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CS409 Project #1 Report

Five different method has been applied to find the root of the non-linear equation $f(x) = x^3 + 2x^2 + 10x - 20$

These methods are:

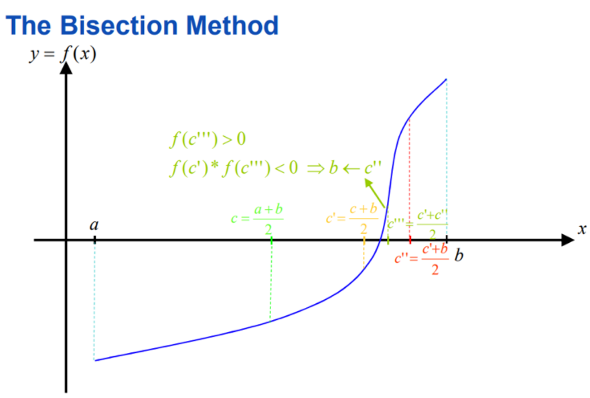
- Bisection Method
- False Position Method
- Modified False Position Method
- Secant Method
- Newton Method

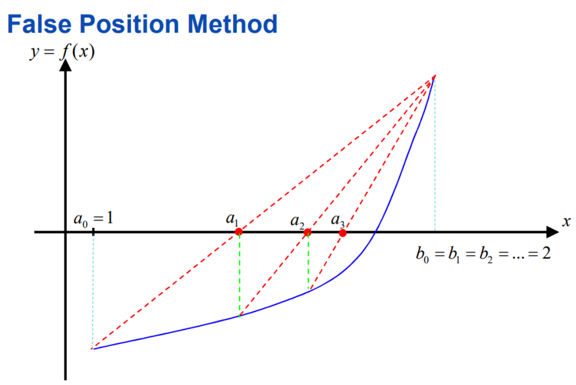
Bisection Method

For a given function *f(x)*, the Bisection Method algorithm works as follows:

1. two bounds are chosen as a and b for which *f(a) > 0* and *f(b) < 0* (vice versa)
2. a midpoint *c* is calculated as the mean of *a* and *b*, *c = (a + b) / 2*
3. the function *f* is evaluated for the value of *c*
4. if *f(c) = 0* means that we found the root of the function, which is *c*
5. if *f(c) ≠ 0* we check the sign of *f(c)*:
   - if *f(c)* has the same sign as *f(a)* we replace *a* with c, and we keep the same value for *b*
   - if *f(c)* has the same sign as *f(b)*, we replace *b* with *c,* and we keep the same value for *a*
6. we go back to step 2. and recalculate *c* with the new value of *a* or *b*

The algorithm ends when the values of *f(c)* is less than a defined tolerance (e.g. 10^{-6}). In this case we say that *c* is close enough to be the root of the function for which *f(c) ~= 0*.
In order to avoid too many iterations, we can set a maximum number of iterations (e.g. 200) and even if we are above the defined tolerance, we keep the last value of *c* as the root of our function.





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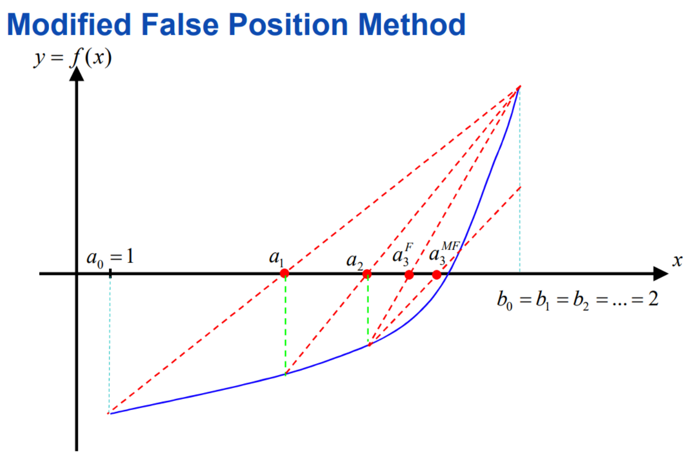
False Position Method

The algorithm is the same with the bisection method except the step 2. In bisection method, we find c value as the mean of and b but for the false position method, c value calculated as weighted average of and b. (c = (|f(b)|*a + |f(a)|*b) / (|f(a)| + |f(b)|))

Modified False Position Method

The algorithm is the same with the false position method but with one difference. The difference is that modified false position algorithm can detect when one of the bounds is stuck and if this occurs, the function at the bound can be divided into half. We add a counter to determine if one of the bounds stay fixed for two iterations and if the counter reached to 2 then divide the boundary by 2.



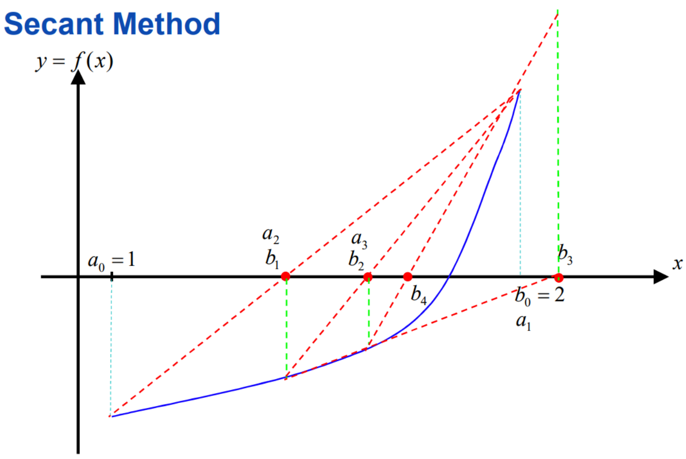
Secant Method

First three methods were closed-bracket techniques, but secant method and newton method are open-bracket techniques, so they do not necessarily enclose the solution, in other words they do not worry about the signs of bounds.

The secant method is defined by the following recurrence relation:

$$x_n = x_{n-1} - f(x_{n-1})\frac{x_{n-1} - x_{n-2}}{f(x_{n-1}) - f(x_{n-2})} = \frac{x_{n-2} f(x_{n-1}) - x_{n-1} f(x_{n-2})}{f(x_{n-1}) - f(x_{n-2})}.$$

In the false position method, the latest estimate of the root replaces the original value yielded a function value with the same sign as the estimate. Consequently, the two estimates bracket the unknown pole, therefore, the method converges for practical purposes. However, in secant method the values are replaced in strict sequence, regardless of the sign. As a result, two values can lie on the same side of the root. For certain cases, this can lead to divergence.



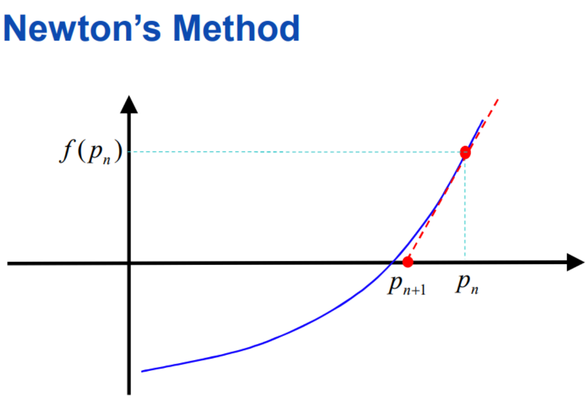
Newton Method

The idea is starting with an initial guess which is reasonably close to the true root, then to approximate the function by its tangent line using calculus, and finally to compute the x-intercept of this tangent line by elementary algebra. This x-intercept will typically be a better approximation to the original function's root than the first guess, and the method can be iterated.

The Newton method is defined by the following recurrence relation:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Newton's method is a general procedure that can be applied to many diverse situations. However, there are situations where it performs poorly. Especially for the cases with multiple roots it may have poor performance.



