EEE 391

Homework 2

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1.

a) $y[n] = x[n] + 3x[n-2] + x[n-4]$

Take z-transform

$$Y(z) = X(z) + 3X(z)z^{-2} + X(z)z^{-4}$$

$$Y(z) = X(z)(1 + 3z^{-2} + z^{-4})$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + \frac{3}{z^2} + \frac{1}{z^4}$$

Let $z = e^{jw}$ (frequency domain)

$$H(e^{jw}) = 1 + \frac{3}{(e^{jw})^2} + \frac{4}{(e^{jw})^4}$$

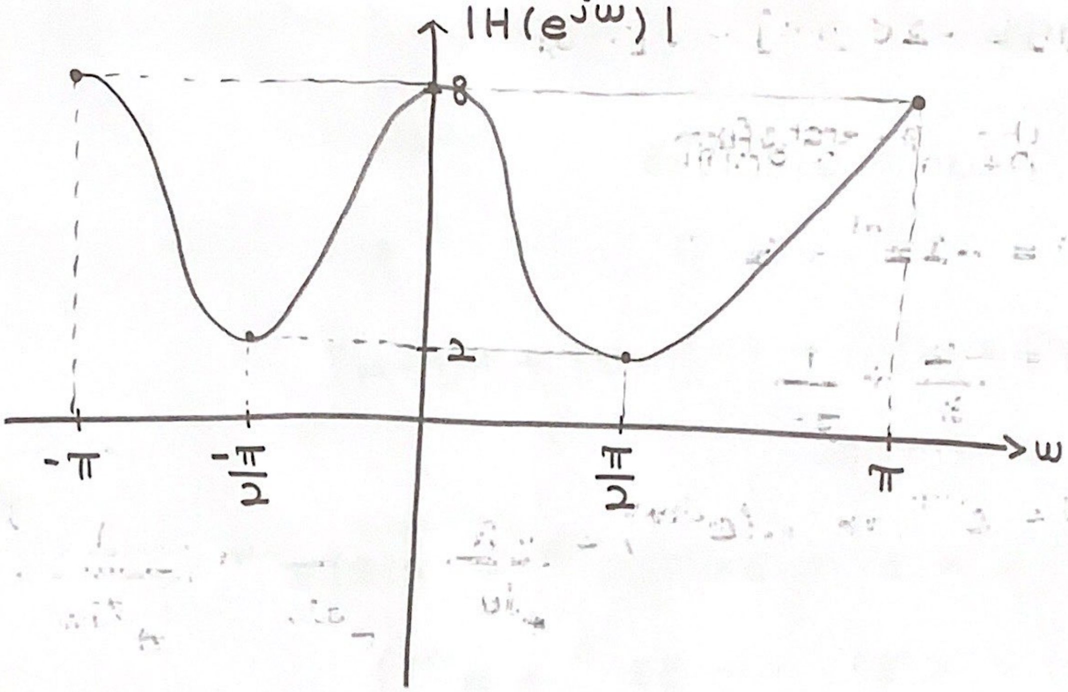
$$= 1 + \frac{3}{e^{2jw}} + \frac{4}{e^{4jw}} = \frac{4 + 3e^{2jw} + e^{4jw}}{e^{4jw}}$$

b) $$|H(e^{jw})| = \frac{|4 + 3e^{2jw} + e^{4jw}|}{|e^{4jw}|}$$

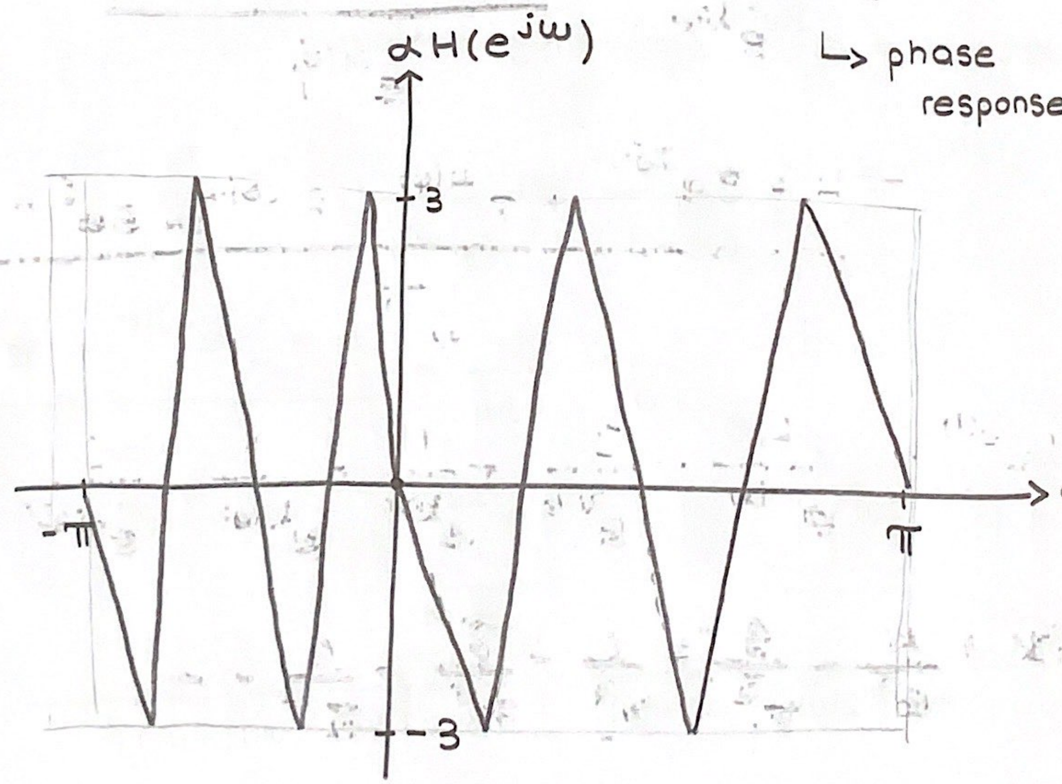
$$= \sqrt{(4 + 3\cos(2w) + \cos(4w))^2 + (3\sin(2w) + \sin(4w))^2}$$

$$= \sqrt{2(8\cos^2(2w) + 15\cos(2w) + 9)}$$ $\Rightarrow$ magnitude response

$\longrightarrow$



$$\measuredangle H(e^{j\omega}) = \arctan\left[\frac{3\sin(2\omega) + \sin(4\omega)}{4 + 3\cos(2\omega) + \cos(4\omega)}\right] - 4\omega$$



