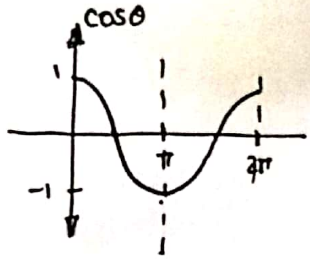
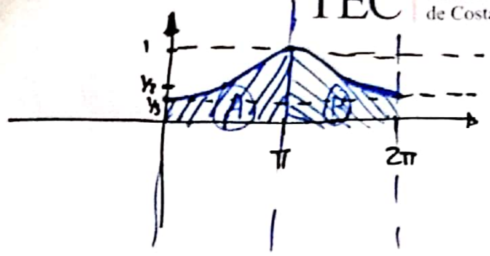
$$\int_0^{2\pi} \frac{1}{2+\cos(\theta)}\, d\theta = \frac{2\pi}{\sqrt{3}}$$





El Área de la región (A) y la región (B) son iguales, eso al ser interpretada la integral

$$\therefore \int_0^{\pi} \frac{1}{2+\cos\theta}\, d\theta = \frac{\pi}{\sqrt{3}}$$

Resolviendo la integral

$$\int_0^{\pi} \frac{1}{2+\cos\theta}\, d\theta \rightarrow \text{cambio de variable } \alpha = 2\theta$$

$$\frac{d\alpha}{2} = d\theta$$

$$\int_0^{2\pi} \frac{1}{2+\cos\left(\frac{\alpha}{2}\right)} \cdot \frac{1}{2} \cdot d\alpha$$

cambio de variable polar a compleja

$$z = e^{j\alpha}$$

$$\frac{dz}{d\alpha} = j e^{j\alpha}$$

$$d\alpha = dz\left(j e^{j\alpha}\right)^{-1}$$

$$\boxed{d\alpha = \frac{dz}{jz}}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\cos\frac{\alpha}{2} = \frac{e^{j\frac{\alpha}{2}} + e^{-j\frac{\alpha}{2}}}{2}$$

$$\cos\frac{\alpha}{2} = \frac{z^{1/2} + z^{-1/2}}{2}$$

$$\therefore \oint_C \frac{1}{2 + \frac{z^{1/2} + z^{-1/2}}{2}} \cdot \frac{1}{2} \cdot \frac{dz}{jz} \qquad C: |z| = 1$$

$$\frac{1}{j}\oint_C \frac{1}{4 + z^{1/2} + z^{-1/2}} \cdot \frac{dz}{z}$$

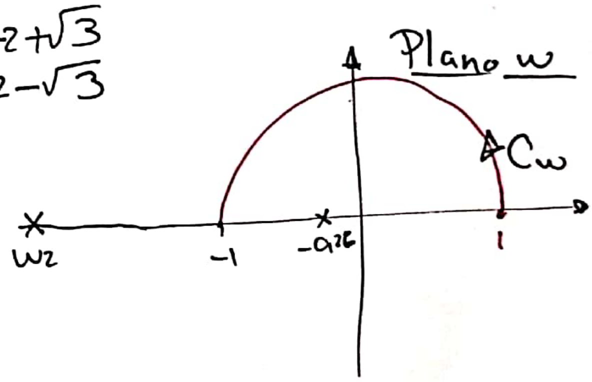
$$\left.\begin{aligned} z^{1/2} &= w \\ z &= w^2 \\ \frac{dz}{dw} &= 2w \end{aligned}\right\} \text{nuevo cambio de variable.}$$

$$\frac{1}{j}\oint \frac{1}{4 + w + w^{-1}} \, \frac{2\cancel{w}\,dw}{w^{2}}$$

$$\frac{1}{j}\oint \frac{1}{w^2+4w+1} \cdot 2dw$$

$$\frac{2}{j}\oint \frac{1}{w^2+4w+1}\, dw \qquad \begin{aligned} w_1 &= -2+\sqrt{3} \\ w_2 &= -2-\sqrt{3} \end{aligned}$$

$$\frac{2}{j}\oint \frac{1}{(w-w_1)(w-w_2)}\, dw$$



Ahora hay que mapear

$$C: |z|=1 \xrightarrow[z=w^2]{} C_w \qquad w = z^{1/2} = \sqrt{r_z}\cdot e^{j\frac{\theta_z}{2}}$$

$$z = r_z e^{j\theta_z} \qquad w = r_w e^{j\theta_w}$$

$$\underset{r_w=1}{r_w^2}\cdot e^{j\theta_w\cdot 2} = \underset{r=1}{r_z} e^{j\theta_z} \qquad \theta_z \in [0, 2\pi[$$

$$\theta_w = [0, \pi[$$

Curva abierta no de Jordan

∴ Por lo tanto con la intención de ubicar los polos al hacer el cambio $z=w^2$, se vuelve a llegar a una curva abierta, no de Jordan, y no se podría aplicar los teoremas de integración conocidos.