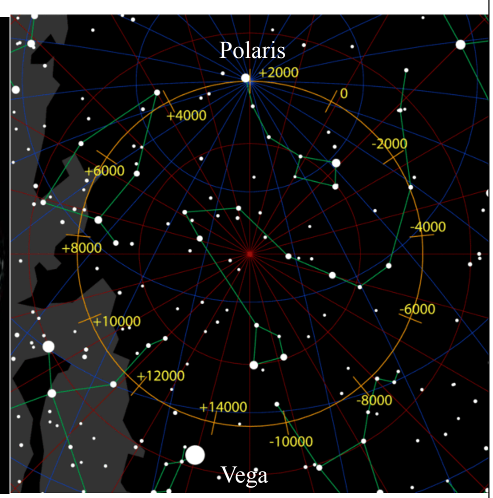
Figure 3-25. Precession and Nutation of the Earth's Equatorial Plane. This figure shows the combined effects of perturbing forces on the Earth. The general motion of the equinox moves west. Planetary and nutation effects are small compared to the luni-solar effects.

Precession and Nutation

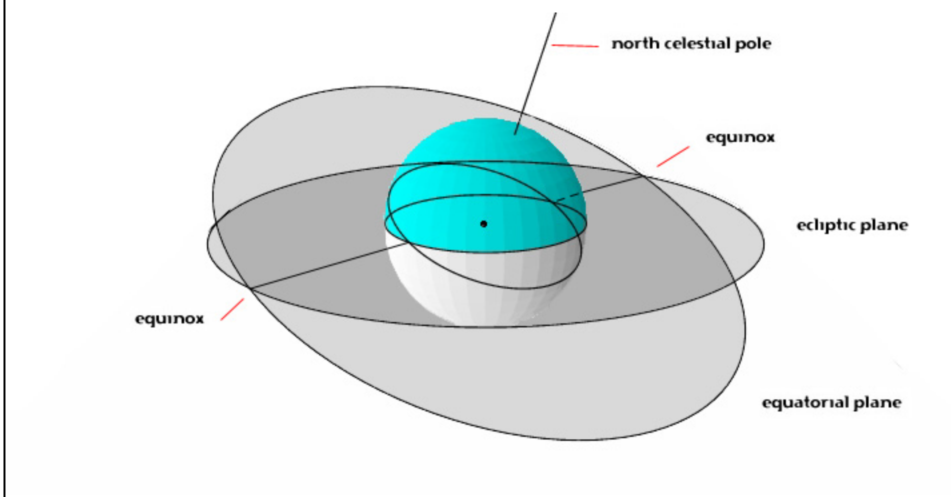
- Luni-Solar Precession – 50” per year, period of 26,000 years (due to the torques of the Moon and the Sun on the Earth’s equatorial bulge).
- Planetary Precession – precession of 12”/century and decrease the obliquity of the ecliptic of 47”/century (due to the planetary perturbations on the Earth’s orbit, causing changes in the ecliptic).
- Nutation – amplitude 9”, occurs at orbital periods of the Sun and the Moon (13.7 days, 27.6 days, 6 months, 1 year, 18.6 years, etc.) 18.6 year motion is largest – 20“ amplitude (0.5 km).

