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MUSIC AND ACOUSTIC ENGINEERING

SOUND ANALYSIS, SYNTHESIS AND PROCESSING

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Sound Synthesis and Spatial Processing Homework
Development of a WDF implementation

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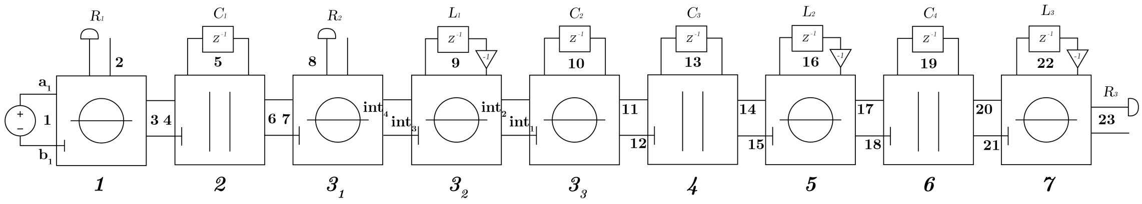
WDF scheme



The aim of this report is to model the electro-mechano-acoustic behavior of a piezoelectric MEMS loudspeaker in the Wave Digital (WD) domain. In order to do that, a Wave Digital Filter that recreates the behavior of the reference circuit has been implemented.
The WDF scheme is the one in figure at the top of the page.
Firstly the ideal voltage generator F_{in} has been chosen as the root, since it is the only element which can not be adapted.
The next step is to evaluate the reference resistances of the passive 1-port elements (resistors, capacitors and inductors), according to the following rules:

$$Z_R = R \qquad Z_C = \frac{C}{2T_s} \qquad Z_L = \frac{2T_s}{L} \qquad \text{with } T_s \text{ sampling period}$$

and also evaluate them for the ports between the junctions through the adaptation process. The adaptation process consists in defining the unknown reference resistance of a junction as a function of the known ones. Since the majority of the junctions have more than one unknown, the process is iterated starting from the only junction for which this is possible and it uses this result for the following ports. In this case the process starts on junction 7 and goes from right to left, adapting the left port of each junction, up to the root.

$$Z_{parallel} = \frac{Z_{right} Z_{up}}{Z_{right} + Z_{up}} \qquad Z_{serie} = Z_{right} + Z_{up}$$

All the values for the reference resistances in the circuit are found in the table on the right.
Now the scattering matrices have to be defined, using the formulas:

$$[S_{par}] = 2[Q]^T([Q][Z]^{-1}[Q]^T)^{-1}[Q][Z]^{-1} - [I]$$
$$[S_{ser}] = [I] - 2[Z][B]^T([B][Z][B]^T)^{-1}[B]$$

where $[Z]$ is a diagonal matrix containing all the reference resistances of the ports attached to the junction, while $[B]$ and $[Q]$ are matrices representing respectively the currents and voltages signs in the ports obtained through tree-cotree decomposition.

$Variable$	$Value$
Z_1	$Z_2 + Z_3$
Z_2	R_1
Z_3	Z_4
Z_4	$\frac{Z_6 Z_5}{Z_6 + Z_5}$
Z_5	$\frac{T_s}{2C_1}$
Z_6	Z_7
Z_7	$Z_8 + Z_{int_4}$
Z_8	R_2
Z_{int_4}	Z_{int_3}
Z_{int_3}	$Z_9 + Z_{int_2}$
Z_9	$\frac{2L_1}{T_s}$
Z_{int_2}	Z_{int_1}
Z_{int_1}	$Z_{10} + Z_{11}$
Z_{10}	$\frac{T_s}{2C_2}$
Z_{11}	Z_{12}
Z_{12}	$\frac{Z_{14} Z_{13}}{Z_{14} + Z_{13}}$
Z_{13}	$\frac{T_s}{2C_3}$
Z_{14}	Z_{15}
Z_{15}	$Z_{17} + Z_{16}$
Z_{16}	$\frac{2L_2}{T_s}$
Z_{17}	Z_{18}
Z_{18}	$\frac{Z_{20} Z_{19}}{Z_{20} + Z_{19}}$
Z_{19}	$\frac{T_s}{2C_4}$
Z_{20}	Z_{21}
Z_{21}	$Z_{22} + Z_{23}$
Z_{22}	$\frac{2L_3}{T_s}$
Z_{23}	R_3

