In [ ]:

```

```

This is a fun but challenging problem set. It will test your python skills, as well as your understanding of the material in class and in the readings. Start early and debug often! Some notes:

- Part 1 is meant to be easy, so get through it quickly.
- Part 2 (especially 2.1) will be difficult, but it is the lynchpin of this problem set to make sure to do it well and understand what you've done. If you find your gradient descent algorithm is taking more than a few minutes to complete, debug more, compare notes with others, and go to the Lab sessions (especially the sections on vectorized computation and computational efficiency).
- Depending on how well you've done 2.1, parts 2.3 and 4.3 will be relatively painless or incredibly painful.
- Part 4 (especially 4.3) will be computationally intensive. Don't leave this until the last minute, otherwise your code might be running when the deadline arrives.
- Do the extra credit problems last. This can help you increase your scores

---

Introduction to the assignment

As with the last assignment, you will be using the Boston Housing Prices Data Set.

In [1]:

```python
import IPython
import numpy as np
import scipy as sp
import pandas as pd
import matplotlib
import sklearn

%matplotlib inline
import matplotlib.pyplot as plt
import statsmodels.api as sm
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error
import statsmodels.formula.api as smf
```

In [2]:

```python
# Load you data the Boston Housing data into a dataframe
# MEDV.txt containt the median house values and data.txt the other 13 features
# in order ["CRIM", "ZN", "INDUS", "CHAS", "NOX","RM","AGE","DIS","RAD","TAX","P
# Your code here

data = np.loadtxt('data.txt')
target = np.loadtxt('target.txt')
col = ['CRIM', 'ZN', 'INDUS', 'CHAS', 'NOX', 'RM', 'AGE', 'DIS', 'RAD', 'TAX', '
 'B', 'LSTAT']
```

```
df = pd.DataFrame(data, columns = col)
df['MEDV'] = target
```

In [3]:

```
print(df.head())
print(df.isnull().sum())
df.dtypes
```

```
       CRIM    ZN     INDUS  CHAS       NOX        RM   AGE       DIS  RAD  \
0  0.218960  18.0  2.629288   0.0  0.869420  6.875396  65.2  4.347275  1.0
1  0.141576   0.0  7.315612   0.0  0.549711  6.499894  78.9  5.315684  2.0
2  0.380457   0.0  7.340354   0.0  0.697928  7.263489  61.1  5.356935  2.0
3  0.313563   0.0  2.562407   0.0  0.599629  7.209732  45.8  6.103983  3.0
4  0.330105   0.0  2.497337   0.0  0.476077  7.184111  54.2  6.264372  3.0

     TAX    PTRATIO           B     LSTAT  MEDV
0  307.0  15.534711  397.462329  5.715647  24.0
1  255.0  17.914131  397.012611  9.338417  21.6
2  243.0  17.919989  396.628236  4.142473  34.7
3  226.0  18.979527  398.564784  3.239272  33.4
4  234.0  18.708888  399.487766  6.115159  36.2
CRIM       0
ZN         0
INDUS      0
CHAS       0
NOX        0
RM         0
AGE        0
DIS        0
RAD        0
TAX        0
PTRATIO    0
B          0
LSTAT      0
MEDV       0
dtype: int64
```

Out[3]:

```
CRIM       float64
ZN         float64
INDUS      float64
CHAS       float64
NOX        float64
RM         float64
AGE        float64
DIS        float64
RAD        float64
TAX        float64
PTRATIO    float64
B          float64
LSTAT      float64
MEDV       float64
dtype: object
```

Part 1: Getting oriented

1.1 Use existing libraries

Soon, you will write your own gradient descent algorithm, which you will then use to minimize the squared error cost function. First, however, let's use the canned versions that come with

Python, to make sure we understand what we're aiming to achieve.

Using the same Boston housing prices dataset, use the Linear Regression class from sklearn or the OLS class from SciPy to explore the relationship between median housing price and number of rooms per house. Do the following:

(a) Regress the housing price on the number of rooms per house. Draw a scatter plot of housing price (y-axis) against rooms (x-axis), and draw the regression line in blue. You might want to make the dots semi-transparent if it improves the presentation of the figure.

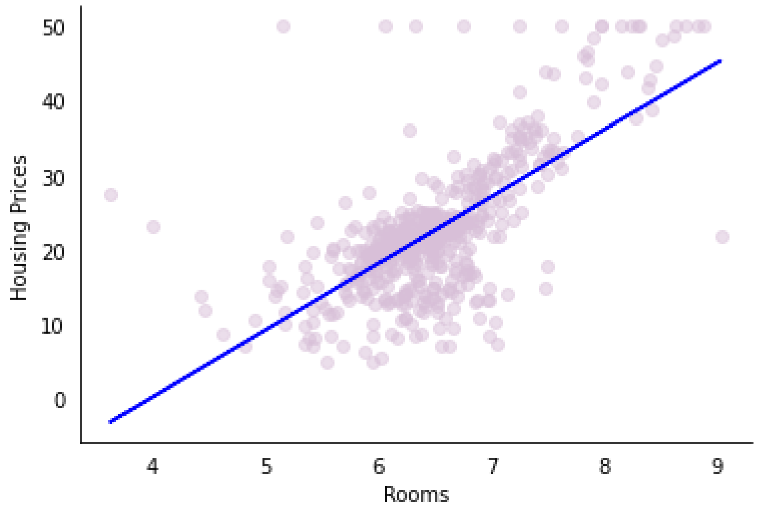
(b) Regress the housing price on the number of rooms per house and the (number of rooms per house) squared. Show the (curved) regression line in green.

(c) Interpret your results.

In [4]:

```
# (a) Regress the housing price on the number of rooms per house. Draw a scatter
# against rooms (x-axis), and draw the regression line in blue.
# You might want to make the dots semi-transparent if it improves the presentati

# setting up the model
X1 = np.array(df[['RM']])
y1 = np.array(df[['MEDV']])
model1 = sklearn.linear_model.LinearRegression()
model1.fit(X1, y1)
print("Coef_: ",model1.coef_)
print("Intercept_: ",model1.intercept_)

# plotting the line
plt.scatter(X1, y1,color='thistle', alpha=0.5)
plt.plot(X1, model1.predict(X1),color='b')
plt.xlabel('Rooms')
plt.ylabel('Housing Prices')
ax = plt.gca()
ax.spines['right'].set_color('none')
ax.spines['top'].set_color('none')
ax.yaxis.set_ticks_position('none')
ax.xaxis.set_ticks_position('none')
plt.show()
```

```
Coef_:  [[8.95992721]]
Intercept_:  [-35.57620687]
```



