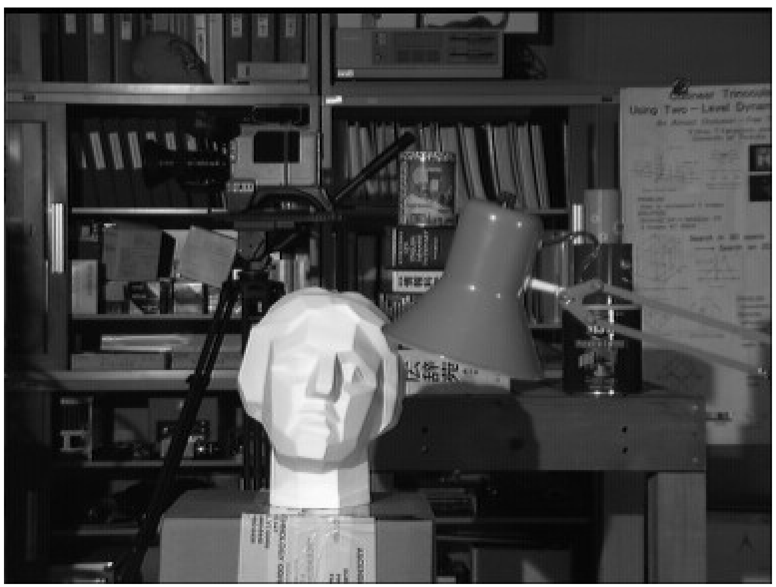
# Disparity Map Computation: Global-Style

**Presentation by Scott Grauer-Gray**

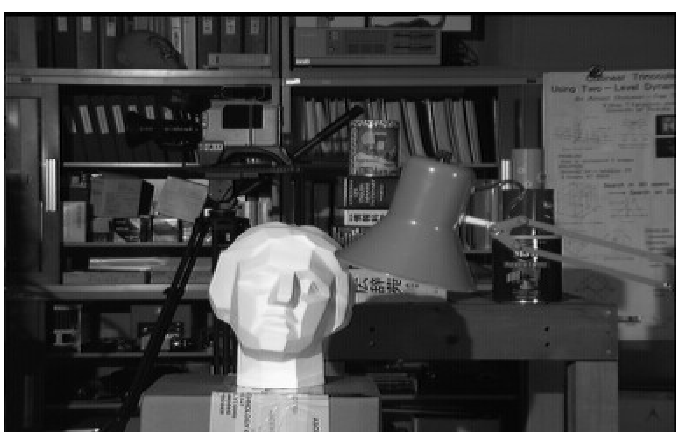


# Stereo Overview

- Given a reference image and matching (test) image
- Goal is to find the disparity between each pixel in the reference image and the corresponding pixel in the matching (test) image
- Disparity is inversely proportional to depth
  - Objects with greater disparity --> closer to “cameras”/ “eyes” (or wherever the image is from)
  - Objects with smaller disparity --> farther from “cameras” / “eyes” (or wherever the image is from)

# Calculating the Disparity Map

- Many algorithms/papers published on topic
- **Overview and evaluation of algorithms**: A Taxonomy and Evaluation of Dense Two-Frame Stereo Correspondence Algorithms
  - Current evaluation at http://vision.middlebury.edu/stereo/



# Stereo Assumptions

- **Surface assumptions**
  - Surfaces in image are Lambertian...appearance does not vary with viewpoint
  - Surfaces are piece-wise smooth; disparity of a single surface does not randomly “jump” around
- **Camera calibration/epipolar geometry**
  - Pair of rectified images given as input

# Calculating Disparity Maps

- Most stereo correspondence algorithms use all or some of following steps
  - Matching cost computation
  - Cost (support) aggregation
  - Disparity computation/optimization
  - Disparity refinement

# Matching Cost Computation

- **Matching Cost Computation** – cost of matching point (x1, y1) in reference image to point (x2, y2) in test image
  - In SSD algorithm, matching cost = squared difference of intensity values of pixels at disparity d
  - Other matching costs: sum of absolute difference (SAD), normalized cross-correlation (NCC), Birchfield-Tomasi

# Cost (support) aggregation

- **Cost (support) aggregation** – summing the costs of matching pixels in a given region (possibly using weights)
  - In SSD algorithm, aggregation is performed by averaging together matching costs for each pixel within a window at each given disparity using a box filter
  - Aggregation can be performed using a Gaussian/ binomial filter to provide greater weight to pixels near center of window

# Disparity Computation/optimization

- Retrieves calculated disparity at each pixel in reference image, method varies by algorithm
  - In SSD algorithm, uses "winner-take-all" method
  - Inspects aggregated cost associated with each disparity via window centered around pixel
  - Disparity with the smallest aggregated cost is selected
  - Step can be performed "in parallel" for every pixel in the reference image