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All methods were implemented by using MATLAB. All codes are added to the folder. I examined the function. $f(x) = x^3 + 2x^2 + 10x - 20$

Findings:

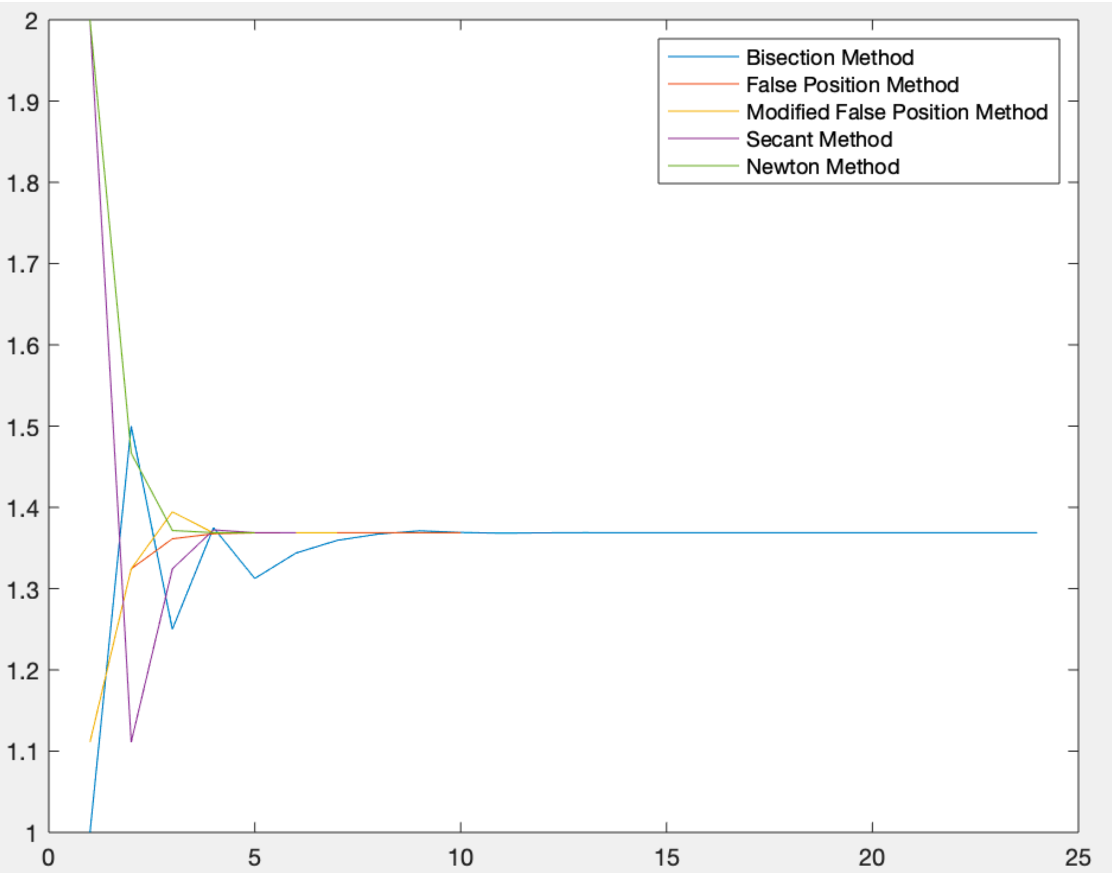
- Increasing the tolerance usually also increases the number of iterations.
- If the bounds are 0 and 2, bisection method is the worse. However, if we change the bounds to 0 and 10, false position method is the worse.
- Secant method is the best method for this function for all cases, because it had the minimum number of iterations for the all scenarios.
- The root is around 1.3688081078

All cases (a,b,c,d,e,f) were examined and their iteration vs x, iteration vs f(x) and iteration vs error graphs are prepared.

