Transformada z

$$y(n) = \left(\tfrac{1}{2}\right)^n u(n) - 2^n u(-n)$$

$$y(n) = \left(\tfrac{1}{2}\right)^n u(n) - 2^n u(-n-1) - \delta(n)$$

$$Y(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-2z^{-1}} - 1$$

$$Y(z) = \frac{(1-2z^{-1}) + (1-\frac{1}{2}z^{-1}) - (1-\frac{1}{2}z^{-1})(1-2z^{-1})}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

$$Y(z) = \frac{\cancel{1-2z^{-1}} + \cancel{1-\frac{1}{2}z^{-1}} - [\cancel{1-2z^{-1}-\frac{1}{2}z^{-1}} + z^{-2}]}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

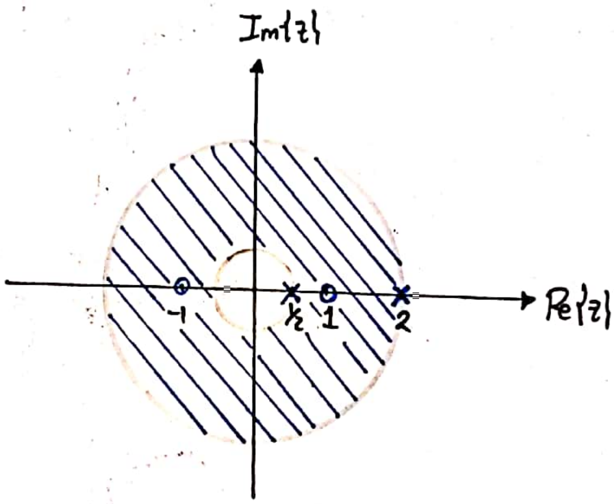
$$\boxed{Y(z) = \frac{1-z^{-2}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}}$$

$$Y(z) = \frac{1-z^{-2}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})} \cdot \frac{z^2}{z^2}$$

$$Y(z) = \frac{z^2-1}{(z-\frac{1}{2})(z-2)}$$

polos: $\begin{cases} z = 1/2 \\ z = 2 \end{cases}$

ceros: $\begin{cases} z = 1 \\ z = -1 \end{cases}$



ROC: $\frac{1}{2} < |z| < 2$

Transformada z inversa

① Método de Integración compleja.

$$x(n) = \frac{1}{2\pi j} \oint_c X(z) z^{n-1} dz$$

$$(z-\tfrac{1}{2})(z-2) = z^2 - 2z - \tfrac{1}{2}z + 1$$

$$(z-\tfrac{1}{2})(z-2) = z^2 - \tfrac{5}{2}z + 1$$

$$y(n) = \frac{1}{2\pi j} \cdot \oint_c \underbrace{\left[\frac{z^2-1}{(z-\frac{1}{2})(z-2)}\right]}_{\text{simplificar}} \cdot z^{n-1} \cdot dz$$

$$\begin{array}{l|l} z^2 - 1 & z^2 - \frac{5}{2}z + 1 \\ \hline -(z^2 - \frac{5}{2}z + 1) & 1 \\ \hline 0 + \frac{5}{2}z - 2 & \end{array}$$

$$1 + \frac{\frac{5}{2}z - 2}{(z-\frac{1}{2})(z-2)} = 1 + \frac{A}{z-\frac{1}{2}} + \frac{B}{z-2}$$

$$A = \lim_{z \to \frac{1}{2}} \cancel{(z-\tfrac{1}{2})} \cdot \frac{\frac{5}{2}z - 2}{\cancel{(z-\frac{1}{2})}(z-2)} = \frac{\frac{5}{2} \cdot \frac{1}{2} - 2}{\frac{1}{2} - 2} = \frac{\frac{5}{4} - 2}{-\frac{3}{2}} = \frac{\frac{5-8}{4}}{-\frac{3}{2}} = \frac{+\frac{3}{4}}{+\frac{3}{2}} = \frac{1}{2}$$

