

عزم دریں سینوں و مسینوں سے سرو ششم

خاکش سری فریب سینوں و مسینوں کی تعداد کسٹم در زمان

جواب سوال ۱)

$$a) \sin\left(\frac{4\pi n}{4}\right) \cos\left(\frac{n\pi}{4}\right)$$

$$= \frac{1}{4} (\sin\left(\frac{4\pi n}{4}\right) + \sin\left(\frac{\pi n}{4}\right))$$

$$N=12 \quad \omega_r = \frac{2\pi}{12} = \frac{\pi}{6}$$

$$x[n] = \frac{1}{4j} (e^{j\frac{4\pi}{6}n} - e^{-j\frac{4\pi}{6}n} + e^{j\frac{\pi}{6}n} - e^{-j\frac{\pi}{6}n})$$

$$\Rightarrow a_1 = \frac{1}{4j}, \quad a_{\omega} = -\frac{1}{4j}, \quad a_v = \frac{1}{4j}, \quad a_{11} = -\frac{1}{4j}$$

لئے خارج درج کردہ ضرب صفر ہستے۔ (11 صفحہ ۱۱)

$$b) \quad N=4, \quad \omega_r = \frac{2\pi}{4} = \frac{\pi}{2}$$

(روز صفر گور در نظر کر دئے)

$$a_k = \frac{1}{4} \sum_{k=0}^{\omega} x[n] e^{-jk\frac{\pi}{4}n} = \frac{1}{4} (0 + (-1)e^{-jk\frac{\pi}{4}} + 2e^{-jk\frac{2\pi}{4}} + e^{-jk\frac{3\pi}{4}}$$

$$+ 2e^{-jk\frac{4\pi}{4}} + (-1)^4)$$

$$\Rightarrow a_k = \frac{1}{4} [(-1)^k + 2 \cos(k\frac{\pi}{4}) + (-1) \cos(k\frac{\pi}{4})]$$

$$a_0 = a/\omega \quad a_1 = -\frac{1}{4} \quad a_v = 0 \quad a_F = \frac{\omega}{4} \quad a_F = 0 \quad a_{\omega} = -\frac{1}{4}$$

جواب سوال ۳)

$$x[n] = \cos\left(\frac{\pi}{4}n\right) + 2 \cos\left(\frac{\pi}{4}n\right)$$

$$= \frac{1}{2} (e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n} + 2e^{j\frac{2\pi}{4}n} + 2e^{-j\frac{2\pi}{4}n}) \quad N=4 \quad \omega_r = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a_1 = \frac{1}{2}, \quad a_v = \frac{1}{2}, \quad a_F = \frac{1}{2}, \quad a_{\omega} = \frac{1}{2}$$

$$y[n] - \frac{1}{F} y[n-1] = x[n]$$

$$\Rightarrow b_k = \frac{1}{F} e^{-jk\frac{\pi}{4}} b_k = a_k \Rightarrow b_k = \frac{a_k}{1 - \frac{1}{F} e^{-jk\frac{\pi}{4}}}$$

$$b_0 = b_\varphi = b_F = b_{\omega_0} = 0$$

$$b_1 = \frac{j/\omega}{1 - \alpha/\omega e^{-j\frac{\pi}{F}}} \quad b_\varphi = \frac{j/\omega}{1 - \alpha/\omega (-1)}$$

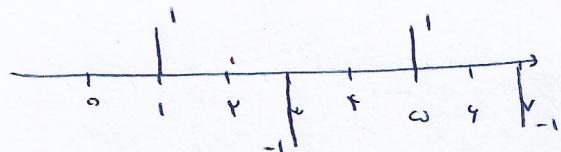
$$b_q = \frac{j/\omega}{1 - \alpha/\omega (-1)} \quad b_F = \frac{j/\omega}{1 - \alpha/\omega e^{j\frac{\pi}{F}}}$$

(حذف شد)

$$N = \Lambda$$

$$a_k = -a_{k-\tau} \Rightarrow x[n] = -(-1)^n x[n] \Rightarrow n = \tau q \rightarrow x[2q] = 0$$

$$x[\tau n + 1] = (-1)^n$$



تبديل دوري سينك موجة درجة

a)  $x[n] = u[n+1] - u[n-\omega]$

جواب سؤال 1

$$= \sum_{n=-\infty}^{+\infty} (1) e^{-jn\omega} - \sum_{n=\omega}^{+\infty} (1) e^{-jn\omega} = \frac{(e^{-j\omega})^{-1}}{1 - e^{j\omega}} - \frac{(e^{-j\omega})^{\omega}}{1 - e^{-j\omega}}$$

$$\rightarrow X(e^{j\omega}) = \frac{e^{j\omega} - e^{-\omega j\omega}}{1 - e^{-j\omega}}$$

b)  $x[n] = \sin(\frac{\pi}{q}n) + \cos n$

$$x[n] = \sin(\omega_n n) \xrightarrow{F.T} X(e^{j\omega}) = \frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$$

$$x[n] = \cos(\omega_n n) \xrightarrow{F.T} X(e^{j\omega}) = \pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