↳ phase response

c) $x[n] = -2\sigma[n-1] + \sigma[n-3]$

Take the z - transform

$$X(z) = -2z^{-1} + z^{-3}$$

$$X(z) = \frac{-2}{z} + \frac{1}{z^3}$$

Let $z = e^{j\omega} \Rightarrow X(e^{j\omega}) = \dfrac{-2}{e^{j\omega}} + \dfrac{1}{e^{3j\omega}} = \dfrac{1-2e^{3j\omega}}{e^{3j\omega}}$

$$y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$$

$$y(e^{j\omega}) = \frac{1-2e^{3j\omega}}{e^{3j\omega}} \cdot \frac{4 + 3e^{2j\omega} + e^{4j\omega}}{e^{4j\omega}}$$

$$= \frac{4 + 3e^{2j\omega} + e^{4j\omega} - 8e^{3j\omega} - 6e^{5j\omega} - 2e^{7j\omega}}{e^{7j\omega}}$$

$$y(e^{j\omega}) = \frac{4}{e^{7j\omega}} + \frac{3}{z^{5j\omega}} + \frac{1}{e^{3j\omega}} - \frac{8}{e^{4j\omega}} - \frac{6}{e^{2j\omega}} - 2$$

$$y(z) = \frac{4}{z^7} + \frac{3}{z^5} + \frac{1}{z^3} - \frac{8}{z^4} - \frac{6}{z^2} - 2$$

$$y[n] = 4\sigma[n-7] + 3\sigma[n-5] + \sigma[n-3] - 8\sigma[n-4] - 6\sigma[n-2]$$
$$- 2\sigma[n]$$

$\longrightarrow$

d) $y_3[n] = \left(x[n] * h[n-1] \right) * h[n]$

$$y_3[n] = x[n] * \left(h[n-1] * h[n] \right)$$

$$Y_3(z) = X(z) \cdot \left[\frac{H(z)}{z} \cdot H(z) \right]$$

Let $z = e^{jw}$

$$Y_3(e^{jw}) = X(e^{jw}) \left(\frac{H(e^{jw})}{e^{jw}} \cdot H(e^{jw}) \right)$$

$$x[n] = 3\underset{\substack{\uparrow\\ \text{given}}}{\sigma}[n-1] \Rightarrow X(z) = \frac{3}{z} \Rightarrow X(e^{jw}) = \frac{3}{e^{jw}}$$

$$Y_3(e^{jw}) = \frac{3}{e^{jw}} \left(\frac{4 + 3e^{2jw} + e^{4jw}}{e^{5jw}} \cdot \frac{4 + 3e^{2jw} + e^{4jw}}{e^{4jw}} \right)$$

$$Y_3(e^{jw}) = \frac{3(4 + 3e^{2jw} + e^{4jw})^2}{e^{10jw}}$$

$$= \frac{3\,(16 + 9e^{4jw} + e^{8jw} + 24e^{2jw} + 8e^{4jw} + 6e^{6jw})}{e^{10jw}}$$

$$Y_3(e^{jw}) = \frac{48 + 51e^{4jw} + 3e^{8jw} + 72e^{2jw} + 18e^{6jw}}{e^{10jw}}$$

$\longrightarrow$

$$y_3(e^{j\omega}) = \frac{48}{e^{10j\omega}} + \frac{3}{e^{2j\omega}} + \frac{51}{e^{6j\omega}} + \frac{18}{e^{4j\omega}} + \frac{72}{e^{8j\omega}}$$

$$y_3(z) = \frac{48}{z^{10}} + \frac{3}{z^2} + \frac{51}{z^6} + \frac{18}{z^4} + \frac{72}{e^8}$$

$$y_3[n] = 48\sigma[n-10] + 3\sigma[n-2] + 51\sigma[n-6] + 18\sigma[n-4] + 72\sigma[n-8]$$

2.

