$\dot{x} = ax\left(1 - \frac{x}{k}\right)$:

$$\frac{dx}{x\left(1-\frac{x}{k}\right)} = a\,dt \qquad \frac{1}{x\left(1-\frac{x}{k}\right)} = \frac{A}{x} + \frac{B}{1-\frac{x}{k}}$$

$$\boxed{A = 1}$$

$$-\frac{A}{k} + B = 0$$

$$B = \frac{1}{k}$$

$$\frac{dx}{x\left(1-\frac{x}{k}\right)} = dx\left[\frac{1}{x} + \frac{1}{k}\,\frac{1}{1-\frac{x}{k}}\right]$$

$$\ln x - \ln\left(1-\frac{x}{k}\right) = \ln\frac{x}{1-\frac{x}{k}} = at$$

$$x = \left(1-\frac{x}{k}\right)e^{at} \times \frac{x_0}{1-\frac{x_0}{k}}.$$

(1) $\boxed{x = x_0\,\frac{e^{at}}{1+\frac{1}{k}e^{at}}}$ $\qquad \cancel{x_0} \qquad \frac{e^{at}}{\cancel{\frac{1}{x_0}\left(1-\frac{x_0}{k}\right)+\frac{1}{k}e^{at}}}$

$$\frac{1}{x_0}\left(1-\frac{x_0}{k}\right)+\frac{1}{k}e^{at}$$

(2). $\boxed{x = x_0\,e^{-dt}}$

$$\frac{e^{a\tau/2}}{\frac{1}{x_t}\left(1-\frac{x_t}{k}\right)+\frac{1}{k}e^{a\tau/2}} \cdot e^{-\gamma\tau/2} = x_{t+1}$$

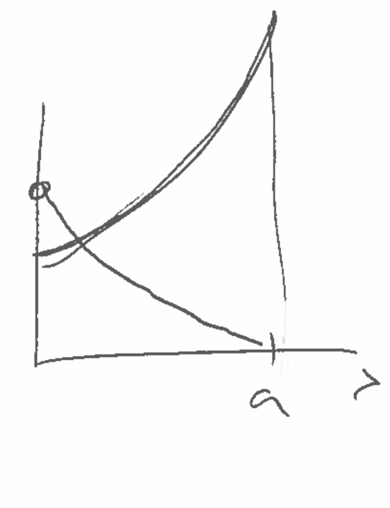
$$e^{(a-\gamma)\tau/2} = x_{t+1}\left[\frac{1}{x_t}\left(1-\frac{x_t}{k}\right)+\frac{1}{k}e^{a\tau/2}\right]$$

$$= \left[1-\frac{x^*}{k}+\frac{x^*}{k}e^{a\tau/2}\right]$$

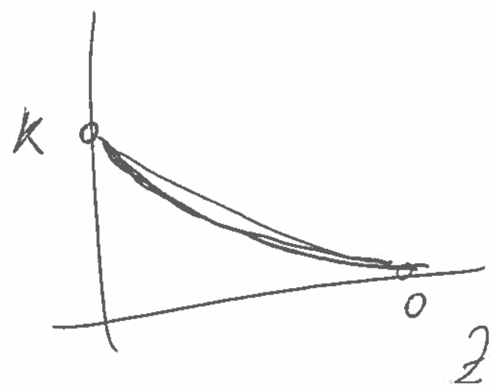
$$x^* = \frac{1}{k}\cancel{\left(1-e^{a\tau/2}\right)}\left[e^{(a-\gamma)\tau/2}+1\right].$$

$$x^* = \frac{1}{k}\cancel{\left(e^{a\tau/2}-1\right)}\left(e^{(a-\gamma)\tau/2}-1\right).$$

$$\boxed{x^* = k\,\frac{e^{(a-\gamma)\tau/2}-1}{e^{(a-m)\tau/2}-1}}$$



$$x^* = k\,\frac{e^{(a-\gamma)\tau/2}-1}{e^{a\tau/2}-1}$$



$$X_2^* = K_1 \frac{e^{(a-\gamma)\tau/2} - 1}{e^{a\tau/2} - 1}$$

$$\frac{e^{at}}{\frac{K}{K x_0}\left(1 - \frac{x_0}{K}\right) + \frac{x_0}{x_0 K} e^{at}}$$