$$\rightarrow X(e^{j\omega}) = \frac{\pi}{j} [\delta(\omega - \frac{\pi}{q}) - \delta(\omega + \frac{\pi}{q})] + \pi [\delta(\omega - 1) + \delta(\omega + 1)]$$

c)  $x[n] = (n-1) (\frac{1}{\rho})^{|n|} = n (\frac{1}{\rho})^{|n|} - (\frac{1}{\rho})^{|n|}$

$$n s[n] \xrightarrow{F.T} j \frac{d}{d\omega} S(e^{j\omega})$$

$$S(e^{j\omega}) = \sum_{n=0}^{+\infty} (\frac{1}{\rho})^n e^{-jn\omega} = \sum_{n=0}^{+\infty} (\frac{1}{\rho} e^{-j\omega})^n = \frac{1}{1 - \frac{1}{\rho} e^{-j\omega}}$$

$$X(e^{j\omega}) = j \frac{d}{d\omega} \left( \frac{1}{1 - \frac{1}{\rho} e^{-j\omega}} \right) = \frac{1}{1 - \frac{1}{\rho} e^{-j\omega}} =$$

$$- \frac{j}{\rho} e^{-j\omega} \left( 1 - \frac{1}{\rho} e^{-j\omega} \right)^{-1} - \left( 1 - \frac{1}{\rho} e^{-j\omega} \right)^{-1} =$$

$$\left( 1 - \frac{1}{\rho} e^{-j\omega} \right)^{-1} \left[ - \frac{j}{\rho} e^{-j\omega} \left( 1 - \frac{1}{\rho} e^{-j\omega} \right)^{-1} - 1 \right]$$

a)  $X(e^{j\omega}) = \cos^2 \omega + \sin^2 \omega$

جواب سؤال 2

$$= \left( \frac{1}{2} e^{j\omega} + \frac{1}{2} e^{-j\omega} \right)^2 + \left( \frac{1}{2j} e^{qj\omega} - \frac{1}{2j} e^{-qj\omega} \right)^2$$

$$= \frac{1}{4} + \frac{1}{4} e^{j\omega q} + \frac{1}{4} e^{-j\omega q} - \frac{1}{4} e^{qj\omega} + \frac{1}{4} e^{-qj\omega} + \frac{1}{4}$$

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$$\rightarrow x[n] = \delta[n] + \frac{1}{\pi} [\delta[n+\gamma] + \delta[n-\gamma] - \delta[n+q] + \delta[n-q])$$

b)  $X(e^{j\omega}) = e^{-j\frac{\omega}{\gamma}} \quad -\pi \leq \omega \leq \pi$

$$\begin{aligned} \rightarrow x[n] &= \int_{-\pi}^{\pi} e^{-j\frac{\omega}{\gamma}} e^{jn\omega} d\omega \rightarrow \\ x[n] &= \frac{1}{j\pi} \int_{-\pi}^{\pi} e^{-j\frac{\omega}{\gamma} + jn\omega} d\omega = \frac{1}{j\pi} \int_{-\pi}^{\pi} e^{jn\omega(-\frac{1}{\gamma} + n)} d\omega = \\ (\frac{1}{j\pi}) \frac{1}{j(-\frac{1}{\gamma} + n)} e^{jn\omega(-\frac{1}{\gamma} + n)} &\Big|_{-\pi}^{\pi} \rightarrow x[n] = \frac{1}{j(-\frac{1}{\gamma} + n)} [(-1)^{-\frac{1}{\gamma} + n} - (-1)^{\frac{1}{\gamma} + n}] (\frac{1}{j\pi}) \\ &= \frac{1}{j(-\frac{1}{\gamma} + n)} [(-1)^{-\frac{1}{\gamma} + n} - (-1)^{\frac{1}{\gamma} + n}] (\frac{1}{j\pi}) \end{aligned}$$

c)  $X(e^{j\omega}) = \frac{e^{-j\omega} - \alpha/2}{1 - \alpha/2 e^{-j\omega}} = \frac{e^{-j\omega}}{1 - e^{-j\omega} \times \alpha/2} - \frac{\alpha/2}{1 - \alpha/2 e^{-j\omega}}$

$$\rightarrow x[n] = \alpha/2 u[n-1] - \alpha \frac{1}{2} \cos(\alpha/2)$$

جواب سؤال ٣

a)  $x[1-n] + x[-1-n]$

ابدأ فرض:  $x[n] \xrightarrow{\text{F.T.}} X_1(e^{j\omega})$

$$x[-n] \xrightarrow{\text{F.T.}} X(e^{-j\omega})$$

$$\begin{aligned} x[-n-1] &\xrightarrow{\text{F.T.}} e^{jn\omega} X(e^{-j\omega}), \quad x[-n+1] \xrightarrow{\text{F.T.}} e^{-jn\omega} X(e^{j\omega}) \\ \Rightarrow x[n] &= x[-n+1] + x[-n-1] \xrightarrow{\text{F.T.}} e^{-jn\omega} X(e^{-j\omega}) + e^{jn\omega} X(e^{j\omega}) = \\ &\quad + X(e^{-j\omega}) \cos \omega \end{aligned}$$

b)  $x[n](n-1)^q$

$$n x[n] \xrightarrow{\text{F.T.}} j \frac{dX(e^{j\omega})}{d\omega} \quad \text{خواص خطي}$$

$$n^q x[n] \xrightarrow{\text{F.T.}} - \frac{d^q X(e^{j\omega})}{d\omega^q} \quad \text{خواص قوى}$$