$$B = \lim_{z \to 2} \cancel{(z-2)} \cdot \frac{\frac{5}{2}z - 2}{\cancel{(z-2)}(z-\frac{1}{2})} = \frac{5-2}{2-\frac{1}{2}} = \frac{3}{\frac{3}{2}} = 2$$

$$y(n) = \frac{1}{2\pi j} \cdot \oint_c \left[1 + \frac{\frac{1}{2}}{z-\frac{1}{2}} + \frac{2}{z-2}\right] z^{n-1} dz$$

$$y(n) = \frac{1}{2\pi j} \oint_c 1 z^{n-1} dz + \frac{1}{2\pi j} \oint_c \frac{\frac{1}{2} \cdot z^{n-1}}{z-\frac{1}{2}} dz + \frac{1}{2\pi j} \oint_c \frac{2 z^{n-1}}{z-2} dz$$

$$y(n) = \frac{1}{2\pi j} \oint_c \frac{z^n}{z} dz + \frac{1}{4\pi j} \oint_c \frac{z^n}{z-\frac{1}{2}} \cdot z^{-1} dz + \frac{1}{\pi j} \oint_c \frac{z^n z^{-1}}{z-2} dz$$

$$y(n) = \frac{\cancel{2\pi j}}{\cancel{2\pi j}} \delta(n) + \frac{1}{4\pi j} \cdot \frac{2\pi j}{0!} \left(\frac{1}{2}\right)^{n-1} u(n-1) - \frac{1}{\cancel{\pi j}} \cdot (2)^{n-1} \cdot u(-n) \cdot \frac{\cancel{2\pi j}}{0!}$$

$$y(n) = \delta(n) + \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{n-1} u(n-1) - 2 \cdot (2)^{n-1} u(-n)$$

$$y(n) = \delta(n) + \left(\frac{1}{2}\right)^{n} u(n-1) - 2^{n} u(-n)$$

$$y(n) = \cancel{\delta(n)} + \left(\frac{1}{2}\right)^{n} u(n) - \cancel{\delta(n)} - 2^{n} u(-n)$$

$$\boxed{y(n) = \left(\frac{1}{2}\right)^{n} u(n) - 2^{n} u(-n)}$$

② Expansión en serie de potencias

$$Y(z) = 1 + \frac{1/2}{z - 1/2} + \frac{2}{z-2}$$

$$Y(z) = 1 + \frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} \cdot z^{-n} + -2 \cdot \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} z^{n}$$

$$Y(z) = 1 + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n} z^{-n} - \underbrace{\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n} z^{n}}_{n=-n}$$

$$Y(z) = 1 + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n} z^{-n} - \sum_{n=-\infty}^{0} 2^{n} z^{-n}$$

$$y(n) = \delta(n) + \left(\frac{1}{2}\right)^{n} u(n-1) - 2^{n} u(-n)$$

$$\boxed{y(n) = \left(\frac{1}{2}\right)^{n} u(n) - 2^{n} u(-n)}$$

3 Descomposión en funciones más simples

$$Y(z) = 1 + \frac{1/2}{z - 1/2} \cdot \frac{z^{-1}}{z^{-1}} + \frac{2}{z-2} \cdot \frac{z^{-1}}{z^{-1}}$$

$$Y(z) = 1 + \frac{\frac{1}{2} z^{-1}}{1 - \frac{1}{2} z^{-1}} + \frac{2 z^{-1}}{1 - 2 z^{-1}}$$

$$Y(n) = \delta(n) + \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} u(n-1) + -1 \cdot 2 \cdot 2^{n-1} u(-n)$$

$$\boxed{y(n) = \left(\frac{1}{2}\right)^{n} u(n) - 2^{n} u(-n)}$$

Ejemplo: Transformada z inversa.

