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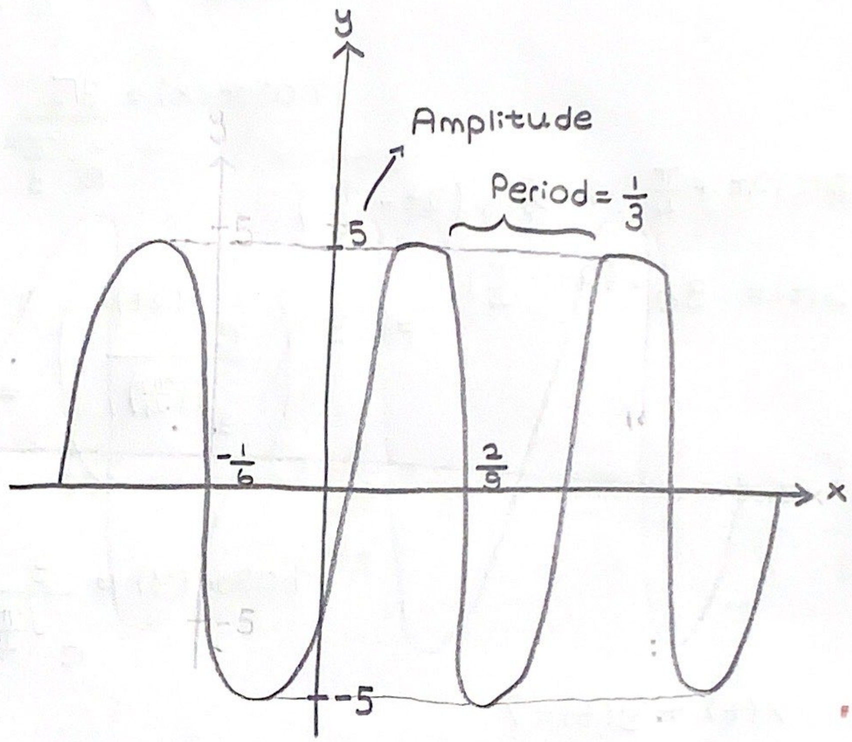
1)

a. $x(t) = 5\sin\left(6\pi t - \frac{\pi}{3}\right)$

$A = 5$

$f = 6\pi \cdot \frac{1}{2\pi} = 3\ Hz$

$T = \frac{1}{3}\ s$



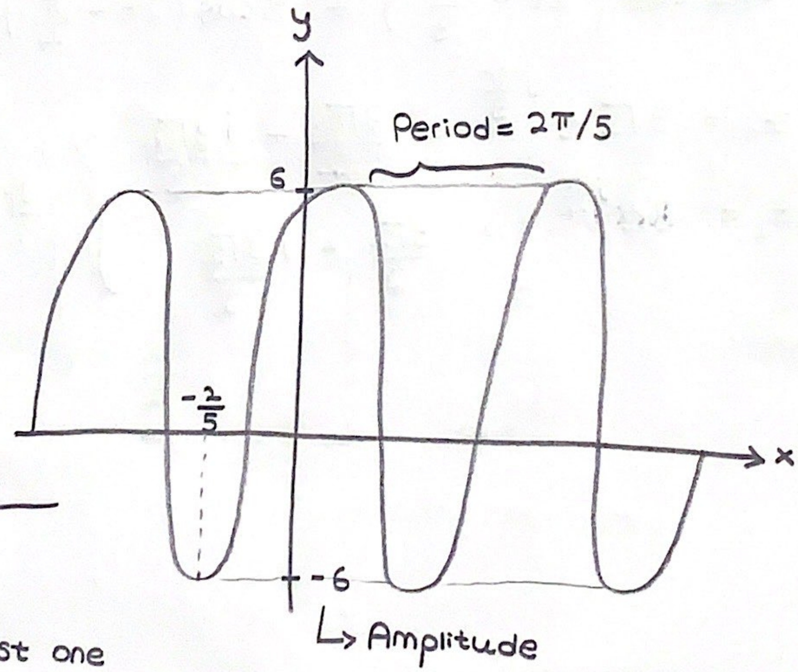
b. $x(t) = -6\cos(5t + 2)$

$A = -6$

$f = 5 \cdot \frac{1}{2\pi} = \frac{5\pi}{2}\ Hz$

$T = \frac{2\pi}{5}\ s$

These waves are more "loose" than the first one ←



2) $x(t) = 3\sqrt{2}\cos\left(2t - \frac{\pi}{3}\right)$

a.