WDF implementation

In order to implement the WDF, a for loop has been utilized, looping on input samples. Inside the loop, for each time instant, five steps are implemented:

- **Time-dependent linear components' behavior**: for inductors and capacitors the v^- of the prior time instants must be propagated to the current v^+, as their physical behavior suggests:

$$L : a[n] = -b[n-1] \qquad C : a[n] = b[n-1]$$

- **Forward scan**: concerned of evaluating the v^- of each port, based on the v^+ of the prior time instant. This procedure is executed from the output to the input, since the output junction is the only one for which all the needed parameters are already known. This is done through the operation:

$$b_k = [S_j](1,:) \cdot \begin{bmatrix} 0 & a_{k+1} & a_{k+2} \end{bmatrix}^T ; a_{k-1} = b_k$$

where b_k is the v^- of the port connected to the $(j-1)^{th}$ junction and a_{k+1} and a_{k+2} are the v^+ of the other two ports in the j^{th} junction (this number convention is related to the scheme utilized in this implementation but can vary based on the names chosen).

- **Local root scattering**: the current value of the input is utilized to compute the v^- as

$$b_{root} = 2F_{in} - a_{root}$$

b_{root} and a_{root} are not part of the sets of the forward scan meaning that their value must be copied from/in those.

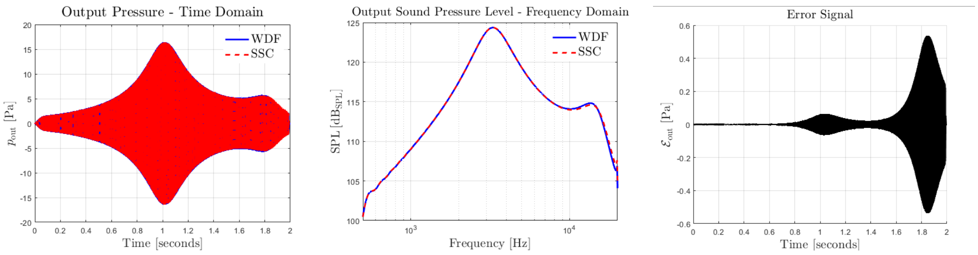
- **Backward scan**: operation similar to the forward scan, computed in the opposite direction (from the root to the output), concerned with evaluating the remaining v^-:

$$b_{k+1:k+2} = [S_j](2:3,:) \cdot \begin{bmatrix} a_k & a_{k+1} & a_{k+2} \end{bmatrix}^T \qquad a_{k+3} = b_{k+2}$$

- **Output sample evaluation**: the output is computed as $F_{out} = \frac{1}{2}(b_{out} + a_{out})$, where in this case *out* corresponds to the 23^{rd} port.

Results and plots

At the end of the for loop, the obtained result looks accurate and consistent with the given ground truth as shown by the plots below. The majority of the difference is found in the last 0.1 second, as can be seen from the third plot.



The Mean Squared Error value obtained is equal to **0.0119** and it confirms the accuracy of the result. From the error plot, it is possible to notice a smaller error in the first seconds, while it becomes bigger around 1.7 seconds.

We also implemented another WDF scheme that uses a single junction for the series $3_1, 3_2, 3_3$, in order to simplify the structure. The obtained MSE was higher (0.0625), diverging primarily around the SPL peak, but it could allow for lower computational complexity and reduced computational time. The WDF scheme of the alternative design is shown in the figure below.