CPSC 202 PSET 9

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A.9.1: Quadrangle Closure

1. Show that every graph G has a unique quadrangle closure.

   a. Define n s.t. there are n paths with 4 vertices:
   $a_{00}a_{01}a_{02}a_{03} \rightarrow$ add $a_{03}a_{00}$
   $a_{10}a_{11}a_{12}a_{13} \rightarrow$ add $a_{13}a_{10}$
   ...
   $a_{n0}a_{n1}a_{n2}a_{n3} \rightarrow$ add $a_{n3}a_{n0}$

   b. Let's define H: $H = G \cup a_{01}, a_{02} \cup a_{11}, a_{12} \cup ... \cup a_{n0}, a_{n3}$

   c. Proof of uniqueness:

      - $\exists H' : \forall i \forall H' : a_{io}a_{i3} \in H'$, therefore, H is either a supergraph or equal to H'.
        Proof by contradiction:
        $G \subseteq H'$ s.t. H' is a supergraph of G, then H' would have to be the closer of G and $H \neq H'$, so $H \neq H' \cap H \nsubseteq H'$
        For this to be true, then $\exists a_{i0}a_{i3} \in H$ s.t. $a_{i0}a_{i3} \neq H'$
        However, then $a_{i0}a_{i1}a_{i2}a_{i3}$ isn't closed in H' is not closure. Therefore H has to be unique.

2. Show that the quadrangle closure of a bipartite graph is bipartite.
   $a_0 \in S, a_1 \in T, a_2 \in S, a_3 \in T$. The edges will always run from $S \rightarrow T$ or $T \rightarrow S$, therefore, the position of the first vertex doesn't matter.

A.9.2 Cycles

Show that G is a cycle.

a. Show G has the cycle C
$C = V_0V_1V_2...V_nV_0$, since G was defined as any two vertices with simple paths with no edges in common.
If C weren't contained in G then there wouldn't be two paths between V_0 and V_1

b. G doesn't have any vertices or edges in C. Proof G=C by contradiction:
If $\exists w_0$ s.t. $w_0 \in G$ and $w_0 \notin C$, then we would have 2 paths that go from any V_i in C to w_0 not in C, but then those paths would share an edge. w_0 cannot exist and C=G.

A.9.3 Deleting a Graph

Show that you can reduce a finite graph G_1 to the empty graph with no vertices by this process $\iff$ G_0 is acrylic.
Prove every vertex of the cycle has degree of at least 2.

a. If acrylic, it can be deleted: $\forall v \in C : d(v) \geq 2$

b. If there are edge points out of the cycle they can be deleted:
Proof by induction on $|v|$:

Base case: $|v| = 1$
It has a degree of 0 so it can be deleted. For an acrylic graph $|v| = n - 1$, there exists one vertex of degree 1.

Induction step: $|v| = 1 \rightarrow |v| = n$
$|E| = |v| - 1$ for an acrylic graph and $2|E| = \sum_{v \in V} d(v)$
If $\forall d(v) \geq 2$, then $2|E| = \sum_{v \in V} d(v) \geq \sum_{v \in v} 2$, so
$2|E| \geq 2|v|$, and $|E| \geq |v|$.
Therefore, $\exists$ at least one vertex of degree 1.