CPSC 202 PSET 4

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A.4.1 Covering a set with itself

Disprove: For any set A, and any surjective function $f : A \to A, f$ is bijective.

a. If A is an infinite set, where $\forall x \in A : x \in \mathbb{N}$

b. f is surjective because for all x, f(x) = f(y) where x and y are elements of the natural numbers. Every element in the codomain is covered.

c. f is not injective because the codomain may be larger than the domain. This is because the infinite set may not have the same cardinality between the domain and codomain.

d. Therefore, the surjective function is not bijective for any set A

A.4.2 More inverses

Let A be a set. Suppose that every function $f : A \to A$ has an inverse function f^{-1}. How many elements can A have?

a. $|A| = 0$. If f is the empty function, then the function is bijective.

b. $|A| = 1$. There is exactly 1 element in the domain and codomain of the function. Therefore, there is a bijection.

c. $|A| = 2$ or more. Suppose our function is $f(x) = x^2$ and $A = 1, -1$. As proven by A.4.1, the function is not bijective for any set A that includes 1 and -1 with this function. Therefore, any set A larger than 2 is does not have an inverse for every function.

d. A can have 0 or 1 elements.

A.4.3 Rational and irrational

Let q and r be rel numbers such that q is rational and r is irrational. Show that there exists a rational q' such that $q < q' < r$

a. Archimedian property: for $0 < x < y : \exists n \in \mathbb{N}, s.t. nx > y$

b. Subtract q from both sides: $0 < r - q$ and q' = q + n, where $n < r - q \in Q$

c. Case 1: $r - q \geq 2 > 1$

- $r - q > 1$
- $r > 1 + q > q$ where $1 + q = q'$

d. Case 2: $0 < r - q < 2$ where $r \neq 0, n \neq 1$

- $n(r - q) > 2$ where n = Q
- q' = q + 2/n