6-) $T(n) = T(n-2) + 2n$

$= T(n-4) + 2n - 4 + 2n$

$= T(n-8) + \underbrace{2n - 8 + 2n - 4 + 2n}_{S}$

$\vdots$

$S = \sum_{i=0}^{k-1} 2(n-2i)$, ① $2 \cdot \sum_{i=0}^{k-1} n - 2i$, ② $2 \cdot \left[kn - 2 \cdot \sum_{i=0}^{k-1} i\right]$,

③ $= 2 \cdot \left[kn - 2 \cdot \left(\frac{k \cdot (k-1)}{2}\right)\right]$, ④ $= 2 \cdot \left[kn - (k^2 - k)\right]$,

Genel

$T(n) = T(n-2k) + 2 \cdot \left[kn - (k^2 - k)\right]$

$n - 2k = 0$,

$2k = n$,

$k = \frac{n}{2}$

$T(n) = T(0) + 2\left[\frac{n}{2} \cdot n - \left(\left(\frac{n}{2}\right)^2 - \frac{n}{2}\right)\right]$

$\boxed{T(n) = T(0) + \frac{n^2}{2} + n}$

$T(0) = 0$ ise

$\boxed{T(n) = \frac{n^2}{2} + n}$ //

5-) $\sum_{i=1}^{n} (i+1) \cdot 2^{i-1}$

$= \frac{1}{2} \sum_{i=1}^{n} (i+1) \cdot 2^{i}$, $= \frac{1}{2} \sum_{i=1}^{n} i \cdot 2^{i} + 2^{i}$

$= \frac{1}{2} \left[\sum_{i=1}^{n} i \cdot 2^{i} + \sum_{i=1}^{n} 2^{i} \right]$

$\sum_{i=1}^{n} i 2^{i} = (n-1) \cdot 2^{n+1} + 2$

$\sum_{i=1}^{n} 2^{i} = \frac{2 \cdot (2^{n}-2)}{2-1} = 2(2^{n}-1) = 2^{n+1} - 2$

$= \frac{1}{2} \left(n-1 \cdot 2^{n+1} + 2 + 2^{n+1} - 2 \right)$

$= \frac{1}{2} \left(2^{n+1} \cdot n \right)$

$f(n) = 2^{n} \cdot n$

$f(n) = 2^{n} \cdot n \in O(n \cdot 2^{n})$

4-2) $f(n) = 2n \lg (n+2)^2 + (n+2)^2 \cdot \log \frac{n}{2}$

$f(n) = \Theta(n^2 \log n)$ ispatla

$c_1 g(n) \leq f(n) \leq c_2 g(n)$ bütün $n \geq n_0$ için

fonksiyon basitleştir:

$$f(n) = 2n \cdot 2 \log(n+2) + (n+2)^2 \cdot (\lg n - \log 2)$$
$$= 4n \cdot \log(n+2) + (n+2)^2 \log n - (n+2)^2 \log 2$$

$(n+2) \approx n$ büyük değerler için

$\log n+2 \approx \log n$

$f(n) \approx 4n \log n + n^2 \log n - n^2 \log 2$

c_1:

$n \geq 1$

$$f(n) \geq 4n \log n + n^2 \log n - n^2 \log 2$$
$$\geq n^2 \log n \cdot (1 + \underbrace{\tfrac{4}{n}}_{\text{ihmal}}) - n^2 \log 2$$

$f(n) \geq n^2 \log n - n^2 \log 2$

$f(n) \geq c_1 \cdot n^2 \log n$ için $c_1 = 1 - \frac{n^2 \log 2}{}$

$c_1 = 1$

c_2:

$n+2 \leq 2n$

$\log n+1 \leq \log 2n$

$$f(n) \leq 4n(\log n + \log 2) + (2n)^2 \log n - n^2 \log 2$$
$$f(n) \leq 4n \log n + 4n \log 2 + 4n^2 \log n - n^2 \log 2$$

($4n^2 \log n$ en hızlı büyüyen)

$f(n) \leq c_2 n^2 \log n$ için $c_2 = 4$

$$n^2 \log n \leq 2n \log(n+2)^2 + (n+2)^2 \log \frac{n}{2} < 4 \cdot n^2 \log n$$

4- $f(n) = 2^{n+1} + 3^{n-1}$

$f(n) = \Theta(3^n)$ ispatla:

$c_1 g(n) \leq f(n) \leq c_2 g(n)$ bütün $n > n_0$ için

fonksiyon basitleştir

$f(n) \overset{1}{=} 2 \cdot 2^n + \frac{3^n}{3}$

$2^n = \left(\frac{2}{3}\right)^n \cdot 3^n$

$\overset{2}{=} 2 \cdot \left(\frac{2}{3}\right)^n \cdot 3^n + \frac{3^n}{3} \overset{3}{=} \left[2 \cdot \left(\frac{2}{3}\right)^n + \frac{1}{3}\right] \cdot 3^n$

c_1: $\left(\frac{2}{3}\right)^n > 0$, $f(n) > \frac{1}{3} 3^n$

böylece

$f(n) \geq c_1 \cdot 3^n$ için $c_1 = \frac{1}{3}$

c_2: $\left(\frac{2}{3}\right)^n < 1$, $\left(\frac{2}{3}\right)^n \cdot 2 \leq 2$

$f(n) < \left(2 + \frac{1}{3}\right) \cdot 3^n = \frac{7}{3} \cdot 3^n$

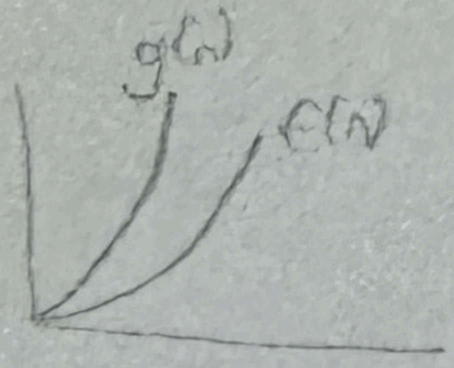
$f(n) \leq c_2 \cdot 3^n$ için $c_2 = \frac{7}{3}$

$g(n) = 3^n$, $c_1 = \frac{1}{3}$, $c_2 = \frac{7}{3}$ ve $n_0 = 1$ için

$$\frac{1}{3} 3^n \leq 2^{n+1} + 3^{n-1} \leq \frac{7}{3} 3^n$$

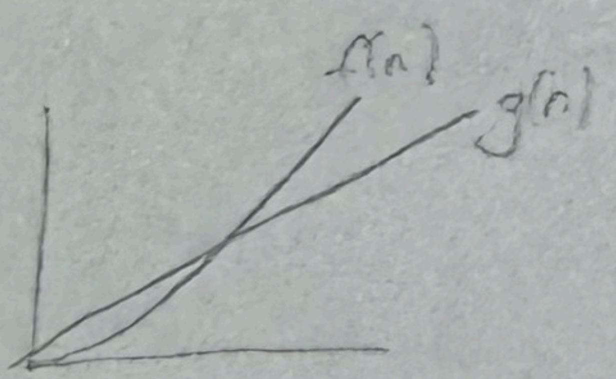
$$f(n) = \Theta(3^n)$$

3-) 1-)



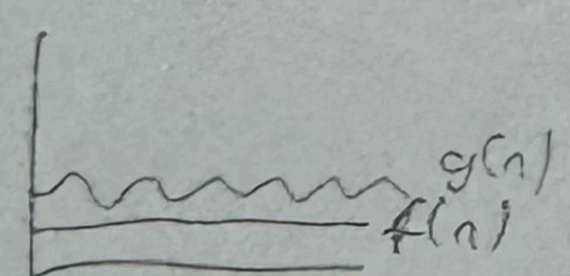
= $f(n) < g(n)$, $f(n) = O(g(n))$

2-)



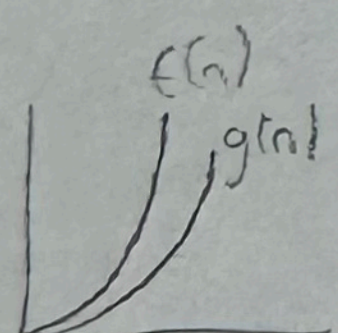
= $f(n) > g(n)$, $f(n) = \Omega(g(n))$

3-)



= çakışma yok X

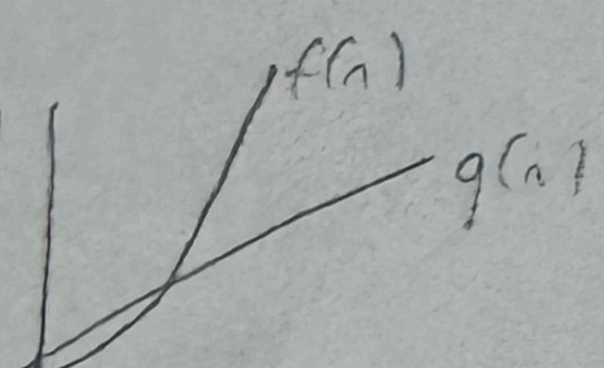
4-)



