

Dataset



Classification → tell

Color
weight

Dog
Dog
1
2
3
n



Learning Method Classification (2)

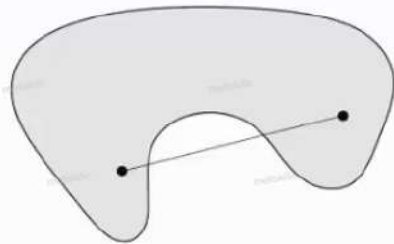
Semi-supervised learning

Reinforcement learning

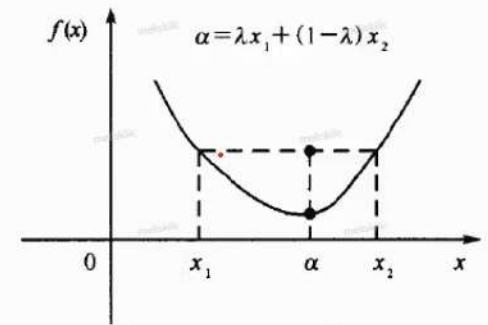
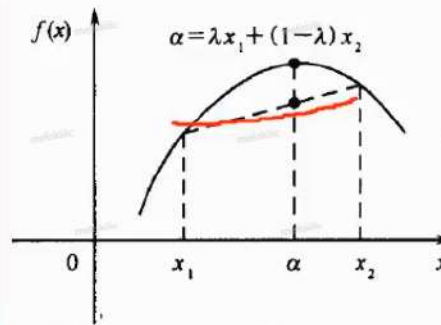
rewards and punishment



Concave and Convex Functions



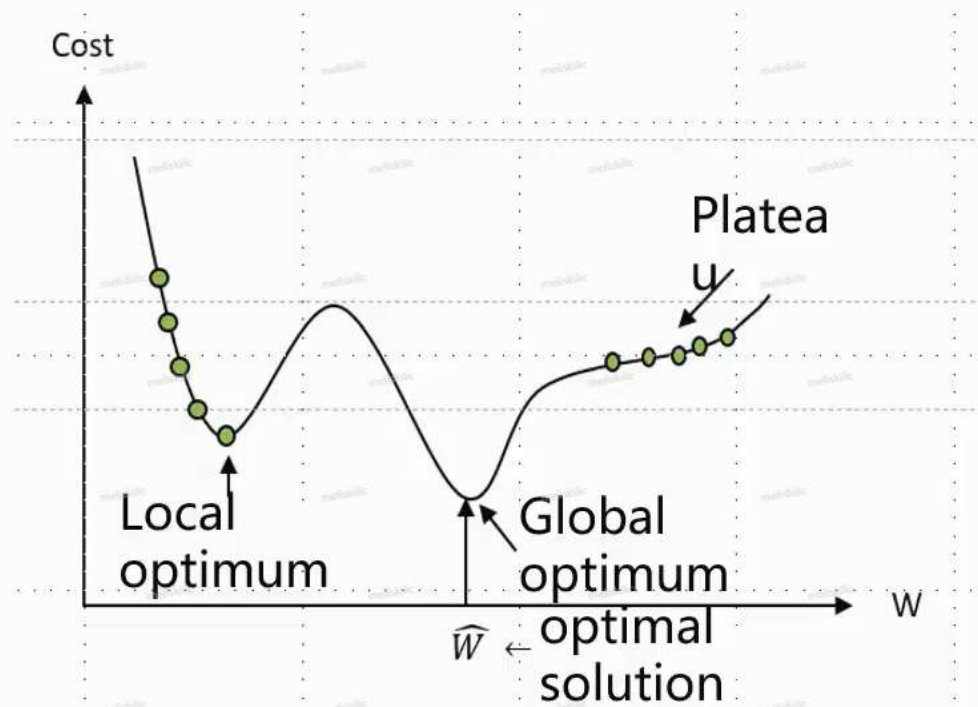
Convex set & Concave set



Concave function & Convex function

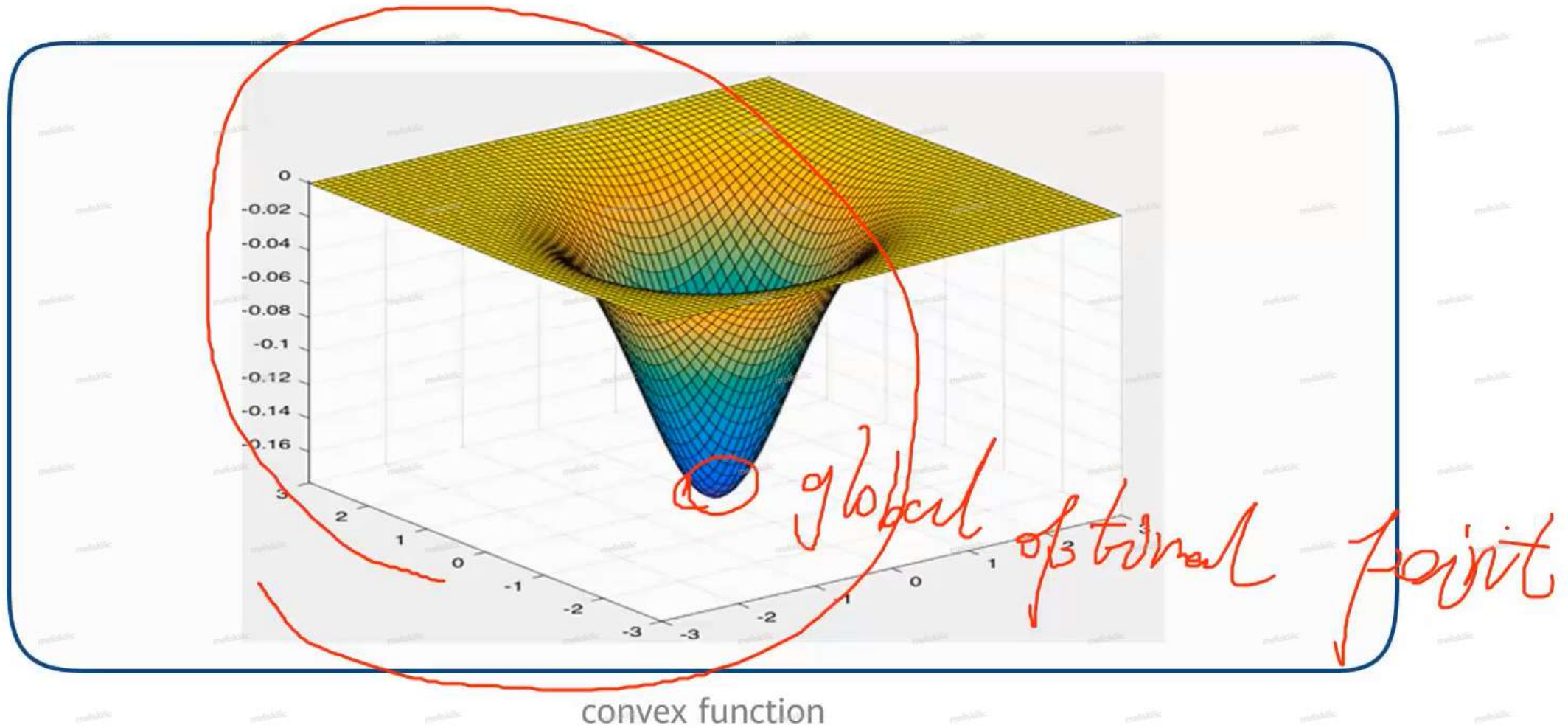
Non-Convex Function

prediction $(=)$ real result

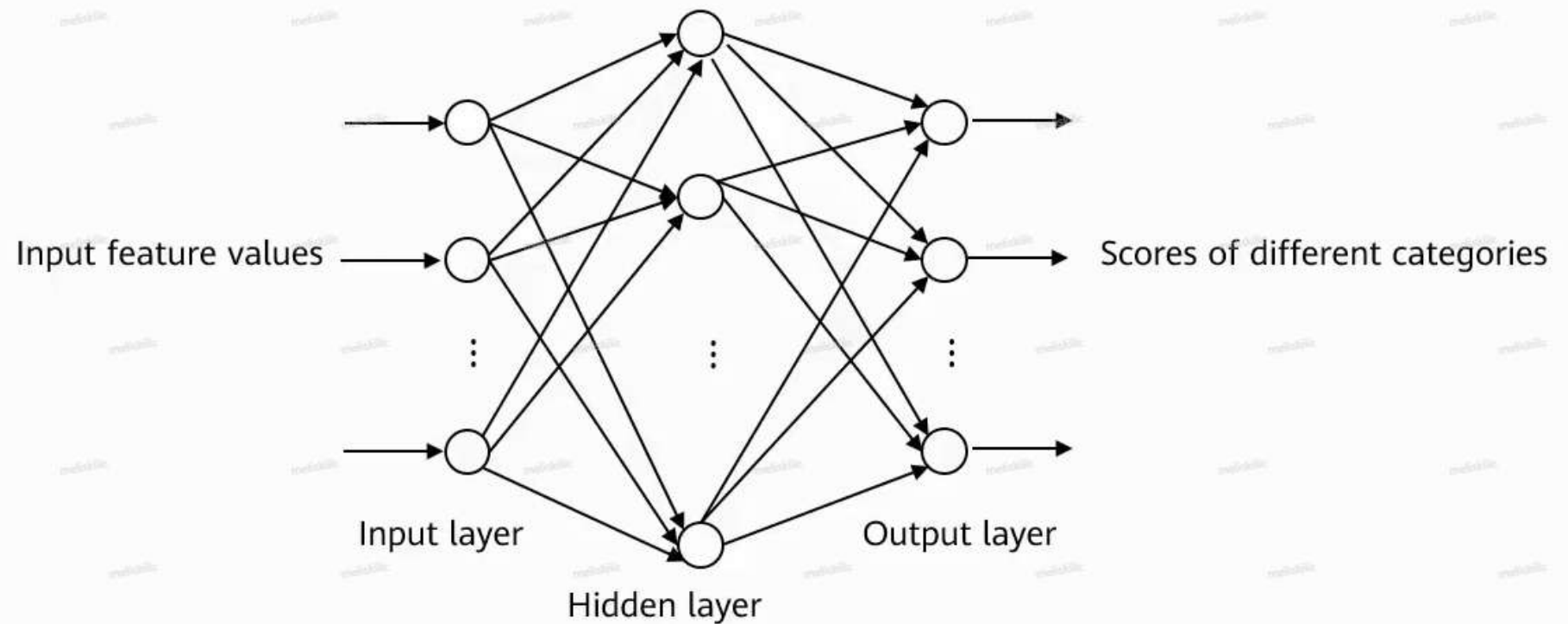


high-order functions

Convex Function



Loss Function



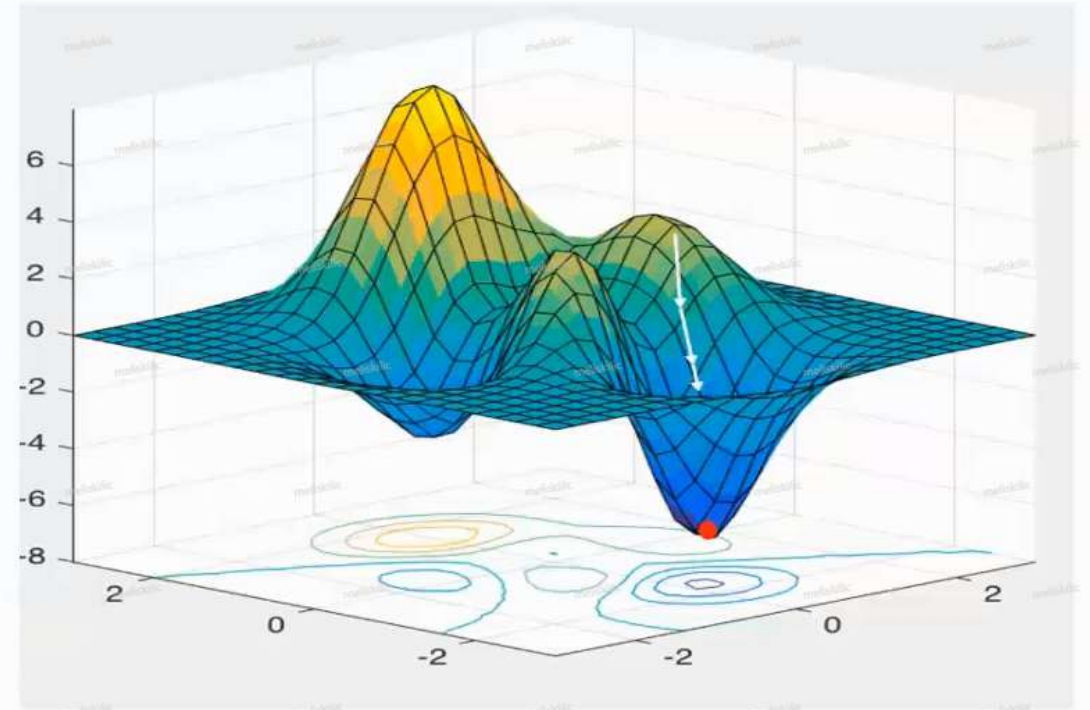
Cross entropy/Softmax Loss Function

$$Loss_i = -\sum_k p_k \log(q_k) = -\sum_k p_k \log\left(\frac{e^{f_k}}{\sum_j e^{f_j}}\right)$$