$$X(z) = e^{1/z}$$

$$e^{z^{1}} = 1 + \frac{z^{1}}{1!} + \frac{z^{2}}{2!} + \cdots + \frac{z^{n}}{n!} + \ldots$$

$$e^{1/z} = 1 + \frac{z^{-1}}{1!} + \frac{z^{-2}}{2!} + \frac{z^{-3}}{3!} + \cdots \frac{z^{-n}}{n!} + \ldots = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\boxed{x(n) = \frac{1}{n!} u(n)}$$

Ejemplo: Transformada z inversa

$$X(z) = \frac{4(1+z^{-1})}{1+2z^{-1}+2z^{-2}} \quad ; \quad \text{ROC: } |z| < \sqrt{2}$$

$$X(z) = \frac{4\cdot(1+z^{-1})}{1+2z^{-1}+2z^{-2}} \cdot \frac{z^2}{z^2}$$

$$X(z) = \frac{4z(z+1)}{z^2+2z+2}$$

$$\frac{X(z)}{z} = \frac{4(z+1)}{z^2+2z+2}$$

$a=1$, $b=2$, $c=2$ | $\Delta = 2^2 - 4\cdot 2$

$\Delta = 4 - 8 = -4$

$$\frac{X(z)}{z} = \frac{A}{z-z_1} + \frac{B}{z-z_2}$$

$$z_{1,2} = \frac{-2 \pm \sqrt{-4}}{2} = \begin{cases} -1+j \\ -1-j \end{cases}$$

$$A = \lim_{z \to z_1} \cancel{(z-z_1)} \cdot \frac{4(z+1)}{\cancel{(z-z_1)}(z-z_2)} = \frac{4(z_1+1)}{z_1-z_2} = \frac{4(-1+j+1)}{-1+j+1+j} = \frac{4j}{2j} = 2$$

$\boxed{B=2}$ $\boxed{A=2}$

$$\frac{X(z)}{z} = \frac{2}{z-z_1}\cdot\frac{z^{-1}}{z^{-1}} + \frac{2}{z-z_2}\cdot\frac{z^{-1}}{z^{-1}}$$

$$X(z) = \frac{2}{1-z_1 z^{-1}} + \frac{2}{1-z_2 z^{-1}}$$

$$x(n) = -2\left(\sqrt{2}\, e^{-j\frac{3\pi}{4}}\right)^n u(-n-1) - 2\left(\sqrt{2}\, e^{j\frac{3\pi}{4}}\right)^n u(-n-1)$$

$$x(n) = -2(\sqrt{2})^n \left[\frac{e^{j\frac{3\pi}{4}n} + e^{-j\frac{3\pi}{4}n}}{2}\right] u(-n-1)\cdot 2 = -4(\sqrt{2})^n \cos\left(\frac{3\pi}{4}n\right) u(-n-1)$$

$$\boxed{x(n) = 2(\sqrt{2})^n \cos\left(\frac{3\pi}{4}n + \pi\right) u(-n-1)}$$

Ejemplo: Transformada z inversa

$$X(z) = \frac{1}{(1+z^{-1})(1-z^{-1})^2} \cdot \frac{z^3}{z^3}$$

$$X(z) = \frac{z^3}{(z+1)(z-1)^2}$$

$$\frac{X(z)}{z} = \frac{z^2}{(z+1)(z-1)^2} = \frac{A}{z+1} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

$$A = \lim_{z \to -1} \cancel{(z+1)} \cdot \frac{z^2}{\cancel{(z+1)}(z-1)^2} = \frac{(-1)^2}{(-1-1)^2} = 1/4$$

$$B = \lim_{z \to 1} \frac{d}{dz} \cancel{(z-1)^2} \cdot \left[\frac{z^2}{(z+1)\cancel{(z-1)^2}}\right] = \lim_{z \to 1} \cdot \left[\frac{2z(z+1) - z^2}{(z+1)^2}\right] = \frac{2 \cdot 2 - 1}{2^2} = \frac{3}{4}$$

$$C = \lim_{z \to 1} \cancel{(z-1)^2} \cdot \frac{z^2}{(z+1)\cancel{(z-1)^2}} = \frac{1}{2}$$

$$\frac{X(z)}{z} = \left[\frac{1/4}{z+1} + \frac{3/4}{z-1} + \frac{1/2}{(z-1)^2}\right]$$