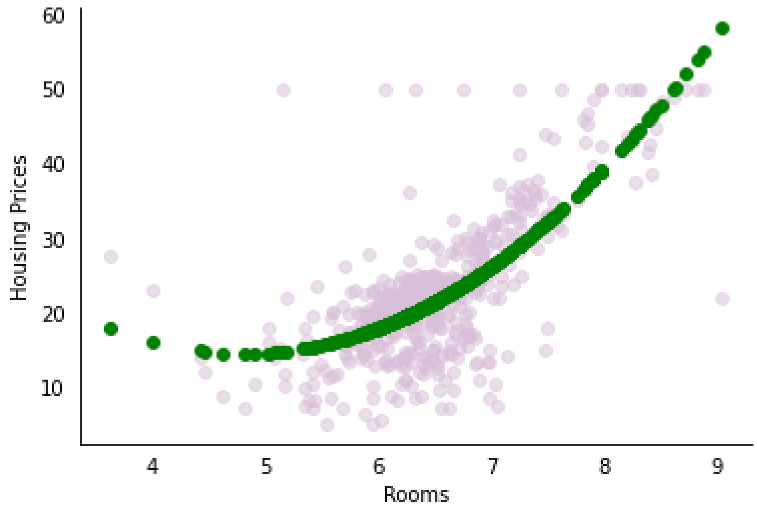
In [5]:

```
from sklearn.linear_model import LinearRegression
```

In [6]:

```
# (b) Regress the housing price on the number of rooms per house and the
# (number of rooms per house) squared. Show the (curved) regression line in gree

# setting up the model
y = np.array(df[['MEDV']])
X = np.array(df[['RM']])
X2 = X**2
x = np.hstack((X,X2))
model2 = LinearRegression().fit(x, y)
model2.score(x, y)
print("Coef_: ",model2.coef_)
print("Intercept_: ",model2.intercept_)

# plotting the curve
plt.scatter(df['RM'], y,color='thistle', alpha=0.5)
plt.scatter(df['RM'], model2.predict(x),color='g')
plt.xlabel('Rooms')
plt.ylabel('Housing Prices')
ax = plt.gca()
ax.spines['right'].set_color('none')
ax.spines['top'].set_color('none')
ax.yaxis.set_ticks_position('none')
ax.xaxis.set_ticks_position('none')
plt.show()
```

```
Coef_:  [[-23.78960283   2.46914488]]
Intercept_:  [71.73632811]
```



c) Interpret your results.

Part a tells us that for every one unit increase in room number, the median home price value increases by $89,599. The line appears to be doing a fairly good job of predicting the data. Part b, however, appears to be doing a better job of fitting the data. This suggests that more complex models could be worth exploring.

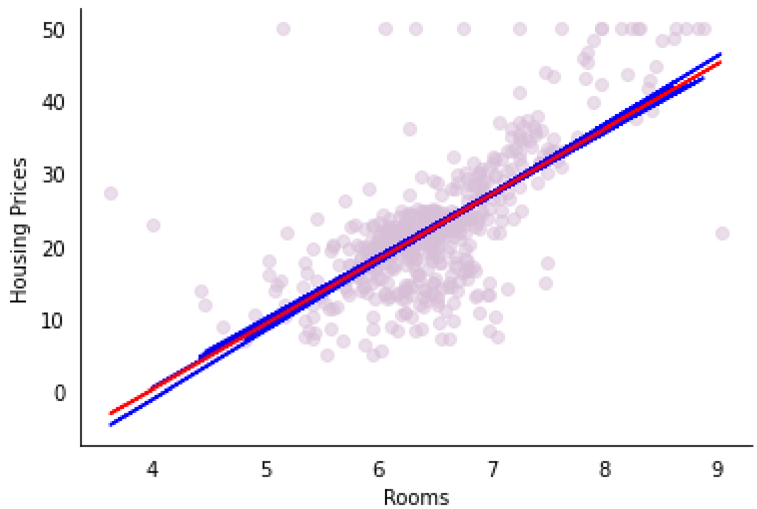
1.2 Training and testing

Chances are, for the above problem you used all of your data to fit the regression line. In some circumstances this is a reasonable thing to do, but often this will result in overfitting. Let's redo the above results the ML way, using careful cross-validation. Since you are now experts in cross-validation, and have written your own cross-validation algorithm from scratch, you can now take a shortcut and use the libraries that others have built for you.

Using the cross-validation functions from scikit-learn, use 5-fold cross-validation to fit the regression model (a) from 1.1, i.e. the linear fit of housing price on number of rooms per house. Each fold of cross-validation will give you one slope coefficient and one intercept coefficient. Create a new scatterplot of housing price against rooms, and draw the five different regression lines in light blue, and the oroginal regression line from 1.1 in red (which was estimated using the full dataset). What do you notice?

In [7]:
```
from sklearn.model_selection import KFold

#setting up the model
X = df['RM']
y = df['MEDV']

model2 = sklearn.linear_model.LinearRegression()

# cross validation
kf = KFold(n_splits = 5, random_state = 1, shuffle = True)
for train_index, test_index in kf.split(X):
    #print(train_index, test_index)
    X_train , X_test = np.array(X.iloc[train_index]),np.array(X.iloc[test_index]
    y_train , y_test = y[train_index] , y[test_index]
```

```
    model2.fit(X_train.reshape(-1, 1),y_train)
    plt.plot(X_test, model2.predict(np.array(X_test).reshape(-1,1)),color='b')

# plotting
plt.scatter(X1, y1,color='thistle', alpha=0.5)
plt.plot(X1, model1.predict(X1),color='r')
plt.xlabel('Rooms')
plt.ylabel('Housing Prices')
ax = plt.gca()
ax.spines['right'].set_color('none')
ax.spines['top'].set_color('none')
ax.yaxis.set_ticks_position('none')
ax.xaxis.set_ticks_position('none')
plt.show()
```



There are variations across all the cross validated lines, but they are all generally around the same. The line using all data appears to be around the center of the cross validated lines, however, it could be overfitted to the data.

Part 2: Gradient descent: Linear Regression

This is where it gets fun!

2.1 Implement gradient descent with one independent variable (average rooms per house)

Implement the batch gradient descent algorithm that we discussed in class. Use the version you implement to regress the housing price on the number of rooms per house. Experiment with 3-4 different values of the learning rate *R*, and do the following:

- Report the values of alpha and beta that minimize the loss function
- Report the number of iterations it takes for your algorithm to converge (for each value of *R*)
- Report the total running time of your algorithm, in seconds
- How do your coefficients compare to the ones estimated through standard libraries? Does this depend on *R*?

Some skeleton code is provided below, but you should feel free to delete this code and start from scratch if you prefer.

