ASEN 5050
SPACEFLIGHT DYNAMICS
Lecture 7: Coordinate Systems

Prof. R. S. Nerem
University of Colorado - Boulder

Coordinate Frames

- Inertial: fixed orientation in space
  - Inertial coordinate frames are typically tied to hundreds of observations of quasars and other very distant near-fixed objects in the sky.
- Rotating
  - Constant angular velocity: mean spin motion of a planet
  - Osculating angular velocity: accurate spin motion of a planet

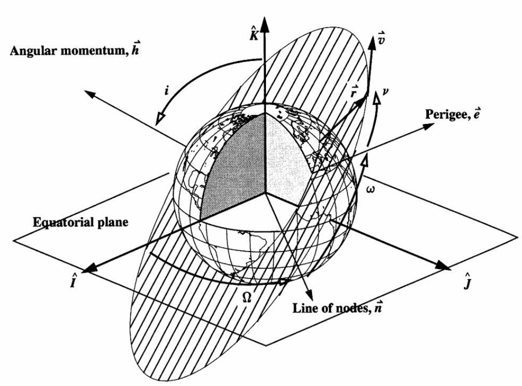
Coordinate Systems

- Coordinate Systems = Frame + Origin
  - Inertial coordinate systems require that the system be non-accelerating.
    - Inertial frame + non-accelerating origin
  - "Inertial" coordinate systems are usually just non-rotating coordinate systems.
- Is the Earth-centered J2000 coordinate system inertial?

Lecture7: Coordinate Systems 3

Coordinate Systems

- Given a full state, with position and velocity known.
- Or, given the full set of coordinate elements.



- What coordinate system is this state represented in?
- Could be any non-rotating coordinate system!
- Earth J2000 or ecliptic J2000 or Mars, etc.

Lecture7: Coordinate Systems 4

Coordinate Systems

Celestial Sphere

- Celestial poles intersect Earth's rotation axis.
- Celestial equator extends Earth equator.
- Direction of objects measured with right ascension (α) and declination (δ).

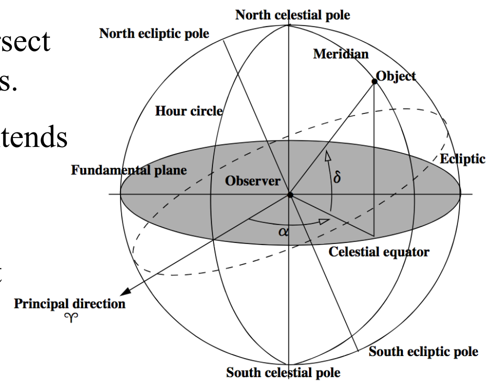


Figure 3-6. Geometry of the Celestial Sphere. The celestial sphere is based on an observer's perceived view of objects in space. A meridian, or hour circle, is any circle that passes through the observer at the center. I've shown the principal direction to help us define right ascension, α, and declination, δ.

Vernal Equinox

The **Vernal Equinox** defines the reference direction. A.k.a. The Line of Aries

The **ecliptic** is defined as the mean plane of the Earth's orbit about the Sun.

The angle between the Earth's mean equator and the ecliptic is called the **obliquity of the ecliptic**, ε~23.5°.

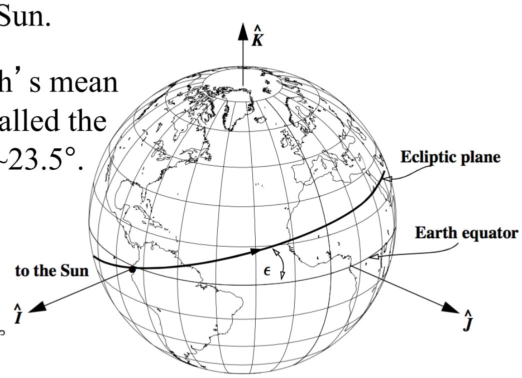


Figure 3-7. Geometry of the Vernal Equinox. The Earth's mean orbit around the Sun forms the ecliptic plane. The Earth's equatorial plane is inclined about 23.5° to the ecliptic. When the Sun is at the intersection of the two planes (has zero declination) and is at the ascending node, as viewed from Earth, it's the first day of spring.

Vernal Equinox

The intersection of the ecliptic and equatorial planes is called the **line of nodes**.

The Sun crosses the line of nodes twice a year at the **equinoxes**:

Ascending node, vernal equinox, March 21

Descending node, autumnal equinox, Sept 23

Latin for equal day and night (Earth experiences equal day and night)

The Vernal Equinox occurs when the Sun's declination is 0° as it approaches the ascending node.

Coordinate Systems

Relationship between right ascension (α) and declination (δ) and latitude (ϕ) and longitude (λ) (*Fig 3-8 – next page*)

Hour Angle – elapsed time in hours since object was directly overhead.

Greenwich Hour Angle (GHA) – Generally angle between vernal equinox and 0° longitude or the Greenwich Meridian. Also called Greenwich Sidereal Time.