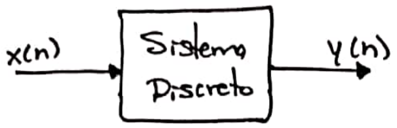
$$X(z) = \frac{1/4 \cdot z}{z+1} \cdot \frac{z^{-1}}{z^{-1}} + \frac{3/4 \cdot z}{z-1} \cdot \frac{z^{-1}}{z^{-1}} + \frac{1/2 \cdot z}{(z-1)^2} \cdot \frac{z^{-2}}{z^{-2}}$$

$$X(z) = \frac{1/4}{1+z^{-1}} + \frac{3/4}{1-z^{-1}} + \frac{1/2\, z^{-1}}{(1-z^{-1})^2}$$

$$x(n) = \frac{1}{4} u(n) \cdot (-1)^n + \frac{3}{4} u(n) + \frac{1}{2} n u(n)$$

$$\boxed{x(n) = \frac{(-1)^n}{4} u(n) + \frac{3}{4} u(n) + \frac{1}{2} n u(n)}$$

Sistemas en tiempo Discreto

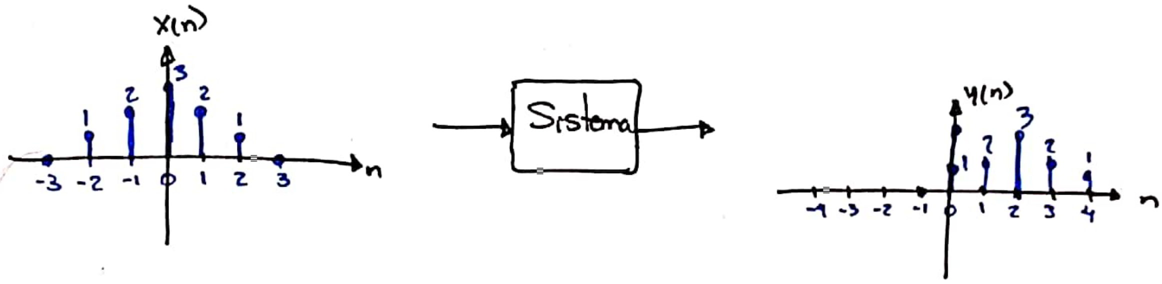


$$y(n) = \mathcal{T}[x(n)]$$

Ejemplo:

$$y(n) = x(n-2)$$

$$x(n) = \begin{cases} 3-|n| & -2 \leq n \leq 2 \\ 0 & \text{en el resto} \end{cases}$$



Ejemplo:

$$y(n) = \frac{1}{3}\left[x(n+1) + x(n) + x(n-1)\right]$$

$$x(n) = \begin{cases} 3-|n| & -2 \leq n \leq 2 \\ 0 & \text{en el resto} \end{cases}$$

n	$x(n)$	$y(n)$
-4	0	0
-3	0	1/3
-2	1	1
-1	2	2
0	3	7/3
1	2	2
2	1	1
3	0	1/3
4	0	0
5	0	0

Ejemplo:

$$y(n) = \sum_{k=-\infty}^{n} x(k) \qquad x(n) = \begin{cases} 3-|n| & -2 \le n \le 2 \\ 0 & \text{en el resto} \end{cases}$$

n	x(n)	y(n)
-5	0	0
-4	0	0
-3	0	0
-2	1	1
-1	2	3
0	3	6
1	2	8
2	1	9
3	0	9
4	0	9
5	0	9
6	0	9

$$\underbrace{y(n) = y(n-1) + x(n)}_{\text{ecuación recursiva.}}$$

Noten: La salida del sistema puede depender de:
- entrada actual
- entradas anteriores
- salidas anteriores

Análisis de condición inicial: ó historia anterior

Ejemplo:

$$y(n) = 2y(n-1) + x(n) - x(n-1)$$

Calcular la salida del sistema para $x(n) = u(n)$ y $y(-1) = 2$

n	-2	-1	0	1	2	3	4	5	6	7	8	9	...
x(n)	0	0	1	1	1	1	1	1	1	1	1	1	...
y(n)	?	2	5	10	20	40	80	160	320	640	1280	2560	...