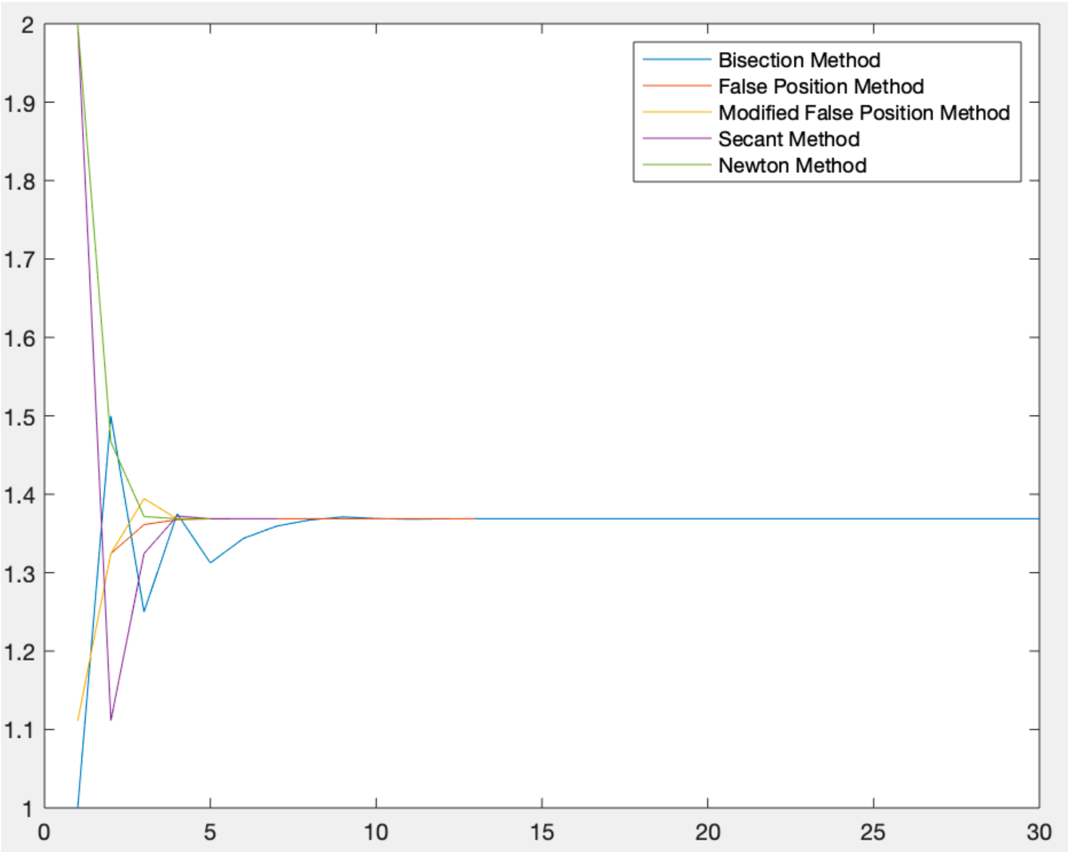
Iteration vs x

x axis stands for number of iteration and y axis stands for value of x

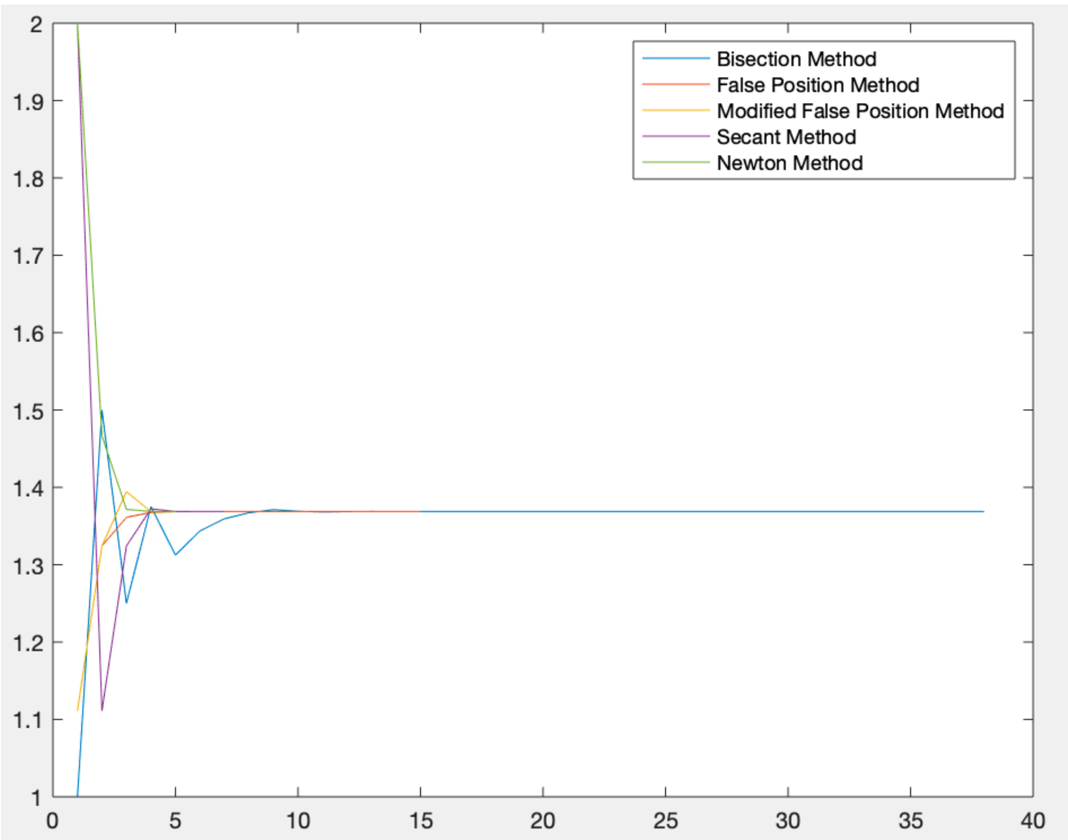
a) Lower value = 0, Upper value = 2, and use the absolute error criteria of 10^{-6}



b) Lower value = 0, Upper value = 2, and use the absolute error criteria of 10^{-8}



c) Lower value = 0, Upper value = 2, and use the absolute error criteria of 10^{-10}



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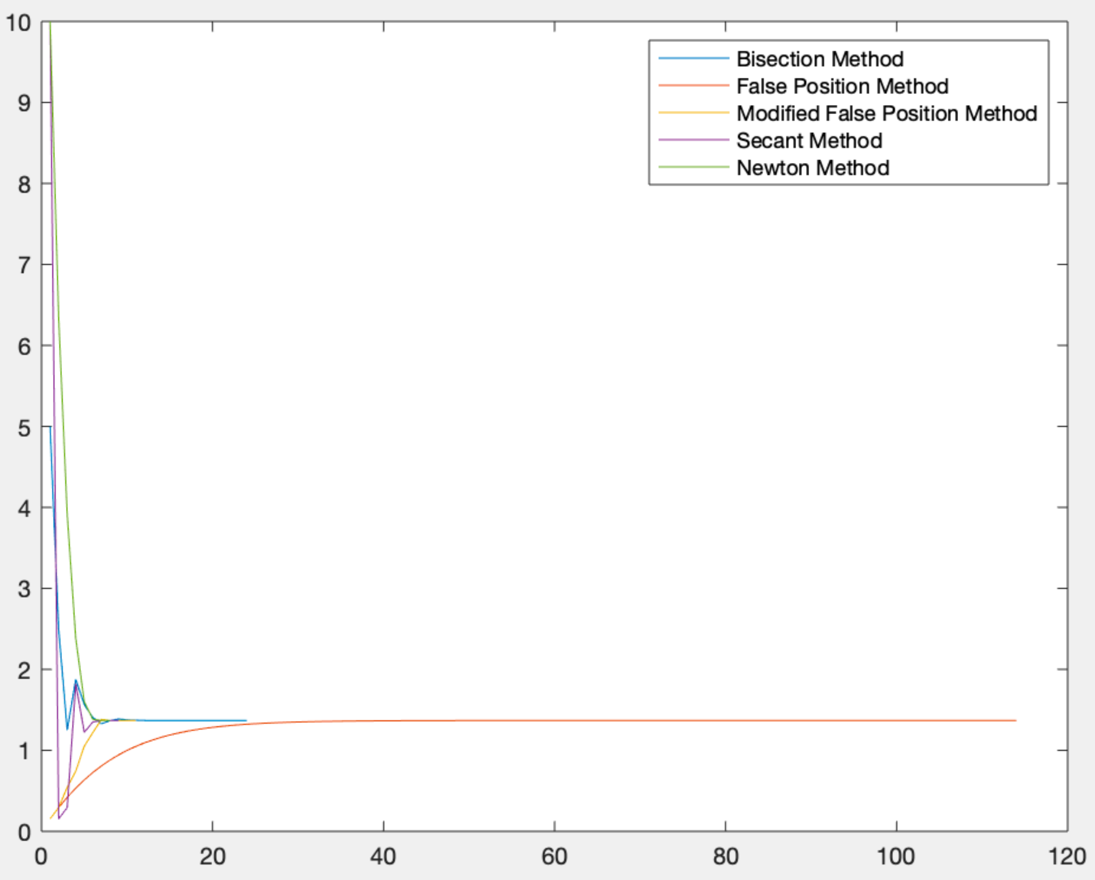
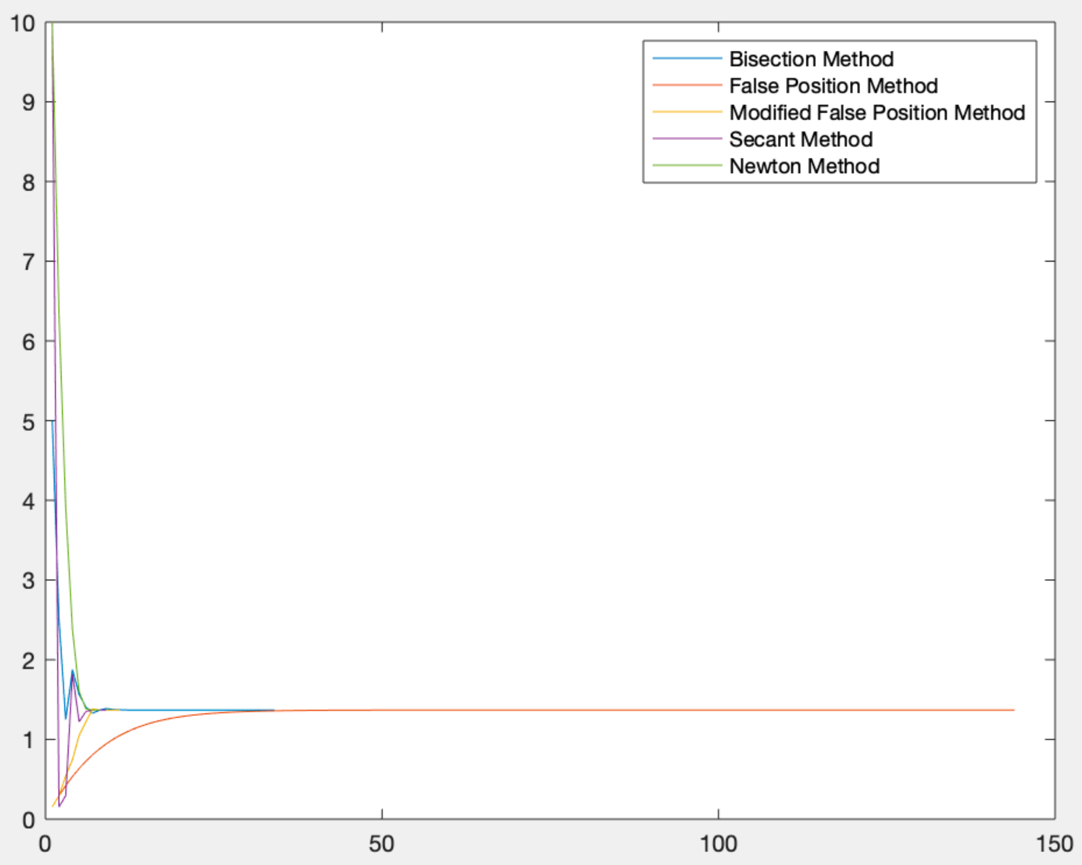
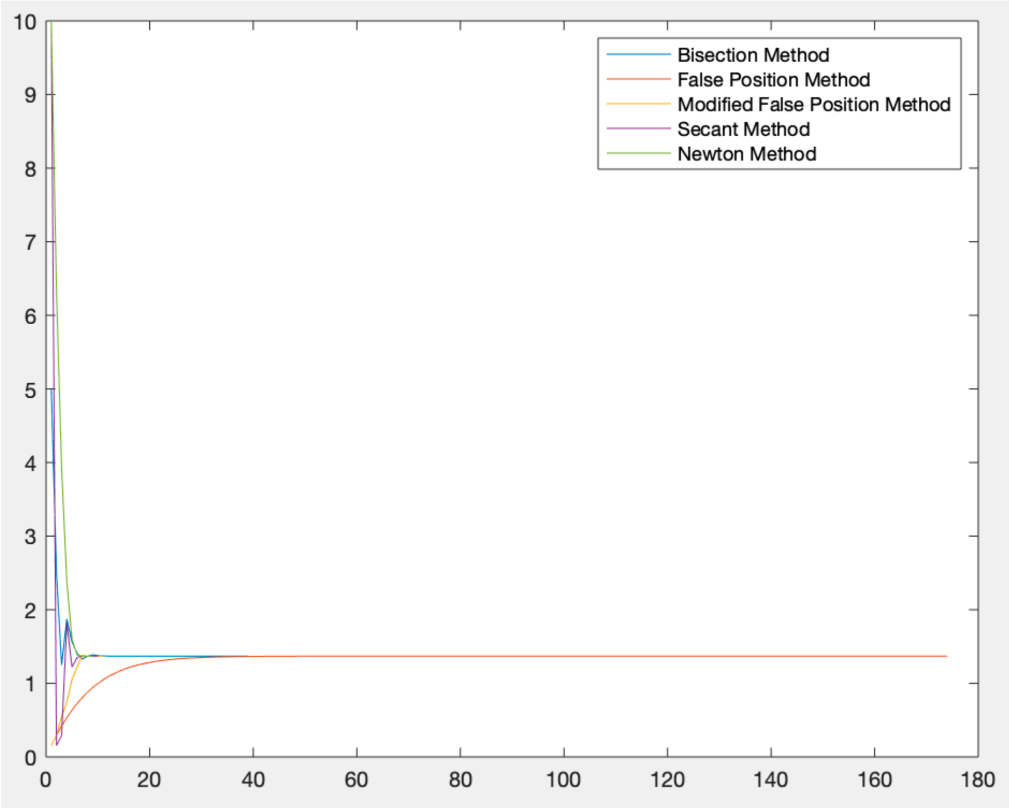
d) Lower value = 0, Upper value = 10, and use the absolute error criteria of 10^{-6}