In the preceding function, p_k is the probability that x_i belongs to class k :

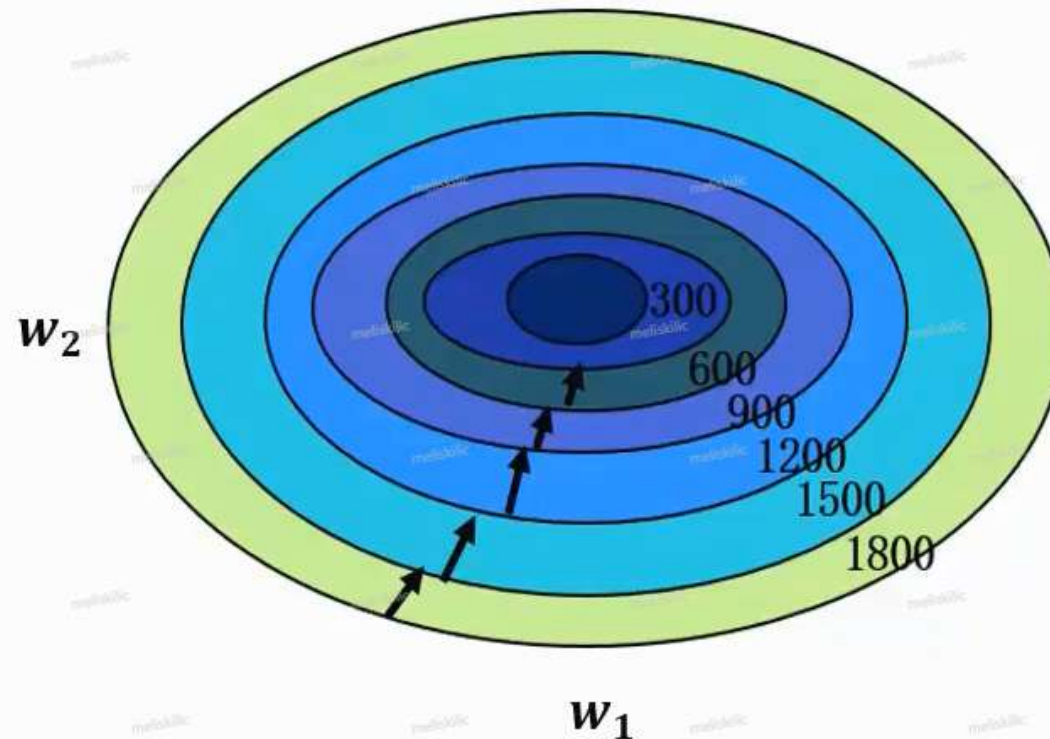
$$p_{k=y_i} = 1, p_{k \neq y_i} = 0.$$

Gradient Descent (1)



Gradient Descent (2)

$$w^+ = w - \eta * \frac{\partial Loss}{\partial w}$$





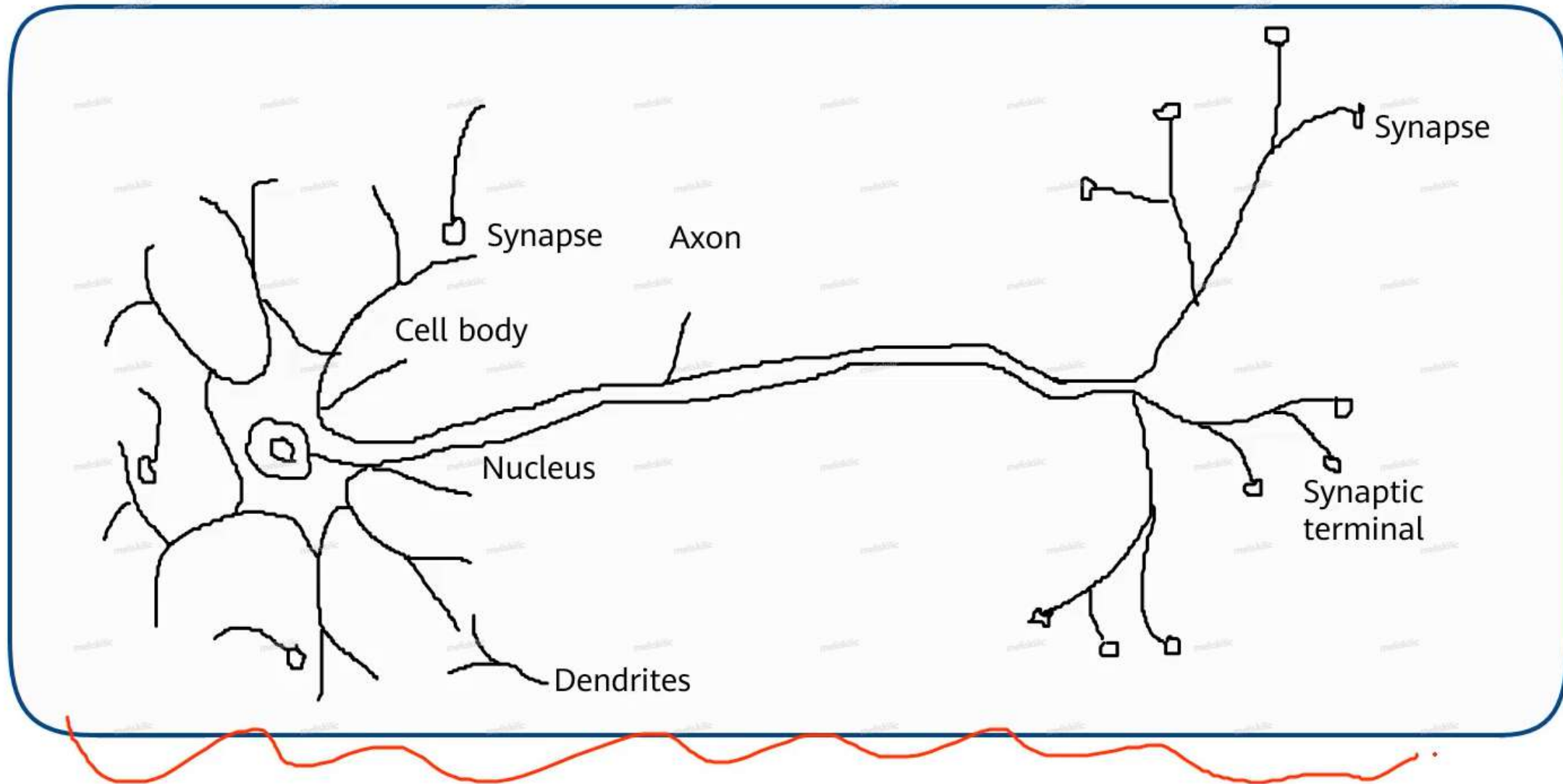
Gradient Descent (3)

Batch gradient descent (BGD)

Stochastic gradient descent (SGD)

Mini-batch gradient descent (MBGD)

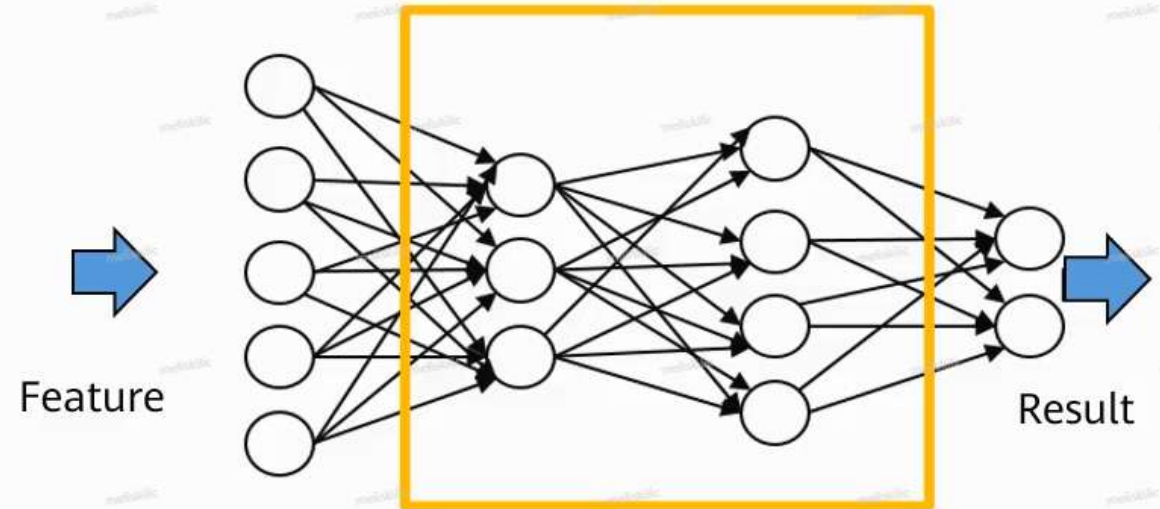
Biological Neural Networks



Artificial Neural Networks



Information Thinking Result



Feature

Result

Input
Layer

Hidden
Layer

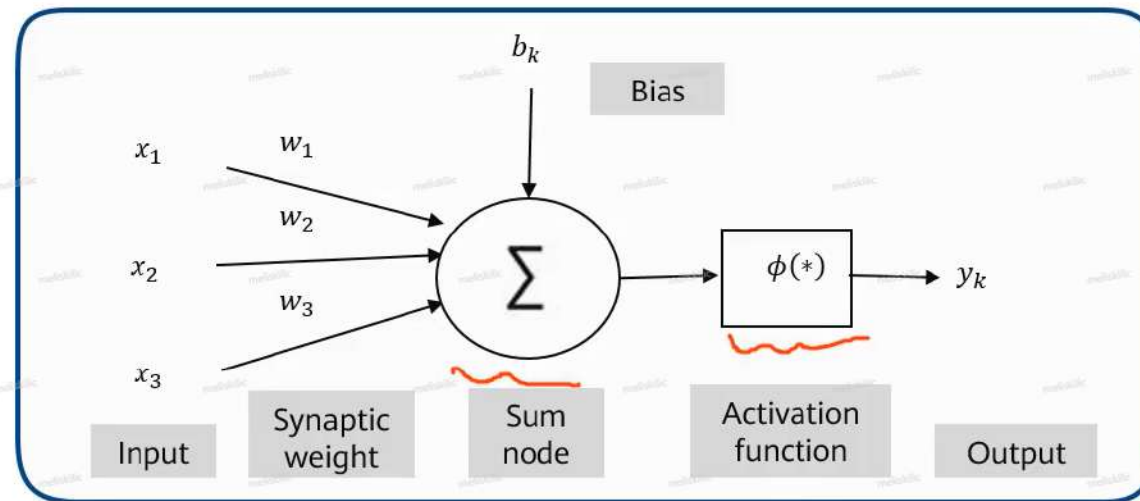
Output
Layer

Dendrites *neuron*

axon



Neuron (1)