$$X_1^* = K_2$$

Kx_0 $\quad K - Kx_0 + x_0 e^{at}$

$K - x_0$ $\quad K + x_0(e^{at} - 1)$

$$\Delta(x_1 - x_1) = (K_1 - K_2) \frac{e^{(a-\gamma)\tau/2} - 1}{e^{a\tau/2} - 1}.$$

$$\frac{K \frac{x_0}{K} e^{a\tau/2}}{1 - \frac{x_0}{K} + \frac{x_0}{K} e^{a\tau/2}} = K \frac{\frac{x_0}{K} e^{a\tau/2}}{1 + \frac{x_0}{K}(e^{a\tau/2} - 1)}.$$

$$K \frac{\left(e^{(a-\gamma)\tau/2} - 1\right) e^{a\tau/2} / \left(e^{a\tau/2} - 1\right)}{1 + \frac{\left(e^{(a-\gamma)\tau/2} - 1\right)}{\left(e^{a\tau/2} - 1\right)} \cdot \left(e^{a\tau/2} - 1\right)} \qquad \frac{Kx_0 e^{a\tau/2}}{K + x_0(e^{a\tau/2} - 1)}.$$

$$K \frac{\left(e^{(a-\gamma)\tau/2} - 1\right) e^{a\tau/2}}{\underbrace{e^{a\tau/2} - 1 + \left(e^{(a+\gamma)\tau/2} - 1\right)\left(e^{a\tau/2} - 1\right)}_{\left(e^{a\tau/2} - 1\right)\left(1 + e^{(a-\gamma)\tau/2}\right)}}$$

73

$$x_1^* = K\frac{\left(e^{(a-\lambda)\tau/2}-1\right)\cancel{e^{a\tau/2}}}{\left(e^{a\tau/2}-1\right)\cancel{e^{(a-\lambda)\tau/2}}} = K\,\frac{e^{(a-\lambda)\tau/2}-1}{e^{a\tau/2}-1}\,e^{\lambda\tau/2}$$

$$\frac{x_1^*}{x^*} = \frac{e^{a\tau/2}}{e^{(a-\lambda)\tau/2}} = \underline{\underline{e^{\lambda\tau/2}}}.$$

$$\Delta = x_1^* - x^* = K\,\frac{e^{(a-\lambda)\tau/2}-1}{e^{a\tau/2}-1}\left[e^{\lambda\tau/2}-1\right]$$

$$\dot{x} = ax\left(1 - \frac{x}{K}\right) - bx \qquad \Gamma = \frac{K(a-b)}{a}$$

$$\dot{x} = (a-b)x\left(1 - \frac{ax}{K(a-b)}\right) = (a-b)x\left(1 - \frac{x}{\Gamma}\right)$$

$$\boxed{x^* = \Gamma.}$$

$$\lambda = (a-b) - 2\frac{a-b}{\Gamma}x^* = (a-b) - 2\frac{(a-b)\,\Gamma}{\cancel{K(a-b)}\;\Gamma} = -(a-b)$$

$$\boxed{\lambda = -(a-b)}$$

$$\left.\begin{aligned} \dot{x} &= \frac{a}{2} z\left(1 - \frac{z}{K}\right) - bx - cx \\ \dot{y} &= cx + \frac{a}{2} z\left(1 - \frac{z}{K}\right) - by. \end{aligned}\right\} \quad \left\{\boxed{\begin{aligned} z^* &= \Gamma \\ \lambda &= -(a-b). \end{aligned}}\right\}$$

$$(b+c)x = (b-c)y \qquad z = x + \frac{b+c}{b-c}x = \frac{2b}{b-c}x$$

$$\left[a\frac{2b}{b-c} - (b+c)\right] = \frac{a}{K}\frac{2b}{b-c}x$$

$$x^* = \frac{\frac{2ab}{b-c} - (b+c)}{\frac{a}{K}\frac{2b}{b-c}} = K\frac{2ab - b - c}{2ab}$$

