Convolución

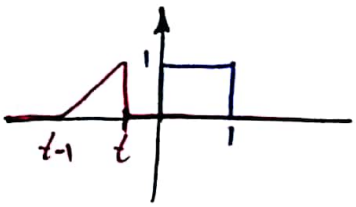
Sea $x(t)$ y $h(t)$ la entrada y la salida o respuesta al impulso de un sistema τ.

Si estas se definen como

$$x(t)=\begin{cases}0 & t\leq 0\\ 1 & 0<t<1\\ 0 & t\geq 1\end{cases} \qquad h(t)=\begin{cases}0 & t\leq 0\\ -t+1 & 0\leq t\leq 1\\ 0 & t\geq 1\end{cases}$$

Calcular $y(t)=x(t)*h(t)=\int_{-\infty}^{t} x(\tau)\,h(-\tau+t)\,d\tau$

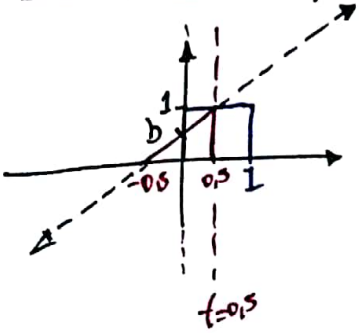
$t\leq 0$

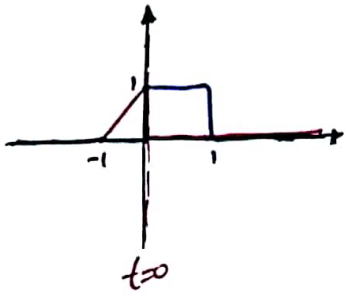


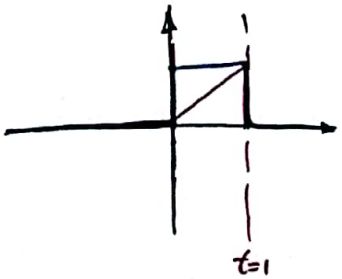
$y(t)=0$

$0<t<1$

$y(\tau)=\tau+\underbrace{1-t}_{b}$ ← intersección



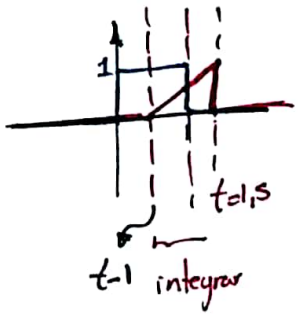




$$y(t)=\int_{0}^{t} 1.(\tau+1-t)\,d\tau=\frac{\tau^2}{2}+\tau-t\tau\Big|_{0}^{t}$$

$$y(t)=\frac{t^2}{2}+t-t^2=-\frac{t^2}{2}+t$$

$1\leq t\leq 2$



$$y(t)=\int_{t-1}^{1} 1.(\tau+1-t)\,d\tau$$

$$y(t)=\frac{\tau^2}{2}+\tau-t\tau\Big|_{t-1}^{1}$$

$$y(t)=\left(\frac{1}{2}+1-t\right)-\left(\frac{(t-1)^2}{2}+(t-1)-t(t-1)\right)$$

$$y(t)=\frac{3}{2}-t-\left(\frac{t^2-2t+1}{2}+t-1-t^2+t\right)$$

$$y(t)=\frac{3}{2}-t-\frac{t^2}{2}+t-\frac{1}{2}-t+1+t^2-t$$

$$y(t)=\frac{t^2}{2}-2t+2$$

$$y(t)=\begin{cases}0 & t<0\\ -\frac{t^2}{2}+t & 0<t<1\\ \frac{t^2}{2}-2t+2 & 1<t<2\\ 0 & t>2\end{cases}$$