Coordinate Systems

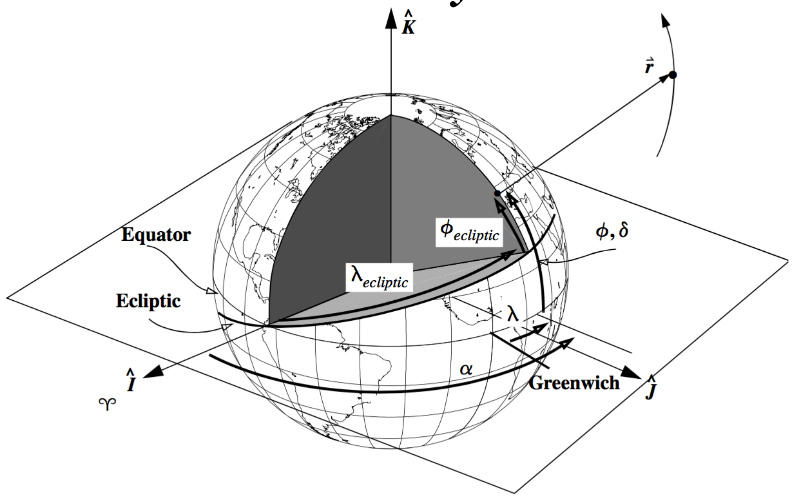


Figure 3-8. Using Right Ascension and Declination or Longitude and Latitude to Measure Location. Notice that both terrestrial latitude and longitude, ϕ and λ, and right ascension and declination, α and δ, reference the Earth's equator, whereas ecliptic latitude and longitude are celestial coordinates that reference the ecliptic. Declination and terrestrial latitude are the same, but right ascension and terrestrial longitude reference different starting locations.

Heliocentric Coordinate System

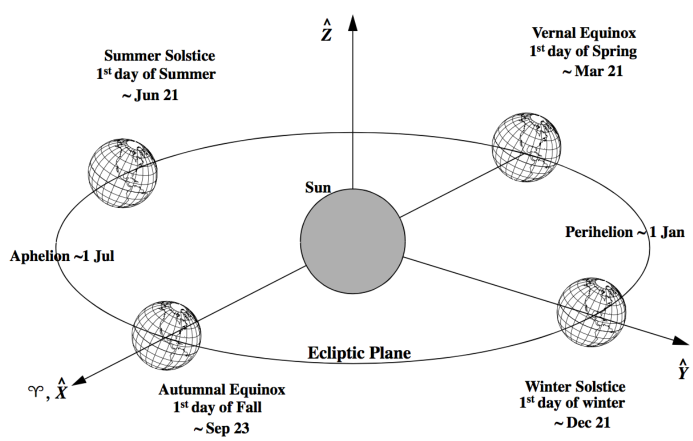


Figure 3-10. Heliocentric Coordinate System, XYZ. The primary direction for this system is the intersection of the ecliptic and equatorial planes. The positive sense (denoted ♈) is given by the location of the Sun with respect to the Earth as it crosses the equatorial plane on the first day of spring. All seasonal dates are for the Northern Hemisphere. The revolution of the Earth about the Sun doesn't match the Gregorian calendar exactly. Only after about 28 years are the dates approximately the same. This affects the dates for the seasons. Seidelmann (1992:477) lists the earliest seasonal dates that occur in 2096 (March 19, June 20, September 21, and December 20). The latest seasonal dates occurred in 1903 (March 21, June 22, September 24, and December 23).

Geocentric Inertial Coordinate System

Geocentric Inertial Coordinate System (IJK)

- aka: Earth Centered Inertial (ECI), or the Conventional Inertial System (CIS)
- J2000 – Vernal equinox on Jan 1, 2000
- non-rotating

Intersection of ecliptic and celestial eq

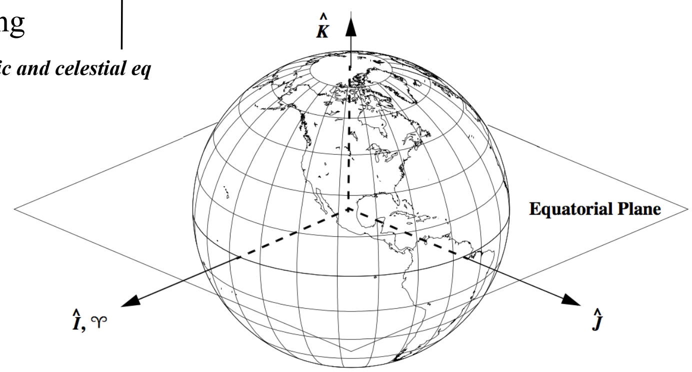


Figure 3-11. Geocentric Equatorial System (IJK). This system uses the Earth's equator and the axis of rotation to define an orthogonal set of vectors. The vernal equinox direction is fixed at a specific epoch for most applications.