①

$$\Delta = (b+c)\cancel{x} = by - cx$$

$$\underline{(b+2c)x = by} \Rightarrow z = x + y = \cancel{(b+2c)}x + \frac{b+2c}{b}x$$

$$\boxed{z = 2\frac{(b+c)}{b}x = 2\frac{b+c}{b+2c}y}$$

$$\frac{a}{2}2\frac{(b+c)}{b} - (b+c) = \frac{a}{k}\frac{4}{2}\frac{(b+c)^2}{b^2}x^*$$

$$\frac{\cancel{2}a(\cancel{b+c}) - b(\cancel{b+c})}{\cancel{b}} = \frac{\cancel{2}a\,(b+c)^2}{k b^2}x^*$$

$$x^* = \frac{k b(\cancel{2}a - b)}{\cancel{2}a(b+c)}$$

$$y^* = \frac{k\cancel{b}(\cancel{2}a-b)}{\cancel{2}a(b+c)}\frac{(b+2c)}{\cancel{b}} = \frac{k(\cancel{2}a-b)(b+2c)}{\cancel{2}a(b+c)}.$$

$$z^* = \frac{k(\cancel{2}a-b)[b+b+2c]}{\cancel{2}a(b+c)} = \frac{k(\cancel{2}a-b)}{\cancel{4}a}$$

$$\frac{a}{2}(x+y) - \frac{a}{2k}(x+y)^2 - bx - \cancel{bx}$$

$$cx - by + \frac{a}{2}(x+y) - \frac{a}{2k}\underbrace{(x+y)^2}_{x^2+2xy+y^2}$$

$$\begin{bmatrix} \frac{a}{2} - b - c - \underbrace{\frac{a}{k}x^* - \frac{a}{k}y^*}_{(a-b)} & \frac{a}{2} - \frac{a}{k}x^* - \frac{a}{k}y^* \\ c + \frac{a}{2} - \frac{a}{k}x^* - \frac{a}{k}y^* & -b + \frac{a}{2} - \frac{a}{k}x^* - \frac{a}{k}y^* \end{bmatrix}$$

$$\left(\frac{a}{2} - \cancel{b} - c - a + \cancel{b} - \lambda\right)\left(-\cancel{b} + \frac{a}{2} - a + \cancel{b} - \lambda\right) - \left(\frac{a}{2} - a + b\right)(c + \frac{a}{2} - a + b)$$

$$\left(+c + \frac{a}{2} + \lambda\right)\left(+\frac{a}{2} + \lambda\right) - \left(b - \frac{a}{2}\right)\left(b + c - \frac{a}{2}\right)$$

$$\lambda^2 + (c+a)\lambda + \frac{a}{2}\left(c + \frac{a}{2}\right) + \frac{a}{2}\left(b + c + b - \frac{a}{2}\right) - \left(b(b+c)\right)$$

$$\frac{a}{2}(b+c) - b(b+c) = (a-b)(b+c$$

$$\lambda^2 + (a+c)\lambda + (a-b)(b+c) = 0.$$

$$y = \frac{-(a+c) \pm \sqrt{(a+c)^2 - 4(a+b)(b+c)}}{2}$$

$(2b-a+c)^2$

$b-a$

$-(b+a-2c)$

$a^2 + \cancel{2ac} + c^2 - 4ab - \cancel{2ac} + 4b^2 + 4bc$

$b^2 - 2ab + a^2 + a^2 + 2ac + c^2$

$2bc + 2bc - 2a^2 - 2ac$

$(b-a)$

$(b-a) + (a+c) - (b+c)$

$((b-a) + (a+c))^2$

$2(b-a) + a + c$

$(2b - a + c)^2$

$c + \frac{a}{2} + b - a$

$a^2 + 2ac + c^2$

$-4ab$

$-4ac$

$4bc$

$4b^2$

$\left(c + b - \frac{a}{2}\right)\left(b - \frac{a}{2}\right)$

$a^2 + c^2 + 4b^2 - 2ac - 4ab - 4bc.$

$b^2 - \cancel{2ab} + \cancel{a^2} + \cancel{ab} + \cancel{bc} - \cancel{a^2} - \cancel{ac} + \cancel{ab} + \cancel{ac} - \cancel{bc} - c^2$