- *Hint 1: Don't forget to implement a stopping condition, so that at every iteration you check whether your results have converged. Common approaches to this are to (a) check to see if the loss has stopped decreasing; and (b) check if both your current parameter esimates are close to the estimates from the previous iteration. In both cases, "close" should not be ==0, it should be <=epsilon, where epsilon is something very small (like 0.0001).*
- *Hint 2: Some people like to include a MaxIterations parameter in their gradient descent algorithm, to prevent divergence.*

In [8]:

```
import time
import random

"""
Function
--------
bivariate_ols
    Gradient Decent to minimize OLS. Used to find co-efficients of bivariate OLS

Parameters
----------
xvalues, yvalues : narray
    xvalues: independent variable
    yvalues: dependent variable

R: float
    Learning rate

MaxIterations: Int
    maximum number of iterations


Returns
-------
alpha: float
    intercept

beta: float
    co-efficient
"""



def bivariate_ols(xvalues, yvalues, R=0.01, MaxIterations=1000):
    # initialize the parameters
    start_time = time.time()
    xvalues = np.array(xvalues)
    yvalues = np.array(yvalues)
    alpha = random.random()
    beta = random.random()
    i = 0


    alphas = [np.inf, alpha]
    betas = [np.inf, beta]
```

```
    # gradient descent
    while i <= MaxIterations:
        if np.abs(alphas[-2] - alphas[-1]) > 0.0001 or np.abs(betas[-2] - betas[
            beta -= R*(2/len(xvalues))*np.sum((np.multiply(xvalues,beta)+alpha-y
            alpha -= R*(2/len(xvalues))*np.sum(np.multiply(xvalues,beta)+alpha-y
            alphas.append(alpha)
            betas.append(beta)
        else:
            break
        i += 1
    alpha = alphas[-1]
    beta = betas[-1]
    print("Time taken: {:.2f} seconds".format(time.time() - start_time))
    print("Iteration number: " + str(i))
    return alpha, beta

# example function call
# print(bivariate_ols(X, Y, 0.01, 100000))
```

In [9]:

```
X = df[['RM']]
Y = df[['MEDV']]

# running function with R = .01
t,u = bivariate_ols(X, Y, 0.01, 100000)
print("Alpha: " + str(t))
print("Beta: " + str(u))
```

```
Time taken: 0.31 seconds
Iteration number: 19088
Alpha: -35.14701971606561
Beta: 8.894505588179126
```

In [10]:

```
# running function with R = .001
t,u = bivariate_ols(X, Y, 0.001, 100000)
print("Alpha: " + str(t))
print("Beta: " + str(u))
```

```
Time taken: 1.46 seconds
Iteration number: 92235
Alpha: -31.20692745328819
Beta: 8.293914758781714
```

In [11]:

```
# running function with R = .0001
t,u = bivariate_ols(X, Y, 0.0001, 100000)
print("Alpha: " + str(t))
print("Beta: " + str(u))
```

```
Time taken: 0.01 seconds
Iteration number: 663
Alpha: 0.6343864261889308
Beta: 3.4302155985777203
```

How do your coefficients compare to the ones estimated through standard libraries? Does this depend on R?

Using a standard library and all of the data, we found that beta was 8.96 and alpha was -35.58. Our alpha and beta values using learning rate .01 was very close to our standard library, but became continuously farther away from our standard library values as learning rate decreased.

2.2 Data normalization (done for you!)

Soon, you will implement a version of gradient descent that can use an arbitrary number of independent variables. Before doing this, we want to give you some code in case you want to standardize your features.

In [12]:
```
def standardize(raw_data):
    return ((raw_data - np.mean(raw_data, axis = 0)) / np.std(raw_data, axis = 0
```

2.3 Implement gradient descent with an arbitrary number of independent variables

Now that you have a simple version of gradient descent working, create a version of gradient descent that can take more than one independent variable. Assume all independent variables will be continuous. Test your algorithm using TAX and RM as independent variables. Standardize these variables before inputting them to the gradient descent algorithm.

As before, report and interpret your estimated coefficients, the number of iterations before convergence, and the total running time of your algorithm. Experiment with 2-3 different values of R.

- *Hint 1: Be careful to implement this efficiently, otherwise it might take a long time for your code to run. Commands like* `np.dot` *can be a good friend to you on this problem*

In [13]:
```
"""
Function
--------
multivariate_ols
    Gradient Decent to minimize OLS. Used to find co-efficients of bivariate OLS

Parameters
----------
xvalue_matrix, yvalues : narray
    xvalue_matrix: independent variable
    yvalues: dependent variable

R: float
    Learning rate

MaxIterations: Int
    maximum number of iterations


Returns
-------
alpha: float
    intercept
```

```
beta_array: array[float]
    co-efficient
"""

def multivariate_ols(xvalue_matrix, yvalues, R=0.01, MaxIterations=1000):
    # initialize the parameters
    start_time = time.time()
    xvalue_matrix = np.array(xvalue_matrix)
    yvalues = np.array(yvalues)
    alpha = random.random()
    N = len(yvalues)
    shape = np.shape(xvalue_matrix)
    beta_array = np.random.rand(shape[1],1)

    # gradient descent
    for i in range(MaxIterations):
        y_hat = np.dot(xvalue_matrix,beta_array) + alpha
        alpha_partial = np.sum(y_hat-yvalues)/N
        beta_partial = np.dot(xvalue_matrix.T,(y_hat-yvalues))/N
        new_alpha = alpha - (R*alpha_partial)
        new_beta = beta_array - (R*beta_partial)
        if abs(alpha - new_alpha) <= .0001 and max(abs(beta_array - new_beta)) <
            print("Time taken: {:.2f} seconds".format(time.time() - start_time))
            print("Iteration number: " + str(i))
            return new_alpha, new_beta
        alpha = new_alpha
        beta_array = new_beta
    print("Time taken: {:.2f} seconds".format(time.time() - start_time))
    print("Iteration number: " + str(i))
    return new_alpha, new_beta
```

In [14]:

```
Y = np.array(df[['MEDV']])
X = standardize(df[['RM','TAX']])


# running function with R = .01
t,u = multivariate_ols(X, Y, 0.01, 100000)
print("Alpha: " + str(t))
print("Beta: " + str(u))
```

```
Time taken: 0.01 seconds
Iteration number: 765
Alpha: 22.52290993255893
Beta: [[ 5.53559337]
 [-2.72407888]]
```

In [15]:

```
# running function with R = .001
t,u = multivariate_ols(X, Y, 0.001, 100000)
print("Alpha: " + str(t))
print("Beta: " + str(u))
```

```
Time taken: 0.07 seconds
Iteration number: 5377
Alpha: 22.43295256596492
Beta: [[ 5.51579539]
 [-2.73596294]]
```

In [16]:
```
# running function with R = .0001
t,u = multivariate_ols(X, Y, 0.0001, 100000)
print("Alpha: " + str(t))
print("Beta: " + str(u))
```

```
Time taken: 0.41 seconds
Iteration number: 31064
Alpha: 21.53296571687588
Beta: [[ 5.35611423]
 [-2.73567129]]
```

It appears that as R decreases, alpha decreases and our beta values decrease. Our running time and iteration count increase as R decreases.

2.4 Compare standardized vs. non-standardized results

Repeat the analysis from 2.3, but this time do not standardize your variables - i.e., use the original data. Use the same three values of R (0.1, 0.01, and 0.001). What do you notice about the running time and convergence properties of your algorithm?

