CS 404 – Artificial Intelligence
HW 4 – 2020 Spring
150pt

Objective: Learning about Propositional Logic

1) 20 pt - Decide whether each of the following sentences is valid, unsatisfiable, or satisfiable (not valid, but only satisfiable). **Show how you come to that decision using truth table enumeration or logical equivalence rules.**

- Smoke => Smoke equals ¬Smoke V Smoke **VALID** 10pt

Smoke	Smoke => Smoke	¬Smoke V Smoke
True	True	True
False	True	True

Always True, hence it is valid.

Circle the correct answer: Valid, Satisfiable, Unsatisfiable

- Smoke => Fire **SATISFIABLE** 10pt

Smoke	Fire	Smoke => Fire
True	True	True
True	False	False
False	True	True
False	False	True

It is satisfiable because at least one true result in truth table.

Circle the correct answer: Valid, Satisfiable, Unsatisfiable

2) 40pt – Truth Table Enumeration for Inference

Assume we have the knowledge base **KB: Rain ∧ (Rain ⇒ Wet) ∧ (Snow ⇒ Cold)** and the given propositions are the only ones in the KB.

a) **25 pts – How many *possible worlds*** (truth value assignments to the propositions) **are *models* of the KB?** Show your work by filling the truth table for the KB.

Rain	Wet	Snow	Cold	KB: Rain ∧ (Rain ⇒ Wet) ∧ (Snow ⇒ Cold)
True	True	True	True	**True**
True	True	True	False	False
True	True	False	True	**True**
True	True	False	False	**True**
True	False	True	True	False
True	False	True	False	False
True	False	False	True	False
True	False	False	False	False
False	True	True	True	False
False	True	True	False	False
False	True	False	True	False
False	True	False	False	False
False	False	True	True	False
False	False	True	False	False
False	False	False	True	False
False	False	False	False	False

Since the knowledge base is true for 3 cases. There are 3 possible worlds for given KB.

b) **15pts** – **Extend the above truth table** (use truth table enumeration method) to show whether the knowledge base entails **α=Wet.**

State your answer here: Knowledge base entails α, because α is true for all the cases KB is true.

Rain	Wet	Snow	Cold	KB: Rain ∧ (Rain ⇒ Wet) ∧ (Snow ⇒ Cold)	α=Wet
True	True	True	True	**True**	**True**
True	True	True	False	False	True
True	True	False	True	**True**	**True**
True	True	False	False	**True**	**True**
True	False	True	True	False	False
True	False	True	False	False	False
True	False	False	True	False	False
True	False	False	False	False	False
False	True	True	True	False	True
False	True	True	False	False	True
False	True	False	True	False	True
False	True	False	False	False	True
False	False	True	True	False	False
False	False	True	False	False	False
False	False	False	True	False	False
False	False	False	False	False	False

3) 90pt – AIMA 3rd Ed. Q. 7.2

7.2 (Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is homed.

a) 20pt – First state the English paragraph as a set of Propositional Logic sentences. Please use the given two-letter proposition names below. Be careful about what propositions to use (especially for the the statement "mortal mammal"), considering all of the paragraph.

My: Unicorn is mythical

Im: Unicorn is immortal

Mm: Unicorn is mammal

Hr: Unicorn is horned

Mg: Unicorn is magical

P1. My ⇒ Im

P2. ¬My ⇒ (¬Im ∧ Mm)

P3. (Im ∨ Mm) ⇒ Hr

P4. Hr ⇒ Mg

b) 20pt – Convert the above KB into Conjunctive Normal Form. Show your work clearly.

KB: (My ⇒ Im) ∧ (¬My ⇒ (¬Im ∧ Mm)) ∧ ((Im ∨ Mm) ⇒ Hr) ∧ (Hr ⇒ Mg)

distributivity rule

(My ⇒ Im) ∧ (¬My ⇒ ¬Im) ∧ (¬My ⇒ Mm) ∧ (Im ⇒ Hr) ∧ (Mm⇒ Hr) ∧ (Hr ⇒ Mg)

implication elimination

CNF: (¬My ∨ Im) ∧ (My ∨ ¬Im) ∧ (My ∨ Mm) ∧ (¬Im ∨ Hr) ∧ (¬Mm ∨ Hr) ∧ (¬Hr ∨ Mg)

c) 50pt – See if the KB entails each of the following conclusions: "unicorn is mythical", "unicorn is magical", "unicorn is horned". Use the indicated method, if available.

For each of them, **please indicate the inference method you are using and clearly indicate your conclusion at the beginning:**

I. 10pts - "unicorn is mythical":

Conclusion: Using **simple resolution** we can show that we CANNOT conclude that the unicorn is mythical.

Show your work or state your argument: Unicorn might be mythical or not, we can confirm the knowledgebase by accepting unicorn is mythical or it is not mythical.

II. 30pt - "unicorn is horned":

Use either:

- simple resolution: that is apply resolution many times to see whether you can infr Horned, or

- resolution refutation (aka proof by contradiction): that is, add the negated form of whatever you want to check for entailment (e.g. ¬Horned), and see if you can reach a contradiction. This would show that the KB+negative is unsatisfiable, hence the conclusion can be inferred).

1. My ⇒ Im (take the contrapositive of it)
2. ¬Im ⇒ ¬My (¬My can be written as (¬Im ∧ Mm))
3. ¬Im ⇒ (¬Im ∧ Mm)
4. Im ∨ (¬Im ∧ Mm)
5. (Im ∨ ¬Im) ∧ (Im ∨ Mm) ((Im ∨ ¬Im) always valid)
6. (Im ∨ Mm)

We know that (Im ∨ Mm) ⇒ Hr , so unicorn is horned.

III. 10pt - “unicorn is magical”:

Use Modus Ponens method and state your conclusion.

$My \Rightarrow Im$

$\neg My \Rightarrow (\neg Im \wedge Mm)$

$(Im \vee Mm) \Rightarrow Hr$

$Hr \Rightarrow Mg$

$\therefore Mg$

My can be True or False, in both case unicorn is magical.

If My is True then, Im is True and if Im is true Hr is true and if Hr is true Mg is true.

If My is False, then Mm is true and if Mm is true then Hr is true and if Hr is true Mg is true.