$y(t) = 3\sin\left(2t + \frac{\pi}{4}\right)$

$$z(t) = 3\sqrt{2}\, e^{j\left(2t - \frac{\pi}{3}\right)} = 3\sqrt{2}\left(\frac{e^{j(2t)}}{e^{j\frac{\pi}{3}}}\right) = \frac{3\sqrt{2}}{e^{j\frac{\pi}{3}}}\, e^{2tj}$$

$$\text{Phasor}(x) = \frac{3\sqrt{2}}{e^{j\frac{\pi}{3}}}$$

$$3\sin\left(2t + \frac{\pi}{4}\right) = 3\cos\left(2t - \frac{\pi}{4}\right)$$

$$z(t) = 3e^{j\left(2t - \frac{\pi}{4}\right)} = 3\left(\frac{e^{j(2t)}}{e^{j\left(\frac{\pi}{4}\right)}}\right) = \frac{3}{e^{j\left(\frac{\pi}{4}\right)}}\, e^{2tj}$$

$$\text{Phasor}(y) = \frac{3}{e^{j\frac{\pi}{4}}}$$

b. $x(t) + y(t) = ?$

$$3\sqrt{2}\cos\left(2t - \frac{\pi}{3}\right) + 3\cos\left(2t - \frac{\pi}{4}\right) = ?$$

$$\left.\begin{aligned} z_1 &= 3\sqrt{2}\, e^{j\frac{\pi}{3}} = \frac{3\sqrt{2}}{2} - j\frac{3\sqrt{6}}{2} \\ z_2 &= 3e^{j\frac{\pi}{4}} = \frac{3\sqrt{2}}{2} - j\frac{3\sqrt{2}}{2} \end{aligned}\right\} \quad 3\sqrt{2} - j\,3\left[\sqrt{6} + \sqrt{2}\right]$$

3)

a. $x(t) = 3\sin(3t)\cos\left(\frac{\pi}{5}t + \frac{\pi}{3}\right) - 1$

$f = 3 \cdot \frac{1}{2\pi} = \frac{3}{2\pi}$

$T = \frac{2\pi}{3}$ s

$f = \frac{\pi}{5} \cdot \frac{1}{2\pi} = 0.1$

$T = 10$ s

$\frac{T_1}{T_2} = \frac{2\pi}{3} \cdot \frac{1}{\cancel{10}_{5}} = \frac{\pi}{15} \longrightarrow$ Aperiodic

b. $x(t) = 7\sin(\sqrt{3}t + 5) + 2\cos(\pi t)$

$f = \sqrt{3} \cdot \frac{1}{2\pi} = \frac{\sqrt{3}}{2\pi}$

$T = \frac{2\pi}{\sqrt{3}}$ s

$f = \frac{\cancel{\pi}}{2\cancel{\pi}} = \frac{1}{2}$

$T = 2$ s

$\frac{T_1}{T_2} = \frac{\cancel{2}\pi}{\sqrt{3}} \cdot \frac{1}{\cancel{2}}$

$= \frac{\pi}{\sqrt{3}}$

$\downarrow$

Aperiodic

c. $x(t) = \cos(12t) - 3\cos(16t)$

$f = \cancel{12}^{6} \cdot \frac{1}{\cancel{2}\pi} = \frac{6}{\pi}$

$T = \frac{\pi}{6}$ s

$f = \cancel{16}^{8} \cdot \frac{1}{\cancel{2}\pi} = \frac{8}{\pi}$

$T = \frac{\pi}{8}$ s

$\frac{T_1}{T_2} = \frac{\cancel{\pi}}{6} \cdot \frac{8}{\cancel{\pi}} = \frac{4}{3}$