In [17]:
```
Y = np.array(df[['MEDV']])
X = (df[['RM','TAX']])
print(type(X))

# running function with R = .01
t,u = multivariate_ols(X, Y, 0.01, 100000)
print(t, u)
```

```
<class 'pandas.core.frame.DataFrame'>
<ipython-input-13-13b5787c16a3>:45: RuntimeWarning: invalid value encountered in subtract
  new_beta = beta_array - (R*beta_partial)
Time taken: 1.31 seconds
Iteration number: 99999
nan [[nan]
 [nan]]
```

In [18]:
```
# running function with R = .001
t,u = multivariate_ols(X, Y, 0.001, 100000)
print(t, u)
```

```
<ipython-input-13-13b5787c16a3>:45: RuntimeWarning: invalid value encountered in subtract
  new_beta = beta_array - (R*beta_partial)
Time taken: 1.31 seconds
Iteration number: 99999
nan [[nan]
 [nan]]
```

In [19]:
```
# running function with R = .0001
t,u = multivariate_ols(X, Y, 0.0001, 100000)
print(t, u)
```

```
<ipython-input-13-13b5787c16a3>:45: RuntimeWarning: invalid value encountered in subtract
  new_beta = beta_array - (R*beta_partial)
Time taken: 1.35 seconds
```

```
Iteration number: 99999
nan [[nan]
 [nan]]
```

As R decreases, the running time seems to increase. Our algorithm does not converge with any value of R when our data is not standardized, but it does converge in 2.3 when we standardize.

3. Prediction

Let's use our fitted model to make predictions about housing prices. Make sure to first standardize your features before proceeding.

3.1 Cross-Validation

Unless you were careful above, you probably overfit your data again. Let's fix that. Use 5-fold cross-validation to re-fit the multivariate regression from 2.3 above, and report your estimated coefficients (there should be three, corresponding to the intercept and the two coefficients for TAX and RM). Since there are 5 folds, there will be 5 sets of three coefficients -- report them all in a 5x3 table.

In [20]:

```
from tabulate import tabulate
import math
```

In [21]:

```
Y = np.array(df[['MEDV']])
X = standardize(df[['RM','TAX']])

coef_alpha = []
coef_betas = []

# cross validation
kf = KFold(n_splits = 5, random_state = 1, shuffle = True)
for train_index, test_index in kf.split(X):
    X_train , X_test = np.array(X.iloc[train_index]),np.array(X.iloc[test_index]
    y_train , y_test = np.array([y[train_index]]).T , np.array([y[test_index]]).
    t, u = multivariate_ols(X_train,y_train, 0.01, 100000)
    coef_alpha.append(t)
    coef_betas.append(u)

# collecting all the coefficients to make a table
d = []
for i in range(5):
    d.append((coef_alpha[i],coef_betas[i][0],coef_betas[i][1]))
# printing a table
print(tabulate(d, headers=["Intercept", "RM Coef", "TAX Coef"]))

print("Average Intercept: " + str(np.mean(coef_alpha)) + ", Average RM Coefficie
```

```
Time taken: 0.01 seconds
Iteration number: 776
Time taken: 0.01 seconds
Iteration number: 773
Time taken: 0.01 seconds
Iteration number: 770
Time taken: 0.01 seconds
```

```
Iteration number: 764
Time taken: 0.01 seconds
Iteration number: 763
  Intercept    RM Coef    TAX Coef
-----------  ---------  ----------
    22.5541    5.25989    -2.76852
    22.445     5.86443    -2.67853
    22.7348    5.45004    -2.56557
    22.4274    5.68236    -2.80136
    22.4595    5.43218    -2.78532
Average Intercept: 22.524158721907597, Average RM Coefficient: [5.53778027], Ave
rage TAX Coefficient: [-2.71985996]
```

As we can see from the table, cross validation produces slight variation in the coefficients. The averages are reported above.

3.2 Predicted values and RMSE

Let's figure out how accurate this predictive model turned out to be. Compute the cross-validated RMSE for each of the 5 folds above. In other words, in fold 1, use the parameters estimated on the 80% of the data to make predictions for the 20%, and calculate the RMSE for those 20%. Repeate this for the remaining folds. Report the RMSE for each of the 5-folds, and the average (mean) RMSE across the five folds. How does this average RMSE compare to the performance of your nearest neighbor algorithm from the last problem set?

In [22]:

```python
def compute_rmse(predictions, yvalues):
    # taking the difference between the 2 arrays
    diffs = (np.array(yvalues)-np.array(predictions))
    # squaring the differences
    squares = np.square(diffs)
    # summing the squares
    s = np.sum(squares)
    # dividing by the length
    inside = s/len(diffs)
    # taking the square root
    rmse = math.sqrt(inside)
    return rmse
```

In [23]:

```python
# to be used for prediction
def model(alpha, beta_array, xvalue_matrix):
    pred = np.dot(xvalue_matrix,beta_array) + alpha
    return pred
```

In [24]:

```python
Y = (df[['MEDV']])
X = standardize(df[['RM','TAX']])

# cross validating and keeping track of RMSE
rmses = []
i = 0
kf = KFold(n_splits = 5, random_state = 1, shuffle = True)
for train_index, test_index in kf.split(X):
    X_train , X_test = np.array(X.iloc[train_index]),np.array(X.iloc[test_index]
    y_train , y_test = np.array([y[train_index]]).T , np.array([y[test_index]]).
    predictions = np.dot(X_test,coef_betas[i]) + coef_alpha[i]
    rmse = compute_rmse(predictions,y_test)
```

```
    rmses.append(rmse)
    i += 1
```

In [25]:

```
from statistics import mean
# printing entire list of RMSEs
print(rmses)
# printing the average RMSE
print(mean(rmses))
```

```
[5.943839700003763, 6.987924014364593, 5.119632335085662, 6.638631824792226, 6.0
78273481162675]
6.153660271081784
```

NN Test RMSE: 7.11504450215995 Our average RMSE for this method (6.15) is lower than the average RMSE for Nearest Neighbors (7.11), so this algorithm performs better than nearest neighbors. It must be noted that we are using different features across problem sets, so we cannot accurately compare these two measures. I use the same model as in PS 3 below. We find that the average RMSE using this algorithm on the old model is 6.2, which is better than 7.11, so it appears that this gradient descent algorithm is better than nearest neighbors for this model and dataset.