Linear function: If the input is the n-dimensional feature vector X , the calculation formula is as follows:

$$\underline{f(\underline{X}, \underline{W}, b)} = \underline{W}\underline{X} + b = \sum_n w_i x_i + b = [W; b][X; 1]^T$$

Activation function: $0 = \text{sign}(\text{net}) = \begin{cases} 1, \text{net} > 0, \\ -1, \text{otherwise.} \end{cases}$

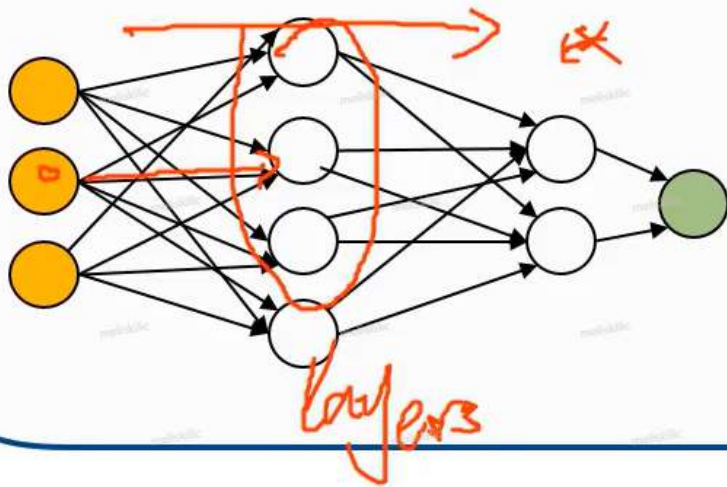
Neuron (2)

**Neural Network
Topology**

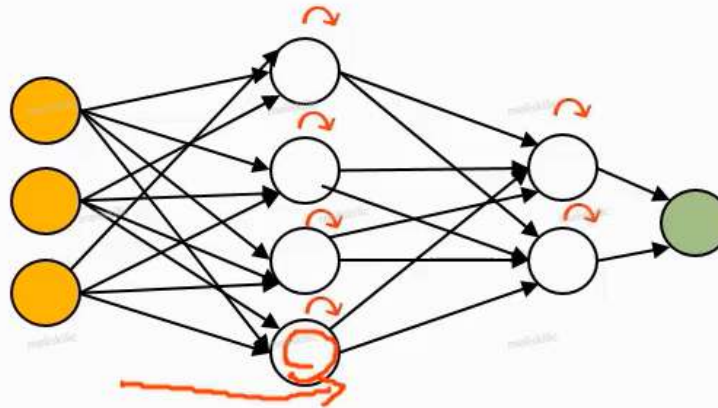
**Activation
Rule**

**Learning
Algorithm**

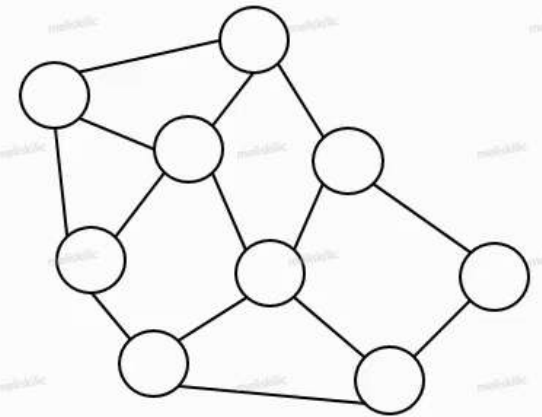
Neural Network Topologies



Feedforward neural network



Feedback neural network



Graph neural network

2009



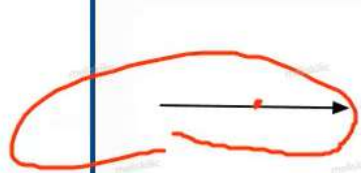
1957

Perceptron — Algorithm Formula

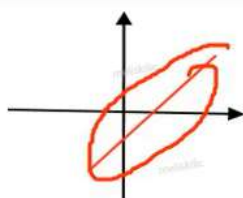
Input value

$$Z = \underline{W_1 X_1} + \underline{W_2 X_2} + \dots + \underline{W_n X_n} = \sum_{i=1}^n \underline{W_i X_i} = \underline{WX}$$

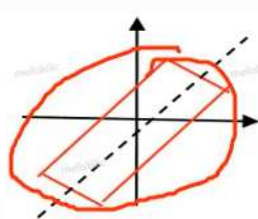
Activation function: $\text{sign}(Z) = \begin{cases} 1 & Z \geq \theta \\ -1 & Z \leq \theta \end{cases}$, where θ is the threshold.



Separating point
hyperplane
 $Ax+B=0$



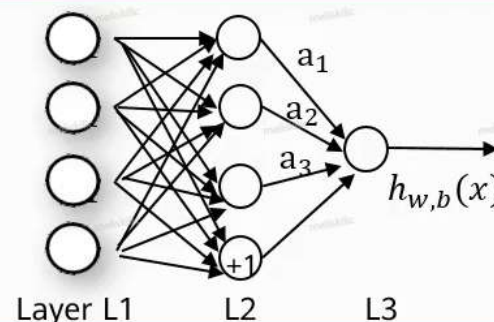
Separating line
 $Ax+By+C=0$



Separating plane
 $Ax+By+Cz+D=0$



Separating
 $WX+b=0$



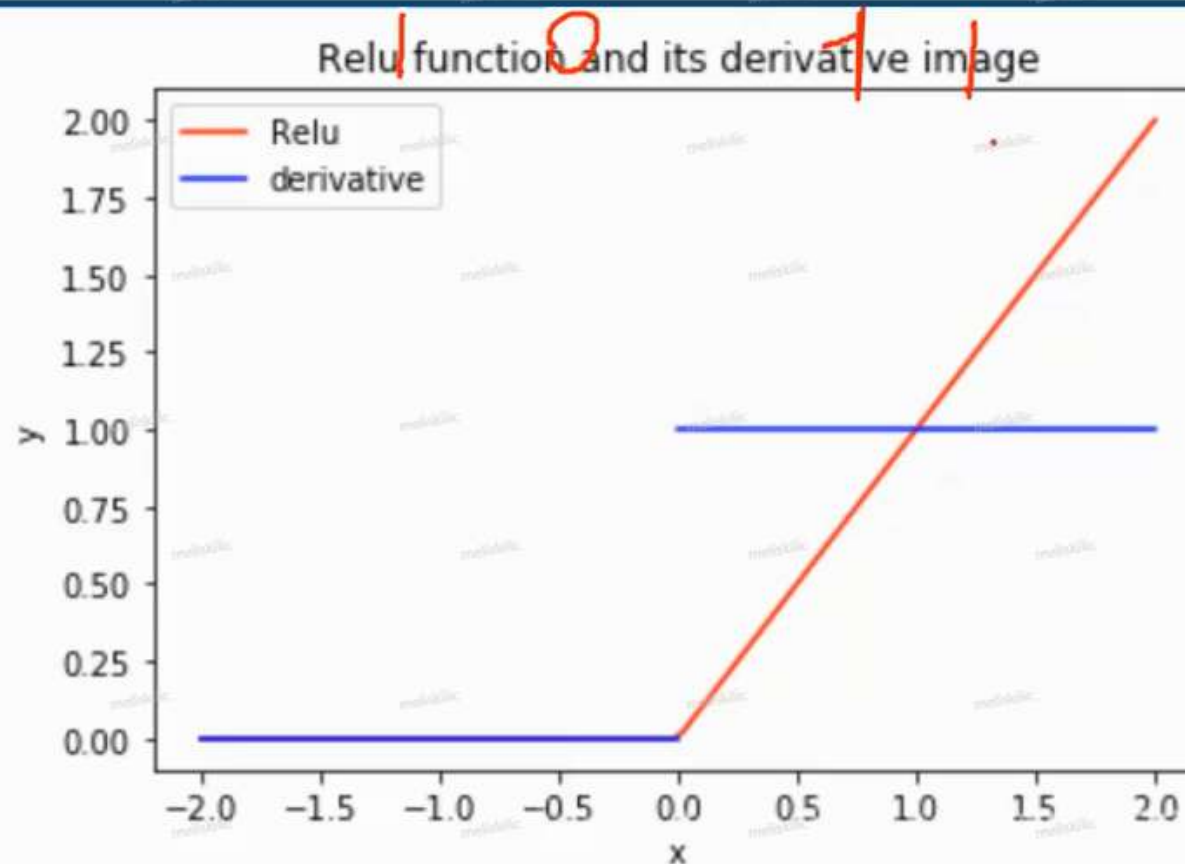
Perceptron

Perceptron — Loss Function

$$L(w, b) = - \sum_{X_i \in M} y_i (w * x_0 + b)$$

ReLU Function

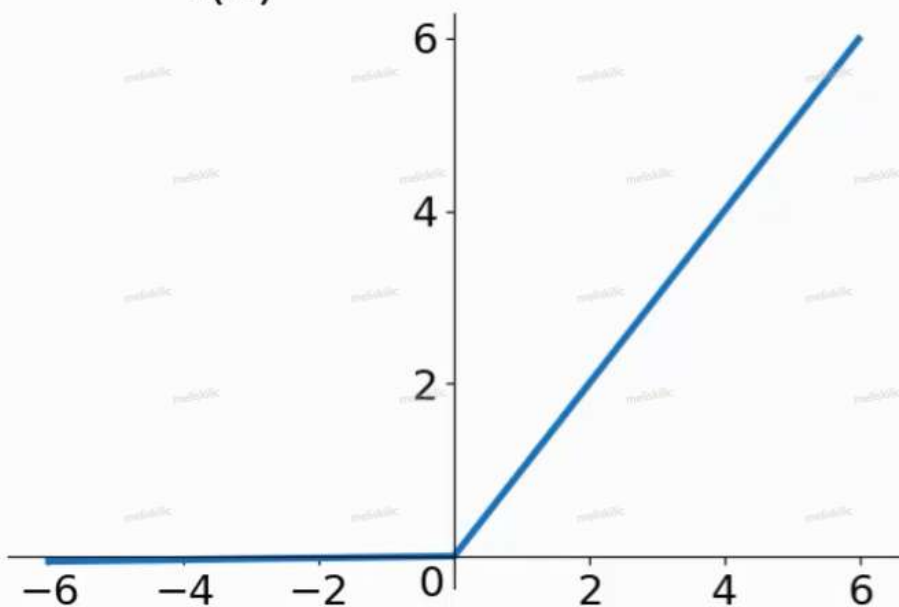
$$\text{ReLU}(x) = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases} = \max(0, x)$$