a) $H(z) = 2 + 3z^{-1} + z^{-2}$

i) $H(z) = 2 + \frac{3}{z} + \frac{1}{z^2} = \frac{2z^2 + 3z + 1}{z^2}$

$$H(z) = \frac{y(z)}{x(z)} = \frac{2z^2 + 3z + 1}{z^2}$$

$$z^2 y(z) = (2z^2 + 3z + 1)\, x(z)$$

$$z^2 y(z) = 2z^2 x(z) + 3z\, x(z) + x(z)$$

$$y[n+2] = 2x[n+2] + 3x[n+1] + x[n]$$

$$y[n] = 2x[n] + 3x[n-1] + x[n-2]$$

<u>LTI</u>

ii) $x[n] = \sigma[n] - \sigma[n-1]$

$$x(z) = 1 - \frac{1}{z} = \frac{z-1}{z}$$

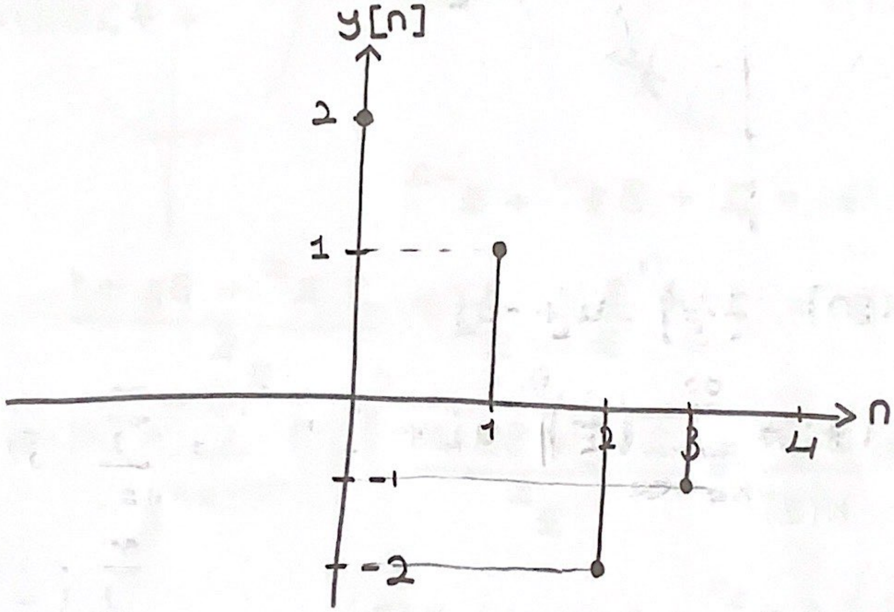
$$y(z) = x(z)\,H(z) = \frac{z-1}{z}\left(\frac{2z^2 + 3z + 1}{z^2}\right)$$

→

$$y(z) = \frac{2z^3 + 3z^2 + z - 2z^2 - 3z - 1}{z^3}$$

$$y(z) = \frac{2z^3 + z^2 - 2z - 1}{z^3} = 2 + \frac{1}{z} - \frac{2}{z^2} - \frac{1}{z^3}$$

$$y[n] = 2\sigma[n] + \sigma[n-1] - 2\sigma[n-2] - \sigma[n-3]$$



b) i) $a^n u[n]$

$$x[n] = a^n u[n]$$

$$x(z) = \sum_{n=-\infty}^{+\infty} a^n u[n] z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n$$

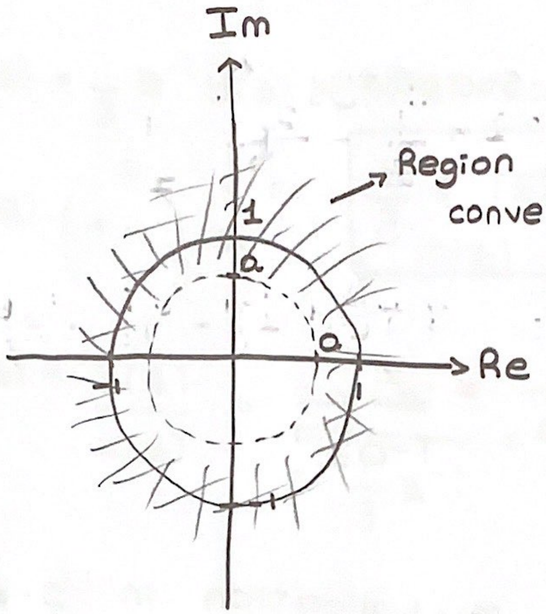
For convergence of $x(z)$ we require that $\sum_{n=0}^{\infty} |az^{-1}|^n < \infty$

Therefore, Region of convergence is $|az^{-1}| < 1$

$$|z| > |a|$$

$$x(z) = \sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1-az^{-1}} = \frac{z}{z-a} \quad \text{for } |z| > |a|$$

For stability Region of convergence should contain unit circle



For stability;

$\boxed{|a| < 1} \Leftrightarrow |z| > |a|$

$\boxed{0 \leq z < 1}$

ii) $x[n] = \left(\frac{1}{5}\right)^n u[n-3]$

$$x(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{5}\right)^n u[n-3]\, z^{-n} = \sum_{n=3}^{\infty} \left(\frac{1}{5}\right)^n z^{-n}$$

$$= \sum_{n=3}^{\infty} \left(\frac{1}{5} z^{-1}\right)^n$$

$$x(z) = \sum_{n=0}^{\infty} \left(\frac{1}{5} z^{-1}\right)^n - 1 - \left(\frac{1}{5} z^{-1}\right) - \left(\frac{1}{5} z^{-1}\right)^2$$

$$x(z) = \frac{1}{1 - \frac{1}{5} z^{-1}} - \frac{1}{5z} - \frac{1}{25z^2} - 1$$

$$x(z) = \frac{5z}{5z - 1} - \frac{1}{5z} - \frac{1}{25z^2} - 1$$

$$x(z) = \frac{\cancel{625z^4} - \cancel{125z^3} + \cancel{25z^2} - \cancel{25z^2} + 5z - \cancel{625z^4} + \cancel{125z^3}}{(5z-1)(5z)(25z^2)}$$

$\longrightarrow$

$$X(z) = \frac{5z}{625z^4 - 125z^3} = \frac{z}{125z^4 - 25z^3} = \frac{z^{-3}}{125 - 25z^{-1}}$$

$$X(z) = \frac{z^{-3}}{1 - \left(\frac{1}{5}\right)z^{-1}}$$

For stability;

$$\boxed{|z| > \frac{1}{5}}$$

c) i) $X(z) = \frac{1}{z-a} = \frac{z^{-1}}{1-az^{-1}}$

$$\frac{z^{-1}}{1-az^{-1}} = a^n u[n]$$

multiplication in z-domain corresponds to time shift!

