

## Homework #4 – ASEN 5050

**Due: Tuesday, 9/29/2015**

Note: Use Appendix D of the book for all constants not given in the problem. Show the steps you take to arrive at your answers, unless otherwise noted.

1. **(25 pts) Hohmann Transfer:** A satellite is in an orbit about the Earth with a periaapse altitude of 250 km and an apoapse altitude of 600 km. We need the satellite in a new orbit that has a periaapse altitude of 2000 km and an apoapse altitude of 5000 km. We decide to perform a two-impulse tangent- $\Delta V$  transfer (e.g., a Hohmann Transfer). Answer the following:
  - a) **(6 pts)** Build a transfer that departs the initial orbit at **periaapse (250 km alt)** and arrives at the target orbit at its **apoapse (5000 km altitude)**. Please provide the magnitude of each  $\Delta V$  and the total  $\Delta V$  for the transfer.
  - b) **(6 pts)** Build a transfer that departs the initial orbit at **periaapse (250 km alt)** and arrives at the target orbit at its **periaapse (2000 km alt)**. Please provide the magnitude of each  $\Delta V$  and the total  $\Delta V$  for the transfer.
  - c) **(6 pts)** Build a transfer that departs the initial orbit at **apoapse (600 km alt)** and arrives at the target orbit at **apoapse (5000 km alt)**. Please provide the magnitude of each  $\Delta V$  and the total  $\Delta V$  for the transfer.
  - d) **(6 pts)** Build a transfer that departs the initial orbit at **apoapse (600 km alt)** and arrives at the target orbit at **periaapse (2000 km alt)**. Please provide the magnitude of each  $\Delta V$  and the total  $\Delta V$  for the transfer.
  - e) **(1 pt)** Which transfer requires the least  $\Delta V$ ?

The following table may be used to present your results, making sure to show enough work for us to know how you got these answers. Show 2 or 3 digits past the decimal.

Transfer Option	$\Delta V_1$ (m/s)	$\Delta V_2$ (m/s)	Total $\Delta V$ (m/s)
a. Peri-Apo			
b. P-P			
c. A-A			
d. A-P			

2. **(25 pts)** Write a function (Matlab, etc) that converts an ECI position into an ECEF position given the Greenwich Sidereal Time:

$$\bar{r}_{ECEF} = ROT3(\theta_{GST})\bar{r}_{IJK}$$

This function call should look something like this:

$$\text{function } [pos\_ecef] = \text{eci2ecef}(pos\_eci, theta\_GST)$$

where  $pos\_ecef$  and  $pos\_eci$  are the ECEF and ECI position vectors. Use this function to compute the ECEF position vector given the following ECI position vector:

$$\bar{r} = \begin{bmatrix} -5634 \\ -2645 \\ 2834 \end{bmatrix} \text{ km}$$

and  $\theta_{GST} = 82.75^\circ$ . Compute the geocentric latitude, longitude, and altitude (relative to a sphere of radius 6378.1363 km) of this position. Note that geocentric latitude ( $\phi$ ), longitude ( $\lambda$ ), and radius ( $r = \text{altitude} + 6378.1363$  km) are related to the  $xyz$  ECEF position components as:

$$x = r \cos \phi \cos \lambda$$

$$y = r \cos \phi \sin \lambda$$

$$z = r \sin \phi$$

Please include your (commented) code with this assignment.

3. **(25 pts)** Write a function that does the opposite, i.e., it converts an ECEF position into an ECI position given the Greenwich Sidereal Time. This function call should look something like this:

$$\text{function } [pos\_eci] = \text{ecef2eci}(pos\_ecef, theta\_GST)$$

Use this function to compute the ECI position of Boulder, Colorado ( $\phi = 40.01^\circ$ ,  $\lambda = 105.57^\circ$ ,  $h = 1615$  m) given a  $\theta_{GST} = 103^\circ$ .

4. **(25 pts)** Write a function that computes the range, elevation, and azimuth of a satellite given an ECEF satellite position and the latitude, longitude, and altitude of the tracking station. This function call should look something like this:

function  $[pos\_topo] = ecef2topo(pos\_ecef, lat, lon, alt)$

Use this routine to compute the azimuth, elevation, and range relative to Boulder ( $\phi=40.01^\circ$ ,  $\lambda=254.83^\circ$ ,  $alt=1615$  m) of the following ECEF satellite position:

$$\bar{r} = \begin{bmatrix} -1681 \\ -5173 \\ 4405 \end{bmatrix} \text{ km}$$

Please include your (commented) code with this assignment.