e) Lower value = 0, Upper value = 2, and use the absolute error criteria of 10^{-8}



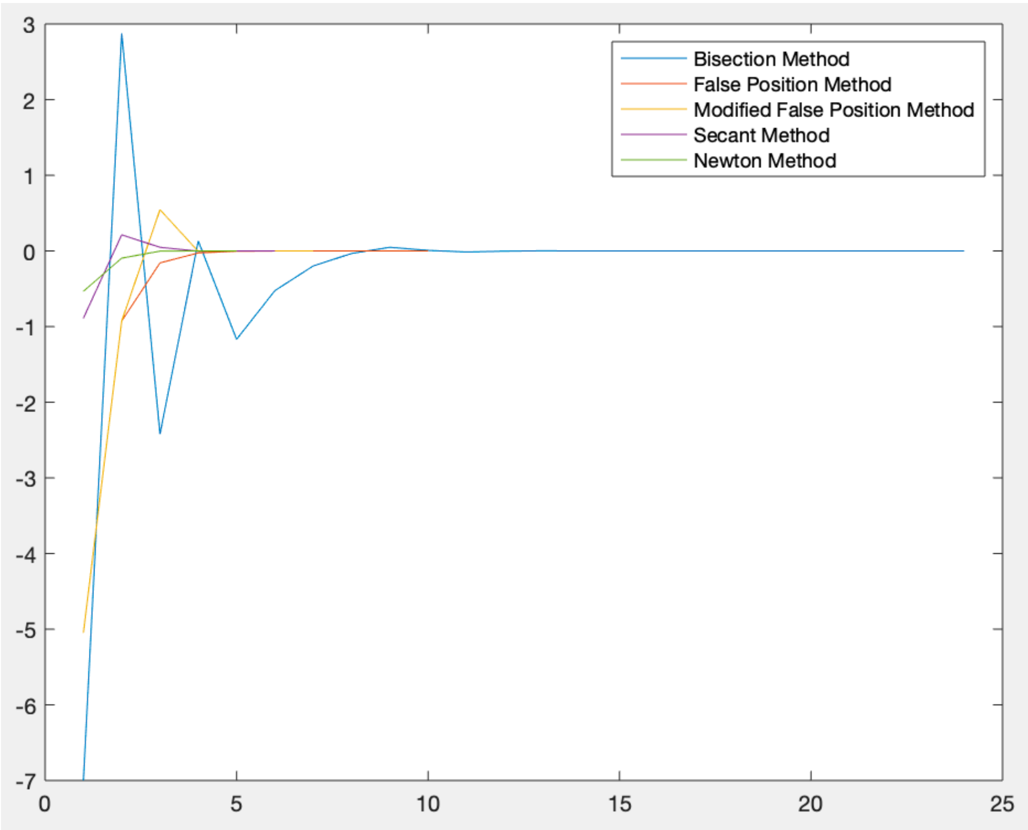
f) Lower value = 0, Upper value = 2, and use the absolute error criteria of 10^{-10}



Iteration vs f(x)

x axis stands for number of iteration and y axis stands for value of f(x).

a) Lower value = 0, Upper value = 2, and use the absolute error criteria of 10^{-6}