= $f(n) > g(n)$; $f(n) = \Omega(g(n))$

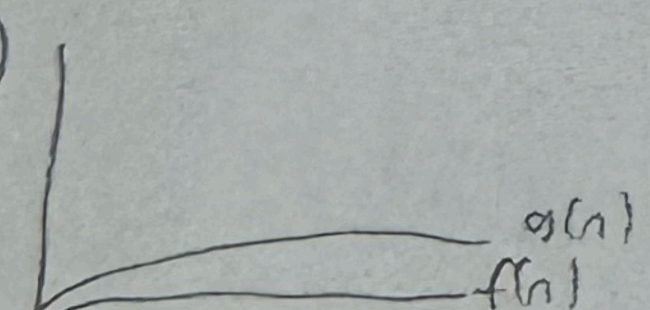
5- 4^n tabanına çevirirsek, $f(n) = \Theta(g(n))$ olur

6-)



= $f(n) > g(n)$, $f(n) = \Omega(g(n))$

7-)



$f(n) < g(n)$, $f(n) = O(g(n))$

8-) benzer hızda büyürler, $f(n) = \Theta(g(n))$

	f(n)	g(n)
O	n^2	n^3
Ω	$n \lg n$	n
X	1	$3 + \sin n$
Ω	3^n	2^n
Θ	4^{n+4}	2^{2n+2}
Ω	$n \lg n$	$n^{105/100}$
O	$\lg \sqrt{10n}$	$\lg n^3$
Θ	nb	$(n+1)!$

2)

f1-) Döngü N kez çalışıyor ve her adımda sabit işlem var.

f1 = O(N)

f2-) Dış döngü N kere, iç döngü i kere işlem ise O(j) olur.

$$\text{işlem} = \sum_{i=0}^{N-1} \sum_{j=0}^{i-1} O(j)$$

① iç toplam $= \sum_{j=0}^{i-1} j = \frac{(i-1).i}{2}$, ② yerine koy: $\frac{1}{2} \sum_{i=0}^{N-1} i^2 - i$

③ parçala $= \frac{1}{2}\left(\sum_{i=0}^{N-1} i^2 - \sum_{i=0}^{N-1} i\right)$, ④ $\sum_{i=0}^{N-1} i = \frac{(N-1).N}{2}$, $\sum_{i=0}^{N-1} i^2 = \frac{(N-1).N.(2N-1)}{6}$

⑤ $\frac{1}{2}\left(\frac{(N-1).N.(2N-1)}{6} - \frac{(N-1).N}{2}\right)$ buradan en yüksek dereceli terim N^3 olur.

$$f(2) = O(N^3)$$

f3-) çıkış durumu sabit işlem o yüzden O(1),

recursive durum $\underbrace{N. f3(n-1)}_{\text{N döngüde çağırıyor}} + O(1)$

$$T(N) \approx N * T(N-1)$$
$$\approx N * (N-1) * T(N-2)$$
$$\approx N * (N-1) * (N-2) * T(N-3)$$
$$\vdots$$
$$\approx N! * \underbrace{T(0)}_{O(1)} , \quad T(N) = N!$$

f4-) Çıkış durum sabit işlem, O(1),

recursive durum $T(N) = 2 * T(\frac{N}{2}) + O(3n)$ önemsiz

$$T(N) = 2. T(N/2) + O(N)$$

master theorem:

$a=2$, $b=2$, $d=1$

$1 = \log_2 2$, $T(N) = O(N \log n)$

1-) Master Theorem

$$T(n) = aT(n/b) + O(n^d)$$

$$T(n) = \begin{cases} O(n^d) & , d > \log_b a \\ O(n^d \log n) & , d = \log_b a \\ O(n^{\log_b a}) & , d < \log_b a \end{cases}$$

a) $a = 9$, $b = 4$, $d = 2$

$2 > \log_4 9$, $T(n) = O(n^2)$

b) $a = 3$, $b = 3$, $d = 1$

$1 = \log_3 3$, $T(n) = O(n \lg n)$

c) $a = 3$, $b = 2$, $d = 1$

$1 < \log_2 3$, $T(n) = O(n^{\log_2 3})$