

# Beta distribution

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To set the costs and benefits parameters we chosed a **beta distribution**. This distribution was chosed for some reasons:

- The values are all positive. As do not exist negative cost or negative benefit, it fit to our data;
- The values are defined on the interval  $[0,1]$ . It could be used as the percentage of costs and benefits;
- As the values are bounded between 0 and 1, we know the maximum value of benefits for one species, that is  $B \leq k_i$ , where  $k$  is the degree of  $i$ ;
- The shape of the curve assume diffent forms as a function of the paramters. In nature we really do not know the shape of the distributions of costs and benefits;
- It have a solution for ou case, which the other option of distribution (lognormal) do not.

The beta distribution have two parameters  $\alpha$  abd  $\beta$  and both control the shpae of the distribution. Therefore,

$$X \approx Beta(\alpha, \beta)$$

where,  $\alpha > 0$  and  $\beta > 0$ . Both parameters control de “influence” of of the shape bias. Higher  $\alpha$  values, higher the concentration towards the right side of the curve. Higher  $\beta$  values, higher the concentration towards the left side of the curve.

The mean and variance of Beta depends on  $\alpha$  and  $\beta$  parameters, as:

$$\bar{x} = \frac{\alpha}{\alpha + \beta}$$
$$\text{Var}(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

and

$$E[X] = \frac{\alpha}{\alpha + \beta}$$