# Disparity Refinement

- Disparity estimates generally in discretized space (such as integer pixel values)
- Some algorithm have refinement step to compute sub-pixel disparities after initial computations
- Methods include iterative gradient descent and fitting a curve to matching costs at discrete disparity levels
- Alternative is starting with more disparity levels

# Stereo Algorithms

- Most stereo algorithms can be placed one of two categories
  - **Local** – disparity computation dependent on intensity values within finite window in reference and matching (test) image; smoothness assumption is implicit with aggregating support
  - **Global** – stereo matching problem converted to global function; goal to optimize this global function that (likely) combines matching cost and smoothness cost terms (and possibly others...); smoothness assumption encoded explicitly

# Local Stereo

- **Example**: SSD using fixed windows
- One problem: setting correct size of window
  - **Small window:** may not be enough intensity variation; signal to noise ratio low
  - **Large window:** may cover region with multiple disparities
  - Paper by Kanade referenced in previous lecture goes into more detail about this...
- **Another problem: what about texture-less regions?** Aggregated matching cost near 0 for multiple disparities

# Global Stereo

- **Goal is to retrieve disparity map that optimizes a global function**
  - Global function can vary across different global algorithms/implementations
  - Matching cost of corresponding pixels in ref/test images given disparity often encoded into function
  - Function often contains a “smoothness cost” - explicitly encodes the “piecewise smooth” assumption
    - Smoothness cost compares computed disparities of neighboring pixels in disparity map (greater difference in disparity -> greater smoothness cost)

# Global Stereo: MRF

- **Global method**: formulate stereo matching problem as a Markov network
  - Markov network - Probabilistic graph model
    - Undirected graph of n nodes with pairwise potentials (given by compatibility function...)
  - State of each node i represented as x_i
  - Given some “evidence” Y
  - Joint compatibility function: ϕ(x_s, Y)
    - Output can be considered “evidence” for x_s given Y; greater if x_s is more likely
  - Compatibility function: ψ(x_s, x_t)
    - Encodes “pairwise potential”/compatibility between neighboring nodes x_s, x_t; small if node pair not “compatible”

# Global Stereo: MRF

- **Goal**: retrieve "most likely" set of nodes $\{ x_1, x_2, ..., x_n\}$ given the evidence Y and the compatibility between neighboring nodes
  - Joint probability distribution function of n nodes:
    - $P( x_1, x_2, ..., x_n \mid Y) = \underbrace{\prod}_{\text{All nodes } s} \phi(x_s, Y) \underbrace{\prod}_{\text{All "neighboring" nodes } s,\, t} \psi(x_s, x_t)$

- **Target**: retrieve set of nodes that maximizes joint probability distribution

# Global Stereo: MRF

- **Target**: turn stereo matching problem into Markov random field problem
  - **Given**: stereo set of images
    - Color/intensity values of pixels in stereo images can be viewed as the “evidence”
    - **Current goal**: Find the disparity map that maximizes P(disparity map | stereo set)
      - No obvious solution...
      - However, you do have some idea of P( stereo set | disparity map) and P(disparity map)
      - How can you use this information?

# Global Stereo: MRF

- **Bayes rule**: P( X | Y) = (P( Y | X) * P(X)) / P(Y)
  - Using Bayes rule...
    - **P(disparity map | stereo set)** = (P(stereo set | disparity map) * P(disparity map)) / (P(stereo set))
    - Given stereo set --> P(stereo set) can be set to 1.0f
  - Now, P(disparity map | stereo set) = P(stereo set | disparity map) * P(disparity map)
    - **New Goal**: retrieve disparity map that maximizes P(stereo set | disparity map) * P(disparity map)
      - One of these terms can be viewed as encoding the "matching" cost/probability with the other one encoding smoothness of the disparity map...
      - Which one is which?

# Global Stereo: MRF

- **Target**: retrieve P(stereo set | disparity map)
  - Probability represents total matching cost across all pixels in a stereo set given the disparity map
    - Greater total matching cost --> lower P(stereo set | disparity map)
    - If matching cost of every pixel is 0 given the current disparity map, then P(stereo set | disparity map) = 1
    - matching costs of pixels increase -> P(stereo set | disparity map) decreases
    - If matching cost of any pixel is infinity --> assume P(stereo set | disparity map) = 0
      - Can use property to rule out certain disparity maps

# Global Stereo: MRF

- P(stereo set | disparity map) =

  **Π** (e^((-1) * matching cost of s given d_s in disp. map))

  All pixels s in
  disparity map

  - Value of P(stereo set | disparity map) is between 0-1 inclusive
  - If matching cost of all pixels is 0, P(stereo set | disparity map) = 1 since e^0 = 1
  - If matching cost of any pixel is infinity P(stereo set | disparity map) = 0 since e^(-infinity) = 0
  - As matching costs of pixel(s) increase, P(stereo set | disparity map) decreases