Earth Fixed Coordinate Systems

Earth-Centered Earth-Fixed Coordinates (ECEF)

Topocentric Horizon Coordinate System (SEZ)

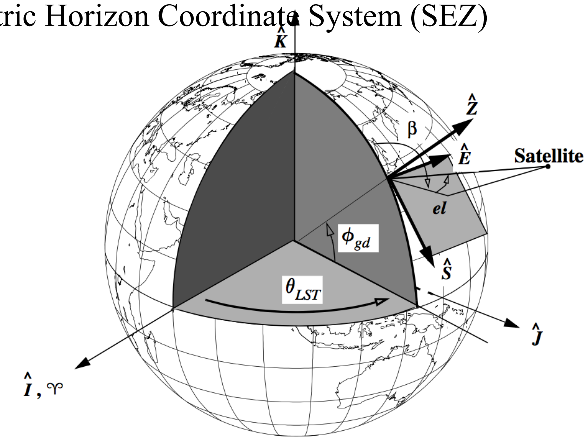
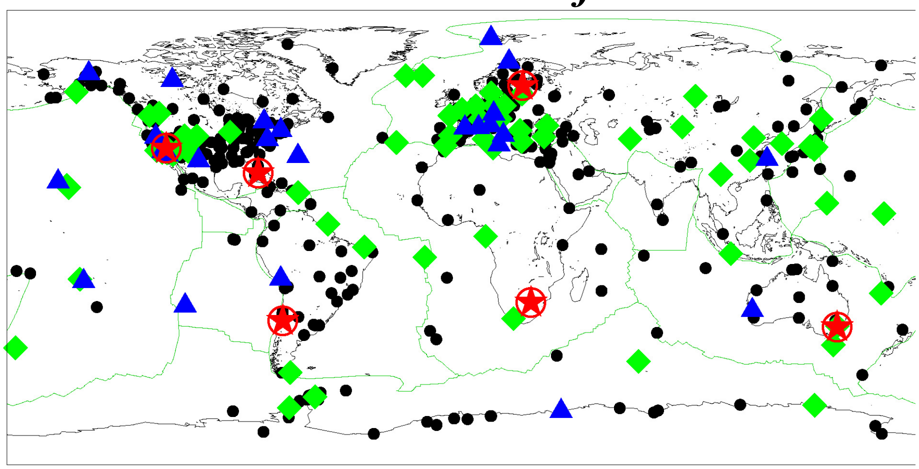


Figure 3-12. Topocentric Horizon Coordinate System, SEZ. This system moves with the Earth, so it's not fixed like the ITRF. θ_{LST} is required to orient the SEZ system to a fixed location. Note that for precise applications, θ_{AST} and longitude replace θ_{LST}. The SEZ system is used extensively in radar observations and often involves azimuth, β, and elevation, *el*. Geodetic latitude is common, ϕ_{gd}, but astronomical latitude is also used in many systems.

ECEF:ITRF
International Terrestrial Reference Frame



Geometry for Sidereal Time

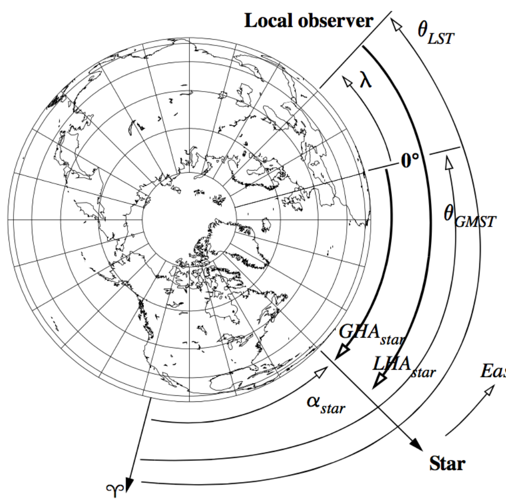


Figure 3-9. **Geometry for LHA and GHA.** GHA, and LHA are always measured positive to the west *from* the observer to the object of interest. Longitude, λ, is positive to the east. Both GHA_{star} and LHA_{star} are positive here. We use subscripts to avoid confusion when referring to specific objects. θ_{LST} and θ_{GMST} will be discussed shortly.

Reference Frame and Time Links

http://www.iers.org/ International Earth Rotation Service

http://tycho.usno.navy.mil/ USNO Time Service Department

http://igscb.jpl.nasa.gov/ International GPS Service

http://ilrs.gsfc.nasa.gov/ International Laser Ranging Service

http://ivscc.gsfc.nasa.gov/ International VLBI Service

http://ids.cls.fr/ International DORIS Service

Satellite Coordinate Systems

Satellite Coordinate System (RSW) -- (Radial-Transverse-Normal)