$\downarrow$

Periodic

Fundamental frequency $= \frac{2}{\pi}$

Fundamental period $= \frac{\pi}{2}$

$\frac{6/\pi}{2/\pi} =$ 3rd harmonic

$\frac{8/\pi}{2/\pi} =$ 4th harmonic

d. $x(t) = \sin^2(60\sqrt{2}t) + \cos^2(24\sqrt{2}t + \frac{\pi}{4})$

$\cos 2\theta = 1 - \underbrace{2\sin^2\theta}_{\frac{1}{2}[1-\cos 2\theta]}$

$\sin^2(60\sqrt{2}t) = \frac{1}{2}\left[1 - \cos 120\sqrt{2}t\right]$

$f = 120\sqrt{2} \cdot \frac{1}{2\pi} = \frac{60\sqrt{2}}{\pi}$

$T = \frac{\pi}{60\sqrt{2}}$

$\cos 2\theta = \underbrace{2\cos^2\theta}_{\frac{1}{2}[\cos 2\theta - 1]} - 1$

$\cos^2(24\sqrt{2}t + \frac{\pi}{4})$

$\to \frac{1}{2}\left[\cos 48\sqrt{2}t + \frac{\pi}{2} - 1\right]$

$f = 48\sqrt{2} \cdot \frac{1}{2\pi} = \frac{24\sqrt{2}}{\pi}$

$T = \frac{\pi}{24\sqrt{2}}$

$\frac{T_1}{T_2} = \frac{\cancel{\pi}}{60\cancel{\sqrt{2}}} \cdot \frac{24\cancel{\sqrt{2}}}{\cancel{\pi}}$

$= \frac{2}{5}$

$\to$ Periodic

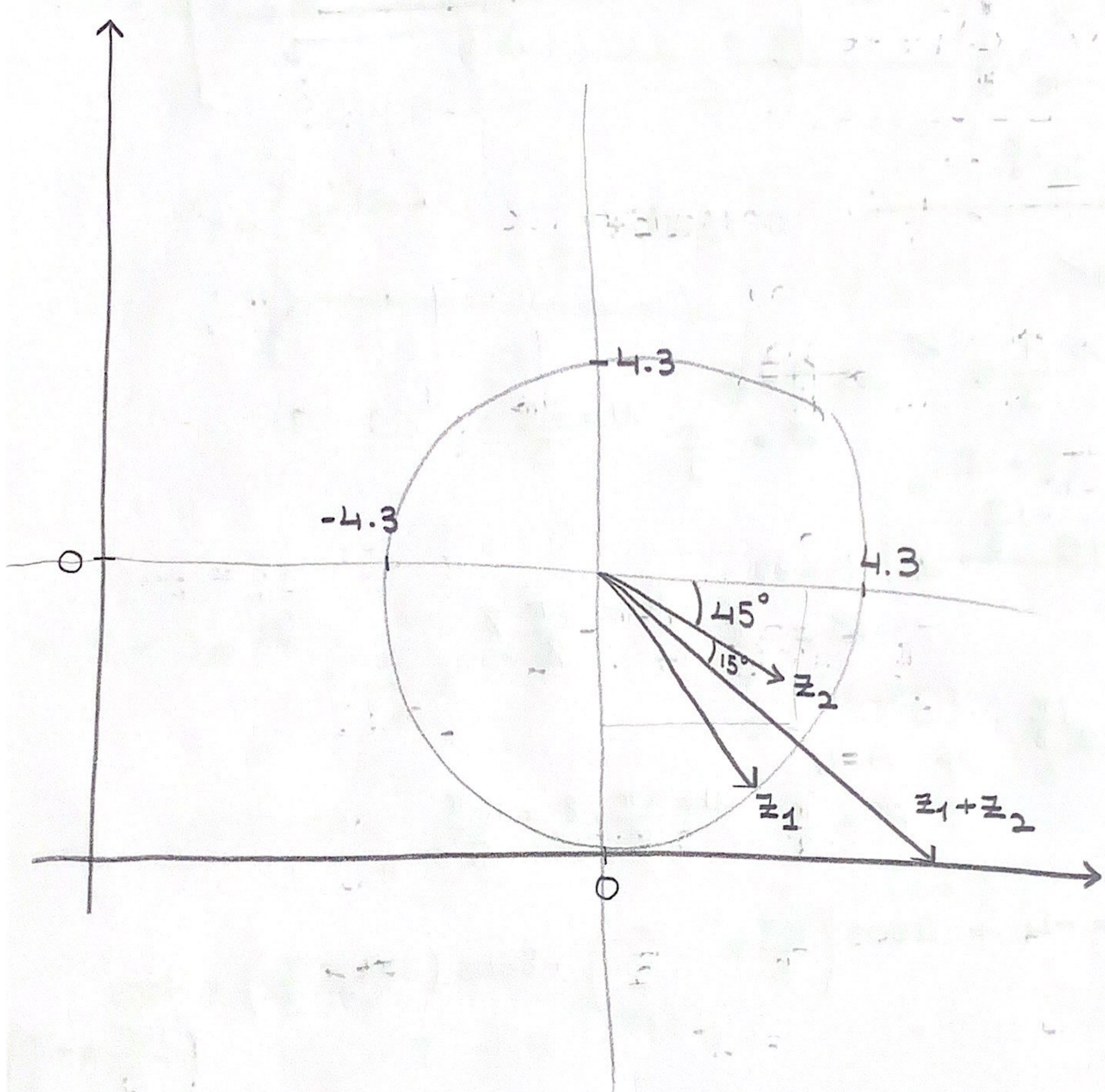
Fundamental frequency $= \frac{12\sqrt{2}}{\pi}$