# Global Stereo: MRF

- **Target**: retrieve P(disparity map)
  - Represents total smoothness cost of disparity map
    - Smoothness cost and P(disparity map) are inversely related (why...remember goal is to minimize smoothness cost)
    - Assume that pixels near each other have the same disparity --> smoothness cost increases when this condition is violated
    - Case where all pixels have same disparity -> total smoothness cost is 0 -> P(disparity map) = 1
    - Smoothness cost approaches infinity -> P(disparity map) approaches 0

# Global Stereo: MRF

- How to compute smoothness cost?
  - **One method**: use function that takes disparities of neighboring pixels in disparity map (generally 4-connected neighbors used)
  - If neighboring pixels have same disparity -> cost is 0
    - Cost increases as change in disparity (between neighboring pixels) increases
    - What to do about discontinuities?
      - May want to account for them in some manner
      - Could truncate smoothness cost at some point...prevent large jumps in disparity from being over-penalized
      - Could use segmentation (in pre-processing) to encode discontinuities and set smoothness cost to 0 where discontinuities expected...(this goes beyond basic stereo)

# Global Stereo: MRF

- **P(disparity map)** =

  **Π** (e^((-1) * smoothness cost between s and t given d_s and d_t))

  All 4-connected
  neighboring pixels
  s, t in disparity map

  - If smoothness cost of all sets of neighboring pixels is 0, P(disparity map) = 1
  - If smoothness cost of any set of neighboring pixels is infinity --> P(disparity map) = 0
  - Note that stereo image set has nothing to do with this probability
    - Disparity of all pixels in disparity map = constant c --> P(disparity map) = 1 (regardless of stereo set...)

# Global Stereo: MRF

- **Original goal**: maximize P(disparity map | stereo set)
  - Used Bayes to set P(disparity map | stereo set) = P(stereo set | disparity map) * P(disparity map)
  - Using new info...
    - **P(disparity map | stereo set)** =

**Π** (e^((-1) * matching cost of s given d_s in disp. map and stereo set)) *

All pixels s in
disparity map

**Π** (e^((-1) * smoothness cost between s and t given d_s and d_t))

All 4-connected
neighboring pixels
s, t in disparity map

# Global Stereo: MRF

- **Models for matching cost**
  - Same as local window: SAD, SSD, NCC, Birchfield-Tomasi -> use corresponding pixels in ref/test images for given disparity to compute cost
- **Models for smoothness cost**
  - Linear model commonly used: analogous to SAD for matching cost
    - Smoothness cost between neighboring pixels on disparity map = absolute difference in disparity
    - Linear model often truncates disparity difference at a given value to allow for discontinuities without too large of a penalty
    - Other models: Potts model, quadratic model

# Global Stereo: MRF

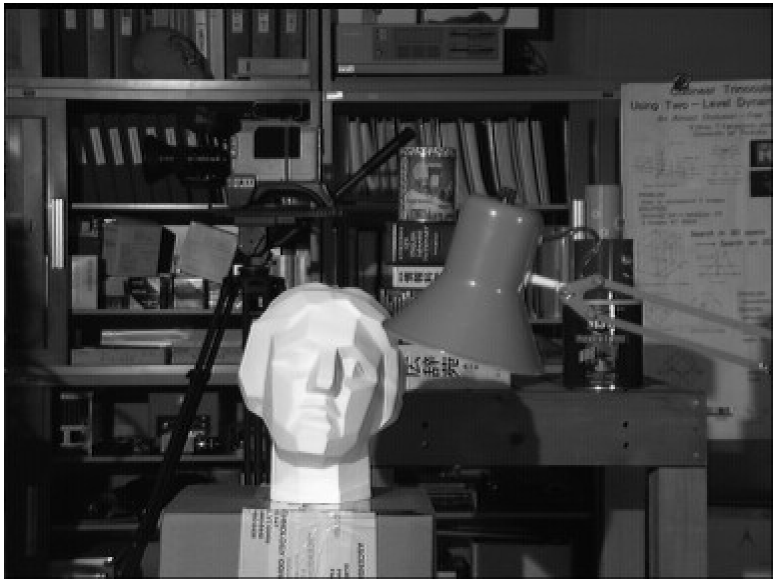
- **Retrieving P( stereo set | disparity map ) and P(disparity map) in “toy” stereo sets**
  - See next few slides...
  - Assume that SAD model used for matching cost computation
  - Assume linear model in smoothness cost computation
- What would be the “simplest” possible stereo set?

# Global Stereo: MRF

- Toy stereo set #1:
  - Two all “black” images given as stereo set
    - What will be the P(stereo set | disparity map) when all disparities are 0?
    - What will be P(disparity map) when all disparities are 0?
    - What is P(disparity map | stereo set) when disparities = 0?
    - Will P(stereo set | disparity map) change if disparity map changes?