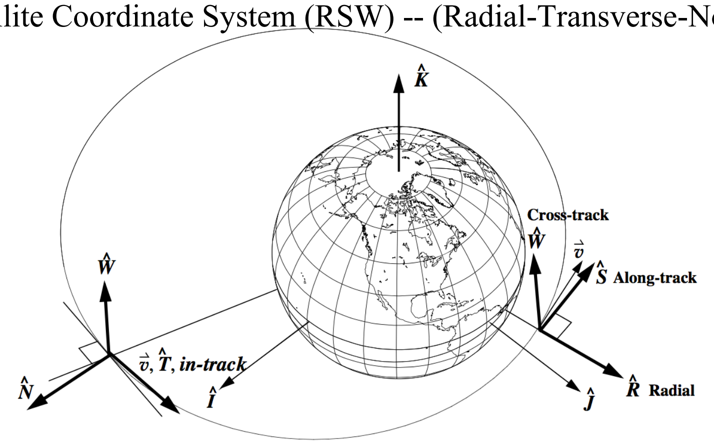


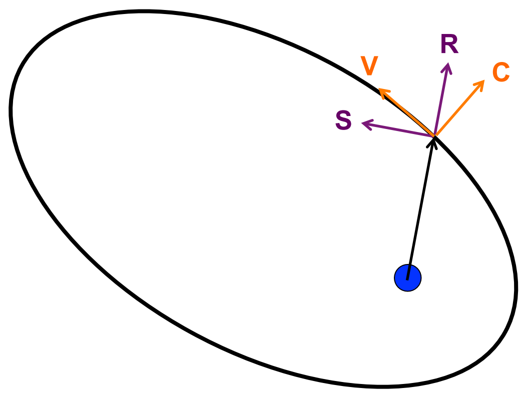
Figure 3-15. Satellite Coordinate Systems, RSW and NTW. These coordinate systems move with the satellite. The *R* axis points out from the satellite along the geocentric radius vector, the *W* axis is normal to the orbital plane (*not* usually aligned with the *K* axis), and the *S* axis is normal to the position vector and positive in the direction of the velocity vector. The *S* axis is aligned with the velocity vector *only* for circular orbits. In the NTW system, the *T* axis is always parallel to the velocity vector. The *N* axis is normal to the velocity vector and is *not* aligned with the radius vector, except for circular orbits, and at apogee and perigee in elliptical orbits.

Satellite Coordinate Systems

Satellite Coordinate Systems:

RSW – Radial-Transverse-Normal

NTW – Normal-Tangent-Normal; VNC is a rotated version



Perifocal Coordinate System (PQW)

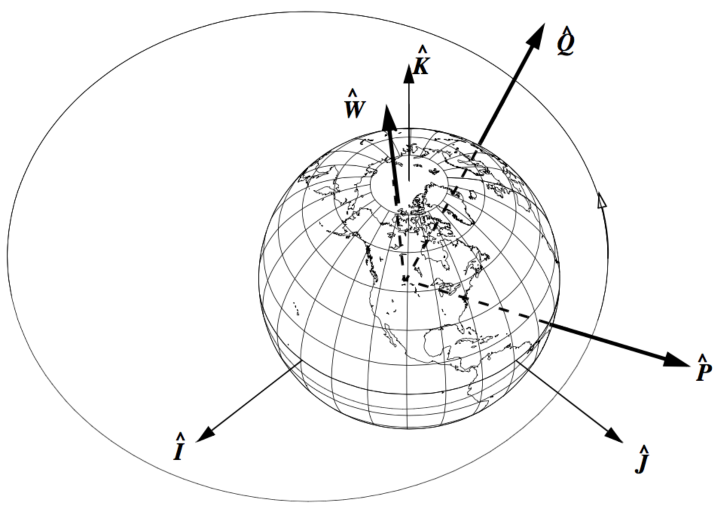


Figure 3-14. Perifocal Coordinate System, PQW. This system points towards perigee. Satellite motion is in the *P*-*Q* plane. The *W* axis is normal to the orbit plane. Although this particular orbit is near equatorial (the *J* axis is 10° up from the *P*-*Q* plane), it doesn't have to be. We often view this system looking down from the *W* axis.

Inertial Celestial Coordinate Systems

- ICRF
- International Celestial Reference Frame, a realization of the ICR System.

- Defined by IAU (International Astronomical Union)
- Tied to the observations of a selection of 212 well-known quasars and other distant bright radio objects.
  - Each is known to within 0.5 milliarcsec
- Fixed as well as possible to the observable universe.
- Motion of quasars is averaged out.
  - Coordinate axes known to within 0.02 milliarcsec
- Quasi-inertial reference frame (rotates a little)
- Center: Barycenter of the Solar System

Inertial Celestial Coordinate Systems

- ICRF2
- Second International Celestial Reference Frame, consistent with the first but with better observational data.

- Defined by IAU in 2009.
- Tied to the observations of a selection of 295 well-known quasars and other distant bright radio objects (97 of which are in ICRF1).
  - Each is known to within 0.1 milliarcsec
- Fixed as well as possible to the observable universe.
- Motion of quasars is averaged out.
  - Coordinate axes known to within 0.01 milliarcsec
- Quasi-inertial reference frame (rotates a little)
- Center: Barycenter of the Solar System

Earth-Centered Inertial Coordinate Systems

- EME2000 / J2000 / ECI
- Earth-centered Mean Equator and Equinox of J2000
  - Center = Earth
  - Frame = Inertial (very similar to ICRF)
    - X = Vernal Equinox at 1/1/2000 12:00:00 TT (Terrestrial Time)
    - Z = Spin axis of Earth at same time
    - Y = Completes right-handed coordinate frame

Earth-Centered Inertial Coordinate Systems

- EMO2000
- Earth-centered Mean Orbit and Equinox of J2000
  - Center = Earth
  - Frame = Inertial
    - X = Vernal Equinox at 1/1/2000 12:00:00 TT (Terrestrial Time)
    - Z = Orbit normal vector at same time
    - Y = Completes right-handed coordinate frame
  - This differs from EME2000 by ~23.4393 degrees.