Fundamental period $= \frac{\pi}{12\sqrt{2}}$

$\frac{\frac{24\sqrt{2}}{\pi}}{\frac{12\sqrt{2}}{\pi}} = $ 2nd harmonic

$\frac{\frac{60\sqrt{2}}{\pi}}{\frac{12\sqrt{2}}{\pi}} = $ 5th harmonic

Phasor Diagram



4) $a_0 = -4$

$a_1 = -j$

$a_5 = -4e^{-j\frac{\pi}{3}}$

$a_8 = 2e^{j\frac{\pi}{3}}$

$$A\left[\frac{e^{j(\omega_0 t + \varnothing)} + e^{-j(\omega_0 t + \varnothing)}}{2}\right]$$

$a_0 = -4$ ✓

$a_1 = -j \to \frac{A}{2} e^{-j\varnothing} \Rightarrow A = 2$

$\varnothing = -\frac{\pi}{2}$ $\quad \omega = \frac{2\pi}{5}$

$a_5 = -4e^{-j\frac{\pi}{3}} \Rightarrow A = -8$

$\varnothing = -\frac{\pi}{3}$ $\quad \omega = \frac{2\pi}{\cancel{5}} \cdot \cancel{5}$

$a_8 = 2e^{j\frac{\pi}{3}} \Rightarrow A = 4$

$\varnothing = \frac{\pi}{3}$ $\quad \omega = \frac{2\pi}{5} \cdot 8 = \frac{16\pi}{5}$

$$x(t) = -4 + 2\cos\left(\frac{2\pi}{5}t - \frac{\pi}{2}\right) - 8\cos\left(2\pi t - \frac{\pi}{3}\right) + 4\cos\left(\frac{16\pi}{5}t + \frac{\pi}{3}\right)$$

5)

$$x(t) = 5 - 2e^{-jt} + 3\left[\frac{e^{j(-t+\frac{\pi}{4})} - e^{-j(-t+\frac{\pi}{4})}}{2}\right]$$

$$-4\left[\frac{e^{j(3t+5)} + e^{-j(3t+5)}}{2}\right] + 3\left[\left(\frac{e^{j4t} + e^{-j4t}}{2}\right)\left(\frac{e^{j(5t+\frac{\pi}{2})} + e^{-j(5t+\frac{\pi}{2})}}{2}\right)\right]$$

$$= 5 - 2e^{-jt} + \frac{3}{2}e^{j(-t-\frac{\pi}{4})} - \frac{3}{2}e^{-j(-t-\frac{3\pi}{4})} - 2e^{j(3t+5)}$$

$$- 2e^{-j(3t+5)} + \frac{3}{4}e^{j(9t+\frac{\pi}{2})} + \frac{3}{4}e^{-j(t+\frac{\pi}{2})}$$

$$+ \frac{3}{4}e^{j(t+\frac{\pi}{2})} + \frac{3}{4}e^{-j(9t+\frac{\pi}{2})}$$

a. $a_0 = 5$ (for $k = 0$)

