Analytical approach for the Simple Life Cycle on a Single Site (SLC-SS).

Differential equations of the SLC-SS:

$$\frac{dNe}{dt} = \left(r * N_a * \left(\frac{S_a}{S_{max}}\right) * \left(\frac{K - N_a}{K}\right)\right) - (d_e * N_e) - (g * N_e)$$

$$\frac{dNa}{dt} = (g * N_e) - (d_a * N_a) - (E * N_a) \quad (1)$$

$$\frac{dSa}{dt} = \gamma * S_a * \left(1 - \frac{S_a}{S_{max} * (1 - E)}\right)$$

Estimation of the Jacoban matrix of the SLC-SS system. Partial derivatives with respect to the variables: Ne, Na and Sa. This matrix was obtained by the function Symbolics.jacobian() form the Symbolics package in Julia language.

$$J_{SLC-SS} = \begin{bmatrix} -d_e - g & \left(\frac{S_a * r * (K - 2 * Na)}{K * S_{max}}\right) & \left(\frac{N_a * r * (K - N_a)}{K * S_{max}}\right) \\ g & -E - d_a & 0 \\ 0 & 0 & \gamma * \left(1 + \frac{2 * S_a}{S_{max} * (1 - E)}\right) \end{bmatrix} \quad (2)$$

The determinant of the Jacobian matrix is:

$$Det(J_{SLC-SS}) = \frac{-g * r * \gamma * (K - 2 * Na) * (Sa^2) * \left(1 + \frac{-2Na}{Smax * (1 - E)}\right)}{K * Smax} + Sa * \gamma * (-E - da) * \left(1 + \frac{-2Na}{Smax * (1 - E)}\right) * (-de - g) \quad (3)$$

The determinant is equalled to 0 and the common factors are grouped:

$$Det(J_{SLC-SS}) = \frac{-g * r * \gamma * (K - 2 * Na) * (Sa^2) * \left(1 + \frac{-2Na}{Smax * (1 - E)}\right)}{K * Smax} + Sa * \gamma * (-E - da) * \left(1 + \frac{-2Na}{Smax * (1 - E)}\right) * (-de - g) = 0$$

$$\left(\frac{-g * r * \gamma * (K - 2 * Na) * (Sa^2)}{K * Smax} + Sa * \gamma * (-E - da) * (-de - g)\right) * \left(1 + \frac{-2Na}{Smax * (1 - E)}\right) = 0$$

$$\frac{-g * r * \gamma * (K - 2 * Na) * (Sa^2)}{K * Smax} + Sa * \gamma * (-E - da) * (-de - g) = 0$$

The last step is repeated:

$$\left(\frac{-g * r * (K - 2 * Na) * S_a}{K * Smax} + \gamma * (-E - da) * (-de - g)\right) * S_a * \gamma = 0$$

$$\frac{-g * r * (K - 2 * Na) * S_a}{K * Smax} + \gamma * (-E - da) * (-de - g) = 0$$

S_a is cleared:

$$\gamma * (-E - da) * (-de - g) = \frac{g * r * (K - 2 * Na) * S_a}{K * Smax}$$

$$\gamma * (-E - da) * (-de - g) * K * Smax = g * r * (K - 2 * Na) * S_a$$

$$S_a = \frac{\gamma * (-E - da) * (-de - g) * K * Smax}{g * r * (K - 2 * Na)}$$

$$S_a = \frac{\gamma * (-1) * (E + da) * (-1) * (de + g) * K * Smax}{g * r * (K - 2 * Na)}$$

$$S_a = \frac{\gamma * (E + da) * (de + g) * K * Smax}{g * r * (K - 2 * Na)} \quad (4)$$

Nai is cleared:

$$(K - 2 * Na) = \frac{\gamma * (E + da) * (de + g) * K * Smax}{g * r * S_a}$$

$$2 * Na = K - \frac{\gamma * (E + da) * (de + g) * K * Smax}{g * r * S_a}$$

$$Na = \left(K - \frac{\gamma * (E + da) * (de + g) * K * Smax}{g * r * S_a}\right) * \frac{1}{2}$$

$$Na = \frac{K}{2} * \left(1 - \frac{\gamma * (E + da) * (de + g) * K * Smax}{g * r * S_a}\right) (5)$$