
PCFFT IMPLEMENTATION NOTES

A PREPRINT

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1 Computing the spreading grid

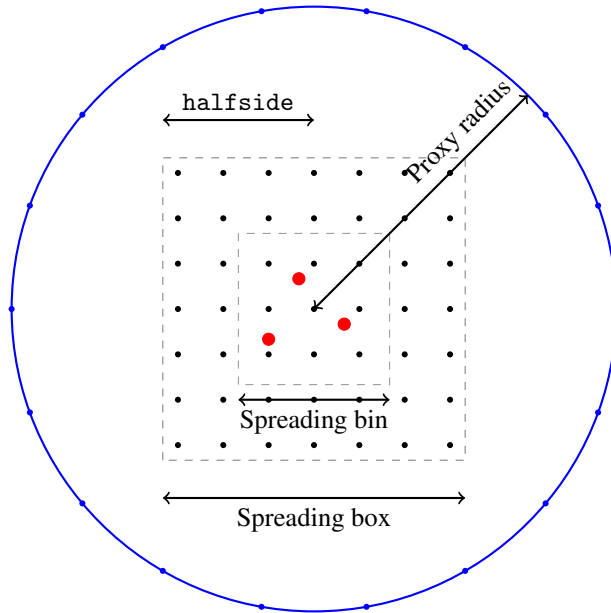


Figure 1: Schematic of the spreading geometry in 2D: sources (red), the regular discretization (black dots), and a proxy ring (blue).

1.1 Computing the spreading box size

The spreading box size is computed by `spread_halfside()`. This is meant to approximately control the number of source points in a particular spreading box.

1.2 Computing the regular grid spacing

This is performed in `dx_nproxy()`. We want to find parameters `dx` and `nproxy`. `dx` is the grid spacing of the regular discretization of the spreading box, which starts at $-\text{halfside} + \frac{dx}{2}$ and ends at $\text{halfside} - \frac{dx}{2}$. `nproxy` is the number of proxy points placed on a proxy ring (or sphere) outside the spreading box.

Notes on the geometry used:

- We put source points in a bin with sidelength $c_bwidth \times \text{halfside}$.

- We place a proxy ring of radius $\sqrt{d} \times \text{halfside} \times \text{crad}$ where d is the dimension (2 or 3) and crad is a constant (default 2.0).
- If we consider breaking the spreading box into nspread cells, the grid points are placed at the center of each cell, so $dx = \frac{2 \times \text{halfside}}{\text{nspread}}$.

Notes on the algorithm used to compute dx and $nproxy$:

- Generate random sources in the spreading bin with side length halfside . Generate target points on a ring/sphere of radius $1.1 \times$ the proxy radius.
- Increase nspread and $nproxy$ until the error tolerance is met.
- Decrease nspread until the error tolerance is no longer met.
- Spreading bin is $dx \times \lfloor \text{nspread}/2 \rfloor$