Tipos de Sistemas en tiempo Discreto

Invarianza temporal — **Linealidad**

$y(n) = \tau[x(n)]$ $\quad$ $y(n-k) = \tau[x(n-k)]$ $\quad$ $\tau[\alpha_1 x_1(n) + \alpha_2 x_2(n)] = \alpha_1 \tau[x_1(n)] + \alpha_2 \tau[x_2(n)]$

Sistemas L.T.I

$\downarrow$

análisis

$$y(n) = x(n) * h(n)$$
$$Y(z) = X(z)\,H(z)$$

- respuesta al impulso
- Función de transferencia

Ecuaciones de diferencias

Ejemplo:

$x(n) = \{1, 2, 3, 1\}$

$h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$

Calcular la salida del sistema LTI

$x(n) \rightarrow \boxed{\text{LTI}} \rightarrow y(n)$

$$y(n) = \{1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1\}$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

$n=0$

$\{\underset{\uparrow}{1}, 2, 3, 1\}$

$\{-1, 1, \underset{\uparrow}{2}, 1\}$

$2+2 = 4$

$n=-1$

$\{\underset{\uparrow}{1}, 2, 3, 1\}$

$\{-1, 1, 2, \underset{\uparrow}{1}\}$

1

$n=1$

$\{\underset{\uparrow}{1}, 2, 3, 1\}$

$\{-1, \underset{\uparrow}{1}, 2, 1\}$

$1+4+3 = 8$

Ejemplo: Calcular la respuesta de un sistema LTI según:

$$h(n) = \left(\tfrac{1}{2}\right)^n u(n)$$

$$x(n) = u(n)$$

Solución:

$$y(n) = h(n) * x(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^k u(k) . u(n-k)$$

$$y(n) = \sum_{k=0}^{n} \left(\tfrac{1}{2}\right)^k = \left[\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right].u(n) = \left[2-\left(\tfrac{1}{2}\right)^n\right].u(n)$$

n	-1	0	1	2	3	4	5	6	....
y(n)	0	1	3/2	7/4	15/8	31/16	63/32	127/64	....
	0	1	1,5	1,75	1,875	1,9375	1,96875	1,984375	

$$y(n) = h(n) * x(n)$$

$$Y(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \cdot \frac{1}{1-z^{-1}} \cdot \frac{z^2}{z^2}$$

$$A = \lim_{z\to 1/2} \cancel{(z-1/2)} . \frac{z}{\cancel{(z-1/2)}(z-1)} = \frac{1/2}{1/2-1} = -1$$

$$\frac{Y(z)}{z} = \frac{A}{z-1/2} + \frac{B}{z-1} = \frac{z}{(z-1/2)(z-1)}$$

$$B = \lim_{z\to 1} \cancel{(z-1)} . \frac{z}{(z-1/2)\cancel{(z-1)}} = \frac{1}{1-1/2} = 2$$

$$Y(z) = \frac{-1.z}{z-1/2} \cdot \frac{z^{-1}}{z^{-1}} + \frac{2.z}{z-1} \cdot \frac{z^{-1}}{z^{-1}} = \frac{-1}{1-\frac{1}{2}z^{-1}} + \frac{2}{1-z^{-1}}$$

$$y(n) = 2u(n) - \left(\tfrac{1}{2}\right)^n u(n) = \left[2-\left(\tfrac{1}{2}\right)^n\right] u(n)$$

Sistemas LTI

Causalidad

$h(n) = 0 \quad n < 0$

ROC de $H(z)$ externa

- ejemplos

Estabilidad (BIBO)

$$\sum_{k=-\infty}^{\infty} |h(n)| < \infty$$

ROC de $H(z)$ incluye $|z| = 1$

$\sigma = 0 \rightarrow z = e^{sT} \rightarrow |z| = 1$

- ejemplos

¿Cuándo un sistema es estable y causal?