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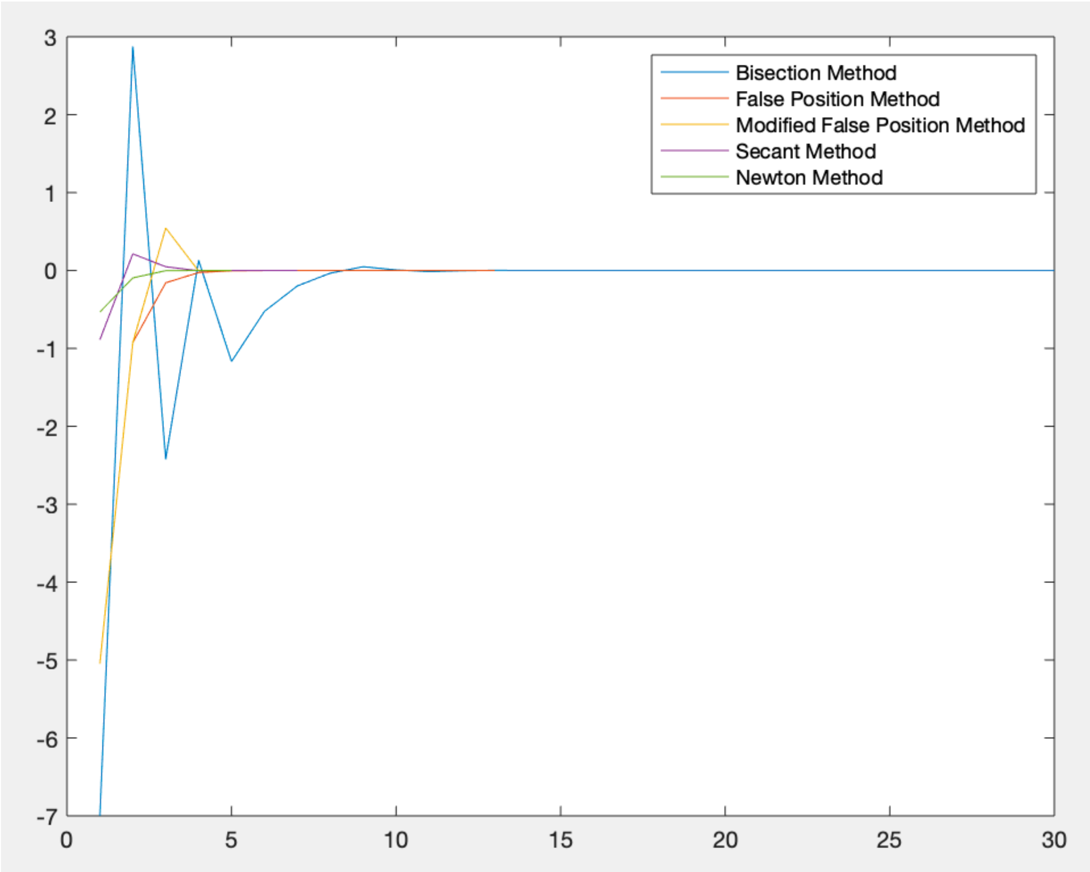

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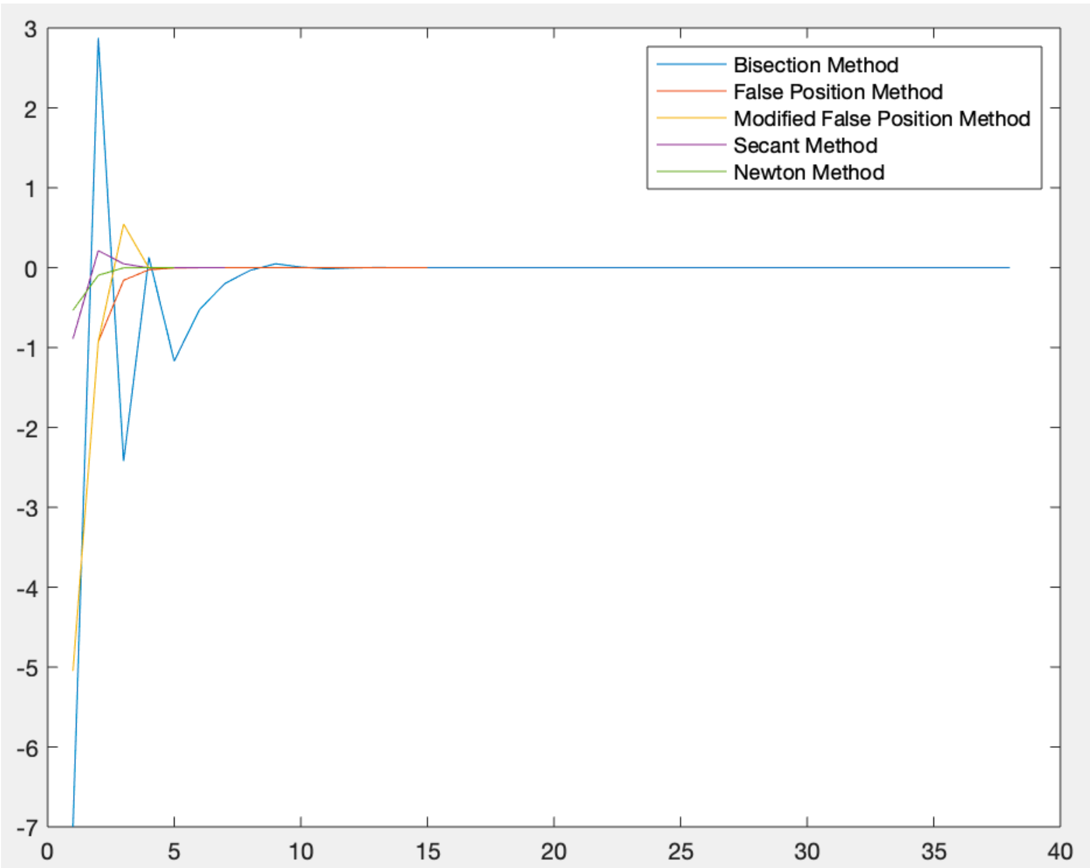

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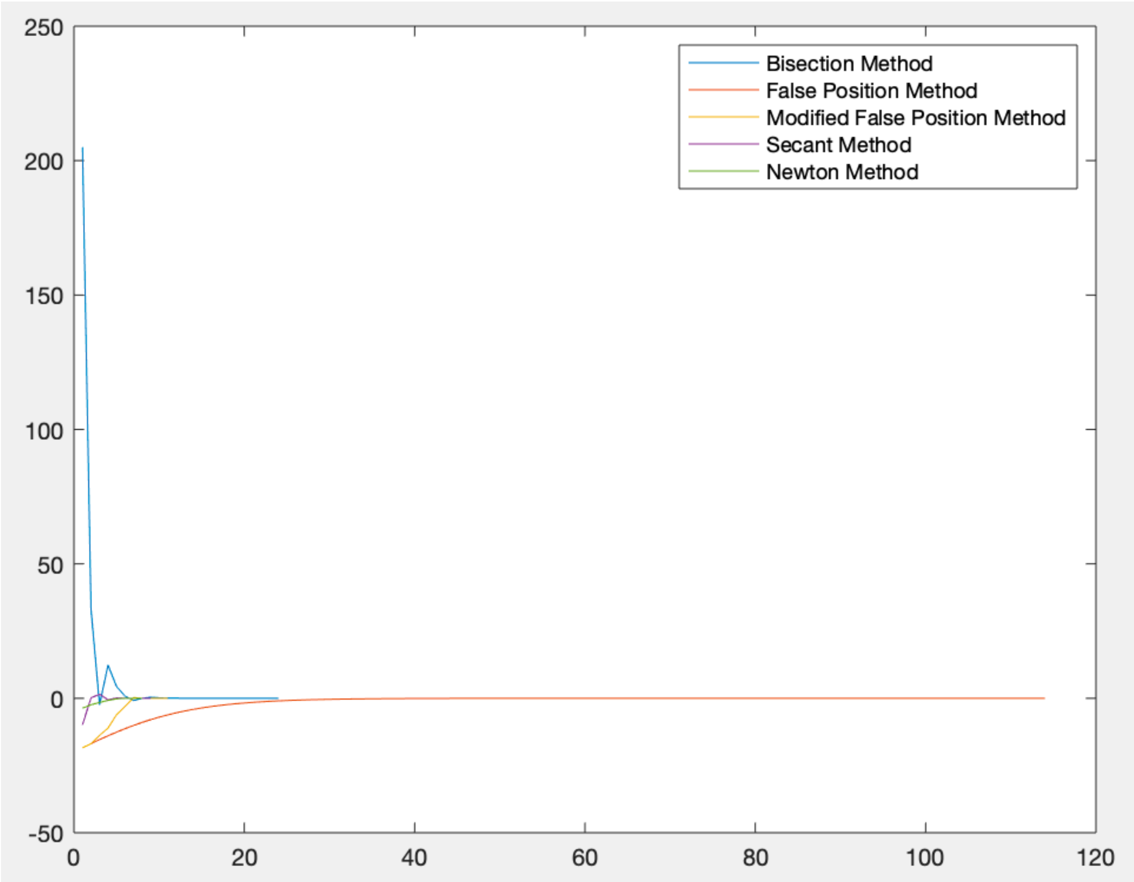