# Global Stereo: MRF

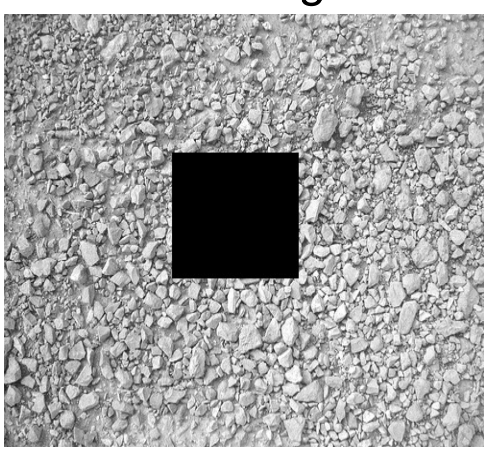
- Toy Stereo Set #2:
  - Two identical images given as stereo set (Tsukuba reference image, as an example)
    - What will be the P(stereo set | disparity map) when all disparities are 0?
    - What will be P(disparity map) when all disparities are 0?
    - Will P(stereo set | disparity map) change if disparity map changes?
    - What happens when all disparities = 1 in disparity map?



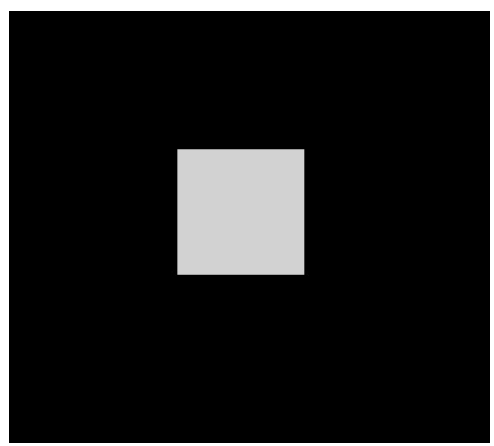
# Global Stereo: MRF

- Toy Stereo set #3: Grayscale stereo set with textured "background" and black object "foreground"
  - Disparity of textured "background" is 0
  - Disparity of "black" object is 5
    - Given ground truth disparity map...

Ref Image



Ground truth disparity map



- What will be the P(stereo set | disparity map) when all disparities are 0?
- What will P(stereo set | disparity map) when all disparities correspond to ground truth?
- What will be P(disparity map) when all disparities are 0?
- What will be P(disparity map) when all disparities correspond to ground truth?

# Global Stereo: MRF

- **Back to the Markov Random Field...**
  - **Markov network** - undirected graph of n nodes with pairwise potentials
  - State of each node i -> x_i
  - Given "evidence" Y
  - Joint probability distribution function of nodes:
    - $P(x_1, x_2, \ldots, x_n \mid Y) = \prod_{\text{All nodes } s} \phi(x_s, Y) \prod_{\text{All "neighboring" nodes } s,\, t} \psi(x_s, x_t)$

# Global Stereo: MRF

- **Stereo...**
  - **Find disparity map to maximize P(disparity map | stereo set) where P(disparity map | stereo set) =**

**Π** (e^((-1) * matching cost of s given d_s in disp. map)) *

All pixels s in
disparity map

**Π** (e^((-1) * smoothness cost between s and t given d_s and d_t))

All 4-connected
neighboring pixels
s, t in disparity map

- **MRF...**
  - **Find set of n nodes with states x_1, x_2, ..., x_n needed to maximize P(x_1, x_2, ..., x_n | Y) =** **Π** ϕ(x_s, Y) **Π** ψ(x_s, x_t)

All nodes s

All "neighboring"
nodes s, t

    - Y = local "evidence"

# Global Stereo: MRF

- **Maximizing P(disparity map | stereo set):** equivalent to maximizing P(x_1, x_2, ..., x_n | Y) in Markov network
  - Set of states x_1, x_2, ..., x_n in Markov network --> set of pixels in disparity map, each with a disparity value (assigned disparity value = “state”)
  - “Evidence” Y in Markov network --> given stereo set of images

- **(A) mission accomplished:** stereo problem turned into Markov network problem
  - Specifically, the stereo problem has been “reduced to” retrieving the maximum a posteriori (MAP) estimation in the Markov network

# Global Stereo: MRF

- **Retrieving the MAP estimation in the Markov network**
  - NP-complete problem; often infeasible to solve using “brute force”
    - Each pixel (“node”) in disparity map can take any value in disparity space (“state”)
  - Methods used to estimate solution in reasonable amount of time
    - Graph cuts
    - Belief propagation

# Global Stereo: MRF

- **Belief Propagation**
  - Iterative inference algorithm that can be used on Markov network problems
    - Works by sending messages through the network for a number of iterations
    - Eventually, the message values at each node will converge and then the message values are used to retrieve the estimated state of the node
  - Retrieves optimal solution in graphs without loops
  - Called loopy belief propagation in graphs with loops (such as graph resulting from stereo problem)
    - No guarantee of optimal solution, but generally gives a good approximation