In [26]:

```
Y = np.array(df[['MEDV']])
X = standardize(df[['CRIM','RM','ZN']])

coef_alpha = []
coef_betas = []

# cross validation
kf = KFold(n_splits = 5, random_state = 1, shuffle = True)
for train_index, test_index in kf.split(X):
    X_train , X_test = np.array(X.iloc[train_index]),np.array(X.iloc[test_index]
    y_train , y_test = np.array([y[train_index]]).T , np.array([y[test_index]]).
    t, u = multivariate_ols(X_train,y_train, 0.01, 100000)
    coef_alpha.append(t)
    coef_betas.append(u)

d = []
for i in range(5):
    d.append((coef_alpha[i],coef_betas[i][0],coef_betas[i][1]))

# cross validating and keeping track of RMSE
rmses = []
i = 0
kf = KFold(n_splits = 5, random_state = 1, shuffle = True)
for train_index, test_index in kf.split(X):
    X_train , X_test = np.array(X.iloc[train_index]),np.array(X.iloc[test_index]
    y_train , y_test = np.array([y[train_index]]).T , np.array([y[test_index]]).
    predictions = np.dot(X_test,coef_betas[i]) + coef_alpha[i]
    rmse = compute_rmse(predictions,y_test)
    rmses.append(rmse)
    i += 1
# printing the average RMSE
print(mean(rmses))
```

```
Time taken: 0.01 seconds
Iteration number: 802
```

```
Time taken: 0.01 seconds
Iteration number: 786
Time taken: 0.01 seconds
Iteration number: 773
Time taken: 0.01 seconds
Iteration number: 760
Time taken: 0.01 seconds
Iteration number: 764
6.232595466719972
```

Extra Credit 1: Logistic Regression

For extra credit, implement logistic regression using gradient descent. Create a new variable (EXPENSIVE) to indicate whether the median housing price is more than $40,000. Use your model a logistic regression of EXPENSIVE on CHAS and RM. Report your results.

In [27]:
```
# Your code here
```

Discuss your results here

4 Regularization

4.1 Get prepped

Step 1: Create new interaction variables between each possible pair of the F_s features. If you originally had *K* features, you should now have K+(K*(K+1))/2 features. Standardize all of your features.

Step 2: Randomly sample 80% of your data and call this the training set, and set aside the remaining 20% as your test set.

In [28]:
```
from sklearn import model_selection, preprocessing
```

In [29]:
```
# creating all features
for i in range(13):
    for j in range(i,13):
        df[str(df.columns[i])+'*'+str(df.columns[j])]=df.iloc[:,i]*df.iloc[:,j]

df1 = standardize(df.loc[:, df.columns != 'MEDV'])
print(df1.head())
```

```
       CRIM        ZN     INDUS      CHAS       NOX        RM       AGE  \
0 -0.416323  0.284830 -1.270520 -0.272599  0.738124  0.552955 -0.120013
1 -0.425331 -0.487722 -0.586750 -0.272599 -1.184627  0.020504  0.367166
2 -0.397524 -0.487722 -0.583140 -0.272599 -0.293242  1.103260 -0.265812
3 -0.405311 -0.487722 -1.280278 -0.272599 -0.884416  1.027034 -0.809889
4 -0.403385 -0.487722 -1.289773 -0.272599 -1.627468  0.990705 -0.511180

        DIS       RAD       TAX  ...   TAX*TAX  TAX*PTRATIO     TAX*B  \
0  0.165247 -0.982843 -0.642280  ... -0.666456    -0.833076 -0.309350
1  0.624852 -0.867883 -0.950995  ... -0.849329    -0.886279 -0.628821
2  0.644430 -0.867883 -1.022237  ... -0.886725    -0.942789 -0.703492
3  0.998977 -0.752922 -1.123163  ... -0.936618    -0.960038 -0.800412
4  1.075097 -0.752922 -1.075668  ... -0.913589    -0.936617 -0.748078
```

```
   TAX*LSTAT  PTRATIO*PTRATIO  PTRATIO*B  PTRATIO*LSTAT       B*B   B*LSTAT  \
0  -0.835367        -1.443290  -0.260551      -1.088942  0.497397 -0.827189
1  -0.714306        -0.413379   0.245572      -0.557697  0.488925 -0.282211
2  -0.979900        -0.410662   0.243110      -1.187464  0.481691 -1.065847
3  -1.032944         0.095261   0.489781      -1.273772  0.518208 -1.199447
4  -0.897918        -0.036731   0.440878      -0.915578  0.535675 -0.762212

   LSTAT*LSTAT
0    -0.783600
1    -0.557487
2    -0.847899
3    -0.875541
4    -0.764004

[5 rows x 104 columns]
```

In [30]:

```python
# splitting the data into train and test sets
X_train , X_test, y_train , y_test = model_selection.train_test_split(df1,df[['M

# checking dimensions
print(np.shape(X_train))
print(np.shape(X_test))
print(np.shape(y_train))
print(np.shape(y_test))
```

```
(404, 104)
(102, 104)
(404, 1)
(102, 1)
```

4.2 Overfitting (sort of)

Now, using your version of multivariate regression from 2.3, let's overfit the training data. Using your training set, regress housing price on as many of those K+(K*(K+1))/2 features as you can (Don't forget to add quadratic terms. Form instance, RM^2.). If you get too greedy, it's possible this will take a long time to compute, so start with 5-10 features, and if you have the time, add more features.

Report the RMSE when you apply your model to your training set and to your testing set. How do these numbers compare to each other, and to the RMSE from 3.2 and nearest neighbors?

In [31]:

```python
# training the model
a, b = multivariate_ols(X_train, y_train, R=0.01, MaxIterations=100000)
# predictions for training set
pred = model(a, b, X_train)
# computing RMSE
rmse1 = compute_rmse(pred, y_train)

# predictions for test set
pred = model(a, b, X_test)
# computing RMSE
rmse2 = compute_rmse(pred, y_test)

print("Training RMSE: " + str(rmse1) + "; Testing RMSE: " + str(rmse2))
```

```
Time taken: 3.73 seconds
```

```
Iteration number: 26794
Training RMSE: 2.719573979435247; Testing RMSE: 4.3016431756475875
```

How do these numbers compare to each other, and to the RMSE from 3.2 and nearest neighbors? 3.2 Testing RMSE: 6.15

My model uses all of the features, and gets a much lower training and testing RMSE. It makes sense that the testing RMSE (4.30) is higher than the training RMSE (2.72), as this data is likely to be overfitted in the absense of cross validation.

Comparing this to the 3.2 testing RMSE (6.15), we can say that our model is much better using all the features instead of just RM and TAX. However, we are not penalizing additional variables in our model, so this could be inaccurate.

Comparing to Nearest Neighbors from PS 3, our testing RMSE was 7.11, which is much higher than the RMSE we get here, which is likely the result of a better model and algorithm.

4.3 Ridge regularization (basic)

Incorporate L2 (Ridge) regularization into your multivariate_ols regression. Write a new version of your gradient descent algorithm that includes a regularization term "lambda" to penalize excessive complexity.

Use your regularized regression to re-fit the model from 4.2 above on your training data, using the value lambda = 0.5. Report the RMSE obtained for your training data, and the RMSE obtained for your testing data.