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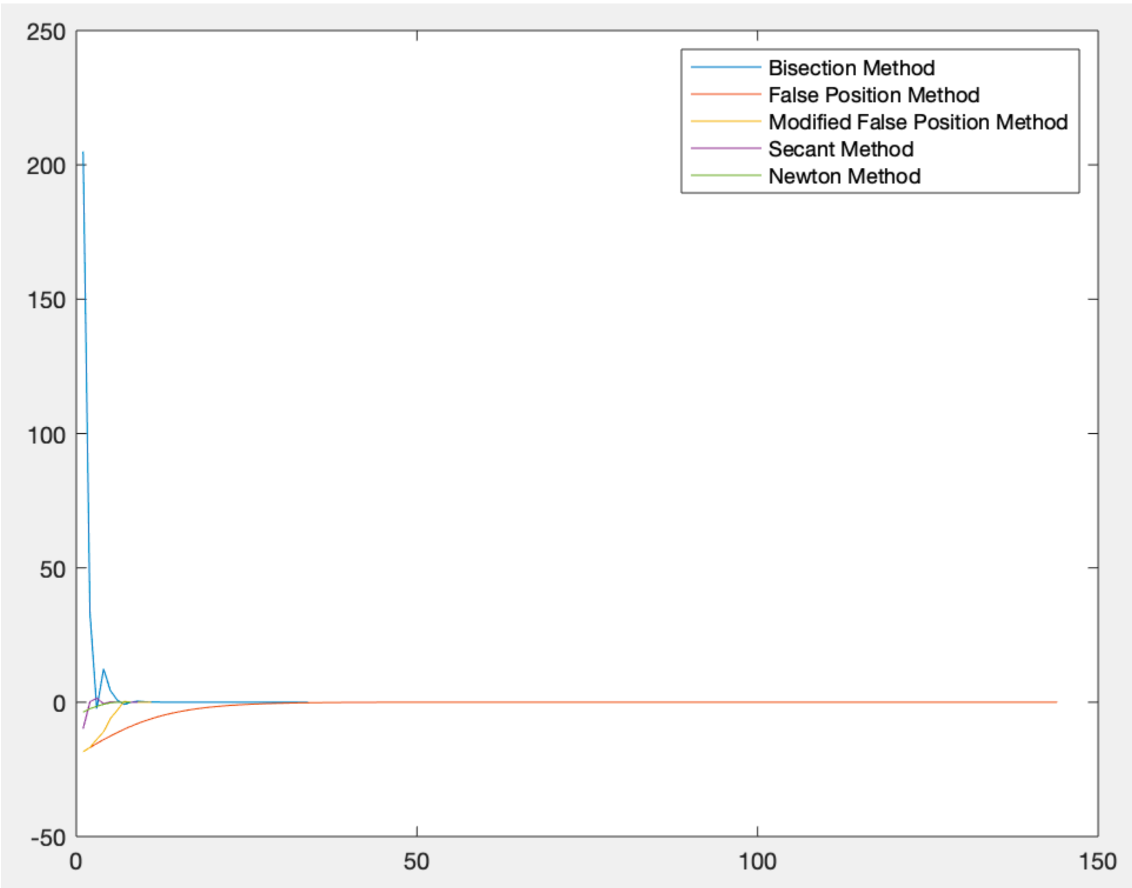

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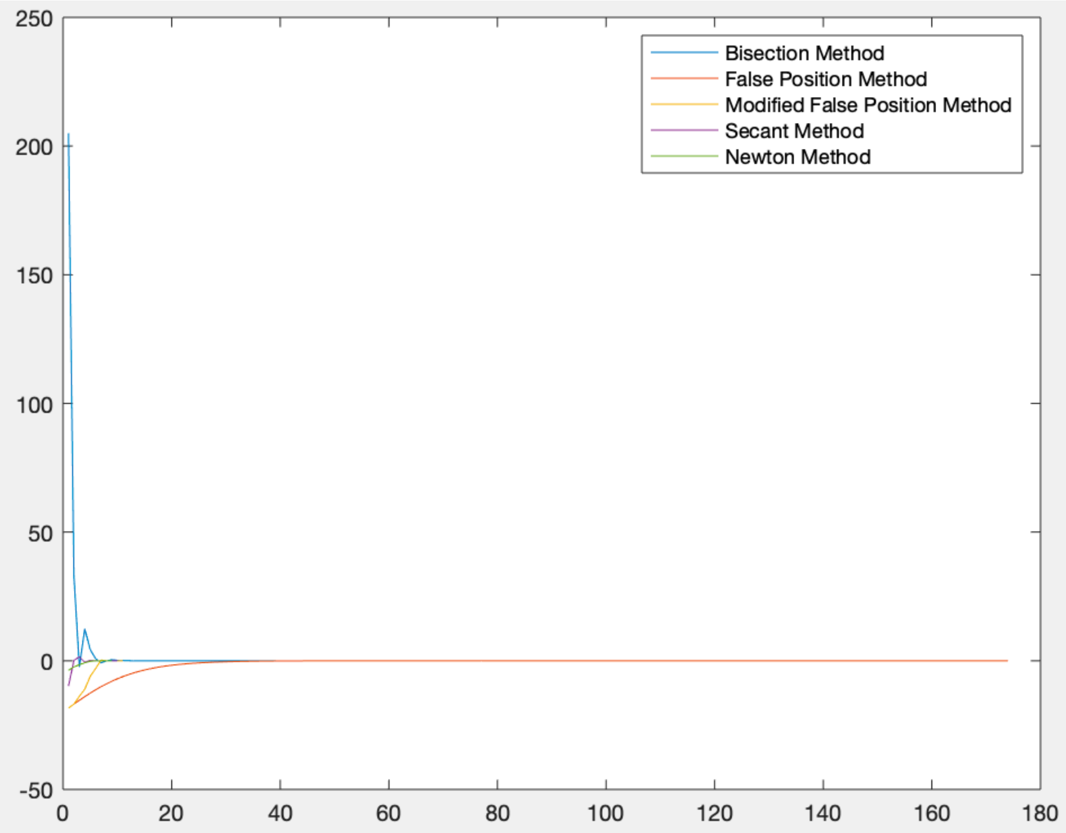
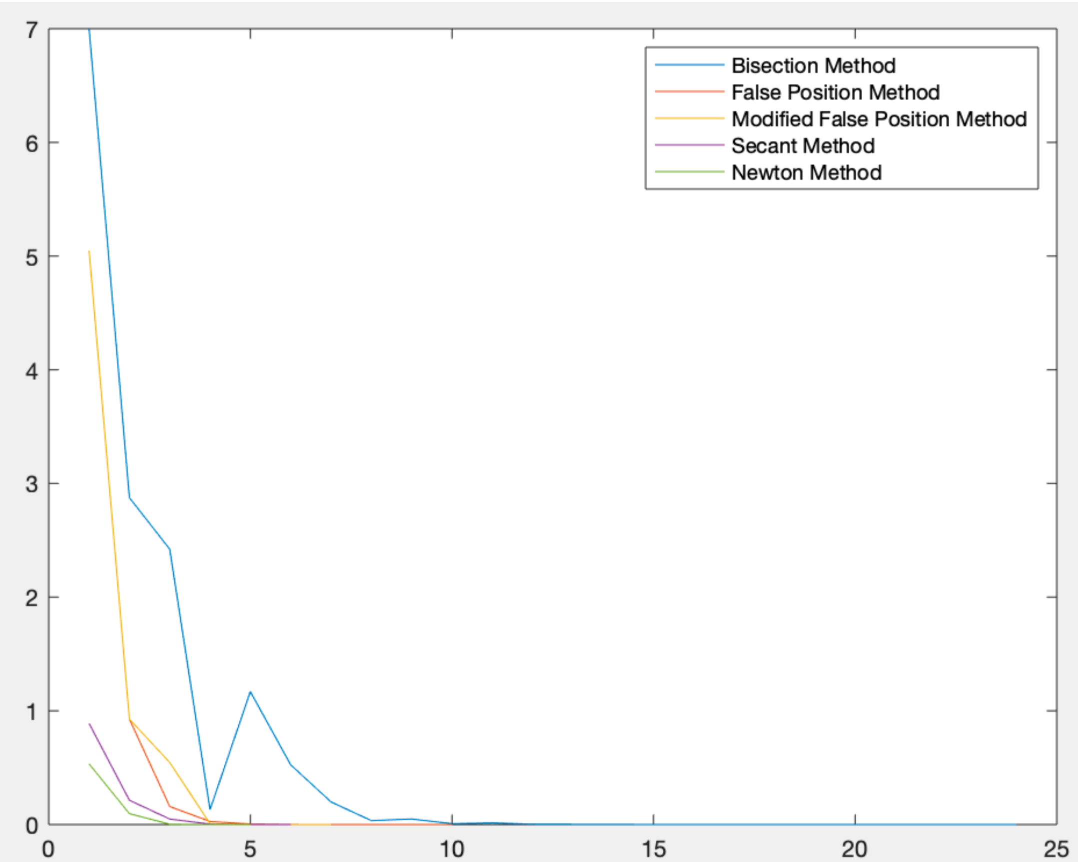