# Belief Propagation

- **Can be used to retrieve the MAP estimation in the Markov network**
  - Each node computes messages to send to four-connected neighbors
    - Each message can be viewed as a vector containing a value for each possible disparity
    - Messages are computed at each pixel (in each iteration) and then passed to four-connected neighbors
    - Messages computed using data cost and message values from neighbors (computed in previous iteration)
    - Higher message value --> higher probability of corresponding disparity

# Belief Propagation: Message Computation

- **Messages initially initialized to 1**
  - Message from pixel s to neighbor t in iteration i+1 corresponding to disparity d_x computed via:

$$M^{i+1}\_st(d\_x) = \max_{d\_y \text{ in disparity space}}( \Psi(d\_x, d\_y) * \phi(d\_x, Y) * \prod_{\text{All neighbors } k \text{ of } s \text{ except } t} M^{i}\_ks(d\_y))$$

**Computational running time for each message at each pixel**: O(D^2), where D is the size of the disparity space

- **Message values will converge after "enough" iterations**

→ Once message values converge, message values (with joint compatibility function) used to compute estimated disparity at each pixel

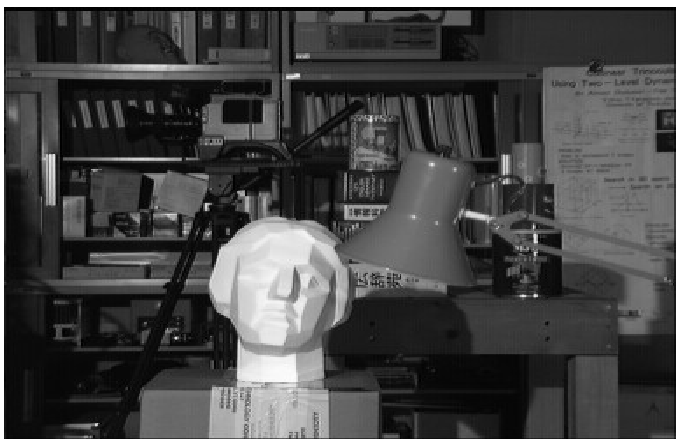
# Belief Propagation

- **After all BP iterations complete...**
  - Compute belief value of each disparity d_x at each pixel s
    - $b\_s(d\_x) = \phi(d\_x, Y) * \prod_{\text{All neighbors k of s}} M\_ks(d\_x)$
  - Disparity value at each pixel in disparity map is set to d_x corresponding to the maximum belief value
  - Resulting disparity map is estimation of desired disparity map that maximizes P(disparity map | stereo set) from the original problem via the MRF formulation and the MAP estimation
  - We are done! (or are we...)

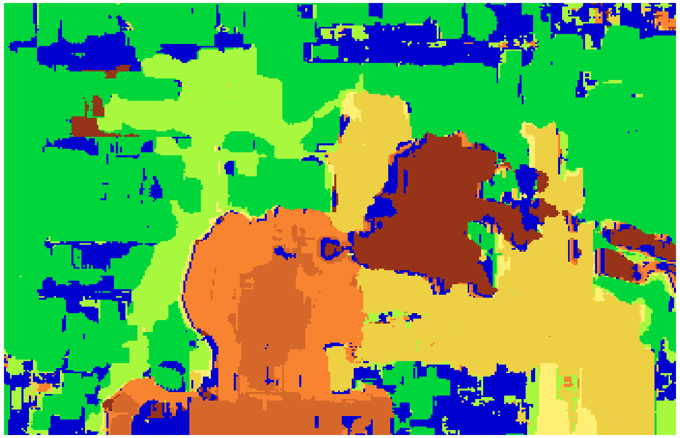
# Belief Propagation: Analysis

- Results for Tsukuba stereo set:

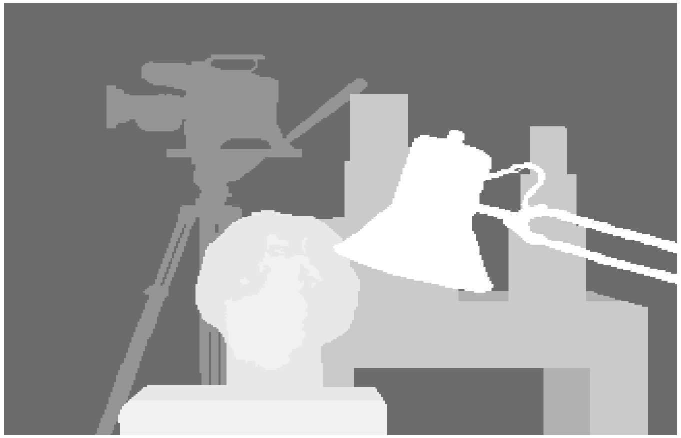
**Reference image:**



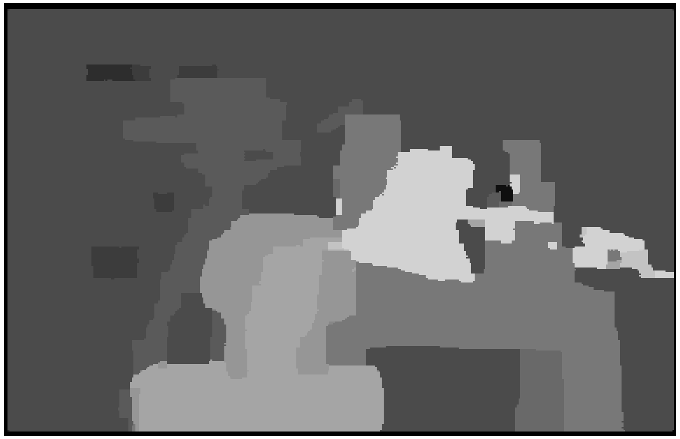
**Result using window-based matching:**



**Ground truth:**



**Result using Belief propagation:**



# Running time of Belief Propagation

- Algorithm runs for I iterations
- D values in disparity space
- Images in stereo set are of size N * M
- **Total Running time (sequential):**
  - Computation of data costs: O(N*M*D)
  - Computation of computing/passing message values in each iteration (naive): O(N*M*D^2)
  - Computation of calculated disparity values: O(N*M*D)
  - Total running time = O(N*M*D) + I * O(N*M*D^2) * O(N*M*D) = O(N*M*I*D^2)
  - Running time if computations performed on all pixels in parallel?

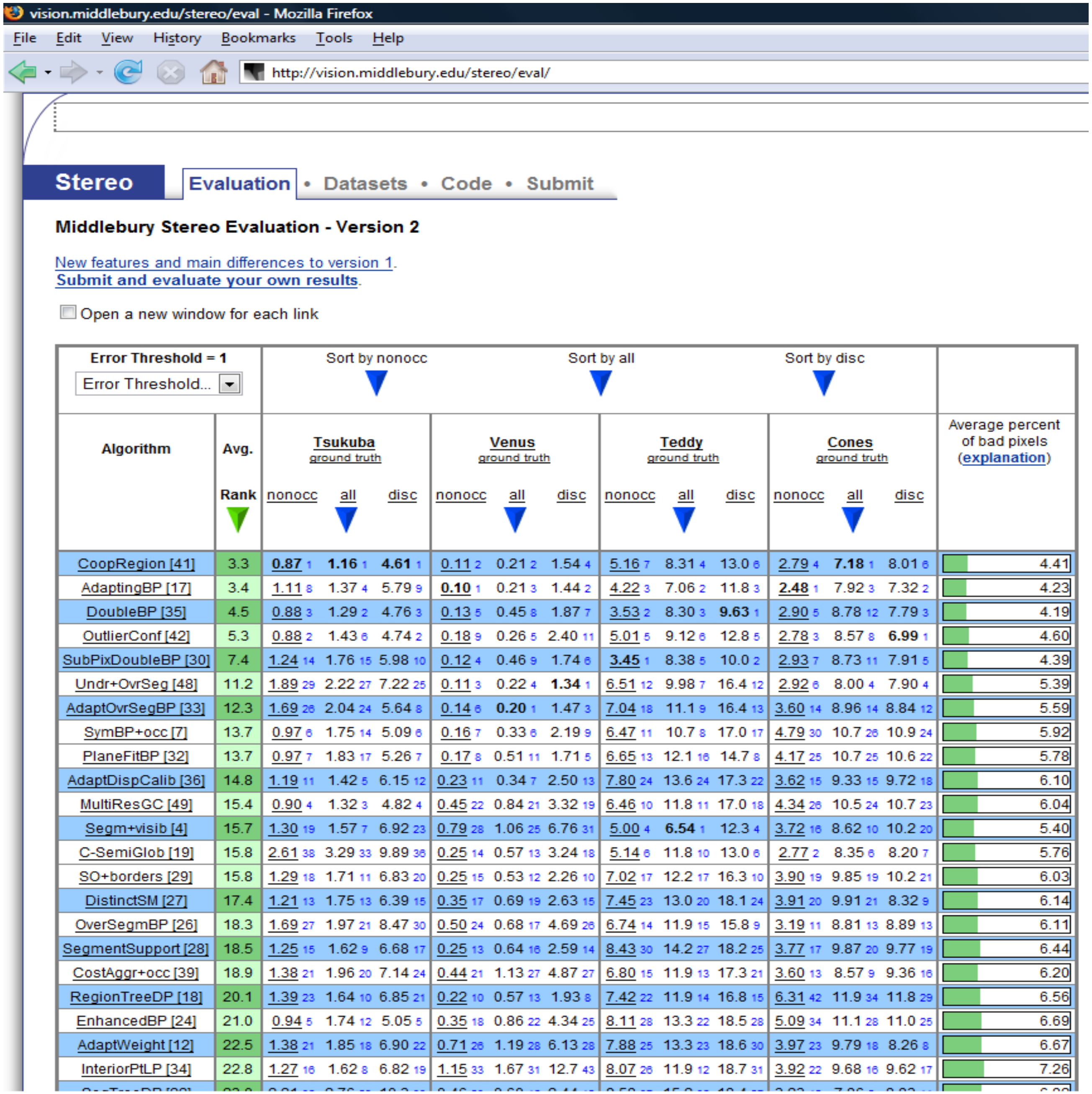
# Storage requirements of Belief Propagation