In [32]:

```
def reg_multivariate_ols(xvalue_matrix, yvalues, R=0.01, MaxIterations=1000, lam
    # initialize the parameters
    start_time = time.time()
    xvalue_matrix = np.array(xvalue_matrix)
    yvalues = np.array(yvalues)
    alpha = random.random()
    N = len(yvalues)
    shape = np.shape(xvalue_matrix)
    beta_array = np.random.rand(shape[1],1)

    # gradient descent
    for i in range(MaxIterations):
        y_hat = np.dot(xvalue_matrix,beta_array) + alpha
        alpha_partial = np.sum(y_hat-yvalues)/N
        beta_partial = np.dot(xvalue_matrix.T,(y_hat-yvalues))
        new_alpha = alpha - (R*alpha_partial)
        # ridge regularization
        new_beta = beta_array - (R/N)*(beta_partial + lam*beta_array)
        if abs(alpha - new_alpha) <= .0001 and max(np.abs(beta_array - new_beta)
            print("Time taken: {:.2f} seconds".format(time.time() - start_time))
            print("Iteration number: " + str(i))
            return new_alpha, new_beta
        alpha = new_alpha
        beta_array = new_beta
    print("Time taken: {:.2f} seconds".format(time.time() - start_time))
    print("Iteration number: " + str(i))
    return new_alpha, new_beta
```

In [33]:

```
# training the model
a, b = reg_multivariate_ols(X_train, y_train, R=0.01, MaxIterations=100000, lam=
# training predictions
pred = np.dot(X_train,b) + a
# computing RMSE
rmse1 = compute_rmse(pred, y_train)

# testing predictions
pred2 = np.dot(X_test,b) + a
# computing RMSE
rmse2 = compute_rmse(pred2, y_test)

print("Training RMSE: " + str(rmse1) + "; Testing RMSE: " + str(rmse2))
```

```
Time taken: 2.41 seconds
Iteration number: 18039
Training RMSE: 2.791560637154338; Testing RMSE: 4.329249888142999
```

Here we can see that our training RMSE (2.79) is lower than our testing RMSE (4.33), which is likely demonstrating overfitting. Compared to 4.2, our training and testing RMSE are slightly higher, which makes sense, as it is demonstrating the result of penalizing additional complexity.

4.4: Cross-validate lambda

This is where it all comes together! Use k-fold cross-validation to select the optimal value of lambda. In other words, define a set of different values of lambda. Then, using the 80% of your data that you set aside for training, iterate through the values of lambda one at a time. For each value of lambda, use k-fold cross-validation to compute the average cross-validated (test) RMSE for that lambda value, computed as the average across the held-out folds. You should also record the average cross-validated train RMSE, computed as the average across the folds used for training. Create a scatter plot that shows RMSE as a function of lambda. The scatter plot should have two lines: a red line showing the cross-validated (test) RMSE, and a blue line showing the cross-validated train RMSE. At this point, you should not have touched your held-out 20% of "true" test data.

What value of lambda minimizes your cross-validated (test) RMSE? Fix that value of lambda, and train a new model using all of your training data with that value of lambda (i.e., use the entire 80% of the data that you set aside in 4.1). Calcuate the RMSE for this model on the 20% of "true" test data. How does your test RMSE compare to the RMSE from 4.3, 4.2, 2.3, and to the RMSE from nearest neighbors? What do you make of these results?

Go brag to your friends about how you just implemented cross-validated ridge-regularized multivariate regression using gradient descent optimization, from scratch. If you still have friends.