Iteration vs error

x axis stands for number of iteration and y axis stands for value of error

a) Lower value = 0, Upper value = 2, and use the absolute error criteria of 10^{-6}



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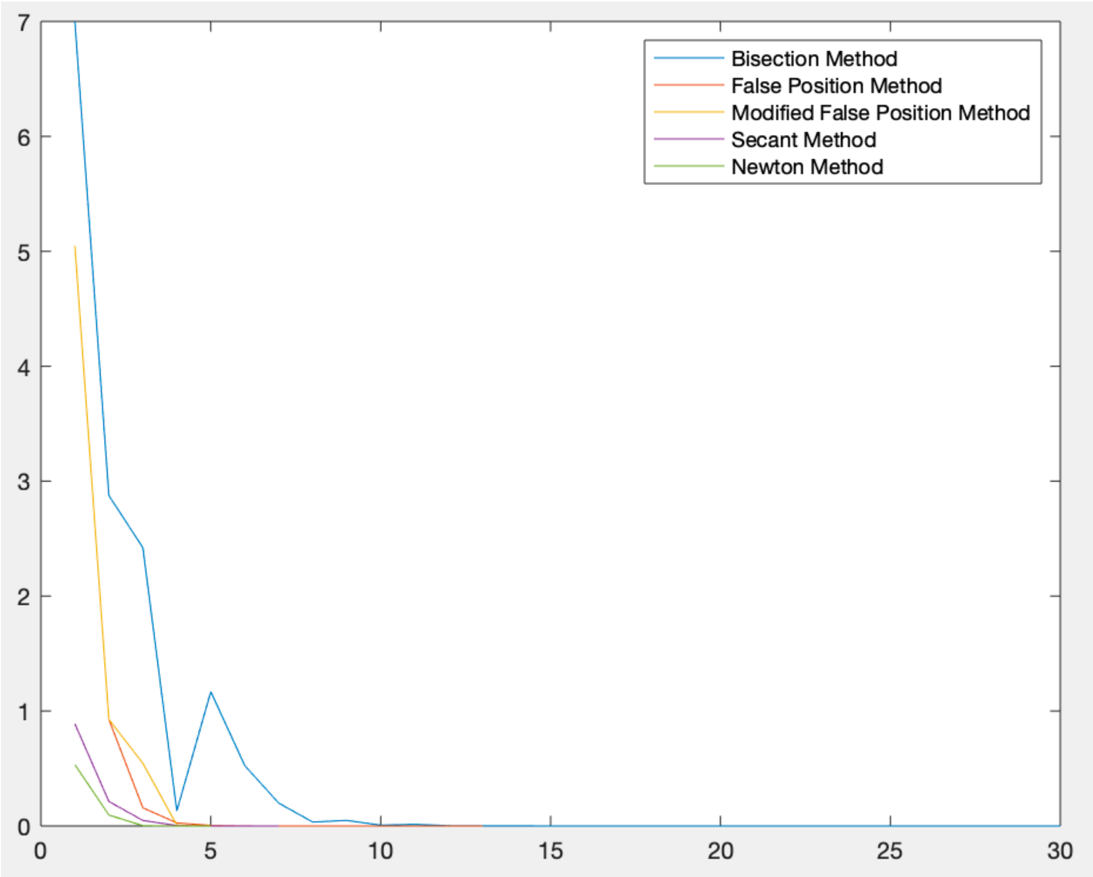
b) Lower value = 0, Upper value = 2, and use the absolute error criteria of 10^{-8}



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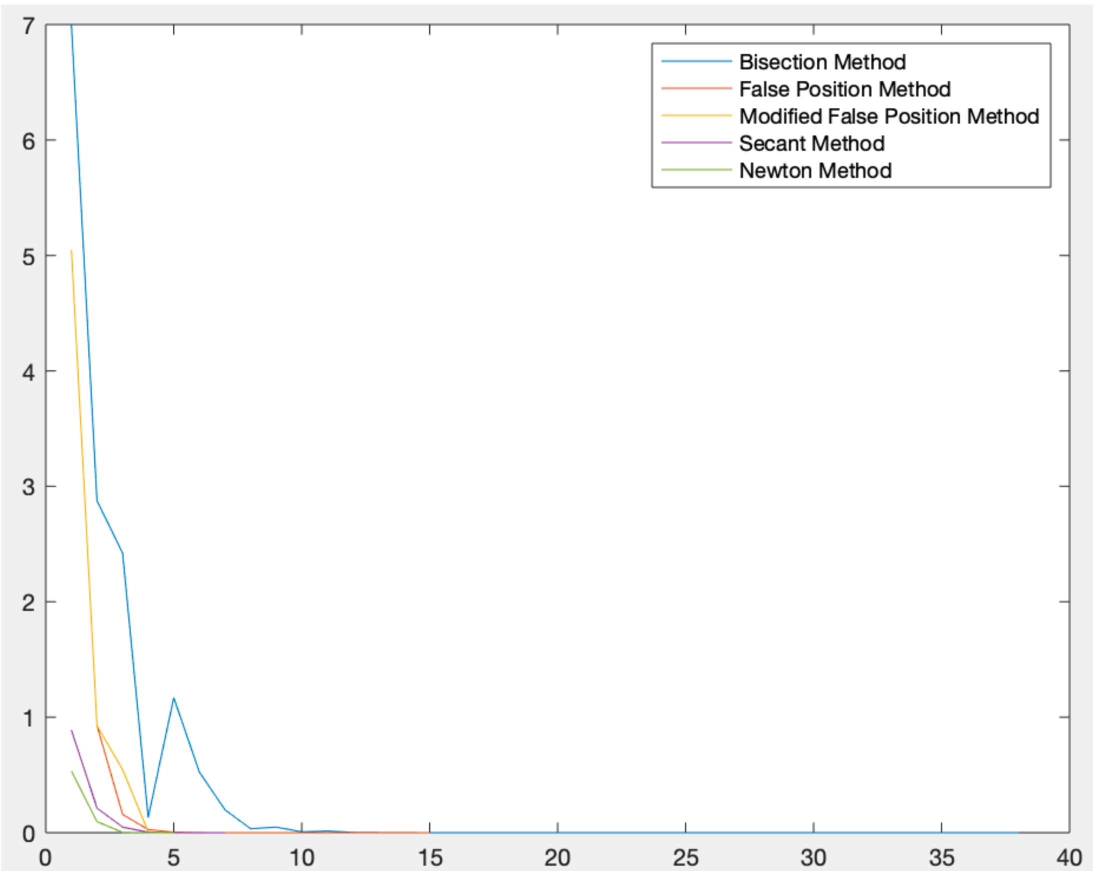