- **Initially**: need to store the 2 N*M images in stereo set: O(2*N*M) (not needed after data costs computed)
- **Matching cost stored for every pixel at each disparity:** O(N*M*D)
- **Four message vectors of size D stored for every pixel:** O(4*N*M*D)
- **Total storage requirement**: O(5*N*M*D)

# Advantages of Belief Propagation

- Resulting disparity map is close to minimization of data and smoothness costs
  - Resulting disparity map relatively accurate in practice
  - Generally better results than local methods such as SSD (even if adaptive windows are used)
- Can be extended to incorporate occlusion, segmentation, and other info to further improve the results
  - The #2 and #3 stereo algorithms according to the Middlebury benchmark are based on belief propagation

# Current Middlebury benchmark stereo results




**Middlebury Stereo Evaluation - Version 2**

New features and main differences to version 1.
**Submit and evaluate your own results.**

Open a new window for each link

Error Threshold = 1 | Error Threshold... | Sort by nonocc | Sort by all | Sort by disc

| Algorithm | Avg. Rank | Tsukuba (ground truth) nonocc | Tsukuba all | Tsukuba disc | Venus (ground truth) nonocc | Venus all | Venus disc | Teddy (ground truth) nonocc | Teddy all | Teddy disc | Cones (ground truth) nonocc | Cones all | Cones disc | Average percent of bad pixels (explanation) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CoopRegion [41] | 3.3 | **0.87** 1 | **1.16** 1 | **4.61** 1 | 0.11 2 | 0.21 2 | 1.54 4 | 5.16 7 | 8.31 4 | 13.0 6 | 2.79 4 | **7.18** 1 | 8.01 6 | 4.41 |
| AdaptingBP [17] | 3.4 | 1.11 8 | 1.37 4 | 5.79 9 | **0.10** 1 | 0.21 3 | 1.44 2 | 4.22 3 | 7.06 2 | 11.8 3 | **2.48** 1 | 7.92 3 | 7.32 2 | 4.23 |
| DoubleBP [35] | 4.5 | 0.88 3 | 1.29 2 | 4.76 3 | 0.13 5 | 0.45 8 | 1.87 7 | 3.53 2 | 8.30 3 | **9.63** 1 | 2.90 5 | 8.78 12 | 7.79 3 | 4.19 |
| OutlierConf [42] | 5.3 | 0.88 2 | 1.43 6 | 4.74 2 | 0.18 9 | 0.26 5 | 2.40 11 | 5.01 5 | 9.12 6 | 12.8 5 | 2.78 3 | 8.57 8 | **6.99** 1 | 4.60 |
| SubPixDoubleBP [30] | 7.4 | 1.24 14 | 1.76 15 | 5.98 10 | 0.12 4 | 0.46 9 | 1.74 6 | **3.45** 1 | 8.38 5 | 10.0 2 | 2.93 7 | 8.73 11 | 7.91 5 | 4.39 |
| Undr+OvrSeg [48] | 11.2 | 1.89 29 | 2.22 27 | 7.22 25 | 0.11 3 | 0.22 4 | **1.34** 1 | 6.51 12 | 9.98 7 | 16.4 12 | 2.92 6 | 8.00 4 | 7.90 4 | 5.39 |
| AdaptOvrSegBP [33] | 12.3 | 1.69 26 | 2.04 24 | 5.64 8 | 0.14 6 | **0.20** 1 | 1.47 3 | 7.04 18 | 11.1 9 | 16.4 13 | 3.60 14 | 8.96 14 | 8.84 12 | 5.59 |
| SymBP+occ [7] | 13.7 | 0.97 6 | 1.75 14 | 5.09 6 | 0.16 7 | 0.33 6 | 2.19 9 | 6.47 11 | 10.7 8 | 17.0 17 | 4.79 30 | 10.7 26 | 10.9 24 | 5.92 |
| PlaneFitBP [32] | 13.7 | 0.97 7 | 1.83 17 | 5.26 7 | 0.17 8 | 0.51 11 | 1.71 5 | 6.65 13 | 12.1 16 | 14.7 8 | 4.17 25 | 10.7 25 | 10.6 22 | 5.78 |
| AdaptDispCalib [36] | 14.8 | 1.19 11 | 1.42 5 | 6.15 12 | 0.23 11 | 0.34 7 | 2.50 13 | 7.80 24 | 13.6 24 | 17.3 22 | 3.62 15 | 9.33 15 | 9.72 18 | 6.10 |