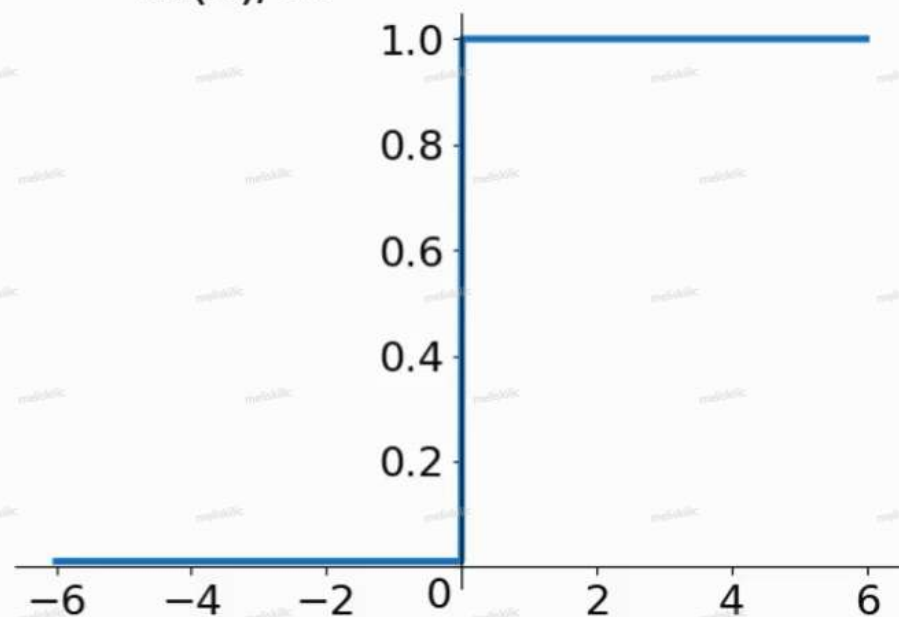
Leaky ReLU Function

$$f(x) = \max(0.01x, x)$$

$f(x)$

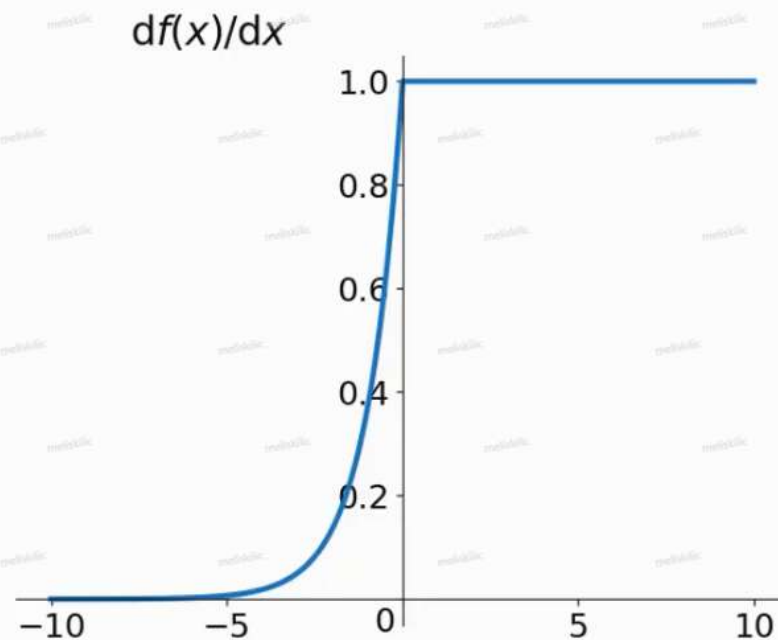
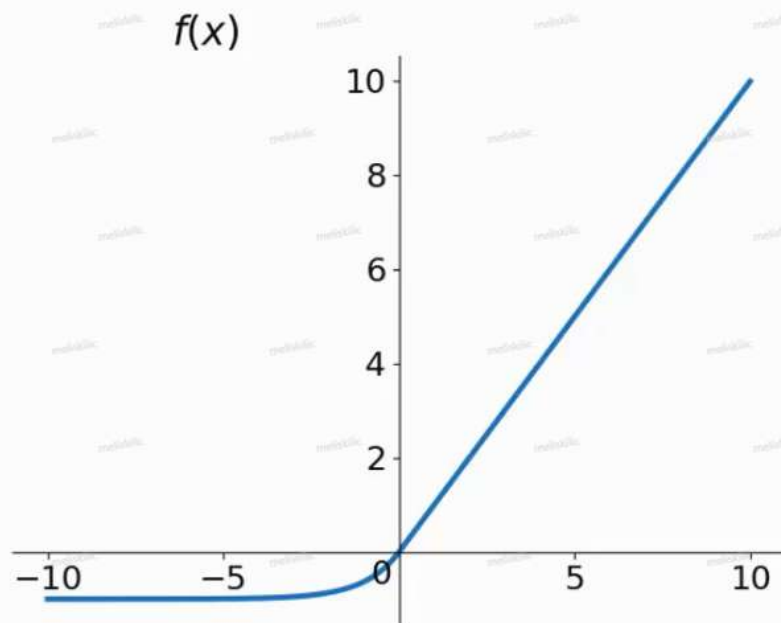


$df(x)/dx$



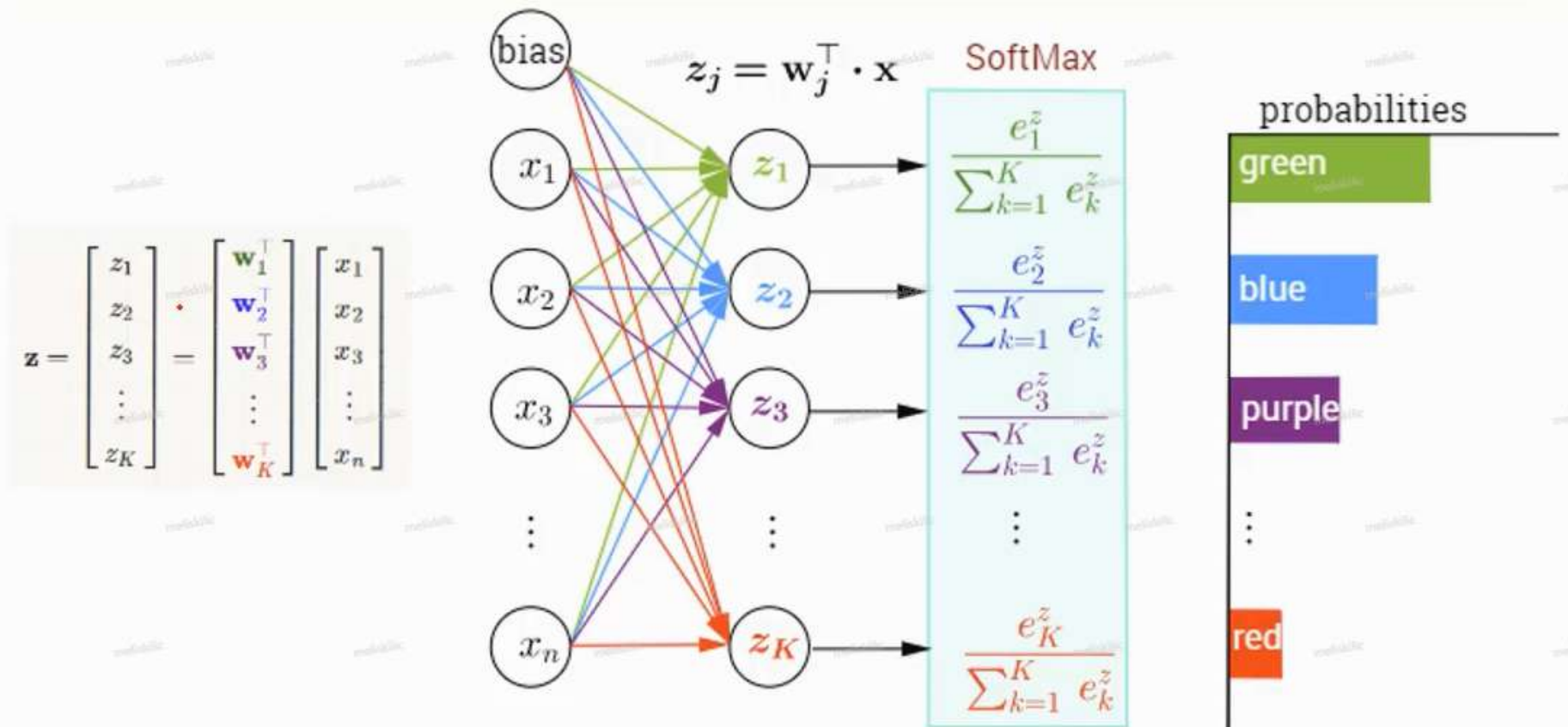
ELU (Exponential Linear Units) Function

$$f(x) = \begin{cases} x, & \text{if } x > 0 \\ \alpha(e^x - 1), & \text{otherwise} \end{cases}$$