In [47]:

```
mean_train = []
mean_test = []
lmbdas = np.logspace(-1, 1, 50)

# looping through values of lambda
```

```
for lmbda in lmbdas:
    rmses_train = []
    rmses_test = []
    i = 0
    # cross validation
    kf = KFold(n_splits = 5, random_state = 1, shuffle = True)
    for train_index, test_index in kf.split(X_train):
        X_train_1 , X_test_1 = np.array(X_train.iloc[train_index]),np.array(X_tr
        y_train_1 , y_test_1 = np.array(y_train.iloc[train_index]) , np.array(y_
        # training the model
        a, b = reg_multivariate_ols(X_train_1, y_train_1, R=0.01, MaxIterations=
        # training predictions
        pred = model(a, b, X_train_1)
        # computing RMSE
        rmse_train = compute_rmse(pred,y_train_1)
        rmses_train.append(rmse_train)
        # testing predictions
        pred2 = model(a, b, X_test_1)
        # computing RMSE
        rmse_test = compute_rmse(pred2,y_test_1)
        rmses_test.append(rmse_test)
        i += 1
    # keeping track of the average RMSE for each value of lambda
    mean_train.append(mean(rmses_train))
    mean_test.append(mean(rmses_test))
```

```
Time taken: 2.89 seconds
Iteration number: 28089
Time taken: 2.35 seconds
Iteration number: 23423
Time taken: 1.56 seconds
Iteration number: 15506
Time taken: 2.35 seconds
Iteration number: 22350
Time taken: 4.10 seconds
Iteration number: 33682
Time taken: 3.28 seconds
Iteration number: 26149
Time taken: 2.68 seconds
Iteration number: 25592
Time taken: 1.66 seconds
Iteration number: 16586
Time taken: 2.31 seconds
Iteration number: 22765
Time taken: 3.27 seconds
Iteration number: 32314
Time taken: 3.65 seconds
Iteration number: 27876
Time taken: 2.39 seconds
Iteration number: 23128
Time taken: 2.32 seconds
Iteration number: 17201
Time taken: 2.57 seconds
Iteration number: 22842
Time taken: 3.32 seconds
Iteration number: 33214
Time taken: 2.73 seconds
Iteration number: 25823
Time taken: 2.50 seconds
Iteration number: 24308
Time taken: 2.46 seconds
Iteration number: 16793
Time taken: 2.26 seconds
```

```
Iteration number: 22281
Time taken: 3.97 seconds
Iteration number: 31628
Time taken: 2.53 seconds
Iteration number: 25010
Time taken: 2.43 seconds
Iteration number: 23587
Time taken: 1.72 seconds
Iteration number: 16925
Time taken: 2.30 seconds
Iteration number: 22810
Time taken: 3.86 seconds
Iteration number: 30825
Time taken: 3.00 seconds
Iteration number: 27571
Time taken: 3.10 seconds
Iteration number: 24218
Time taken: 1.67 seconds
Iteration number: 16617
Time taken: 2.21 seconds
Iteration number: 21818
Time taken: 2.93 seconds
Iteration number: 29128
Time taken: 2.88 seconds
Iteration number: 26839
Time taken: 2.85 seconds
Iteration number: 21195
Time taken: 1.68 seconds
Iteration number: 16535
Time taken: 2.91 seconds
Iteration number: 21778
Time taken: 3.13 seconds
Iteration number: 29656
Time taken: 2.31 seconds
Iteration number: 23350
Time taken: 2.20 seconds
Iteration number: 22213
Time taken: 1.70 seconds
Iteration number: 16258
Time taken: 2.68 seconds
Iteration number: 21968
Time taken: 3.33 seconds
Iteration number: 29796
Time taken: 3.43 seconds
Iteration number: 26635
Time taken: 2.11 seconds
Iteration number: 20778
Time taken: 1.50 seconds
Iteration number: 15313
Time taken: 2.25 seconds
Iteration number: 23082
Time taken: 2.75 seconds
Iteration number: 27436
Time taken: 3.06 seconds
Iteration number: 25580
Time taken: 2.55 seconds
Iteration number: 21592
Time taken: 1.59 seconds
Iteration number: 16012
Time taken: 2.87 seconds
Iteration number: 20893
Time taken: 2.98 seconds
Iteration number: 29843
Time taken: 2.42 seconds
Iteration number: 23743
```

```
Time taken: 2.14 seconds
Iteration number: 20785
Time taken: 1.58 seconds
Iteration number: 15576
Time taken: 2.70 seconds
Iteration number: 21207
Time taken: 2.99 seconds
Iteration number: 26757
Time taken: 2.84 seconds
Iteration number: 21920
Time taken: 2.35 seconds
Iteration number: 21148
Time taken: 1.61 seconds
Iteration number: 15947
Time taken: 2.11 seconds
Iteration number: 19889
Time taken: 2.78 seconds
Iteration number: 27109
Time taken: 2.34 seconds
Iteration number: 22718
Time taken: 2.87 seconds
Iteration number: 21065
Time taken: 1.62 seconds
Iteration number: 15989
Time taken: 2.49 seconds
Iteration number: 19630
Time taken: 2.73 seconds
Iteration number: 24280
Time taken: 2.27 seconds
Iteration number: 22033
Time taken: 2.10 seconds
Iteration number: 20351
Time taken: 1.61 seconds
Iteration number: 15984
Time taken: 1.97 seconds
Iteration number: 19194
Time taken: 3.42 seconds
Iteration number: 26327
Time taken: 1.98 seconds
Iteration number: 19564
Time taken: 2.51 seconds
Iteration number: 19826
Time taken: 1.95 seconds
Iteration number: 16209
Time taken: 1.98 seconds
Iteration number: 19549
Time taken: 2.19 seconds
Iteration number: 22089
Time taken: 1.98 seconds
Iteration number: 19377
Time taken: 1.88 seconds
Iteration number: 18458
Time taken: 1.70 seconds
Iteration number: 15566
Time taken: 2.61 seconds
Iteration number: 19055
Time taken: 2.44 seconds
Iteration number: 24522
Time taken: 2.94 seconds
Iteration number: 21338
Time taken: 1.77 seconds
Iteration number: 17371
Time taken: 1.42 seconds
Iteration number: 14134
Time taken: 1.84 seconds
```

```
Iteration number: 18708
Time taken: 2.27 seconds
Iteration number: 22750
Time taken: 2.13 seconds
Iteration number: 20869
Time taken: 2.47 seconds
Iteration number: 17724
Time taken: 1.72 seconds
Iteration number: 14786
Time taken: 1.82 seconds
Iteration number: 17793
Time taken: 2.74 seconds
Iteration number: 19407
Time taken: 1.95 seconds
Iteration number: 18661
Time taken: 1.91 seconds
Iteration number: 18361
Time taken: 1.48 seconds
Iteration number: 14725
Time taken: 1.78 seconds
Iteration number: 18048
Time taken: 1.98 seconds
Iteration number: 19806
Time taken: 2.04 seconds
Iteration number: 18020
Time taken: 2.12 seconds
Iteration number: 15189
Time taken: 1.36 seconds
Iteration number: 13505
Time taken: 1.95 seconds
Iteration number: 18005
Time taken: 2.51 seconds
Iteration number: 18925
Time taken: 1.87 seconds
Iteration number: 18168
Time taken: 1.71 seconds
Iteration number: 16807
Time taken: 1.39 seconds
Iteration number: 13632
Time taken: 1.66 seconds
Iteration number: 16510
Time taken: 1.76 seconds
Iteration number: 17418
Time taken: 1.87 seconds
Iteration number: 16769
Time taken: 2.15 seconds
Iteration number: 14783
Time taken: 1.46 seconds
Iteration number: 14638
Time taken: 1.63 seconds
Iteration number: 16272
Time taken: 2.44 seconds
Iteration number: 16936
Time taken: 1.74 seconds
Iteration number: 17721
Time taken: 1.42 seconds
Iteration number: 14390
Time taken: 1.27 seconds
Iteration number: 13467
Time taken: 1.72 seconds
Iteration number: 17024
Time taken: 1.70 seconds
Iteration number: 16597
Time taken: 1.73 seconds
Iteration number: 17107
```

```
Time taken: 1.69 seconds
Iteration number: 13925
Time taken: 1.87 seconds
Iteration number: 13097
Time taken: 1.73 seconds
Iteration number: 16431
Time taken: 1.73 seconds
Iteration number: 14974
Time taken: 2.17 seconds
Iteration number: 15431
Time taken: 1.44 seconds
Iteration number: 13979
Time taken: 1.30 seconds
Iteration number: 12623
Time taken: 1.53 seconds
Iteration number: 15330
Time taken: 1.43 seconds
Iteration number: 14664
Time taken: 1.59 seconds
Iteration number: 15729
Time taken: 1.33 seconds
Iteration number: 13122
Time taken: 1.13 seconds
Iteration number: 11417
Time taken: 2.43 seconds
Iteration number: 14830
Time taken: 1.80 seconds
Iteration number: 14857
Time taken: 1.32 seconds
Iteration number: 13229
Time taken: 1.56 seconds
Iteration number: 12396
Time taken: 1.76 seconds
Iteration number: 12263
Time taken: 1.52 seconds
Iteration number: 14895
Time taken: 1.62 seconds
Iteration number: 15492
Time taken: 1.41 seconds
Iteration number: 12975
Time taken: 1.35 seconds
Iteration number: 13300
Time taken: 1.10 seconds
Iteration number: 11347
Time taken: 1.47 seconds
Iteration number: 14111
Time taken: 1.29 seconds
Iteration number: 13168
Time taken: 1.76 seconds
Iteration number: 12799
Time taken: 1.60 seconds
Iteration number: 12369
Time taken: 1.20 seconds
Iteration number: 11689
Time taken: 1.32 seconds
Iteration number: 13259
Time taken: 1.82 seconds
Iteration number: 13007
Time taken: 1.52 seconds
Iteration number: 12652
Time taken: 1.29 seconds
Iteration number: 12728
Time taken: 1.08 seconds
Iteration number: 10818
Time taken: 1.34 seconds
```

```
Iteration number: 12876
Time taken: 1.25 seconds
Iteration number: 12552
Time taken: 1.09 seconds
Iteration number: 11371
Time taken: 1.16 seconds
Iteration number: 11641
Time taken: 0.96 seconds
Iteration number: 9672
Time taken: 1.31 seconds
Iteration number: 12486
Time taken: 1.83 seconds
Iteration number: 12737
Time taken: 1.45 seconds
Iteration number: 11614
Time taken: 1.20 seconds
Iteration number: 11677
Time taken: 1.09 seconds
Iteration number: 10704
Time taken: 1.70 seconds
Iteration number: 12331
Time taken: 1.47 seconds
Iteration number: 11472
Time taken: 1.18 seconds
Iteration number: 11571
Time taken: 1.04 seconds
Iteration number: 10332
Time taken: 0.95 seconds
Iteration number: 9549
Time taken: 1.16 seconds
Iteration number: 11665
Time taken: 1.08 seconds
Iteration number: 10907
Time taken: 0.97 seconds
Iteration number: 9444
Time taken: 1.09 seconds
Iteration number: 10727
Time taken: 0.89 seconds
Iteration number: 8805
Time taken: 1.10 seconds
Iteration number: 11045
Time taken: 1.44 seconds
Iteration number: 10747
Time taken: 1.27 seconds
Iteration number: 8574
Time taken: 1.12 seconds
Iteration number: 10240
Time taken: 0.92 seconds
Iteration number: 9134
Time taken: 1.08 seconds
Iteration number: 10855
Time taken: 1.21 seconds
Iteration number: 9373
Time taken: 1.28 seconds
Iteration number: 8751
Time taken: 1.13 seconds
Iteration number: 10466
Time taken: 0.84 seconds
Iteration number: 8273
Time taken: 1.02 seconds
Iteration number: 10294
Time taken: 0.92 seconds
Iteration number: 9477
Time taken: 0.82 seconds
Iteration number: 8535
```

```
Time taken: 0.96 seconds
Iteration number: 9637
Time taken: 0.83 seconds
Iteration number: 8180
Time taken: 1.02 seconds
Iteration number: 10200
Time taken: 0.81 seconds
Iteration number: 8374
Time taken: 0.83 seconds
Iteration number: 8148
Time taken: 0.92 seconds
Iteration number: 9246
Time taken: 0.95 seconds
Iteration number: 7991
Time taken: 1.37 seconds
Iteration number: 9276
Time taken: 1.04 seconds
Iteration number: 8126
Time taken: 0.77 seconds
Iteration number: 7641
Time taken: 0.85 seconds
Iteration number: 8847
Time taken: 0.79 seconds
Iteration number: 7698
Time taken: 1.09 seconds
Iteration number: 8968
Time taken: 1.11 seconds
Iteration number: 7763
Time taken: 0.99 seconds
Iteration number: 7611
Time taken: 0.81 seconds
Iteration number: 8249
Time taken: 0.74 seconds
Iteration number: 7548
Time taken: 0.79 seconds
Iteration number: 7999
Time taken: 0.67 seconds
Iteration number: 6701
Time taken: 0.72 seconds
Iteration number: 7363
Time taken: 0.84 seconds
Iteration number: 8342
Time taken: 0.73 seconds
Iteration number: 7573
Time taken: 0.78 seconds
Iteration number: 8067
Time taken: 0.75 seconds
Iteration number: 7457
Time taken: 0.67 seconds
Iteration number: 6704
Time taken: 0.76 seconds
Iteration number: 7852
Time taken: 0.61 seconds
Iteration number: 6132
Time taken: 0.79 seconds
Iteration number: 7908
Time taken: 0.79 seconds
Iteration number: 6560
Time taken: 1.00 seconds
Iteration number: 6818
Time taken: 1.06 seconds
Iteration number: 7160
Time taken: 0.59 seconds
Iteration number: 5893
Time taken: 0.81 seconds
```