Precession of the Equinox

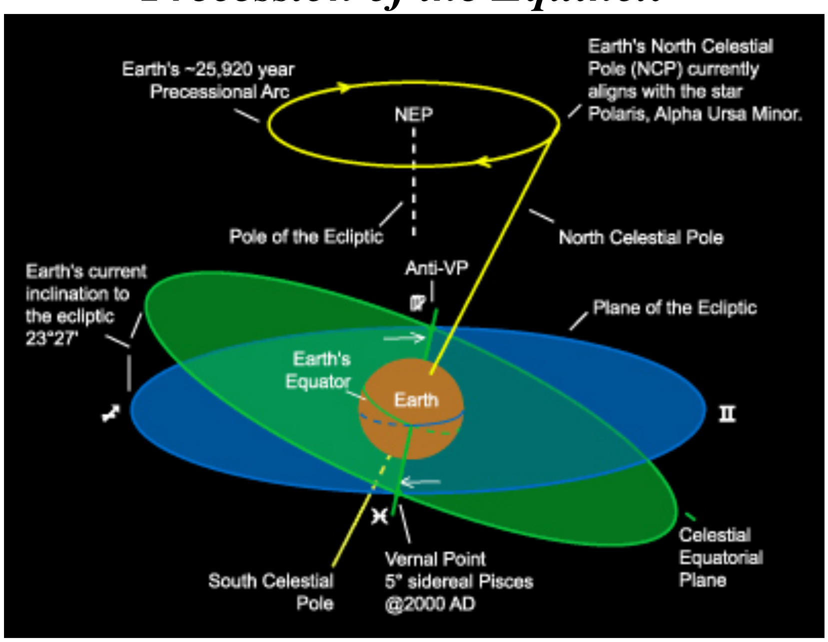




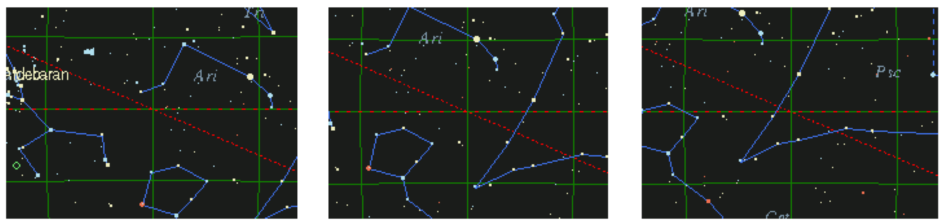
Precession of the Equinox



Precession of the Equinox



Equinox Positions Through Time



1500 BC | 500 BC | AD 150

- The red line is a section of the apparent path traced by the Sun through the Earth's year. The red/green line is a projection of the Earth's equator on to the celestial sphere. The crossing point of these two lines is the spring equinox. In 1500 BC it was near the end of the Aries constellation, in 500 BC it was near the beginning of the Aries constellation and in AD 150 (the time of Ptolemy) it was in the center of the Pisces constellation).

Motion of the Coordinate System

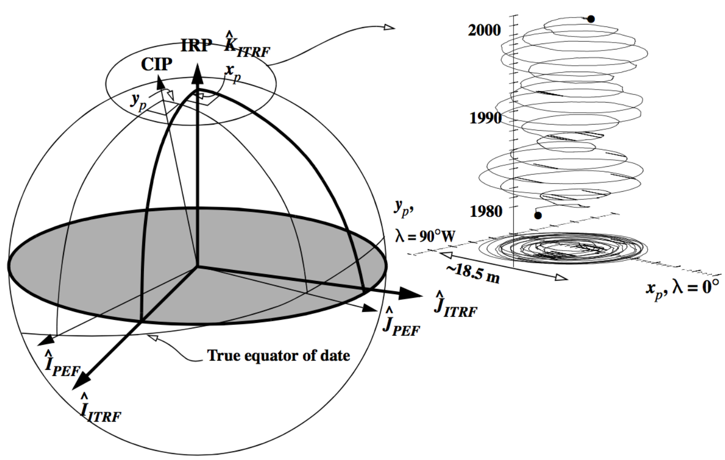


Figure 3-27. Transformation Geometry Due to Polar Motion. This transformation takes into account the actual location of the Celestial Intermediate Pole (CIP) over time. It relates a Pseudo Earth fixed (PEF) system about the CIP, and the ITRF system and the IERS Reference Pole (IRP). The inset plot shows the IRP motion for about 20 years. Notice the slight "structure" to the variations over time. These motions are partially due to the annual (365 d) and Chandler (435 d) periods. The PEF is virtually identical to the TIRS (See Table 3-6), but it's convenient to separate the names as the exact formulae differ.

Polar Motion

- Linear drift of the rotation pole of 3-4 milli arc seconds/year in a direction between Greenland and Hudson Bay (due to post glacial rebound)
- Long period wobble (30 years) of amplitude 30 milli arc seconds (cause unknown)
- Annual Wobble (amplitude of 0.1 arc seconds – 3 meters on the Earth's surface), 75% caused by annual variation in the inertia tensor of the atmosphere, rest by mass variations in snow, ice, ground water, etc.
- Chandler Wobble (430 day period), 6 meters amplitude. Normal mode of the Earth. Caused by atmospheric and oceanic effects.