Time shifting $\Rightarrow$ $\boxed{a^{n-1} u[n-1]}$

ii) $X(z) = z^{-4}\left(\frac{1}{1-\frac{3}{7}z^{-1}}\right)$

$$z^{-1}\left(\frac{1}{1-\frac{3}{7}z^{-1}}\right) = \left(\frac{3}{7}\right)^n u[n]$$

Since multiplication in z-domain corresponds to time shift;

$$z^{-1}\left(z^{-4}\frac{1}{1-\frac{3}{7}z^{-1}}\right) = \boxed{\left(\frac{3}{7}\right)^{n-4} u[n-4]}$$

3.

a) i) $y[n] = \frac{1}{3} y[n-1] + x[n] - 4x[n-2]$

$$Y(z) = \frac{1}{3} z^{-1} Y(z) + X(z) - 4z^{-2} X(z)$$

$$Y(z)\left(1 - \frac{1}{3} z^{-1}\right) = X(z)\left(1 - 4z^{-2}\right)$$

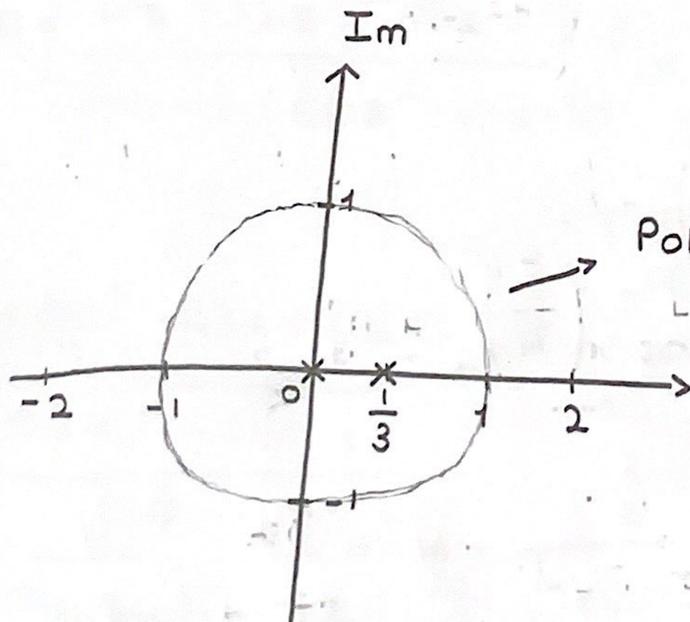
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 4z^{-2}}{1 - \frac{1}{3} z^{-1}} = \frac{1 - \frac{4}{z^2}}{1 - \frac{1}{3z}} = \frac{z^2 - 4}{z^2} \cdot \frac{3z}{3z - 1}$$

$$= \frac{3(z^2 - 4)}{z(3z - 1)}$$

$$H(z) = \frac{3(z-2)(z+2)}{z(3z-1)}$$

Zeros: $z = 2, -2$

Poles: $z = 0, \frac{1}{3}$



Poles are inside the unit circle. System is stable.

→

ii) $$H(z) = \frac{(z+0.3)(z-0.2)}{(z^2 - 0.7z + 0.1)}$$

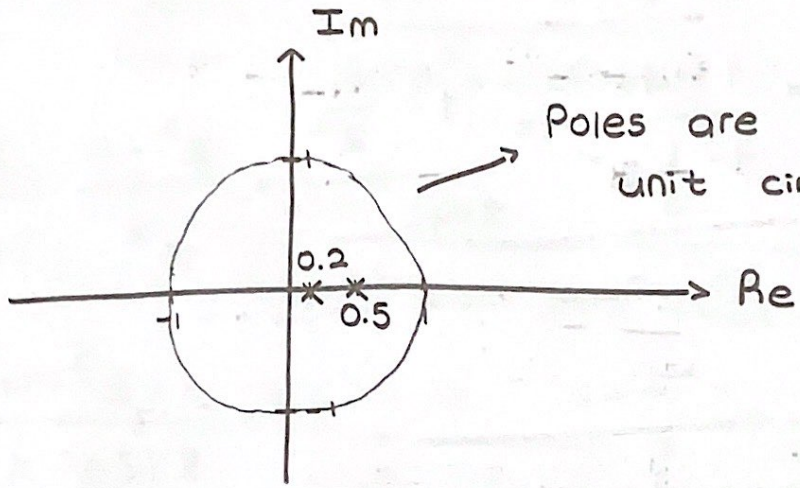
$z \quad -0.5$

$z \quad -0.2$

$$= \frac{(z+0.3)(z-0.2)}{(z-0.5)(z-0.2)}$$

Zeros: $z = -0.3, 0.2$

Poles: $z = 0.5, 0.2$



Poles are inside the unit circle. System is stable.

b) $$y[n] = \frac{1}{4}^n u[n] \xrightarrow{z} \frac{1}{1-\frac{1}{4}z^{-1}}, \quad |z| > |a|$$

$$\frac{1}{1-\frac{1}{4}z^{-1}} = \frac{4z}{4z-1}$$

$$x[n] = \left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{4}\right)^{n-1} u[n]$$

$\downarrow$ z - transform

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{z^{-1}}{1-\frac{1}{4}z^{-1}} + 4 = \frac{3z}{3z-1} - \frac{4}{4z-1} + 4$$

