EEE 391
Basics of Signals and Systems
Spring 2021–2022
Homework 2
due: 12 May 2022, Thursday by 23:00 on Moodle

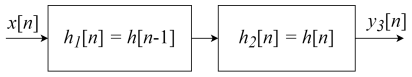
1. (a) Find the frequency response function of the FIR filter described by the difference equation in complex form:

$$y[n] = x[n] + 3x[n-2] + x[n-4]$$

(b) Sketch the magnitude and phase responses of the frequency response you found in (a) for $-\pi \leq \hat{\omega} \leq \pi$.

(c) Find the output $y[n]$ of the system when $x[n] = -2\delta[n-1] + \delta[n-3]$.

(d) Given that the impulse response of the filter in part a) is $h[n]$, find the output $y_3[n]$ of the cascaded system given below when $x[n] = 3\delta[n-1]$.



2. (a) For a linear time-invariant (LTI) system whose system function is

$$H(z) = 2 + 3z^{-1} + z^{-2}$$

i. Find the difference equation that relates $y[n]$ to $x[n]$.

ii. Determine and sketch the output when the input is $x[n] = \delta[n] - \delta[n-1]$.

(b) Determine the z transforms of the given sequences. Indicate the boundaries of z so that the systems are stable.

i. $a^n\, u[n]$, where a is an arbitrary constant.

ii. $\left(\frac{1}{5}\right)^n u[n-3]$

(c) Find the inverse z transforms of the following (Hint: Use the results you found in (b) and properties of the z transform):

i. $\dfrac{1}{(z-a)}$

ii. $\dfrac{z^{-4}}{1-\frac{3}{7}z^{-1}}$

3. (a) Given that they are causal, draw pole-zero diagrams of the following systems and explain if they are stable or not:

i. $y[n] = \dfrac{1}{3}y[n-1] + x[n] - 4x[n-2]$

ii. $H(z) = \dfrac{(z+0.3)(z-0.2)}{(z^2 - 0.7z + 0.1)}$

(b) Find a causal z-domain representation for an LTI system whose output is given by $y[n] = \frac{1}{4}^n u[n]$ when the input is given by

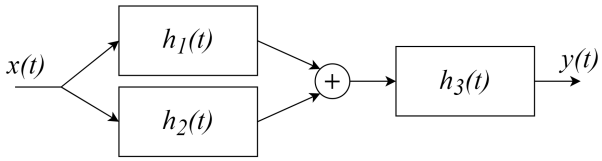
$$x[n] = \left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{4}\right)^{n-1} u[n].$$

What are the pole(s) and zero(s) of the system? Is this system stable?

4. (a) Find the frequency response $X(j\omega)$ of the signals $x(t)$ given below:
   i. $\delta(t-2) - 3\delta(t-3)$
   ii. $e^{-2t}\, u(t)$
   iii. $e^{-3t+12}\, u(t-4)$ (use the result of ii.)
   iv. $e^{-2|t|}\, \cos(t)$

   (b) Find the inverse Fourier transform $x(t)$ of the following functions $X(j\omega)$:
   i. $e^{-j3\omega} + e^{-j5\omega}$
   ii. $2\pi\delta(\omega - 2) + 2\pi\delta(\omega + 2)$
   iii. $\cos(\omega + \frac{\pi}{4})$

5. Consider the system of three LTI sub-systems given below:



For the given cases below, determine the impulse response of the overall system and answer the following questions: Is the overall system causal? Is it stable? Justify.

(a) $h_1(t) = \delta(t-3) \quad h_2(t) = \delta(t-2) \quad h_3(t) = u(t+1)$
(b) $h_1(t) = \delta(t-1) \quad h_2(t) = \delta(t+1) \quad h_3(t) = u(t)$
(c) $h_1(t) = \delta(t-2) \quad h_2(t) = \delta(t+3) \quad h_3(t) = \delta(t-1)$
(d) $h_1(t) = u(t-2) \quad h_2(t) = u(t-1) \quad h_3(t) = \delta(t-1)$

6. Signal $x(t)$ is a periodic CT signal with the Fourier series representation

$$x(t) = \sum_{k=\infty}^{\infty} a_k e^{jkt}$$

where the Fourier series coefficients are given by

$$a_k = \begin{cases} \frac{2}{\pi}, & k = 0 \\ \frac{1}{\pi k^2}, & \text{otherwise} \end{cases}$$

(a) Find and sketch the Fourier transform $X(j\omega)$ for $-4 \leq \omega \leq 4$.

(b) The signal $x(t)$ is provided as input to an LTI system with the following impulse response $h(t)$:

$$h(t) = \frac{\sin(t)\ \cos\left(\frac{5}{2}t\right)}{\pi\, t}.$$

Find the Fourier transform $H(j\omega)$ of $h(t)$. (Hint: You do not need to compute the FT integral here. Instead, express $h(t)$ as the product of a sinc signal and cosine signal, then use the multiplication property of the Fourier transform.)

(c) Let $y(t)$ be the output of the LTI system when $x(t)$ is provided as input. Use the relation $Y(j\omega) = X(j\omega)H(j\omega)$ to find the spectrum $Y(j\omega)$ of the output. Also sketch the spectrum.

(d) By using your result in (c), find the time-domain representation $y(t) = x(t) * h(t)$ of the output signal.

IMPORTANT NOTE:
Please name the pdf file you submit on Moodle as follows using only lower-case English characters for your first name, middle name (if any), and lastname. Please use your full name as it appears on the Bilkent system.

HW1_firstname_middlename_lastname.pdf
filename example for Ayşenur Çiğdem Sürücü:
HW1_aysenur_cigdem_surucu.pdf