b. $f_0 = \frac{1}{2\pi}$ Hz $\rightarrow$ fundamental period $= 2\pi$ s

c. No, because $a_{-1} \neq a_1^*$

6)

a. $y(t) = \underbrace{c}_{\text{constants}}x(t-t_0)$

$$a_k' = \frac{1}{T_0}\int_0^{T_0} c\,x(t-t_0)\,e^{-j\left(\frac{2\pi}{T_0}\right)kt}\,dt = \frac{c}{T_0}\int_0^{T_0} x(t-t_0)\,e^{-j\left(\frac{2\pi}{T_0}\right)kt}\,dt$$

$$\left.\begin{aligned} u &= t-t_0 \\ du &= dt \end{aligned}\right\} = \frac{c}{T_0}\int_{-t_0}^{T_0-t_0} x(u)\,e^{-j\left(\frac{2\pi}{T_0}\right)k(u+t_0)}\,dt$$

$$= \frac{c}{T_0}\,e^{-j\left(\frac{2\pi}{T_0}\right)kt_0}\int_{-t_0}^{T_0-t_0} x(u)\,e^{-j\left(\frac{2\pi}{T_0}\right)ku}\,du$$

$$= c\cdot e^{-j\left(\frac{2\pi}{T_0}\right)kt_0}\cdot\underbrace{\frac{1}{T_0}\int_0^{T_0} x(u)\,e^{-j\left(\frac{2\pi}{T_0}\right)ku}\,du}_{a_k}$$

$$= a_k\,c\,e^{-j\left(\frac{2\pi}{T_0}\right)kt_0}$$

b. $$a_k' = \frac{1}{T_0}\int_0^{T_0}\frac{dx(t)}{dt}\,e^{-j\left(\frac{2\pi}{T_0}\right)kt}\,dt$$

$$= \frac{1}{T_0}\left(\left. e^{-j\left(\frac{2\pi}{T_0}\right)kt}\,x(t)\right|_0^{T_0} + j\left(\frac{2\pi}{T_0}\right)k\underbrace{\int_0^{T_0} x(t)\,e^{-j\left(\frac{2\pi}{T_0}\right)kt}\,dt}_{a_k T_0}\right)$$

$$\left(\begin{aligned} u &= e^{-j\left(\frac{2\pi}{T_0}\right)kt} \\ v &= x(t) \qquad du = -j\left(\frac{2\pi}{T_0}\right)k\,e^{-j\left(\frac{2\pi}{T_0}\right)kt}\,dt \\ dv &= dx(t) \end{aligned}\right)$$