Inertial Coordinate Systems

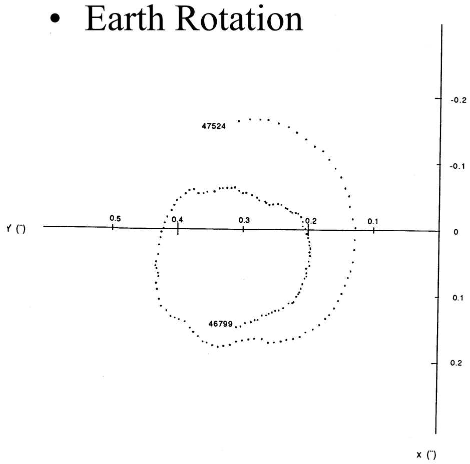
- Note that J2000 is very similar to ICRF and ICRF2
  - The pole of the J2000 frame differs from the ICRF pole by ~18 milliarcsec
  - The right ascension of the J2000 x-axis differs from the ICRF by 78 milliarcsec
- JPL's DE405 / DE421 ephemerides are defined to be consistent with the ICRF, but are usually referred to as "EME2000." They are very similar, but not actually the same.

Earth-Centered Earth-Fixed (ECEF) Coordinate Systems

- ECF / ECEF / Earth Fixed / International Terrestrial Reference Frame (ITRF)
- Earth-centered Earth Fixed
  - Center = Earth
  - Frame = Rotating and osculating (including precession, nutation, etc)
    - X = Osculating vector from center of Earth toward the equator along the Prime Meridian
    - Z = Osculating spin-axis vector
    - Y = Completes right-handed coordinate frame

ECEF Coordinate Systems

- Earth Rotation



- The angular velocity vector $\boldsymbol{\omega}_E$ is not constant in direction or magnitude
  - Direction: polar motion
    - Chandler period: 430 days
    - Solar period: 365 days
  - Magnitude: related to length of day (LOD)
- Components of $\boldsymbol{\omega}_E$ depend on observations; difficult to predict over long periods

Useful Coordinate Systems

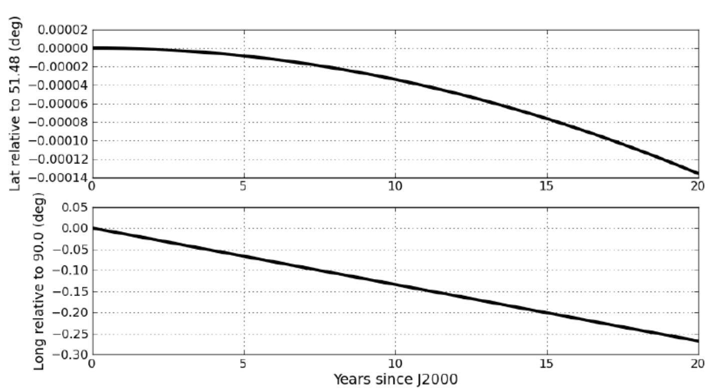
- Principal Axis Frames
- Planet-centered Rotating System
  - Center = Planet
  - Frame:
    - X = Points in the direction of the minimum moment of inertia, i.e., the prime meridian principal axis.
    - Z = Points in the direction of maximum moment of inertia (for Earth and Moon, this is the North Pole principal axis).
    - Y = Completes right-handed coordinate frame

Useful Coordinate Systems

- IAU Systems
- Center: Planet
- Frame: Either inertial or fixed
  - Z = Points in the direction of the spin axis of the body.
    - Note: by convention, all z-axes point in the solar system North direction (same hemisphere as Earth's North).
    - Low-degree polynomial approximations are used to compute the pole vector for most planets wrt ICRF.
  - Longitude defined relative to a fixed surface feature for rigid bodies.

Useful Coordinate Systems

- Example:
  - Latitude and Longitude of Greenwich, England, shown in EME2000.
  - Greenwich defined in IAU Earth frame to be at a constant latitude and longitude at the J2000 epoch.