Softmax Function

$$\sigma(z)_j = \frac{e^{z_j}}{\sum_k e^{z_k}}$$





Factors

Non-linear

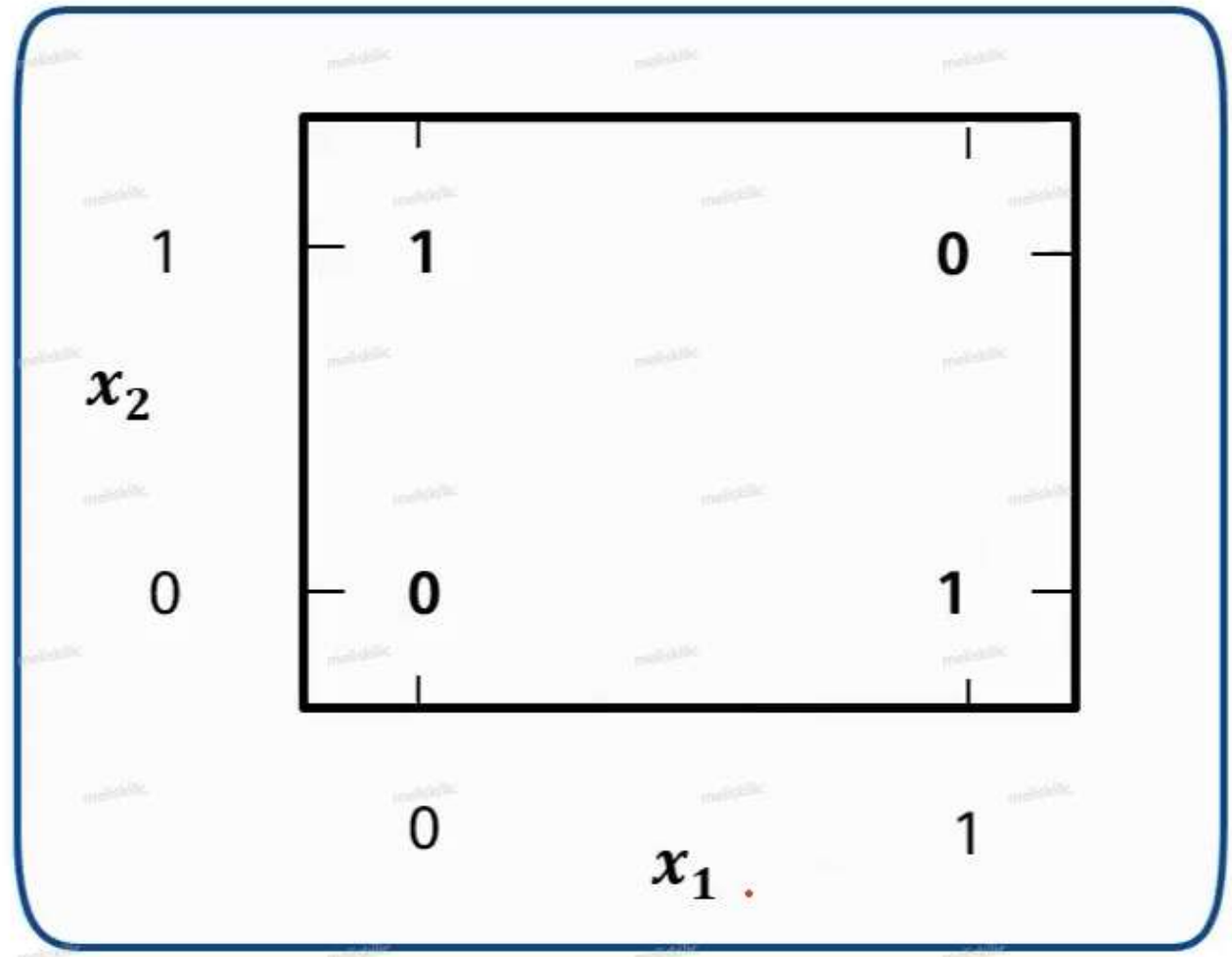
**Continuously
differentiable**

Range

Monotonic

Smooth

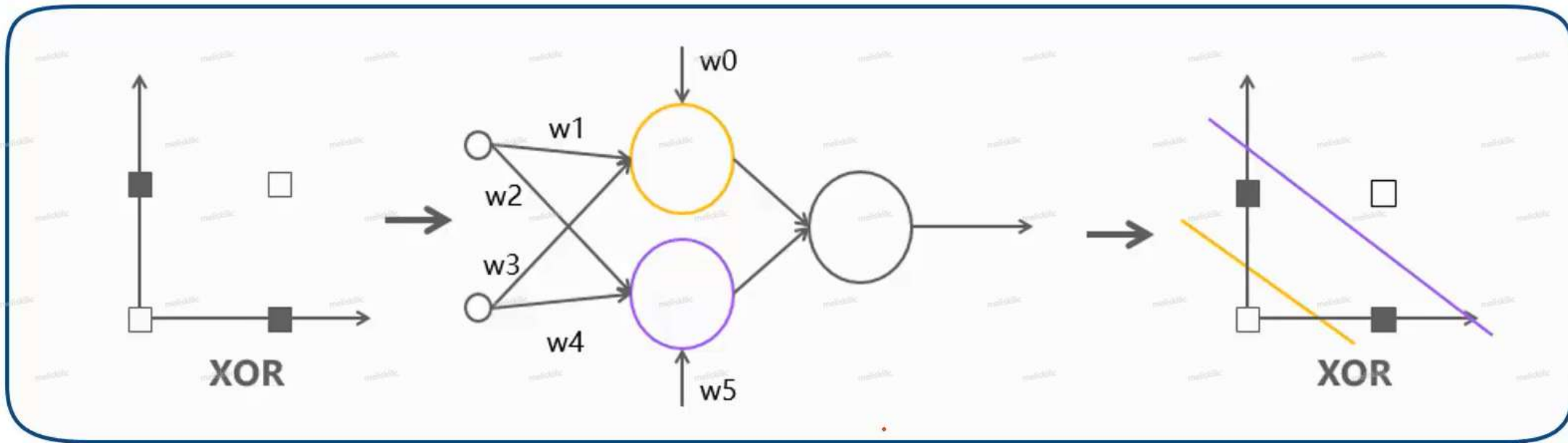
XOR Problem



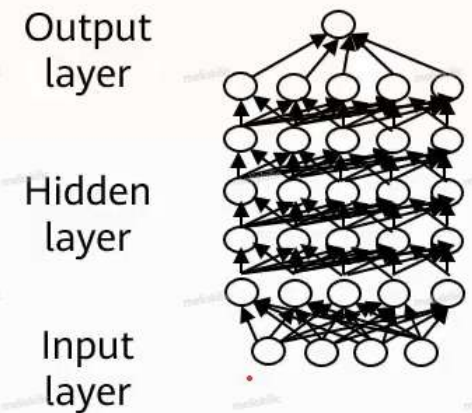
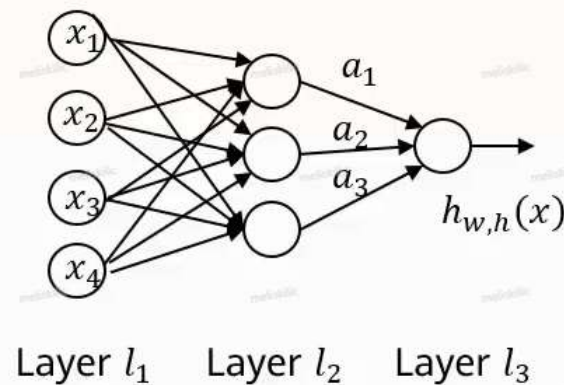
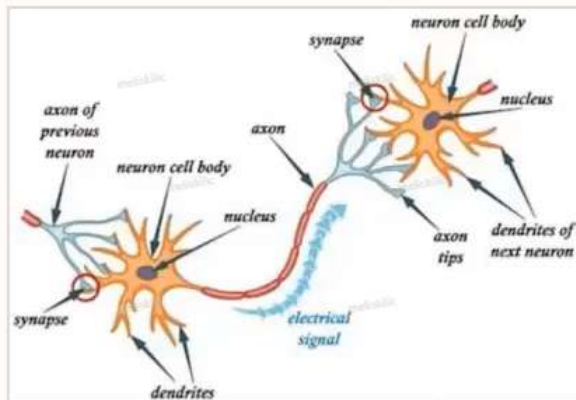
Original x Space



Multi-layer Fully-Connected ANN



Deep Learning

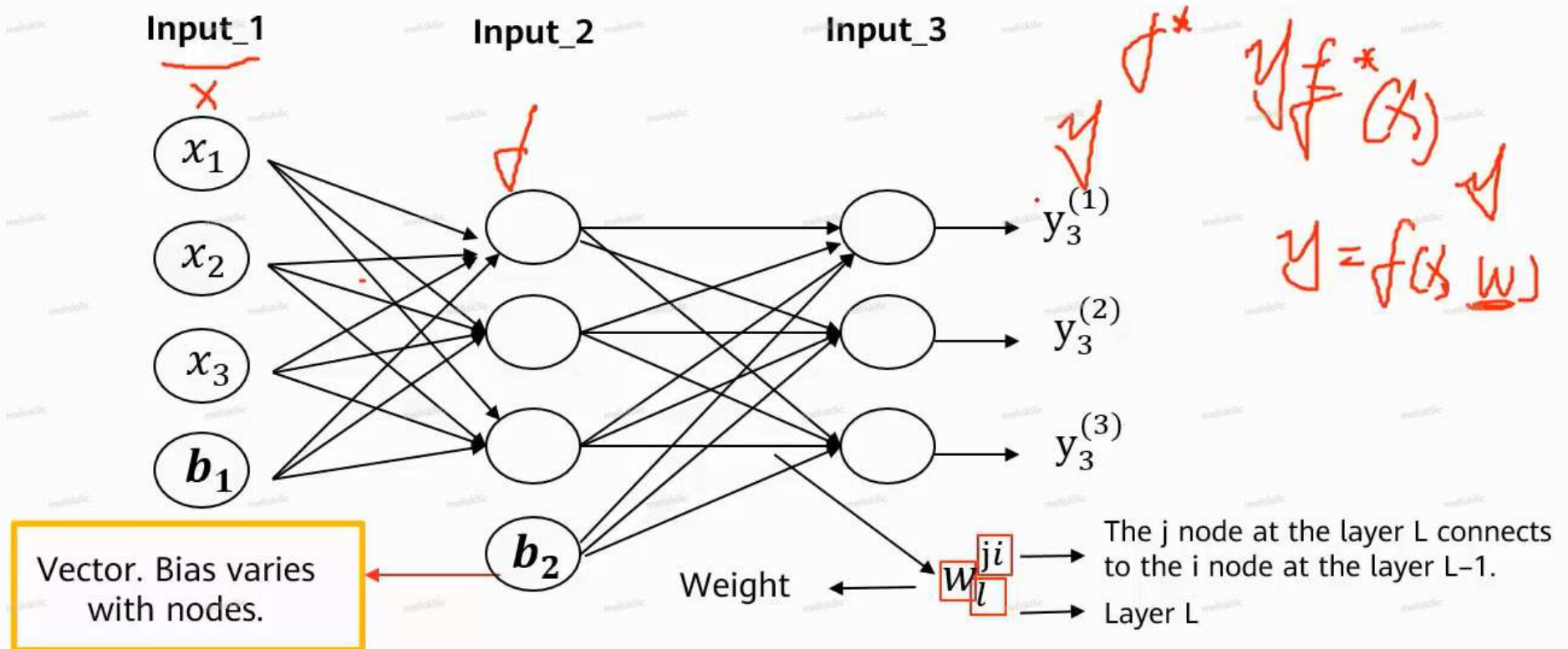


Human neural network

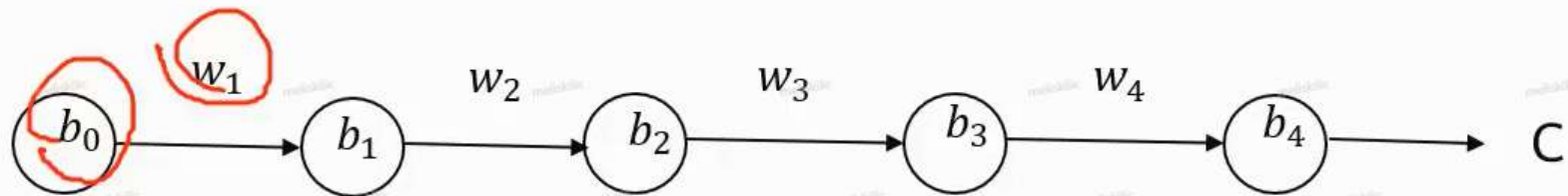
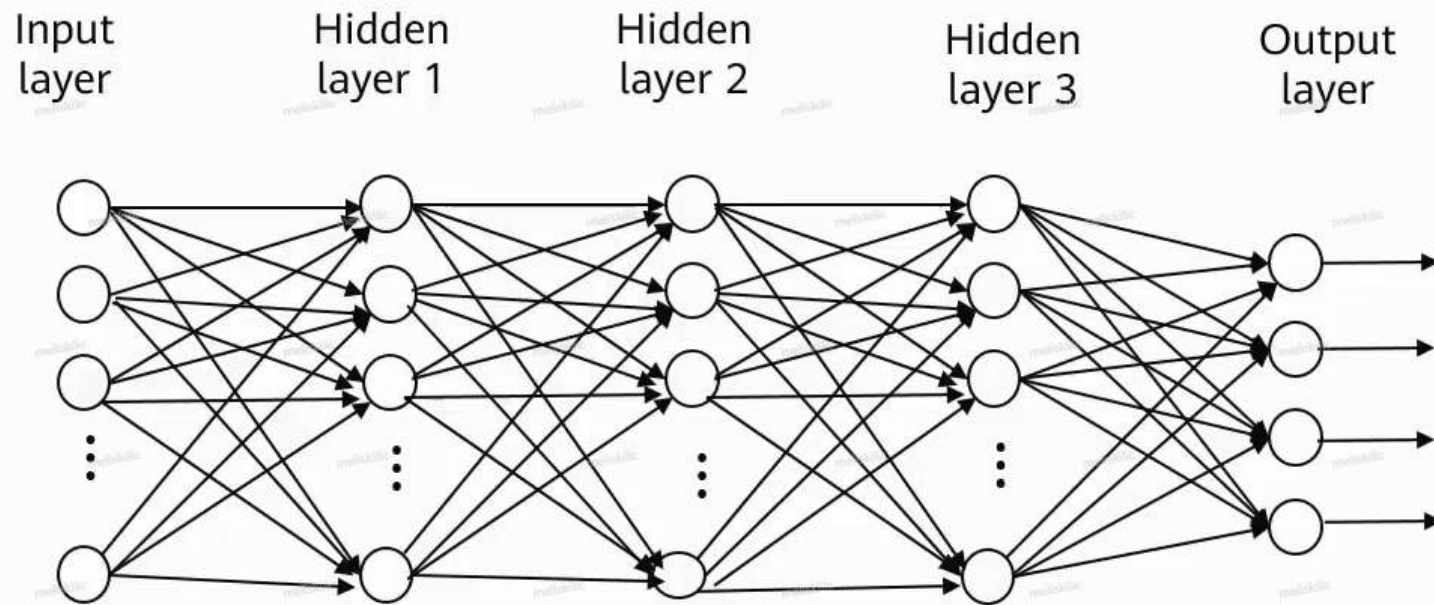
Perceptron