(1 arc min = 1/60 deg; 1 arc sec = 1/60 arc min)

Motion of the Coordinate System

Only concerned with rotation of Earth about Z:

$$\theta_{GST} = \theta_{GST0} + \omega_{\oplus} \cdot \Delta UT1 \qquad \omega_{\oplus} = 7.2921158553 \times 10^{-5} \, rad/s\sec$$

$$\theta_{GST0} = 1.753368560 + 628.3319706889 T_{UT1}$$
$$+6.7707 \times 10^{-6} T_{UT1}^2 - 4.5 \times 10^{-10} T_{UT1}^3$$

T_{UT1} = number of Julian centuries elapsed from epoch J2000

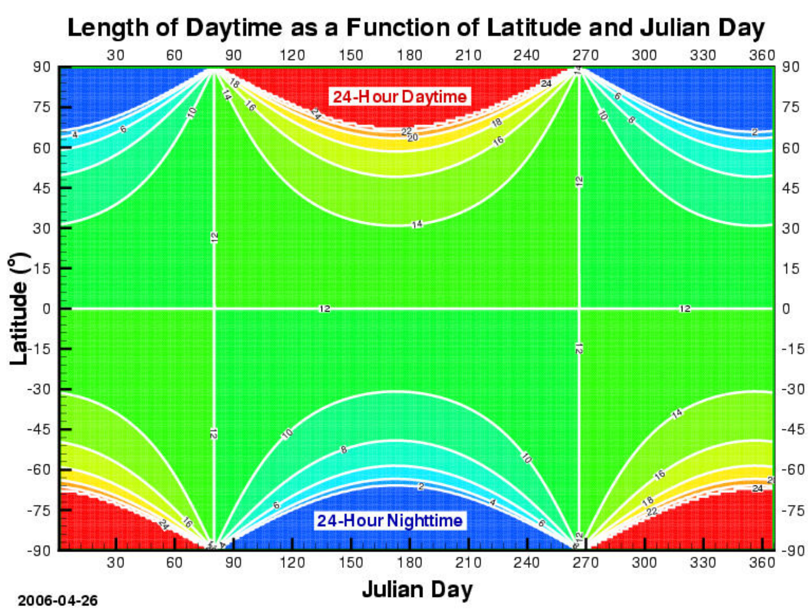
Motion of the Coordinate Systems

$$\vec{r}_{IJK} = ROT3(-\theta_{GST})\vec{r}_{ECEF}$$

$$\vec{v}_{IJK} = ROT3(-\theta_{GST})\vec{v}_{ECEF} + R\dot{O}T3(-\theta_{GST})\vec{r}_{ECEF} + \vec{\omega} \times ROT3(-\theta_{GST})\vec{r}_{ECEF}$$

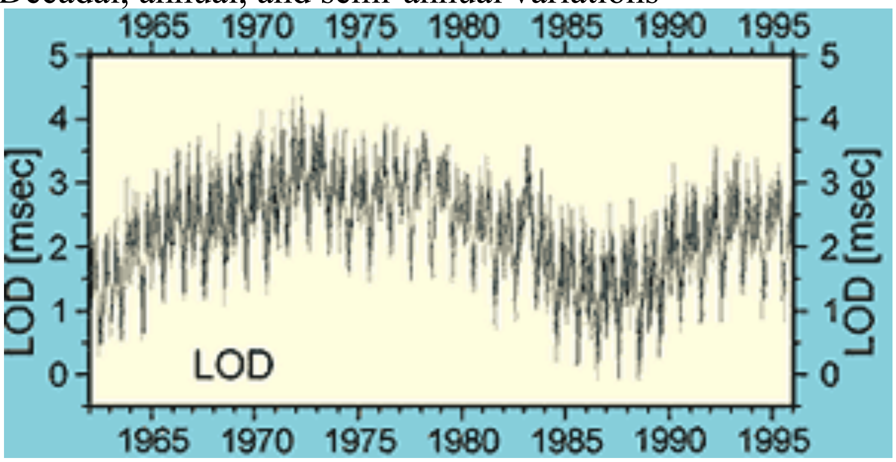
$$ROT3 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R\dot{O}T3 = \begin{bmatrix} -\omega\sin\theta & \omega\cos\theta & 0 \\ -\omega\cos\theta & -\omega\sin\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Length of Day