Ecuaciones de diferencias

- Sistemas LTI descritos por respuestas al impulso infinitas:

IIR : Infinite Impulse Response
FIR : Finite Impulse Response

FIR

$h(n) = \{1, 2, 1\}$

$y(n) = x(n) * h(n)$ ✓

IIR

$h(n) = \left(\frac{1}{2}\right)^n u(n)$

$y(n) = x(n) * h(n)$ X

ecuaciones de diferencias recursivas ✓

ejemplo:

$y(n) = 2y(n-1) + x(n) - x(n-1)$

$$\underbrace{\sum_{k=0}^{N} a_k\, y(n-k) = \sum_{k=0}^{M} b_k\, x(n-k)}$$

a_k, b_k : constantes

- Sistemas LTI

N: orden de la ecuación de diferencias

Ejemplo: Encuentre la función de ecuación de diferencias de un sistema causal con función de transferencia:

$$H(z) = \frac{1 + z^{-1}}{1 + 5/6\, z^{-1} + \frac{1}{6} z^{-2}}$$

$$\frac{Y(z)}{X(z)} = \frac{1 + z^{-1}}{1 + \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}}$$

$$Y(z) \cdot \left[1 + \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}\right] = X(z)\left[1 + z^{-1}\right]$$

$$y(n) + \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2) = x(n) + x(n-1)$$

$$y(n) = -\frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n) + x(n-1)$$

Transformada z unilateral

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

La transformada z unilateral ignora los valores de las muestras para $n < 0$, que es lo mismo que asumir que son igual a 0.

Ejemplo:

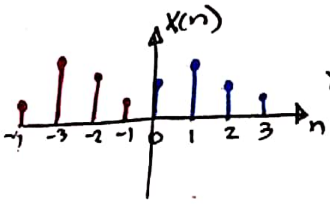
$$x_1(n) = \{1, -2, \underset{\uparrow}{2}, 3\} \circ\!\!-\!\!\bullet\ X_1(z) = 2 + 3z^{-1}$$

Como $x(n) = x(n)u(n)$ en la transformada unilateral por lo que las transformada $X(z)$ tendrá solo regiones de convergencia externas a un círculo de cierto radio. ROC: $|z| > R$.

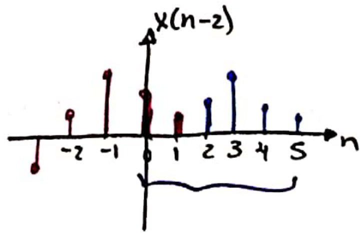
Propiedades de la transformada z unilateral

Retardo temporal

$$x(n-k) \circ\!\!-\!\!\bullet\ z^{-k}X(z) + z^{-k}\sum_{n=1}^{k} x(-n) z^{n}$$



$x(n) \overset{z_u}{\circ\!\!-\!\!\bullet} X(z)$



$$\circ\!\!-\!\!\bullet\ z^{-2}X(z) + \underbrace{x(-1)z^{-1} + x(-2)}_{\text{términos de corrección}}$$

Ejemplo: Un sistema LTI en tiempo discreto está descrito por la ecuación de diferencias:

$$y(n) = -\frac{1}{3}y(n-1) + \frac{2}{9}y(n-2) + x(n) - 2x(n-1)$$

Encuentre la respuesta del sistema ante las condiciones iniciales $y(-1)=0$ y $y(-2)=1$ con la señal de entrada $x(n)=u(n)$.