$\longrightarrow$

$$X(z) = \frac{12z^2 - 3z - 12z + 4 + 48z^2 - 28z + 4}{12z^2 - 7z + 1}$$

$$X(z) = \frac{60z^2 - 43z + 8}{12z^2 - 7z + 1} \Rightarrow H(z) = \frac{Y(z)}{X(z)}$$

$$H(z) = \frac{4z}{4z-1} \cdot \frac{12z^2 - 7z + 1}{60z^2 - 43z + 8}$$

$$H(z) = \frac{48z^3 - 28z^2 + 4z}{240z^3 - 172z^2 + 32z - 60z^2 + 43z - 8}$$

$$= \frac{48z^3 - 28z^2 + 4z}{240z^3 - 232z^2 + 75z - 8} = \frac{48 - 28z^{-1} + 4z^{-2}}{240 - 232z^{-1} + 75z^{-2} - 8z^{-3}}$$

$$H(z) = \frac{4z(12z^2 - 7z + 1)}{(4z-1)(60z^2 - 43z + 8)} = \frac{4z(1-3z)(1-4z)}{(4z-1)(60z^2 - 43z + 8)}$$

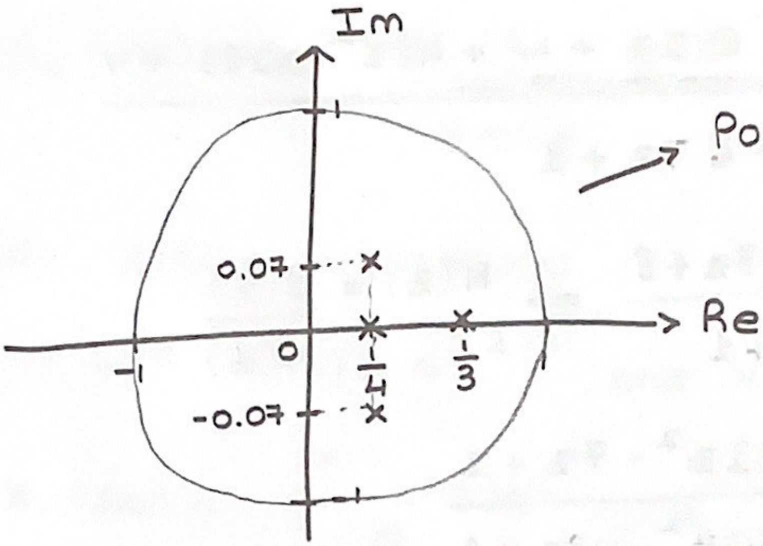
Zeros: $z = 0, \frac{1}{3}, \frac{1}{4}$

Poles: $z = \frac{1}{4},\ 0.358 + 0.07j,\ 0.358 - 0.07j$

$$y = 60z^2 - 43z + 8 = 0$$

$$\frac{43 \pm \sqrt{43^2 - 240 \cdot 8}}{120}$$

$$z_1, z_2 = \frac{43}{120} \pm \frac{j\sqrt{71}}{120}$$



Poles are inside unit circle. System is stable.

4.

a) i) $\delta(t-2) - 3\delta(t-3)$

$\delta(t) \longrightarrow 1$

$x(t-t_0) \longrightarrow e^{-j\omega t_0} X(j\omega)$

$x(t) = \delta(t-2) - 3\delta(t-3) \longrightarrow X(j\omega) = e^{-2j\omega} - 3e^{-3j\omega}$

ii) $x(t) = e^{-2t}u(t) \longrightarrow \int_{-\infty}^{\infty} e^{-2t} u(t) e^{-j\omega t} dt$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-t(2+j\omega)} = \frac{e^{-t(2+j\omega)}}{2+j\omega}\Bigg|_{0}^{\infty} = \frac{1}{2+j\omega}$$

iii) $x_2(t) = e^{-3(t-4)} u(t-4)$

Let $x_1(t) = e^{-3t} u(t)$

$x_2(t) = x_1(t-4) \Rightarrow X_1(j\omega) = \frac{1}{3+j\omega}$

$$X_2(j\omega) = e^{-4j\omega}\frac{1}{3+j\omega} = \frac{e^{-4j\omega}}{3+j\omega}$$

iv) $x(t) = x_1(t)\,x_2(t)$

Let $x_1(t) = e^{-2|t|}$ and $x_2(t) = \cos(t)$

$$X_1(j\omega) = \int_{-\infty}^{\infty} e^{-2|t|} e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{2t} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-2t} e^{-j\omega t}\, dt$$

$$X_1(j\omega) = \left.\frac{e^{t(2-j\omega)}}{2-j\omega}\right|_{-\infty}^{0} + \left.\frac{-e^{-t(2+j\omega)}}{2+j\omega}\right|_{0}^{\infty}$$

$$X_2(j\omega) = \frac{1}{2-j\omega} + \frac{1}{2+j\omega} = \frac{4}{4+\omega^2}$$

$$X_2(j\omega) = \pi\left(\delta(\omega-1) + \delta(\omega+1)\right)$$

$$x(t) = x_1(t) + x_2(t) \longrightarrow X(j\omega) = \frac{1}{2\pi}\left(X_1(j\omega) * X_2(j\omega)\right)$$

$$X(j\omega) = \frac{1}{2\pi}\left[\pi\left(\delta(\omega-1) + \delta(\omega+1)\right) * \frac{4}{4+\omega^2}\right]$$

$\longrightarrow$

$$X(jw) = \frac{1}{2}\left[\left(\delta(w-1) * \frac{4}{4+w^2}\right) + \left(\delta(w+1) * \frac{4}{4+w^2}\right)\right]$$

$$X(jw) = \frac{1}{2}\left[\frac{4}{4+(w-1)^2} + \frac{4}{4+(w+1)^2}\right] = 2\left[\frac{2w^2+10}{w^4+6w^2+25}\right]$$

$$X(jw) = 4\left[\frac{w^2+5}{w^4+6w^2+25}\right]$$

b) i) $X(jw) = e^{-3jw} + e^{-5jw} \longrightarrow x(t) = \delta(t-3) + \delta(t-5)$

using the result from a) i)

ii) $X(jw) = 2\pi\left(\delta(w-2) + \delta(w+2)\right)$

$* \cos(w_0 t) \longrightarrow \pi\delta(w-w_0) + \pi\delta(w+w_0)$

Therefore;