Length of day changes

Decadal, annual, and semi-annual variations

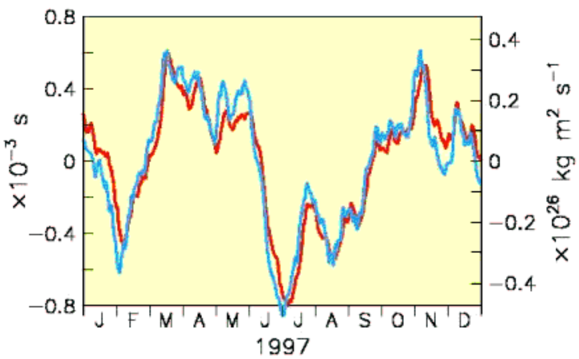


Variations of LOD

- There is a linear increase in the LOD of 1-2 msec/century.
- Increase mainly due to tidal breaking.
- Some decrease due to decreasing moment of inertia (J2, e.g. skater spins faster when hands are drawn in).
- Decadal fluctuations of 4-5 msec (due to transfer of angular momentum between the core and the mantle).
- Short period variability of 2-3 msec at period of less than 5 years (mainly 2 weeks, 1 month, 6 months, 1 year) (due to Earth and ocean tides, the atmosphere, and wind-driven ocean circulation).
- Before modern times, ancient eclipse records have been used to determine variations in LOD

AAM vs LOD changes

- Length of day changes are highly correlated with changes in atmospheric angular momentum (AAM):



Reference Frame Terminology

- Equator – the great circle on the surface of a body formed by the intersection of the surface with the plane passing through the center of the body perpendicular to the axis of rotation.
- Celestial Equator – the projection on to the celestial sphere of the Earth's equator.
- Ecliptic – plane of the Earth's orbit about the Sun, affected by planetary precession.
- True Equator and Equinox of Date – the celestial coordinate system determined by the instantaneous positions of the celestial equator and ecliptic (motion due to precession and nutation).
- Mean Equator and Equinox of Date – the celestial reference system determined by ignoring small variations of short-period (nutation) in the motions of the celestial equator (motion due to only precession).
- Mean Equator and Equinox of J2000.0 – celestial reference system at 12 hours, January 1, 2000.

Reference Frame Transformation

- $\bar{r}_{J2000} = PNSM\ \bar{r}_{ECEF}$
- P = precession transformation (moves state from mean equinox of epoch (J2000) to mean equinox of date)
- N = nutation transformation (moves state from mean equinox to true equinox of date)
- S = Apparent Sidereal Time Transformation (referenced to true equinox)

  = ROT3(θ_{GST}) + $EQ_{equinox}$ EQ = GAST-GMST
- θ_{GST} = Greenwich Mean Sidereal Time
- $EQ_{equinox}$ = "equation of the equinoxes" = difference between mean and true equinoxes
- M = polar motion transformation = $\begin{bmatrix} 1 & 0 & x_p \\ 0 & 1 & -y_p \\ -x_p & y_p & 1 \end{bmatrix}$

What the Rotations Do

- *M – Polar Motion*: rotates the terrestrial (ECEF) frame from the conventional pole to the celestial ephemeris pole.
- *S – Sidereal Time*: rotates the terrestrial frame from the Greenwich Meridian to the true equinox of date.
- *N – Nutation*: rotates the celestial frame from the true equinox of date to the mean equinox of date
- *P – Precession*: rotates the celestial frame from the mean equinox of date to the mean equinox of J2000.0

Announcements

- Homework #4 is posted – due next Thursday.
- No in-class lecture next week – I'll be attending the GRACE Science Team Meeting in Austin, TX.
- Lecture 9 will be pre-recorded tomorrow for you to watch.
- We will also do STK Lab #1:
  - 9/21 Tuesday 11 am (First letter of last name A-K)
  - 9/23 Thursday 11 am (First letter of last name L-Z)