$$\boxed{j\left(\frac{2\pi}{T_0}\right)k\,a_k}$$

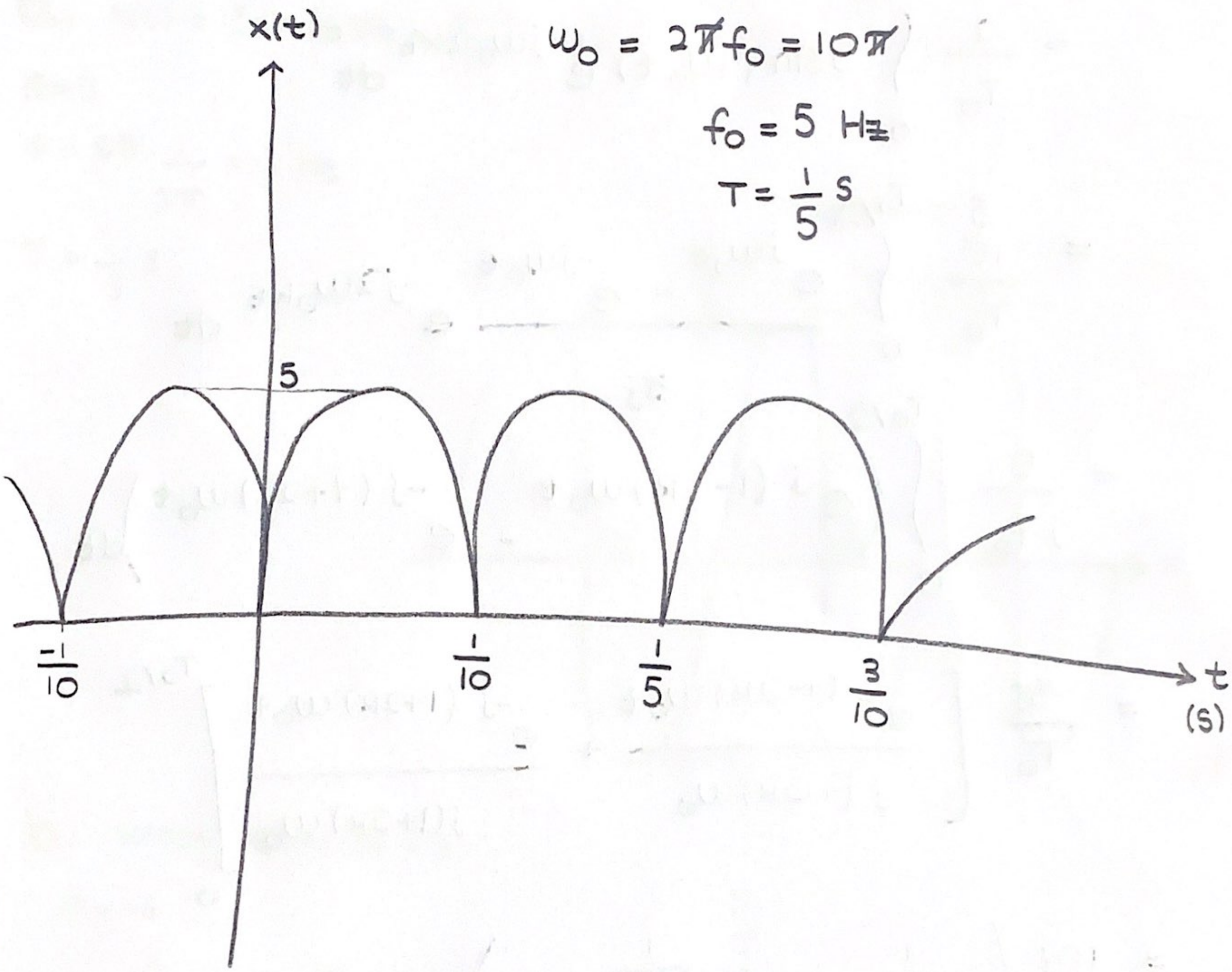
7.

$\omega_o = 10\pi$ rad/sec

$\omega_o = 2\pi f_o = 10\pi$

$f_o = 5$ Hz

$T = \frac{1}{5}$ s



$$x(t) = \begin{cases} 5\sin(\omega_o t), & \text{for } kT_o \leq t \leq kT_o + \frac{T_o}{2} \\ -5\sin(\omega_o t), & \text{for } kT_o + \frac{T_o}{2} \leq t \leq kT_o + T_o \end{cases}$$

for all $k = 0, \pm 2, \pm 4, \ldots$

$$a_k = \frac{1}{T_0'}\int_0^{T_0'} x(t)\, e^{-j\omega_0' k t}\, dt$$

$$= \frac{2}{T_0}\int_0^{T_0/2} 5\sin(\omega_0 t)\, e^{-j\omega_0 2kt}\, dt$$

$$= \frac{\overset{5}{\cancel{10}}}{T_0}\int_0^{T_0/2} \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{\cancel{2}j}\, e^{-j2\omega_0 kt}\, dt$$

$$= \frac{5}{jT_0}\int_0^{T_0/2}\left(e^{j(1-2k)\omega_0 t} - e^{-j(1+2k)\omega_0 t}\right)dt$$

$$= \frac{-5j}{T_0}\left[\frac{e^{j(1-2k)\omega_0 t}}{j(1-2k)\omega_0} + \frac{e^{-j(1+2k)\omega_0 t}}{j(1+2k)\omega_0}\right]_0^{T_0/2}$$

$$= \frac{10j}{T_0}\left(\frac{1}{j(1-2k)\omega_0} + \frac{1}{j(1+2k)\omega_0}\right) = \frac{10j}{T_0}\left(\frac{-j}{(1-2k)\omega_0} + \frac{-j}{(1+2k)\omega_0}\right)$$

$$= \frac{10}{\pi(1-4k^2)}$$