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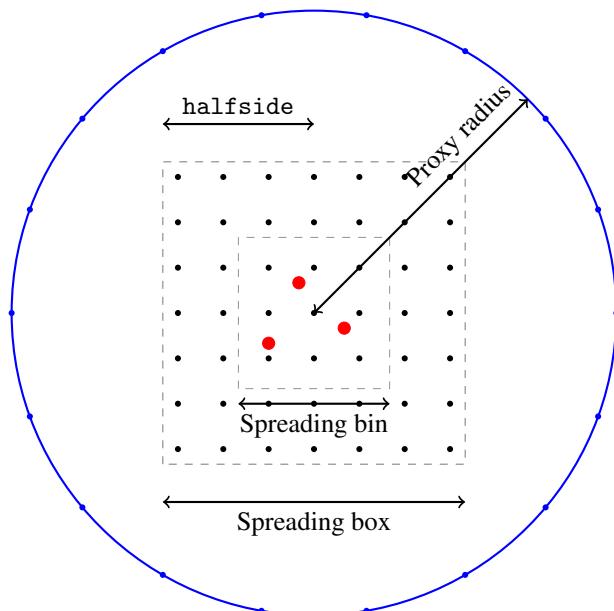
# PCFFT IMPLEMENTATION NOTES

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A PREPRINT

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## 1 Computing the spreading grid



**Figure 1:** Schematic of the spreading geometry in 2D: sources (red), the regular discretization (black dots), and a proxy ring (blue).

### 1.1 Computing the spreading box size

The spreading box size is computed by `spread_halfsize()`. This is meant to approximately control the number of source points in a particular spreading box.

### 1.2 Computing the regular grid spacing

This is performed in `dx_nproxy()`. We want to find parameters `dx` and `nproxy`. `dx` is the grid spacing of the regular discretization of the spreading box, which starts at  $-\text{halfside} + \frac{\text{dx}}{2}$  and ends at  $\text{halfside} - \frac{\text{dx}}{2}$ . `nproxy` is the number of proxy points placed on a proxy ring (or sphere) outside the spreading box.

Notes on the geometry used:

- We put source points in a bin with sidelength `c_bwidth × halfside`.

- We place a proxy ring of radius  $\sqrt{d} \times \text{halfside} \times \text{crad}$  where  $d$  is the dimension (2 or 3) and  $\text{crad}$  is a constant (default 2.0).
- If we consider breaking the spreading box into  $\text{nspread}$  cells, the grid points are placed at the center of each cell, so  $\text{dx} = \frac{2 \times \text{halfside}}{\text{nspread}}$ .

Notes on the algorithm used to compute  $\text{dx}$  and  $\text{nproxy}$ :

- Generate random sources in the spreading bin with side length  $\text{halfside}$ . Generate target points on a ring/sphere of radius  $1.1 \times$  the proxy radius.
- Increase  $\text{nspread}$  and  $\text{nproxy}$  until the error tolerance is met.
- Decrease  $\text{nspread}$  until the error tolerance is no longer met.
- Spreading bin is  $\text{dx} \times \lfloor \text{nspread}/2 \rfloor$