Solución:

Como hay condiciones iniciales distintas de 0 es necesario utilizar una transformación unilateral para z.

$$y(n) = -\frac{1}{3}y(n-1) + \frac{2}{9}y(n-2) + x(n) - 2x(n-1)$$

$$Y(z) = -\frac{1}{3}\left[z^{-1}Y(z) + y(-1)\right] + \frac{2}{9}\left[z^{-2}Y(z) + y(-1)z^{-1} + y(-2)\right] + X(z) - 2\left[z^{-1}X(z) + x(-1)\right]$$

$$Y(z)\cdot\left[1 + \frac{1}{3}z^{-1} - \frac{2}{9}\right] = -\frac{1}{3}y(-1) + \frac{2}{9}y(-1)z^{-1} + \frac{2}{9}y(-2) + X(z)\left[1-2z^{-1}\right] - 2x(-1).$$

$$Y(z) = \underbrace{\frac{X(z)\left[1-2z^{-1}\right] - 2\cancel{x(-1)}^{\,0}}{1+\frac{1}{3}z^{-1} - \frac{2}{9}z^{-2}}}_{\text{Respuesta Forzada}} + \underbrace{\frac{\cancel{y(-1)}^{\,0}\left[-\frac{1}{3} + \frac{2}{9}z^{-1}\right] + \frac{2}{9}\cancel{y(-2)}^{\,1}}{1+\frac{1}{3}z^{-1} - \frac{2}{9}z^{-2}}}_{\text{Respuesta natural}}$$

$$Y(z) = \underbrace{\frac{1-2z^{-1}}{(1-z^{-1})\left(1+\frac{1}{3}z^{-1}-\frac{2}{9}z^{-2}\right)}}_{Y_{zs}(z)} + \underbrace{\frac{2/9}{1+\frac{1}{3}z^{-1}-\frac{2}{9}z^{-2}}}_{Y_{zi}(z)}$$

Respuesta Forzada

$$Y_{zs}(z) = \frac{1-2z^{-1}}{(1-z^{-1})(1+\frac{1}{3}z^{-1}-\frac{2}{9}z^{-2})} \cdot \frac{z^3}{z^3}$$

$$Y_{zs}(z) = \frac{z^2(z-2)}{(z-1)(z^2+\frac{1}{3}z-2/9)}$$

$$\frac{Y_{zs}(z)}{z} = \frac{z(z-2)}{(z-1)(z^2+\frac{1}{3}z-2/9)}$$

$a=1$ ¦ $\Delta = \left(\frac{1}{3}\right)^2 - 4 \cdot -\frac{2}{9}$

$b = 1/3$

$c = -2/9$ ¦ $\Delta = \frac{1}{9} + \frac{8}{9} = 1$

$$z_{1,2} = \frac{-1/3 \pm \sqrt{1}}{2} = \begin{cases} \frac{-\frac{1}{3}+1}{2} = \frac{2/3}{2} = 1/3 \\ \frac{-\frac{1}{3}-1}{2} = \frac{-4/3}{2} = -2/3 \end{cases}$$

$z_1 = 1/3$

$z_2 = -2/3$

$$\frac{Y_{zs}(z)}{z} = \frac{A}{z-1} + \frac{B}{z-1/3} + \frac{C}{z+2/3}$$

$$A = \lim_{z\to 1} \cancel{(z-1)} \cdot \left[\frac{z\cdot(z-2)}{\cancel{(z-1)}(z-1/3)(z+2/3)}\right] = \frac{1\cdot -1}{(1-1/3)(1+2/3)} = \frac{-1}{\frac{2}{3}\cdot\frac{5}{3}} = \frac{-9}{10} \quad \boxed{A = \frac{-9}{10}}$$

$$B = \lim_{z\to 1/3} \cancel{(z-1/3)}\left[\frac{z(z-2)}{(z-1)\cancel{(z-1/3)}(z+2/3)}\right] = \frac{1/3\cdot(1/3-2)}{(1/3-1)(1/3+2/3)} = \frac{1/\cancel{3}\cdot +5/3}{+2/\cancel{3}\cdot 1} = \frac{5}{6} \quad \boxed{B = 5/6}$$