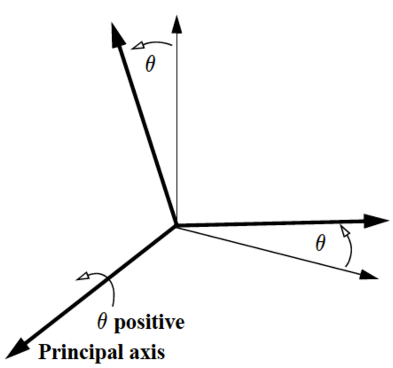


Synodic Coordinate Systems

- Earth-Moon, Sun-Earth/Moon, Jupiter-Europa, etc
  - Center = Barycenter of two masses
  - Frame:
    - X = Points from larger mass to the smaller mass.
    - Z = Points in the direction of angular momentum.
    - Y = Completes right-handed coordinate frame

Coordinate Transformations

Coordinate rotations can be accomplished through rotations about the principal axes.



$$ROT1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$ROT2(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$ROT3(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ECI to ECEF Coordinate Transformations

To convert from the ECI (IJK) system to ECEF, we simply rotate around ***Z*** by the GHA:

$$\vec{r}_{ECEF} = ROT3(\theta_{GST})\vec{r}_{IJK} \qquad \text{GST = Greenwich Sidereal Time}$$

$$\text{or} \quad \vec{r}_{IJK} = ROT3(-\theta_{GST})\vec{r}_{ECEF}$$

ignoring precession, nutation, polar motion, motion of equinoxes.

ECEF to SEZ Coordinate Transformations

To convert from ECEF to SEZ:

$$\vec{r}_{SEZ} = ROT2(90° - \phi)ROT3(\lambda)\vec{r}_{ECEF}$$
$$= ROT2(90° - \phi)ROT3(\theta_{LST} = \lambda + \theta_{GST})\vec{r}_{IJK}$$

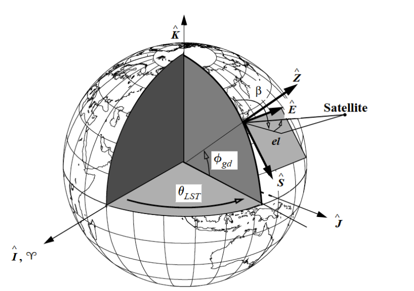


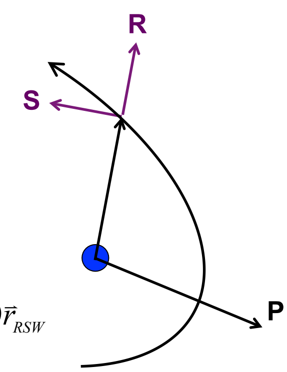
Figure 3-12. Topocentric Horizon Coordinate System, SEZ. This system moves with the Earth, so it's not fixed like the ITRF. θ_{LST} is required to orient the SEZ system to a fixed location. Note that for precise applications, θ_{AST} and longitude replace θ_{LST}. The SEZ system is used extensively in radar observations and often involves azimuth, β, and elevation, *el*. Geodetic latitude is common, ϕ_{gd}, but astronomical latitude is also used in many systems.

RSW to ECI Coordinate Transformations

To convert between IJK and PQW:

$$\vec{r}_{IJK} = ROT3(-\Omega)ROT1(-i)ROT3(-\omega)\vec{r}_{PQW}$$

$$\vec{r}_{PQW} = ROT3(\omega)ROT1(i)ROT3(\Omega)\vec{r}_{IJK}$$



To convert between PQW and RSW:

$$\vec{r}_{RSW} = ROT3(\nu)\vec{r}_{PQW}$$

$$\vec{r}_{PQW} = ROT3(-\nu)\vec{r}_{RSW}$$

Thus, RSW → IJK is:

$$\vec{r}_{IJK} = ROT3(-\Omega)ROT1(-i)ROT3(-u)\vec{r}_{RSW}$$

where $u = \nu + \omega$

Coordinate Transformations

- One of the coolest shortcuts for building transformations from one system to any other, without building tons of rotation matrices:

$$\vec{r}_{IJK} = \left[\hat{S} \mid \hat{E} \mid \hat{Z}\right]\vec{r}_{SEZ}$$

The unit vector in the S-direction, expressed in I,J,K coordinates

(sometimes this is easier, sometimes not)

Coordinate Transformations

- You can use the algorithms and examples in Vallado to write your coordinate transformation code and test it.
- Let's discuss some conceptual purpose for considering different coordinate systems!

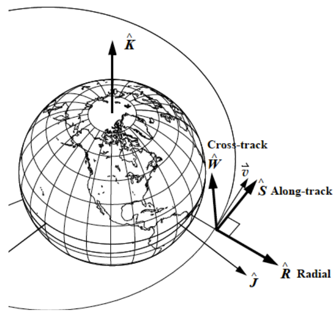
Scenario: Tracking Stations

- Consider a satellite in orbit.
- How long is the satellite overhead, as viewed by a ground station in Goldstone, California?
  - What's the elevation/azimuth time profile of the pass?
- Need: elevation (and azimuth) angles of satellite as viewed by station.
  - Need: satellite's states represented in SEZ coordinates
    - Transform satellite from IJK to ECEF
    - Transform satellite from ECEF to SEZ
    - Compute elevation and azimuth angles

Scenario: Solar Power

- A satellite is nadir-pointed with body-fixed solar panels pointed 90 deg away from nadir. How should the satellite rotate to maximize the energy output of the panels? What is the incidence angle of the Sun over time?