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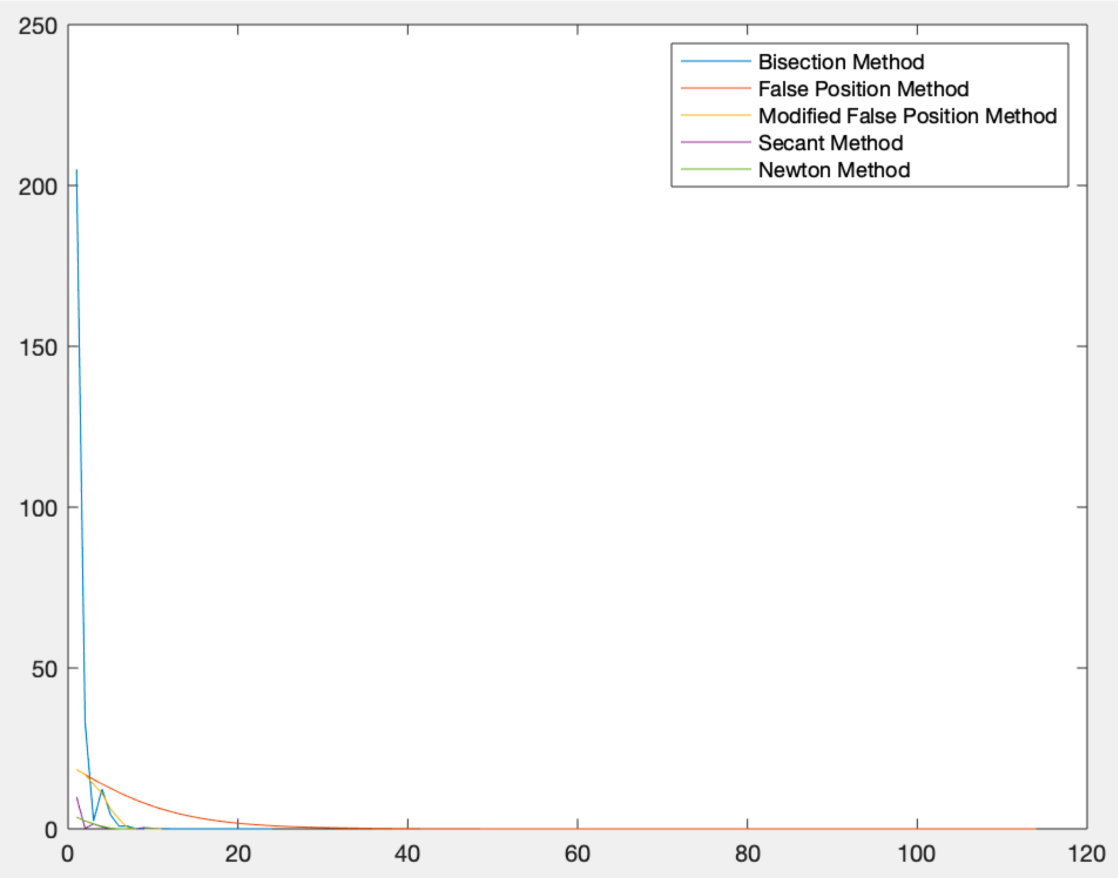

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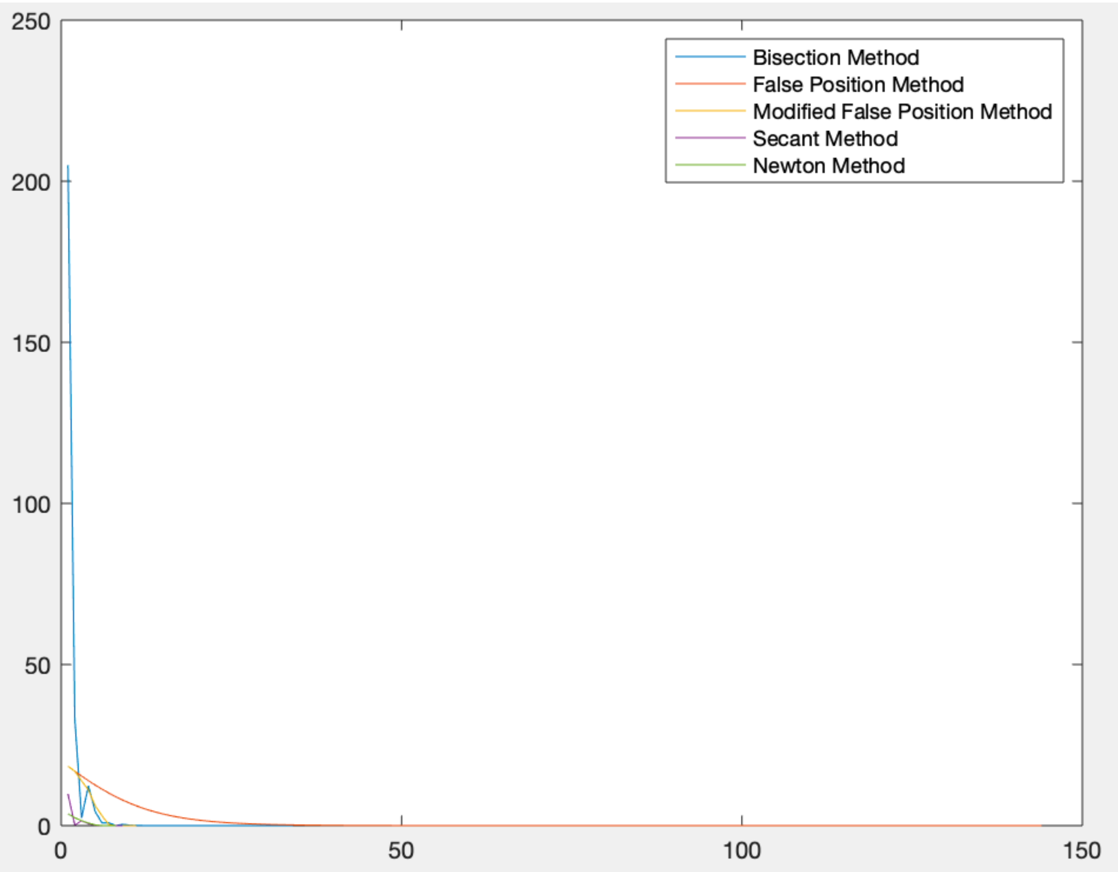

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