$b^2 - c^2 = (b+c)(b-c)$

$b^2 - 2ab + a^2$

4

$c^2 + 3b^2 - 2ac - 2ab - 4bc.$

$$\dot{x} = az\left(1-\frac{z}{k}\right) - bx - cx \qquad z^* = \Gamma$$

$$\dot{y} = cx - by \qquad \lambda = -(a-b)$$

$$cx^* = by^*$$

$$z^* = x^* + y^* = x^* + \frac{c}{b}x^* = \frac{b+c}{b}x^*$$

$$\boxed{z = \frac{b+c}{b}x^* = \frac{b+c}{c}y^*}$$

$$\frac{a}{b}(b+c) \mp b \mp c = \frac{a}{k}\frac{(b+c)^2}{b^2}x^*$$

$$\cancel{(ab + ac - b^2 - bc)}$$

$$\frac{(a-b)\cancel{(b+c)}}{\cancel{b}} = \frac{a}{k}\frac{(b+c)^{\cancel{2}}}{b^{\cancel{2}}}x^*$$

$$x^* = \frac{kb(a-b)}{a(b+c)}$$

$$y^* = \frac{c}{\cancel{b}}\frac{k\cancel{b}(a-b)}{a(b+c)} = \frac{kc(a-b)}{a(b+c)}$$

$$z^* = \frac{k(b+c)(a-b)}{a(b+c)} = \frac{k(a-b)}{a}$$

$$\begin{vmatrix} \underbrace{a - b - c - \frac{2a}{k}x^* - \frac{2a}{k}y^*}_{2(a-b)} & a - \frac{2a}{k}x^* - \frac{2a}{k}y^* \\ c & -b \end{vmatrix}$$

$$\left(a-b-c-2(a-b)-\lambda\right)(-b-\lambda) - c\left(a-2(a-b)\right) =$$

$$\left(-a+b-c-\lambda\right)(-b-\lambda) - c(2b-a) =$$

$$\lambda^2 + (\cancel{b}+a-\cancel{b}+c)\lambda + b(a-b+c) - c(2b-a).$$

$$ab - b^2 + \cancel{bc} - \cancel{2}bc + ca$$

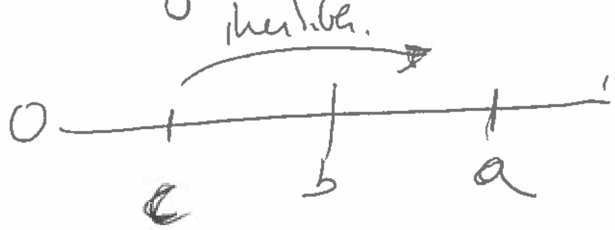
$$\lambda^2 + (a+c)\lambda - \left[(b-a)^2 + (a+c)(b-a)\right]$$