- Need: satellite state represented as RSW
- Compute angles to the Sun in that frame



Geocentric vs Geodetic Latitude

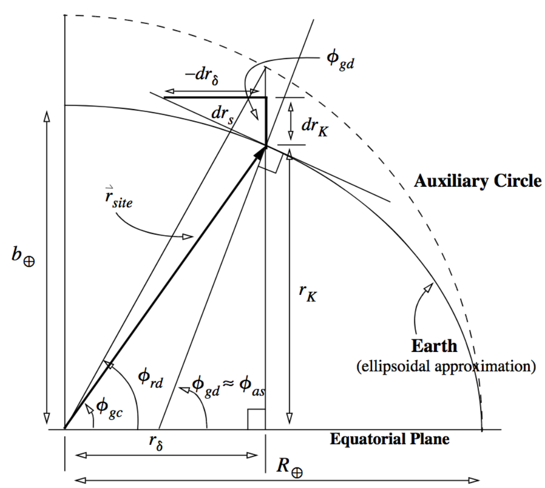


Figure 3-3. Latitude Geometry (exaggerated scale). The geodetic latitude, ϕ_{gd}, makes an angle perpendicular to the surface and the equatorial plane, whereas the geocentric latitude, ϕ_{gc}, is referenced to the center of the Earth. Astronomic latitude, ϕ_{as}, is very close to ϕ_{gd}. The center of the ellipsoid differs slightly for ϕ_{gd} and ϕ_{as}. Reduced latitude, ϕ_{rd}, is used only to derive relations.

Latitude/Longitude

For geodetic latitude use:

where $e_{\oplus}=0.081819221456$

$$\tan\phi_{gd} = \frac{\tan\phi_{gc}}{1 - e_{\oplus}^{2}}$$

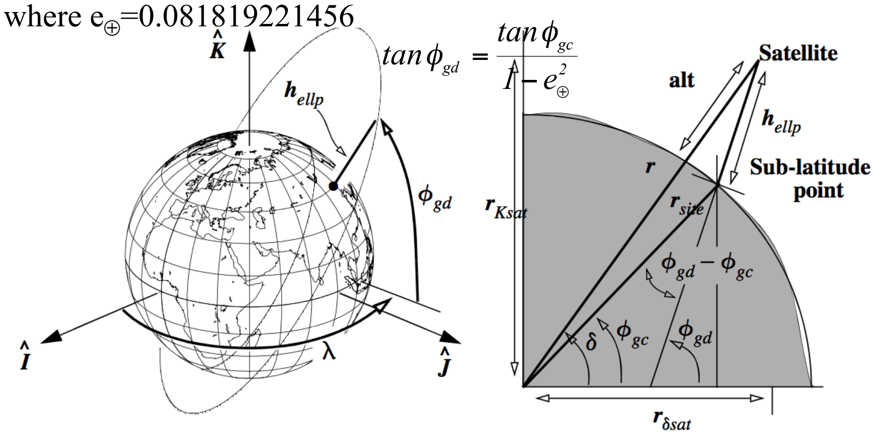


Figure 3-20. Determining a Satellite's Sub-latitude Point. Notice h_{ellp} is used for the altitude, and not H_{MSL}. The difference between the geodetic and geocentric angles is helpful in setting up the iteration for the problem.

Latitude/Longitude

Rotate into ECEF

$$\vec{r}_{ECEF} = ROT3(\theta_{GST})\vec{r}_{IJK}$$

$$\lambda = \tan^{-1}\left(\frac{r_y}{r_x}\right)$$

$$\phi_{gc} = \sin^{-1}\left(\frac{r_z}{r}\right)$$

$$r = \sqrt{r_x^2 + r_y^2 + r_z^2}$$

$$\vec{r}_{ECEF} = \begin{bmatrix} r\cos\phi\cos\lambda \\ r\cos\phi\sin\lambda \\ r\sin\phi \end{bmatrix} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$$

Right Ascension/Declination

$$\vec{r}_{IJK} = \begin{bmatrix} r\cos\delta\cos\alpha \\ r\cos\delta\sin\alpha \\ r\sin\delta \end{bmatrix} = \begin{bmatrix} r_I \\ r_J \\ r_K \end{bmatrix}$$

$$r = \sqrt{r_I^2 + r_J^2 + r_K^2}$$

thus,

$$\delta = sin^{-1}\left(\frac{r_K}{r}\right)$$

$$\alpha = tan^{-1}\left(\frac{r_J}{r_I}\right) \qquad (\lambda = \alpha - \theta_{GST})$$