$$X(jw) = 2\pi\left[\delta(w-2) + \delta(w+2)\right] \rightarrow 2\cos(2t)$$

iii) $X(jw) = \cos\left(w + \frac{\pi}{4}\right) = \cos(w)\cos\left(\frac{\pi}{4}\right) - \sin(w)\sin\left(\frac{\pi}{4}\right)$

$$X(jw) = \frac{1}{\sqrt{2}}\left(\cos(w) - \sin(w)\right)$$

$\longrightarrow$

$$\cos(t) \longrightarrow \pi\left[\delta(\omega-1)+\delta(\omega+1)\right]$$

$$\sin(t) \longrightarrow \frac{\pi}{j}\left[\delta(\omega-1)-\delta(\omega+1)\right]$$

$$X(\omega=t) \longrightarrow 2\pi x(t=-\omega)$$

$$x(t)=\frac{1}{\sqrt{2}}\left[\frac{1}{2\pi}\left(\pi\left(\delta(-t-1)+\delta(-t+1)\right)\right)-\right.$$

$$\left.\frac{1}{2\pi}\left(\frac{\pi}{j}\left(\delta(-t-1)-\delta(-t+1)\right)\right)\right]$$

$$x(t)=\frac{1}{2\sqrt{2}}\left(\delta(-t-1)+\delta(-t+1)\right)-$$

$$\frac{1}{2\sqrt{2}j}\left(\delta(-t-1)-\delta(-t+1)\right)$$

$$x(t)=\frac{1}{2\sqrt{2}}\left(\delta(t-1)+\delta(t+1)\right)-\frac{1}{2\sqrt{2}j}\left(\delta(t-1)-\delta(t+1)\right)$$

$$x(t)=\delta(t-1)\left[\frac{1}{2\sqrt{2}}-\frac{1}{2\sqrt{2}j}\right]+\delta(t+1)\left[\frac{1}{2\sqrt{2}}+\frac{1}{2\sqrt{2}j}\right]$$

$$x(t)=\frac{\delta(t-1)}{2\sqrt{2}}(1+j)+\frac{\delta(t+1)}{2\sqrt{2}}(1-j)$$

5. $y(t) = \Big[\big(x(t) * h_1(t)\big) + \big(x(t) * h_2(t)\big)\Big] * h_3(t)$

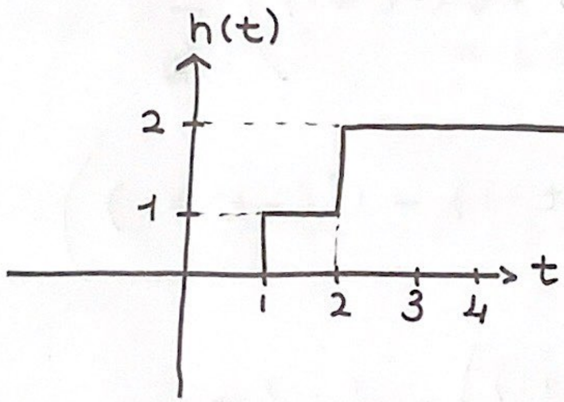
$$h(t) = \Big[\big(\delta(t) * h_1(t)\big) + \big(\delta(t) * h_2(t)\big)\Big] * h_3(t)$$

$$h(t) = \big[h_1(t) + h_2(t)\big] * h_3(t)$$

a) $h(t) = \big[\delta(t-3) + \delta(t-2)\big] * u(t+1)$

$$h(t) = \big[u(t+1) * \delta(t-3)\big] + \big[u(t+1) * \delta(t-2)\big]$$

$$h(t) = u(t-2) + u(t-1)$$



when $t < 0$;

$h(t) = 0$

Causal ✓

$\int_{-\infty}^{\infty} |h(t)|\,dt \rightarrow$ not finite

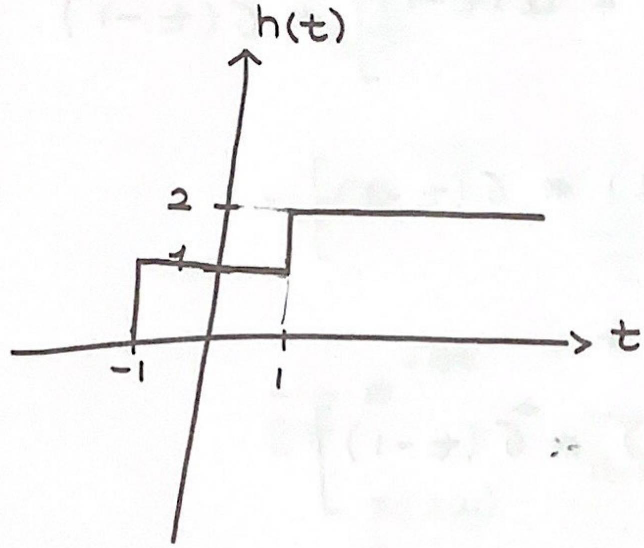
Not Stable ✓

b) $h(t) = \big[\delta(t-1) + \delta(t+1)\big] * u(t)$

$$h(t) = \big[u(t) * \delta(t-1)\big] + \big[u(t) * \delta(t+1)\big]$$

$\longrightarrow$

$h(t) = u(t-1) + u(t+1)$



When $t < 0$;

