CPSC 202 PSET 3

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A.3.1 A powerful problem

If A and B are sets, then A^B is the set of all functions $f : B \to A$. $1 = \{\emptyset\}$
$|1^A| = |A^1|$ then $|A| = 1$

a. for 1, $|1|$ is 1 because it contains exactly one set, the null set.

b. $|1^A|$ is $|1|^{|A|}$ and because $|1|$ is 1, $|1|^{|A|}$ is also 1.

c. This means that $1 = |A^1|$, and we simplify $|A|^{|1|}$ to just $|A|$

d. $1 = |A|$

A.3.2 A correspondence

Prove or disprove: For any sets A, B, and C, there exists a bijective function $f : C^{A\times B} \to (C^B)^A$.

a. $F(f) = g \leftrightarrow \forall a \in A, \forall b \in B, \forall c \in C : f(a, b) = c \to (g(a) = h \wedge h(b) = c)$

b. Prove injective

- F(f) = g and F(f') = g
- $\forall a \forall b \forall c : f(a, b) = c = f'(a, b) \to f = f'$
- Therefore, f = f'

c. Prove surjective by contradiction

- $\exists g : \forall f : F(f) \neq g$
- $\forall f : \exists a \exists b \exists c : f(a, b) \neq c \wedge (g(a) = h \wedge h(b) = c)$
- But fs are all functions $A \times B \to C$: contradiction

A.3.3 Inverses

Show that f is injective and g is surjective.

a. Suppose we have elements x and y in f(x). If f(x) = f(y) then x = y, for f to be injective. Therefore, if g(f(x)) = g(f(y)), x = y

b. For g to be surjective, this means $\forall x \in A : \exists y \in B$ s.t. g(y) = x

- x = g $\circ$ f(x) = g(f(x))
- choose y = f(x)
- g(y) = x