

EEE 391
Basics of Signals and Systems
Spring 2021–2022
Homework 1

due: 15 March 2022, Tuesday by 23:00 on Moodle

1) Plot the following sinusoids accurately. Label both axes and the critical values of the signal such as the minimum and maximum amplitude, period, zero crossings, etc. Time units are in seconds and the arguments are in radians. Include about one period to the left and two periods to the right of the origin.

a) $x(t) = 5 \sin(6\pi t - \frac{\pi}{3})$

b) $x(t) = -6 \cos(5t + 2)$

2) $x(t)$ and $y(t)$ are two sinusoidal signals defined as follows:

$$x(t) = 3\sqrt{2} \cos(2t - \frac{\pi}{3}) \quad y(t) = 3 \sin(2t + \frac{\pi}{4})$$

a) Find the phasors X and Y representing the sinusoids $x(t)$ and $y(t)$, respectively.

b) Find $x(t) + y(t)$ using the phasor addition technique and verify your result by providing a phasor addition diagram, drawn to scale.

3) Determine whether the following signals are periodic or not. If periodic, identify the fundamental frequency and the period and indicate which harmonics exist in the frequency spectrum of the signal.

a) $x(t) = 3 \sin(3t) \cos(\frac{\pi}{5}t + \frac{\pi}{3}) - 1$

b) $x(t) = 7 \sin(\sqrt{3}t + 5) + 2 \cos(\pi t)$

c) $x(t) = \cos(12t) - 3 \cos(16t)$

d) $x(t) = \sin^2(60\sqrt{2}t) + \cos^2(24\sqrt{2}t + \frac{\pi}{4})$

4) A continuous-time signal is real valued and has a fundamental period of $T_o = 5$ sec. The non-zero Fourier series coefficients of $x(t)$ in the interval $0 < k < \infty$ of the synthesis equation are specified as follows:

$$a_0 = -4, \quad a_1 = -j \quad a_5 = -4e^{-j\frac{\pi}{3}} \quad a_8 = 2e^{j\frac{\pi}{3}}$$

Express $x(t)$ in the form:

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(\omega_k t + \phi_k)$$

5) Find all the Fourier series coefficient of the following signal without having to evaluate the analysis integral:

$$x(t) = 5 - 2e^{-jt} + 3\sin(-t + \pi/4) - 4\cos(3t + 5) + 3\cos(4t)\cos(5t + \pi/2)$$

a) What is the average value of $x(t)$?

b) What is the fundamental period of $x(t)$?

c) Is $x(t)$ a real signal?

d) Plot the frequency spectrum of $x(t)$. Label all frequencies and complex amplitudes. Which harmonics exist in the spectrum?

6) If a continuous-time periodic signal has the Fourier series coefficients a_k , $k = 0, \pm 1, \pm 2, \pm 3, \dots$, derive the Fourier series coefficients of the following signals in terms of a_k :

a) $y(t) = cx(t - t_o)$ where c and t_o are constants

b) $y(t) = \frac{dx(t)}{dt}$

7) Find all of the Fourier series coefficients of a full-wave rectified sine waveform with amplitude 5 units and radian frequency $\omega_o = 10\pi$ rad/sec. The phase shift of the waveform is zero.

8) The sinusoid $x(t) = 3\sin(120\pi t - \frac{\pi}{3})$ is sampled using three different sampling frequencies: a) 40 Hz, b) 75 Hz, c) 180 Hz.

Analyze each case in detail by making a digital spectrum diagram with respect to $\hat{\omega}$. In each case, indicate whether the signal is undersampled, oversampled, or sampled at the Nyquist rate and whether folding occurs or not. For each part, find a closed-form expression for the reconstructed signal $y(t)$ and plot it on the same diagram as $x(t)$ using a different color or linestyle.

IMPORTANT NOTE:

Please name the pdf file you submit on Moodle as follows using only lower-case English characters for your first name, middle name (if any), and lastname. Please use your full name as it appears on the Bilkent system.

MA1_firstname_middlename_lastname.pdf
filename example for Ayşenur Çiğdem Sürücü:
MA1_aysenur_cigdem_surucu.pdf