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$$\rightarrow x[n] = (n-1)^r x[n] = n^r x[n] - r n x[n] + 1 \xrightarrow{\text{F.T}} -\frac{d^r X(e^{j\omega})}{d\omega^r} - r j \cdot \frac{d X(e^{j\omega})}{d\omega} +$$

$$x(e^{j\omega})$$

جواب سؤال ۴

$$h[n] = \frac{\sin(n\frac{\pi}{T})}{\pi n} \rightarrow H(e^{j\omega}) = \pi \left( \frac{\omega}{\pi T} \right) = \begin{cases} 1 & |\omega| < \frac{\pi}{T} \\ 0 & \frac{\pi}{T} < |\omega| < \pi \end{cases}$$

$$\frac{\sin(n\frac{\pi}{T})}{\pi n} \xrightarrow{\text{F.T}} \pi \left( \frac{\omega}{\pi T} \right)$$

a)  $x[n] = \delta[n+1] + \delta[n-1] \rightarrow X(e^{j\omega}) = e^{j\omega} + e^{-j\omega}$

$$Y(e^{j\omega}) = H(e^{j\omega}) X(e^{j\omega}) = \begin{cases} e^{j\omega} + e^{-j\omega} & |\omega| < \frac{\pi}{T} \\ 0 & \frac{\pi}{T} < |\omega| < \pi \end{cases}$$

b)  $x[n] = \delta[n-k] + \delta[n-r] + \delta[n-1] + \delta[n-0]$

$$X(e^{j\omega}) = 1 + e^{-jk\omega} + e^{-jr\omega} + e^{-j\omega}$$

$$\rightarrow Y(e^{j\omega}) = H(e^{j\omega}) X(e^{j\omega}) = \begin{cases} 1 + e^{-jk\omega} + e^{-jr\omega} + e^{-j\omega} & |\omega| < \frac{\pi}{T} \\ 0 & \frac{\pi}{T} < |\omega| < \pi \end{cases}$$

جواب سؤال ۵

$$y[n] + \alpha / \omega y[n-1] = x[n]$$

$$\rightarrow Y(e^{j\omega}) + \alpha / \omega e^{-j\omega} Y(e^{j\omega}) = X(e^{j\omega}) \quad (\text{الف})$$

$$\rightarrow H(e^{j\omega}) X(e^{j\omega}) + \alpha / \omega (e^{-j\omega}) H(e^{j\omega}) X(e^{j\omega}) = X(e^{j\omega}) \rightarrow$$

$$H(e^{j\omega}) = \frac{1}{1 + \alpha / \omega e^{-j\omega}}$$

(ب)

a)  $x[n] = (\alpha / \omega)^n u[n] \rightarrow$

$$X(e^{j\omega}) = \frac{1}{1 - \frac{\alpha}{\omega} e^{-j\omega}}$$

$$\rightarrow Y(e^{j\omega}) = H(e^{j\omega}) \times X(e^{j\omega}) = \left( \frac{1}{1 + \gamma/\omega e^{-j\omega}} \right) \left[ \frac{1}{1 - \gamma/\omega e^{-j\omega}} \right] =$$

$$\frac{1}{(1 + \gamma/\omega e^{-j\omega})(1 - \gamma/\omega e^{-j\omega})} = \frac{\frac{1}{\gamma}}{1 - \frac{\gamma}{\omega} e^{-j\omega}} + \frac{\frac{1}{\gamma}}{1 + \frac{\gamma}{\omega} e^{-j\omega}}$$

$$y[n] = \frac{1}{\gamma} \left( \frac{1}{\gamma} \right)^n u[n] + \frac{1}{\gamma} \left( -\frac{1}{\gamma} \right)^n u[n]$$

b)  $x[n] = (-\gamma/\omega)^n u[n] \rightarrow X(e^{j\omega}) = \frac{1}{1 + \frac{\gamma}{\omega} e^{-j\omega}}$

$$\rightarrow Y(e^{j\omega}) = \left( \frac{1}{1 + \gamma/\omega e^{-j\omega}} \right) \left( \frac{1}{1 + \gamma/\omega e^{-j\omega}} \right)$$

$$y[n] = (n+1) \left( -\frac{1}{\gamma} \right)^n u[n]$$

c)  $X(e^{j\omega}) = \frac{1}{1 + e^{-\gamma j\omega}}$

$$Y(e^{j\omega}) = \frac{1}{1 + \gamma/\omega e^{-j\omega}} + \frac{\gamma e^{-\gamma j\omega}}{1 + \gamma/\omega e^{-j\omega}} \Rightarrow y[n] = \left( -\frac{1}{\gamma} \right)^n u[n] + \gamma \left( -\frac{1}{\gamma} \right)^{n-\gamma} u[n-\gamma]$$