Azimuth-Elevation

Compute slant-range vector from site to satellite:

$$\vec{\rho}_{IJK} = \vec{r}_{IJK} - \vec{r}_{siteIJK}$$

Rotate into SEZ

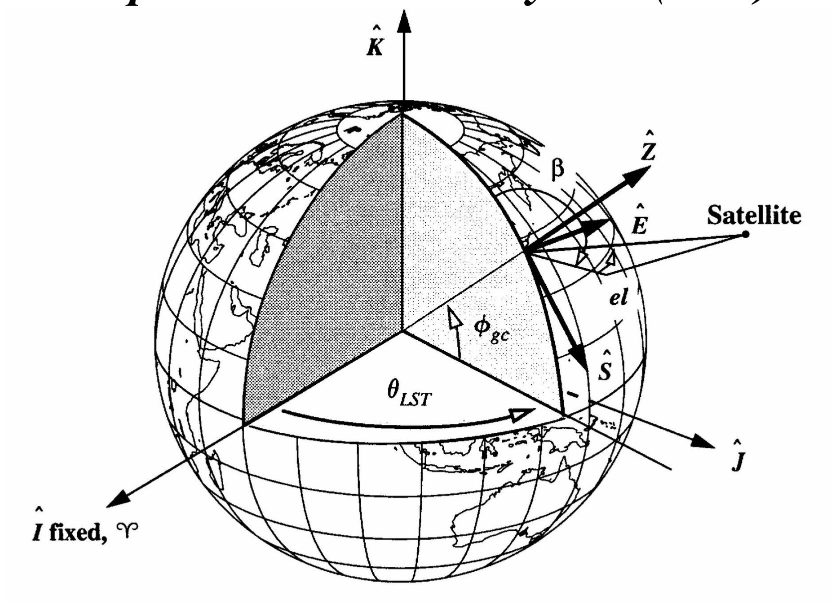
$$\vec{\rho}_{SEZ} = ROT2(90° - \phi)ROT3(\lambda)ROT3(\theta_{GST})\vec{\rho}_{IJK}$$

since $\vec{\rho}_{SEZ} = \begin{bmatrix} -\rho\, cos(el)cos(\beta) \\ \rho\, cos(el)sin(\beta) \\ \rho\, sin(el) \end{bmatrix}$

$$sin(el) = \frac{\rho_Z}{\rho} \qquad cos(el) = \frac{\sqrt{\rho_S^2 + \rho_E^2}}{\rho}$$

$$sin(\beta) = \frac{\rho_E}{\sqrt{\rho_S^2 + \rho_E^2}} \qquad cos(\beta) = \frac{-\rho_S}{\sqrt{\rho_S^2 + \rho_E^2}}$$

Topocentric Horizon System (SEZ)



Azimuth-Elevation

Alternatively:

$$\vec{r}_{ECEF} = ROT3(\theta_{GST})\vec{r}_{IJK}$$

$$\vec{\rho}_{ECEF} = \vec{r}_{ECEF} - \vec{r}_{siteECEF}$$

$$\vec{\rho}_{SEZ} = ROT2(90° - \phi)ROT3(\lambda)\vec{\rho}_{ECEF}$$

Geopotential Reference Surfaces

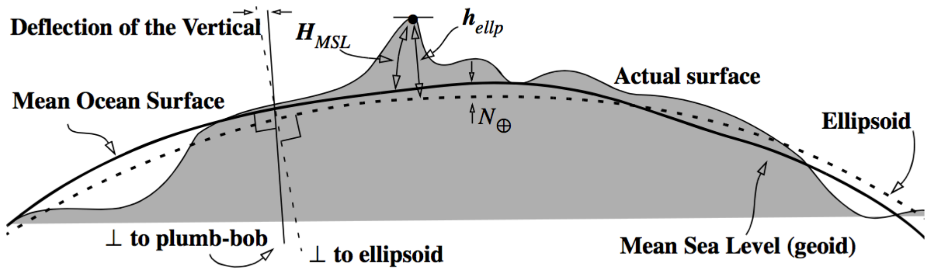


Figure 3-5. **Geopotential Reference Surfaces.** The height above sea level, H_{MSL}, and the geoid's undulation, $N_\oplus$, determine the ellipsoidal height, h_{ellp}. $N_\oplus$ is typically very small. Notice that the ellipsoid is "smooth" whereas the geoid has many small variations. Also, H_{MSL} is measured along a curved surface as shown.

Summary of Coordinate Systems (T3.1)

System	Symbol	Origin	Fundamental Plane	Principal Direction	Example Use
Interplanetary systems					
Heliocentric	XYZ	Sun	Ecliptic	Vernal equinox	Patched conic
Solar system*	$(XYZ)_{ICRF}$	Barycenter	varies	varies	Planetary motion
Earth-based systems					
Geocentric†	IJK	Earth	Earth equator	Vernal equinox	General
Earth	$(IJK)_{GCRF}$	Earth	varies	varies	Perturbations
Body-fixed	$(IJK)_{ITRF\text{-}\#\#}$	Earth	Earth equator	Greenwich meridian	Observations
Earth-Moon (synodic)	$(IJK)_S$	Barycenter	Invariable plane	Earth	Restricted three-body
Topocentric horizon	SEZ	Site	Local horizon	South	Radar observations
Topocentric equatorial	$(IJK)_t$	Site	Parallel to Earth equator	Vernal equinox	Optical observations
Satellite-based systems					
Perifocal**	PQW	Earth	Satellite orbit	Periapsis	Processing
Satellite radial	RSW††	Satellite	Satellite orbit	Radial vector	Relative motion, Perturbations
Satellite normal	NTW	Satellite	Satellite orbit	Normal to velocity vector	Perturbations
Equinoctial	EQW	Satellite	Satellite orbit	Calculated vector	Perturbations