$h(t) \neq 0$

Not causal ✓

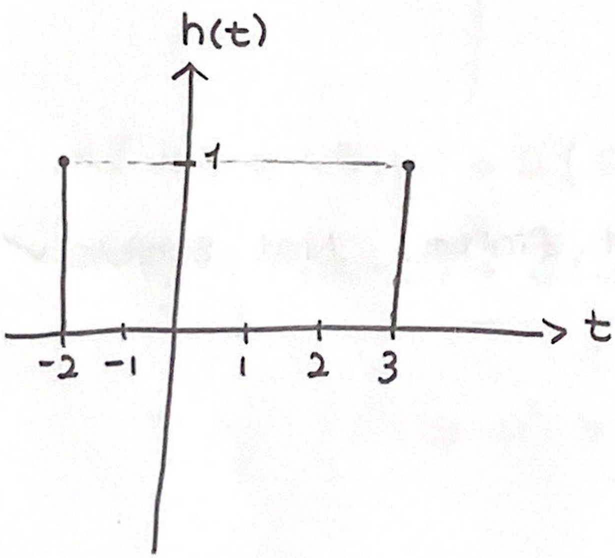
$\int_{-\infty}^{\infty} |h(t)|\,dt \rightarrow$ not finite

Not stable ✓

c) $h(t) = [\delta(t-2) + \delta(t+3)] * \delta(t-1)$

$h(t) = [\delta(t-2) * \delta(t-1)] + [\delta(t+3) * \delta(t-1)]$

$h(t) = \delta(t-3) + \delta(t+2)$



$\int_{-\infty}^{\infty} |h(t)|\,dt = 2 \Rightarrow$ finite

Stable ✓

When $t < 0$;

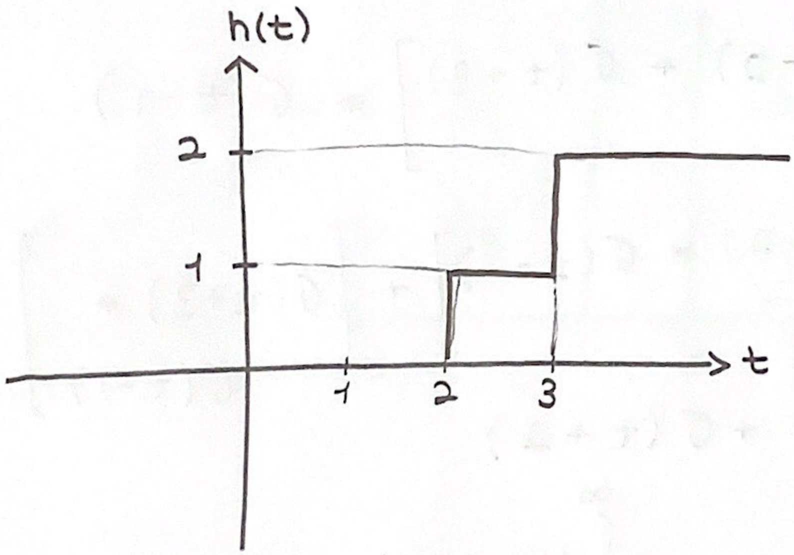
$h(t) = 0 \rightarrow$ Causal ✓

$\longrightarrow$

d) $h(t) = \left[u(t-2) + u(t-1) \right] * \delta(t-1)$

$$h(t) = \left[u(t-2) * \delta(t-1) \right] + \left[u(t-1) * \delta(t-1) \right]$$

$$h(t) = u(t-3) + u(t-2)$$



$$\int_{-\infty}^{\infty} |h(t)|\, dt = \infty$$ (not finite) Not stable ✓

When $t < 0$;

$h(t) = 0$ Causal ✓

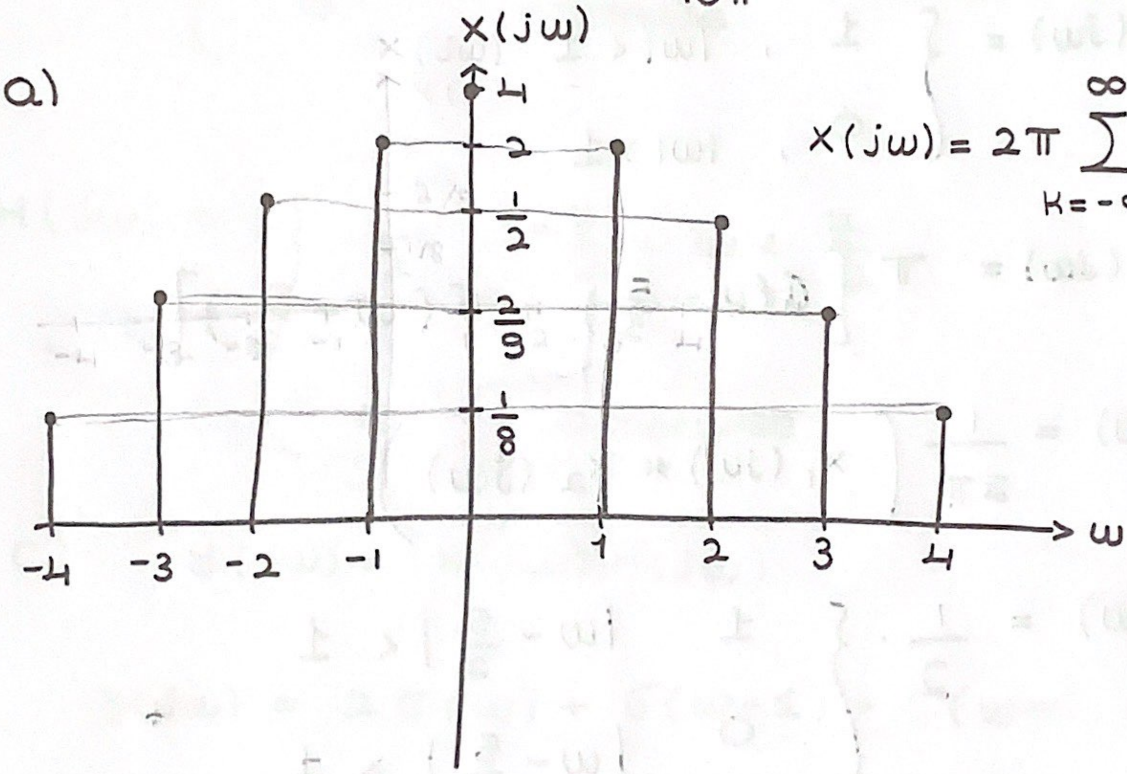
→

6. $x(t) = \frac{1}{16\pi} e^{-j4t} + \frac{1}{9\pi} e^{-j3t} + \frac{1}{4\pi} e^{-j2t} +$