| MultiResGC [49] | 15.4 | 0.90 4 | 1.32 3 | 4.82 4 | 0.45 22 | 0.84 21 | 3.32 19 | 6.46 10 | 11.8 11 | 17.0 18 | 4.34 26 | 10.5 24 | 10.7 23 | 6.04 |
| Segm+visib [4] | 15.7 | 1.30 19 | 1.57 7 | 6.92 23 | 0.79 28 | 1.06 25 | 6.76 31 | 5.00 4 | **6.54** 1 | 12.3 4 | 3.72 16 | 8.62 10 | 10.2 20 | 5.40 |
| C-SemiGlob [19] | 15.8 | 2.61 38 | 3.29 33 | 9.89 36 | 0.25 14 | 0.57 13 | 3.24 18 | 5.14 6 | 11.8 10 | 13.0 6 | 2.77 2 | 8.35 6 | 8.20 7 | 5.76 |
| SO+borders [29] | 15.8 | 1.29 18 | 1.71 11 | 6.83 20 | 0.25 15 | 0.53 12 | 2.26 10 | 7.02 17 | 12.2 17 | 16.3 10 | 3.90 19 | 9.85 19 | 10.2 21 | 6.03 |
| DistinctSM [27] | 17.4 | 1.21 13 | 1.75 13 | 6.39 15 | 0.35 17 | 0.69 19 | 2.63 15 | 7.45 23 | 13.0 20 | 18.1 24 | 3.91 20 | 9.91 21 | 8.32 9 | 6.14 |
| OverSegmBP [26] | 18.3 | 1.69 27 | 1.97 21 | 8.47 30 | 0.50 24 | 0.68 17 | 4.69 26 | 6.74 14 | 11.9 15 | 15.8 9 | 3.19 11 | 8.81 13 | 8.89 13 | 6.11 |
| SegmentSupport [28] | 18.5 | 1.25 15 | 1.62 9 | 6.68 17 | 0.25 13 | 0.64 16 | 2.59 14 | 8.43 30 | 14.2 27 | 18.2 25 | 3.77 17 | 9.87 20 | 9.77 19 | 6.44 |
| CostAggr+occ [39] | 18.9 | 1.38 21 | 1.96 20 | 7.14 24 | 0.44 21 | 1.13 27 | 4.87 27 | 6.80 15 | 11.9 13 | 17.3 21 | 3.60 13 | 8.57 9 | 9.36 16 | 6.20 |
| RegionTreeDP [18] | 20.1 | 1.39 23 | 1.64 10 | 6.85 21 | 0.22 10 | 0.57 13 | 1.93 8 | 7.42 22 | 11.9 14 | 16.8 15 | 6.31 42 | 11.9 34 | 11.8 29 | 6.56 |
| EnhancedBP [24] | 21.0 | 0.94 5 | 1.74 12 | 5.05 5 | 0.35 18 | 0.86 22 | 4.34 25 | 8.11 28 | 13.3 22 | 18.5 28 | 5.09 34 | 11.1 28 | 11.0 25 | 6.69 |
| AdaptWeight [12] | 22.5 | 1.38 21 | 1.85 18 | 6.90 22 | 0.71 26 | 1.19 28 | 6.13 28 | 7.88 25 | 13.3 23 | 18.6 30 | 3.97 23 | 9.79 18 | 8.26 8 | 6.67 |
| InteriorPtLP [34] | 22.8 | 1.27 16 | 1.62 8 | 6.82 19 | 1.15 33 | 1.67 31 | 12.7 43 | 8.07 26 | 11.9 12 | 18.7 31 | 3.92 22 | 9.68 16 | 9.62 17 | 7.26 |

# Drawbacks of Belief Propagation

- Requires many iterations for message values to converge and retrieve an accurate disparity estimate
- High storage requirements

# Drawbacks of Belief Propagation

- Felzenwalb (2004) presents methods to account for these drawbacks
  - Hierarchical scheme to reduce number of iterations -> longer-range interactions between pixels in fewer iterations course levels
  - Checkerboard scheme for message passing
    - Only half of the pixels must compute message values in each iteration
    - Allows BP iterations to be performed in place; cuts storage requirements

# Other Global Methods

- Belief propagation's primary “competitor” is graph cut
  - Either can be used to minimize total data and smoothness costs in global function
  - Tappan (2003) compared the two algorithms using identical parameters
    - Disparity maps retrieved using graph cut had slightly lower energy, but results similar in relation to ground truth
  - Belief propagation appears more popular based on Middlebury benchmark evaluation