Deep feedforward network

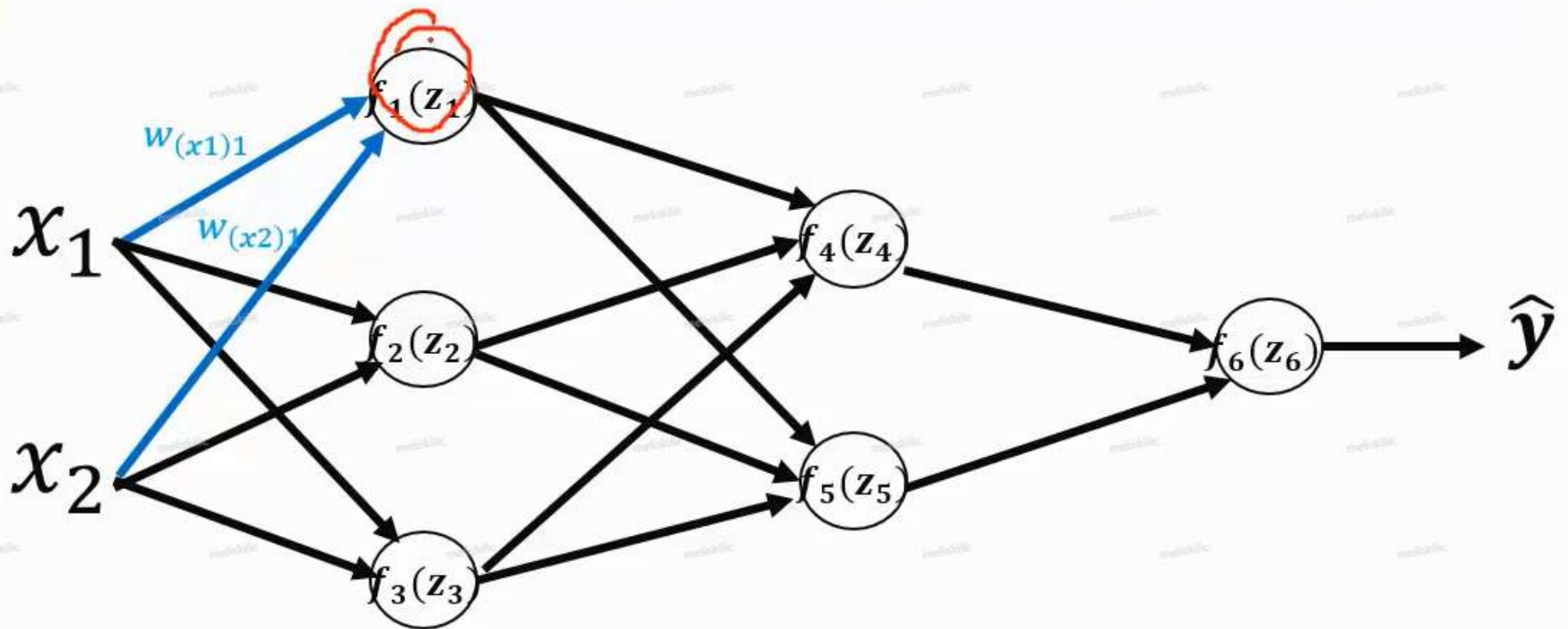
Deep Feedforward Network



Back propagation

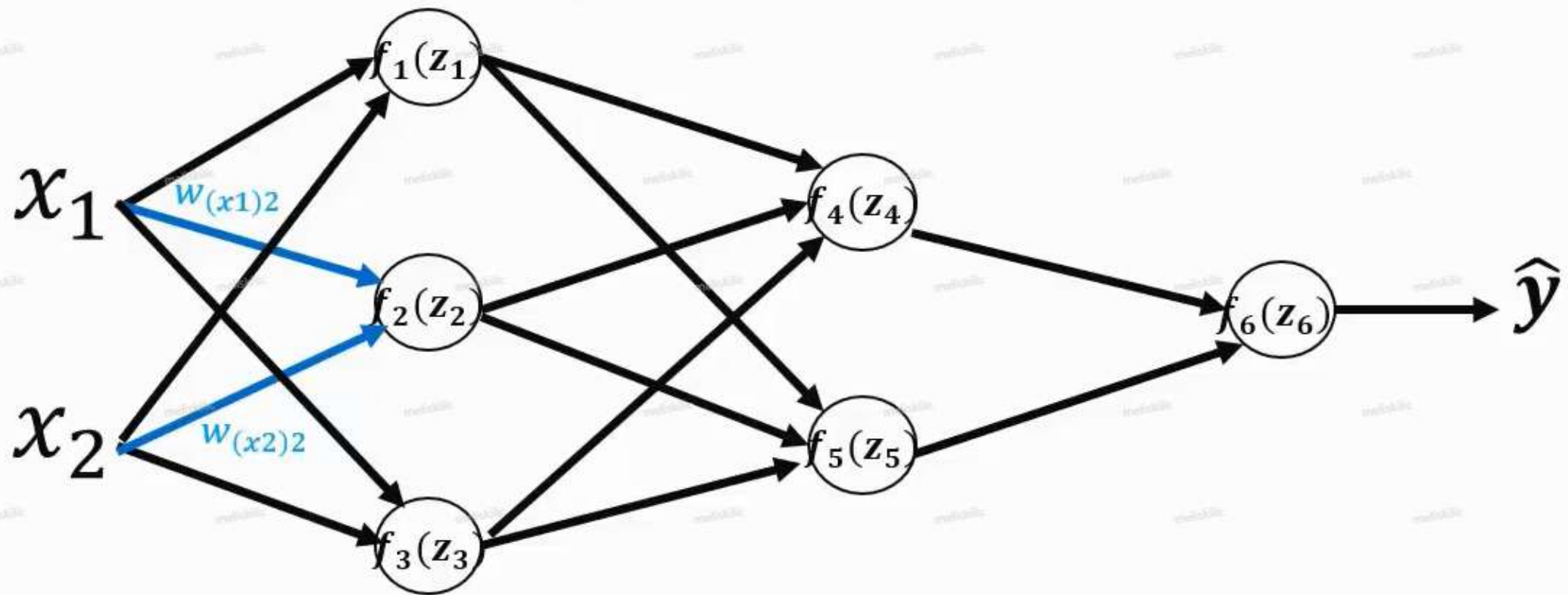


Forward Propagation (1)



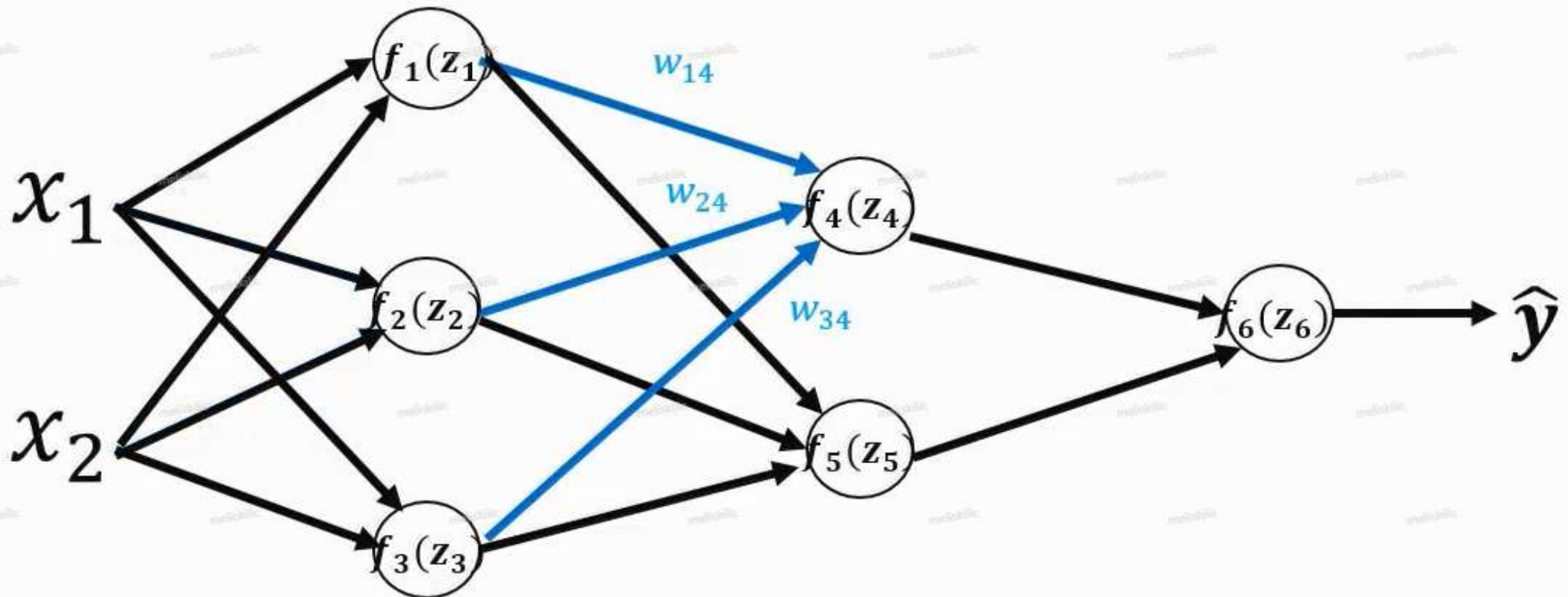
Forward Propagation (2)

$$y_2 = f_2(w_{(x1)2}x_1 + w_{(x2)2}x_2)$$



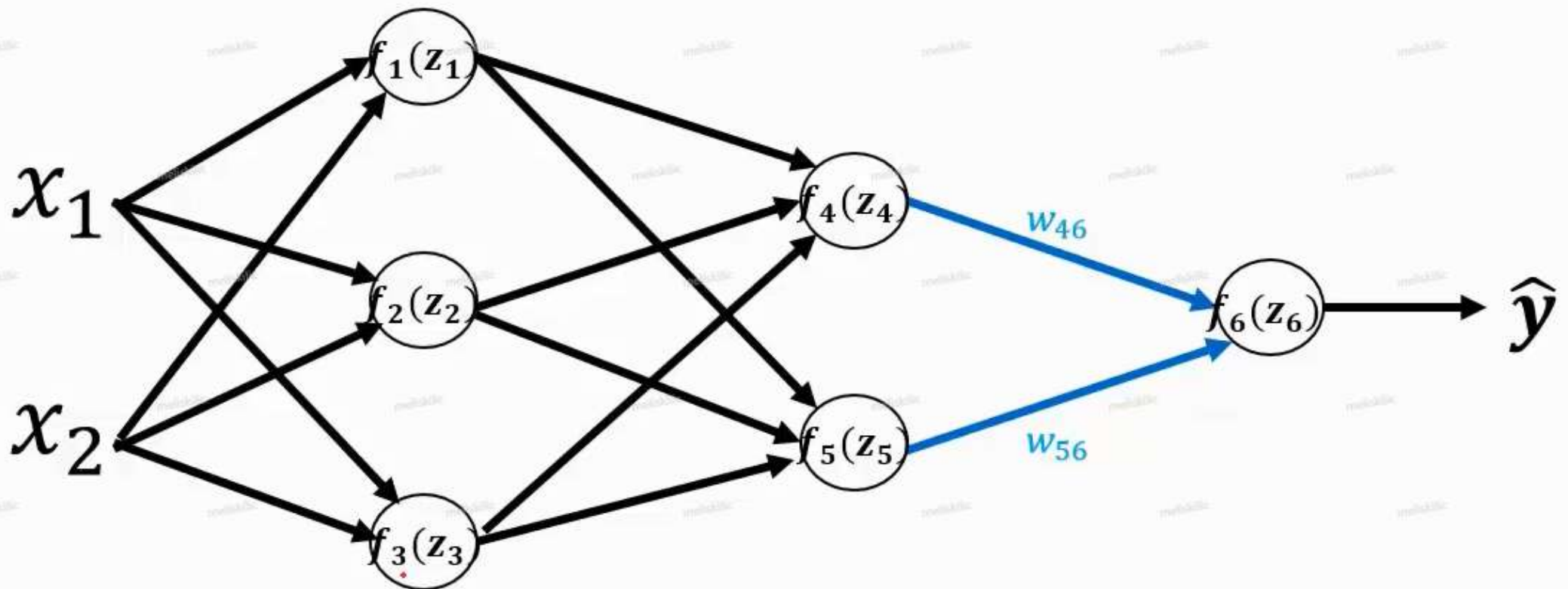
Forward Propagation (3)