$$b^2 - 2ab + a^2 \quad ab \quad +bc \quad -a^2 \quad -ac$$

$$(b-a)\left[b - \cancel{a} + \cancel{a} + c\right]$$

$$\lambda^2 + (a+c)\lambda - (b-a)(b+c) = 0.$$

$\Rightarrow$ igual que antes. A3



B2

$$\dot{x} = a x\left(1 - \frac{x}{K}\right) - bx - cx$$

$$\dot{y} = cx - dy.$$

$$cx^* = dy^* \qquad x^* = \frac{d}{c} y^* \qquad y^* = \frac{c}{d} x^*$$

$$z^* = x^* + y^* = x^* + \frac{c}{d} x^* = \frac{d+c}{d} x^*$$

$$\boxed{z^* = \frac{d+c}{d} x^* = \frac{d+c}{c} y^*}$$

$$a\left(1 - \frac{x^*}{K}\right) = (b+c)$$

$$1 - \frac{x^*}{K} = \frac{b+c}{a} \Rightarrow x^* = \frac{a-b-c}{a} K$$

$$y^* = \frac{c(a-b-c)}{da} K$$

$$\boxed{z^* = K \frac{(a-b-c)[d+c]}{da}}$$

C1

$$a - b - c - \frac{2a}{K} x^*$$

$$a\left(1 - \frac{Y^*}{K}\right) = (b+c)\frac{d}{c}$$

$$\left(1 - \frac{Y^*}{K}\right) = \frac{d}{ca}(b+c)$$

$$Y^* = K\left[1 - \frac{d}{ca}(b+c)\right]$$

$$y^* = K\left(\frac{ca - db - dc}{ca}\right)$$

$$x^* = \frac{Kd}{c^2 a}\left(ca - d(b+c)\right)$$

$$z^* = \frac{K}{c^2 a}\left(ca - d(b+c)\right)(c+d)$$

c2

$$\begin{vmatrix} -(b+c) & a - \frac{2a}{k} y^* \\ c & -d \end{vmatrix} . \qquad = \frac{2\cancel{\alpha}}{\cancel{k}} \frac{\cancel{k}}{c\cancel{\alpha}} \left(ca - d(b+c) \right)$$

$$= \frac{2}{c} \left(ca - d(b+c) \right).$$

$$\left(-(b+c)-\lambda\right)\left(-d-\lambda\right) - c\left(a - \frac{2}{c}\left(ca - d(b+c)\right)\right).$$

$$\lambda^2 + (d+b+c)\lambda + \cancel{d(b+c)} - \cancel{c}a + \cancel{2}ca - \cancel{2}db - \cancel{2}dc$$

$$\lambda^2 + (d+b+c)\lambda - d(b+c) + ca$$

$$\frac{-(d+\Delta) \pm \sqrt{(d+\Delta)^2 + 4(d\Delta - ca)}}{2}.$$

$$d^2 + 2d\Delta + \Delta^2 + 4d\Delta$$

$$d^2 + 6d\Delta + \Delta^2.$$

stable. $\Leftarrow$ $d\Delta - ca < 0.$

$$\boxed{\frac{d(b+c)}{c} < a}$$

$\frac{\partial}{\partial c}:$

$-\frac{db}{c^2} + d$

C3.

$\underline{b=0}$:

$$\lambda = \frac{-(d+\cancel{b}) \pm \sqrt{(d+c)^2 - 4(d\cancel{b} - ca)}}{2}$$

$$\frac{d^2 + \cancel{2}dc + c^2 - \cancel{4}dc + 4ca}{2}.$$

$$\underline{(d-c)^2 + 4ca}$$

$\underline{b=d}$:

$$b + (b+c)$$

$$-(2b+c)^{\cancel{2}} \pm \sqrt{(2b+c)^2 - 4(b(b+c) - ca)}$$

$$\cancel{4b^2} + \cancel{4bc} + c^2 - \cancel{4b^2} - \cancel{4bc} + 4ca.$$

$$c^2 - 4ca.$$

c3.

$$\dot{x} = a y \left(1 - \frac{z}{K}\right) - bx - cx$$

$$\dot{y} = cx - dy.$$

$$a \frac{c}{d} \left(1 - \frac{d+c}{dK} x^*\right) = b + c$$

$$1 - \frac{d+c}{dK} x^* = \frac{d(b+c)}{ac}$$

$$x^* = \frac{dK}{d+c} \left[1 - \frac{d(b+c)}{ac}\right]$$

$$x^* = \frac{dK}{(d+c)ac} \left(ac - d(b+c)\right).$$

$$y^* = \frac{cK}{(d+c)ac} \left(ac - d(b+c)\right).$$

$$\boxed{z^* = \frac{K}{ac} \left(ac - d(b+c)\right)}$$

D1

$$\begin{vmatrix} -(b+c) - \dfrac{a y^*}{k} & a - \dfrac{2a}{k} y^* - \dfrac{a}{k} x^* \\ c & -d. \end{vmatrix} \qquad y\left(1 - \frac{x+y}{k}\right) \qquad y - \frac{xy + y^2}{k}$$

$$\frac{a y^*}{k} = \frac{1}{d+c} \left(ac - d(b+c)\right)$$

$$\frac{a}{k} x^* = \frac{d}{(d+c)c} \left(ac - d(b+c)\right).$$

$$\left(-\Delta - \frac{B}{d+c} - \lambda\right)(-d-\lambda) - c\left(a - \frac{B}{(d+c)}\left(2 + \frac{d}{c}\right)\right).$$

$$bd + bc + cd + c^2 - ac + db + dc$$

$$\frac{(b+c)(d+c) - ac + d(b+c)}{d+c}$$

$$2bd + bc + 2cd + c^2 - ac$$