$$C = \lim_{z\to -2/3} \cancel{(z+2/3)}\left[\frac{z\cdot(z-2)}{(z-1)(z-1/3)\cancel{(z+2/3)}}\right] = \frac{-2/3\cdot(-2/3-2)}{(-2/3-1)(-2/3-1/3)} = \frac{+2/\cancel{3}\cdot +8/3}{+5/\cancel{3}\cdot +1} = \frac{16}{15} \quad \boxed{C = \frac{16}{15}}$$

$$\frac{Y_{zs}(z)}{z} = \frac{-9/10}{z-1}\cdot\frac{z^{-1}}{z^{-1}} + \frac{5/6}{z-1/3}\cdot\frac{z^{-1}}{z^{-1}} + \frac{16/15}{z+2/3}\cdot\frac{z^{-1}}{z^{-1}}$$

$$Y_{zs}(z) = \frac{-9/10}{1-z^{-1}} + \frac{5/6}{1-\frac{1}{3}z^{-1}} + \frac{16/15}{1+\frac{2}{3}z^{-1}}$$

$$Y_{zs}(n) = -\frac{9}{10}u(n) + \frac{5}{6}\left(\frac{1}{3}\right)^n u(n) + \frac{16}{15}\left(\frac{-2}{3}\right)^n u(n)$$

Respuesta Natural

$$Y_{zi}(z) = \frac{2/9}{1+\frac{1}{3}z^{-1}-\frac{2}{9}z^{-2}} \cdot \frac{z^2}{z^2}$$

$$Y_{zi}(z) = \frac{2/9\, z^2}{z^2+\frac{1}{3}z-\frac{2}{9}}$$

$$\frac{Y_{zi}(z)}{z} = \frac{2/9 \cdot z}{(z-1/3)(z+2/3)} = \frac{A}{z-1/3} + \frac{B}{z+2/3}$$

$$A = \lim_{z\to 1/3} \cancel{(z-1/3)} \cdot \left[\frac{2/9 \cdot z}{\cancel{(z-1/3)}(z+2/3)}\right] = \frac{2/9 \cdot 1/3}{\frac{1}{3}+2/3} = \frac{2}{27} \qquad \boxed{A = 2/27}$$

$$B = \lim_{z\to -2/3} \cancel{(z+2/3)} \cdot \left[\frac{2/9 \cdot z}{(z-1/3)\cancel{(z+2/3)}}\right] = \frac{2/9 \cdot -2/3}{-2/3-1/3} = \frac{4}{27} \qquad \boxed{B = 4/27}$$

$$\frac{Y_{zi}(z)}{z} = \frac{2/27}{z-1/3} \cdot \frac{z^{-1}}{z^{-1}} + \frac{4/27}{z+2/3} \cdot \frac{z^{-1}}{z^{-1}}$$

$$Y_{zi}(z) = \frac{2/27}{1-\frac{1}{3}z^{-1}} + \frac{4/27}{1+\frac{2}{3}z^{-1}}$$

$$y_{zi}(n) = \frac{2}{27}\left(\frac{1}{3}\right)^n u(n) + \frac{4}{27}\left(\frac{-2}{3}\right)^n u(n)$$

Ahora la respuesta total:

$$y(n) = y_{zs}(n) + y_{zi}(n)$$

$$y(n) = \frac{-9}{10}u(n) + \frac{5}{6}\left(\frac{1}{3}\right)^n u(n) + \left(\frac{16}{15}\right)\cdot\left[\frac{-2}{3}\right]^n u(n) + \frac{2}{27}\left(\frac{1}{3}\right)^n u(n) + \frac{4}{27}\left(\frac{-2}{3}\right)^n u(n)$$

$$y(n) = \underbrace{\frac{-9}{10}u(n)}_{\text{Respuesta de estado permante}} + \underbrace{\frac{49}{54}\left(\frac{1}{3}\right)^n u(n) + \frac{164}{135}\left(\frac{-2}{3}\right)^n u(n)}_{\text{respuesta transitoria}}$$