$$y_4 = f_4(w_{14}y_1 + w_{24}y_2 + w_{34}y_3)$$



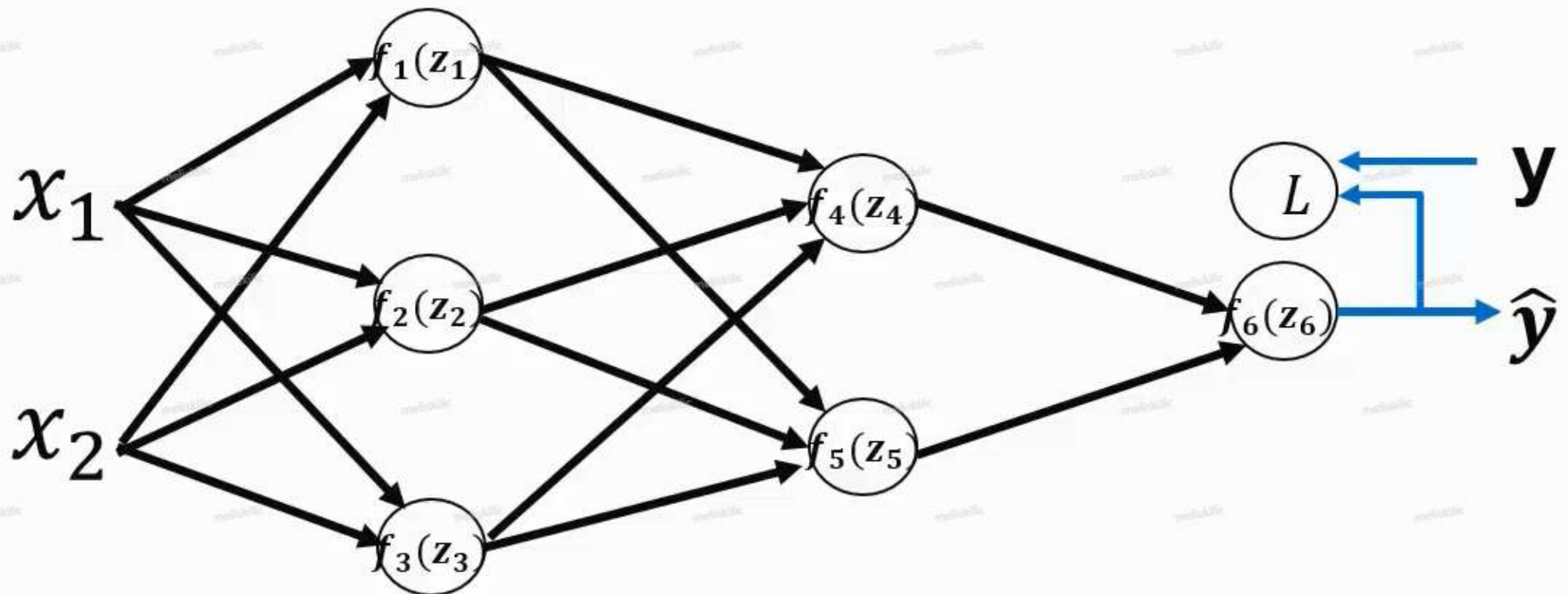
Forward Propagation (5)

$$y = f_6(w_{46}y_4 + w_{56}y_5)$$



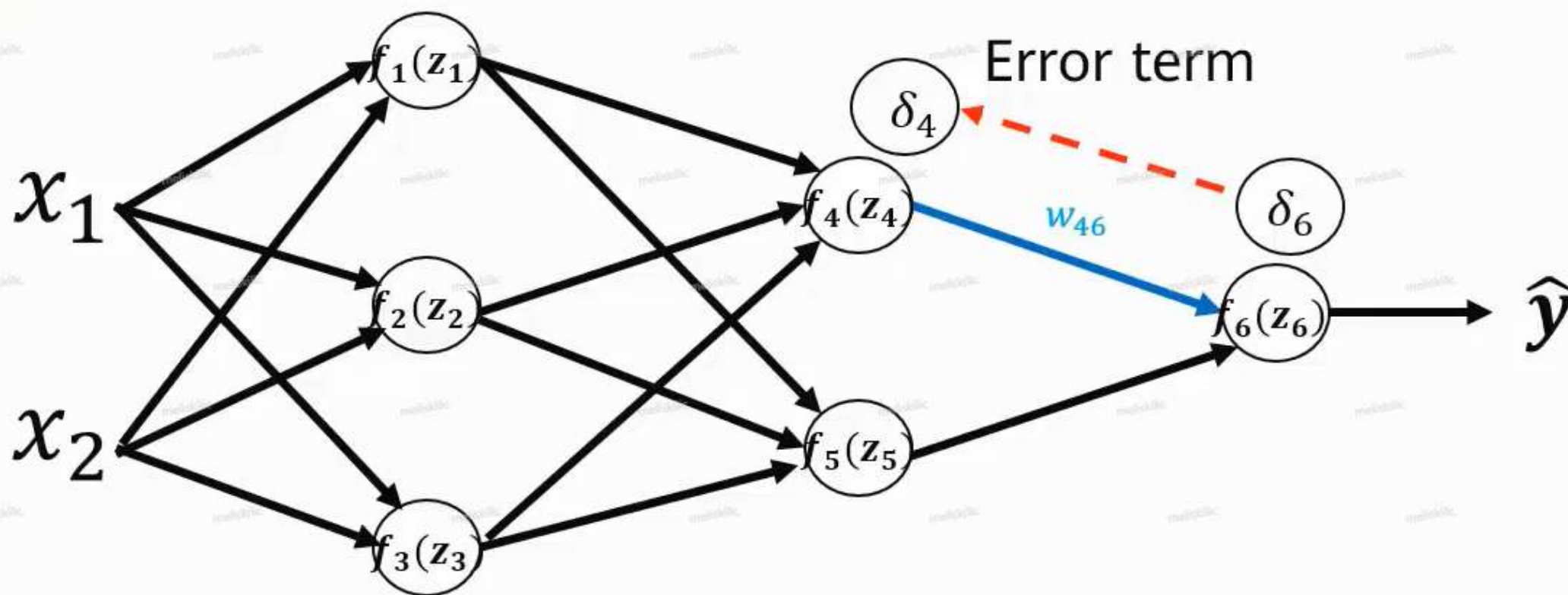
Error Backpropagation (1)

$$\delta_6 = \frac{\partial L(y, \hat{y})}{\partial z_6} = \frac{\partial L(y, f_6(z_6))}{\partial z_6} = \frac{\partial L(y, f_6(z_6))}{\partial f_6(z_6)} * \frac{\partial f_6(z_6)}{\partial z_6}$$



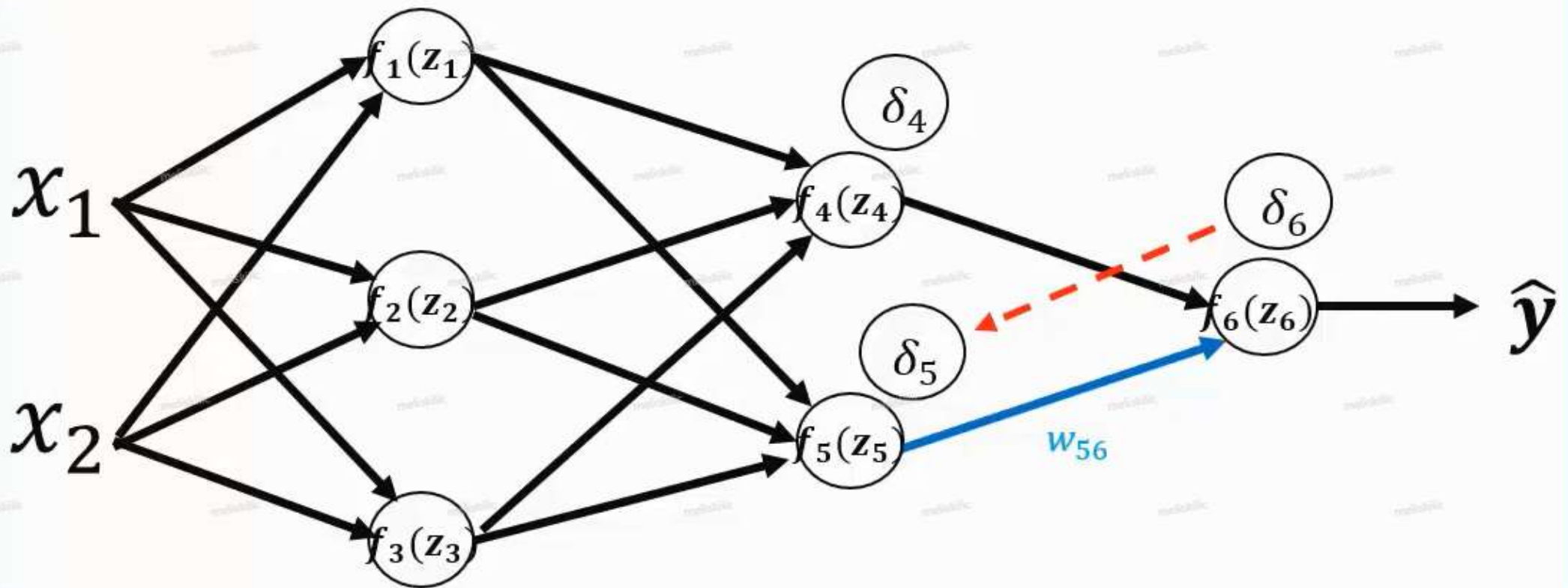
Error Backpropagation (2)

$$\delta_4 = \frac{\partial L}{\partial z_4} = \delta_6 * \underline{w_{46}} * \dot{f_4}(z_4)$$

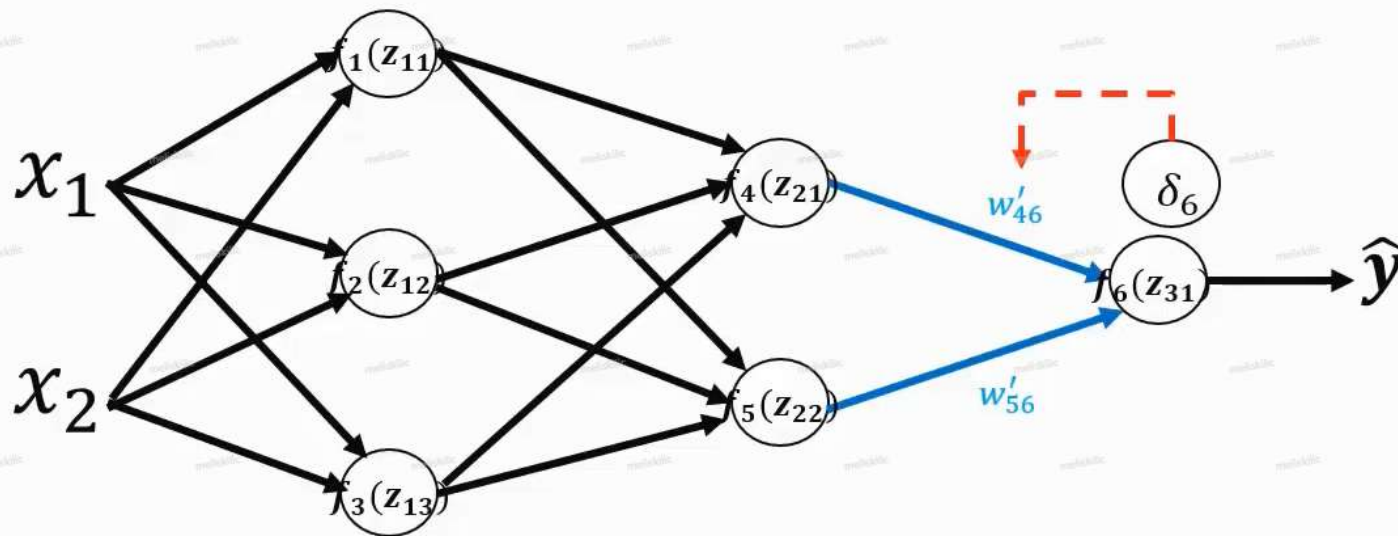


Error Backpropagation (3)