$\frac{1}{\pi} e^{-jt} + \frac{2}{\pi} + \frac{1}{\pi} e^{jt} + \frac{1}{4\pi} e^{-j2t} +$

$\frac{1}{9\pi} e^{j3t} + \frac{1}{16\pi} e^{-j4t} + \dots$

a)



$$X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$$

$$X(j\omega) = 4\delta(\omega) + 2\left(\delta(\omega-1) + \delta(\omega+1)\right) +$$

$$\frac{1}{2}\left(\delta(\omega-2) + \delta(\omega+2)\right) + \frac{2}{9}\left(\delta(\omega-3) + \delta(\omega+3)\right)$$

$$+ \frac{1}{8}\left(\delta(\omega-4) + \delta(\omega+4)\right) + \dots$$

$\longrightarrow$

b) $h(t) = \frac{1}{\pi t} \sin(t) \cos\left(\frac{5}{2} t\right)$

$$= x_1(t)\, x_2(t)$$

Let $x_1(t) = \frac{\sin(t)}{\pi t}$ and $x_2(t) = \cos\left(\frac{5}{2} t\right)$

$$X_1(j\omega) = \begin{cases} 1, & |\omega| < 1 \\ 0, & |\omega| > 1 \end{cases}$$

$$X_2(j\omega) = \pi \left[\delta\left(\omega - \frac{5}{2}\right) + \delta\left(\omega + \frac{5}{2}\right) \right]$$

$$H(j\omega) = \frac{1}{2\pi} \left(X_1(j\omega) * X_2(j\omega) \right)$$

$$H(j\omega) = \frac{1}{2} \cdot \begin{cases} 1 & \left|\omega - \frac{5}{2}\right| < 1 \\ 0 & \left|\omega - \frac{5}{2}\right| > 1 \end{cases} +$$

$$\frac{1}{2} \cdot \begin{cases} 1 & \left|\omega + \frac{5}{2}\right| < 1 \\ 0 & \left|\omega + \frac{5}{2}\right| > 1 \end{cases}$$

$$-1 < \omega - \frac{5}{2} < 1 \Rightarrow -\frac{3}{2} < \omega < \frac{7}{2}$$

$$-1 < \omega + \frac{5}{2} < 1 \Rightarrow -\frac{7}{2} < \omega < -\frac{3}{2}$$

$\rightarrow$

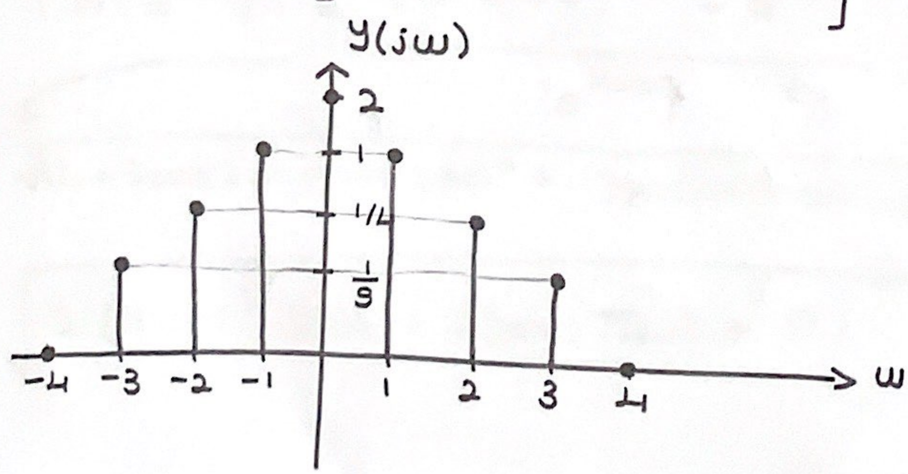
$$H(j\omega) = \begin{cases} \frac{1}{2} & -\frac{3}{2} < \omega < \frac{7}{2} \\ 0 & \text{otherwise} \end{cases} \quad +$$

$$\begin{cases} \frac{1}{2} & -\frac{7}{2} < \omega < -\frac{3}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$H(j\omega) = \begin{cases} \frac{1}{2} & -\frac{7}{2} < \omega < \frac{7}{2} \\ 0 & \text{otherwise} \end{cases}$$

c) $y(j\omega) = x(j\omega)\,H(j\omega)$

$$y(j\omega) = 2\sigma(\omega) + \sigma(\omega-1) + \sigma(\omega+1) + \frac{1}{4}\left[\sigma(\omega-2) + \sigma(\omega+2)\right] + \frac{1}{9}\left[\sigma(\omega-3) + \sigma(\omega+3)\right]$$



d) $Y(j\omega) \longrightarrow y(t)$

$$y(t) = 2 \cdot \frac{1}{2\pi} + \cos(t) \frac{1}{\pi} + \frac{1}{\pi} \frac{1}{4} \cos(2t)$$

$$+ \frac{1}{\pi} \frac{1}{9} \cos(3t)$$

$$y(t) = \frac{1}{\pi} \left(1 + \cos(t) + \frac{1}{4} \cos(2t) + \frac{1}{9} \cos(3t) \right)$$