$$\delta_5 = \frac{\partial L}{\partial z_5} = \delta_6 * w_{56} * f'_5(z_5)$$

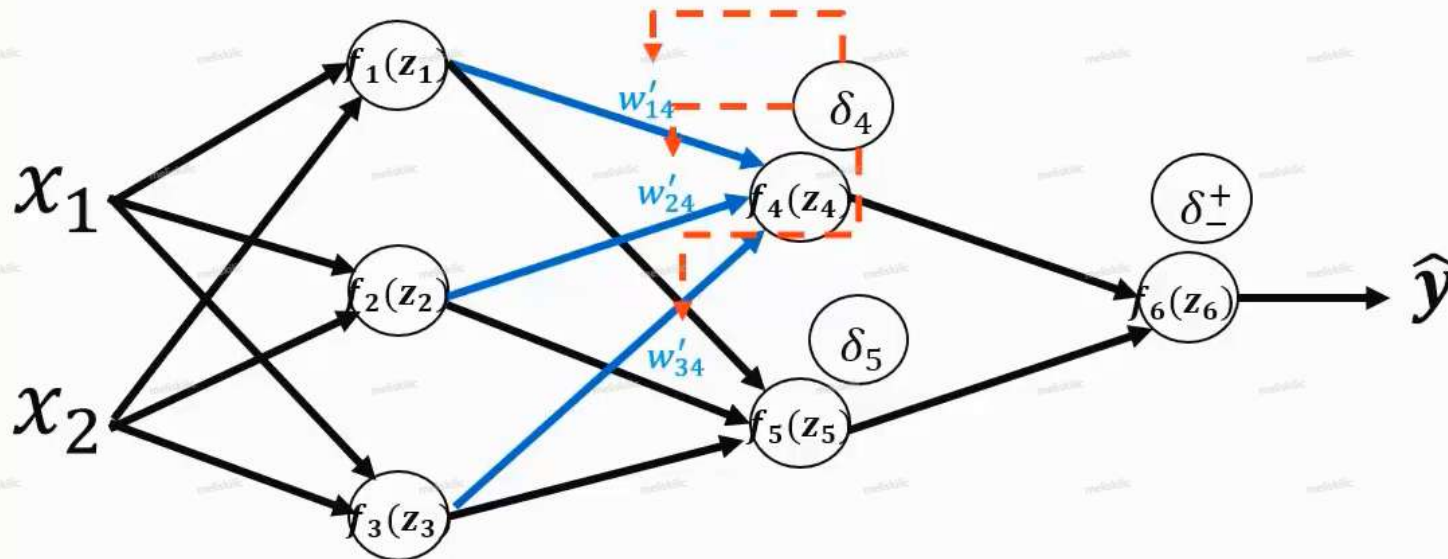


Weight Update (1)



$$\begin{aligned}w'_{46} &= w_{46} - \eta * \frac{\partial L(y, \hat{y})}{\partial w_{46}} \\&= w_{46} - \eta * \frac{\partial L(y, \hat{y})}{\partial z_6} * \frac{\partial z_6}{\partial w_{46}} \\&= w_{46} - \eta * \delta_6 * \frac{\partial (w_{46}y_4 + w_{56}y_5)}{\partial w_{46}} \\&= w_{46} - \eta * \delta_6 * y_4\end{aligned}$$

Weight Update (2)

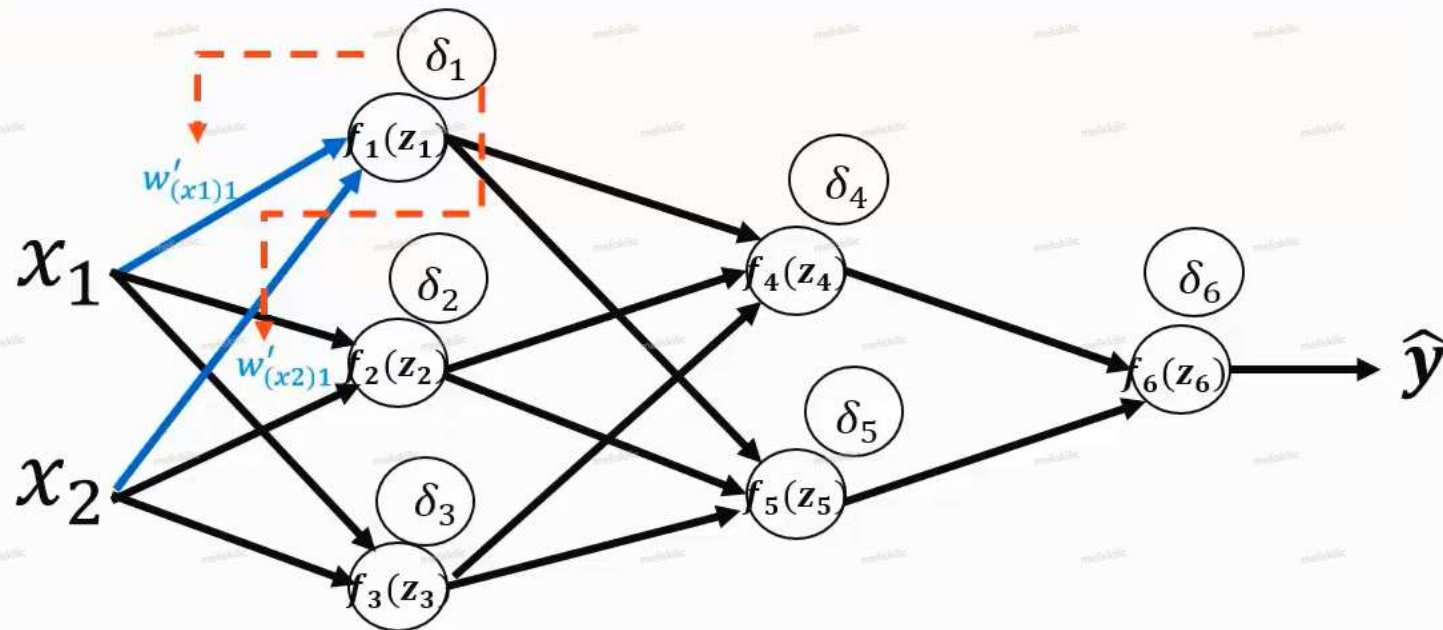


$$w'_{14} = w_{14} - \eta \delta_4 y_1$$

$$w'_{24} = w_{24} - \eta \delta_4 y_2$$

$$w'_{34} = w_{34} - \eta \delta_4 y_3$$

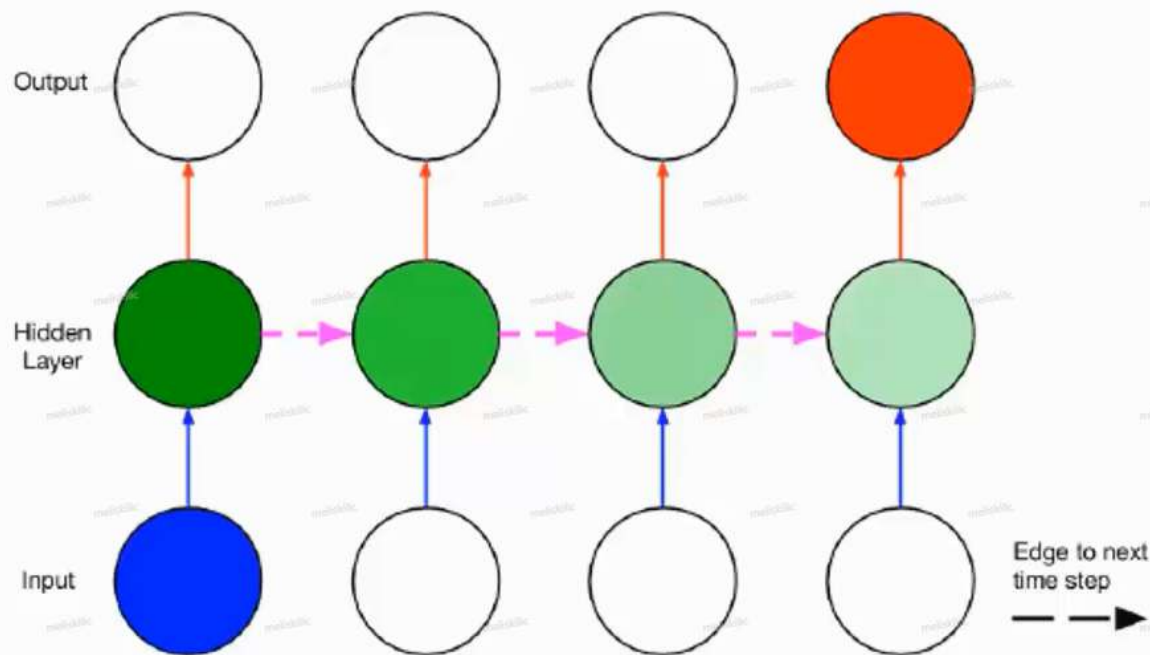
Weight Update (3)



$$w'_{(x1)1} = w_{(x1)1} - \eta \delta_1 x_1$$

$$w'_{(x2)1} = w_{(x2)1} - \eta \delta_1 x_2$$

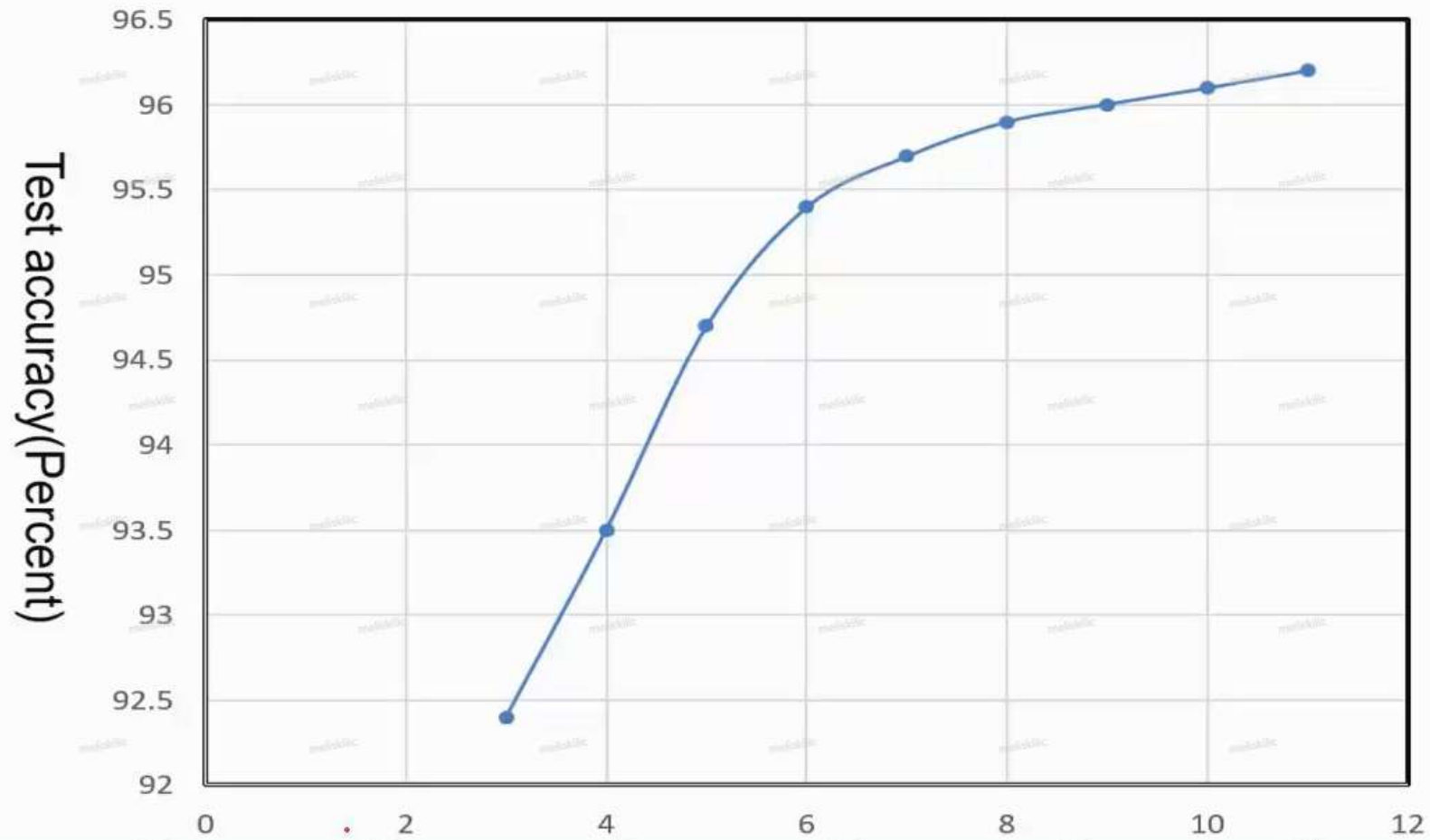
Gradient Problems



W is larger → Exploding

W is small → Vanishing

Effect of Neural Network Depth



Effect of the Number of Neural Network Parameters