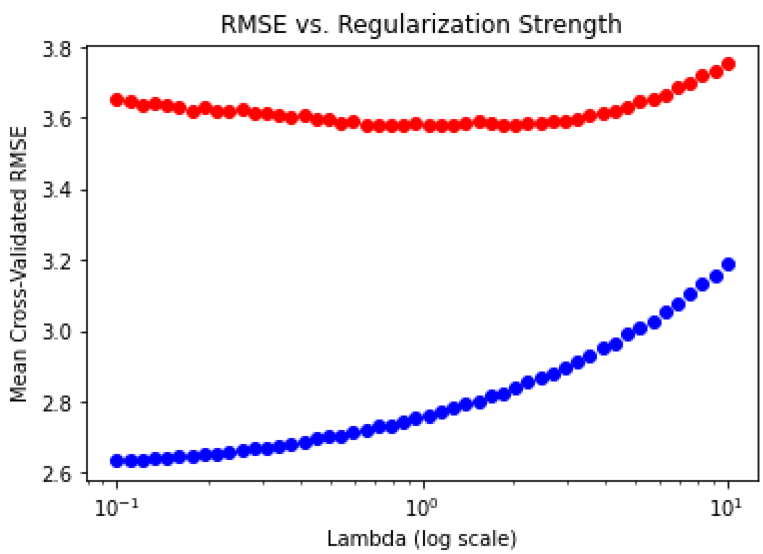
```
Iteration number: 7394
Time taken: 0.65 seconds
Iteration number: 6333
Time taken: 0.65 seconds
Iteration number: 6422
Time taken: 0.78 seconds
Iteration number: 7060
Time taken: 0.89 seconds
Iteration number: 6203
Time taken: 1.06 seconds
Iteration number: 7363
Time taken: 0.73 seconds
Iteration number: 6704
Time taken: 0.61 seconds
Iteration number: 5966
Time taken: 0.60 seconds
Iteration number: 6010
Time taken: 0.57 seconds
Iteration number: 6282
Time taken: 0.70 seconds
Iteration number: 6892
Time taken: 0.55 seconds
Iteration number: 5757
Time taken: 0.58 seconds
Iteration number: 5812
Time taken: 0.70 seconds
Iteration number: 6429
Time taken: 0.58 seconds
Iteration number: 5848
Time taken: 0.65 seconds
Iteration number: 6460
Time taken: 0.62 seconds
Iteration number: 5795
Time taken: 0.54 seconds
Iteration number: 5275
Time taken: 0.60 seconds
Iteration number: 6189
Time taken: 0.49 seconds
Iteration number: 5408
Time taken: 0.56 seconds
Iteration number: 5906
Time taken: 0.50 seconds
Iteration number: 5277
Time taken: 0.49 seconds
Iteration number: 5001
Time taken: 0.56 seconds
Iteration number: 5465
Time taken: 0.79 seconds
Iteration number: 4916
Time taken: 0.83 seconds
Iteration number: 5714
Time taken: 0.76 seconds
Iteration number: 5284
Time taken: 0.53 seconds
Iteration number: 5137
Time taken: 0.55 seconds
Iteration number: 5607
Time taken: 0.51 seconds
Iteration number: 5159
Time taken: 0.47 seconds
Iteration number: 4940
Time taken: 0.46 seconds
Iteration number: 4970
Time taken: 0.43 seconds
Iteration number: 4488
```

```
Time taken: 0.60 seconds
Iteration number: 5006
Time taken: 0.64 seconds
Iteration number: 4543
Time taken: 0.70 seconds
Iteration number: 4916
Time taken: 0.66 seconds
Iteration number: 4822
```

In [48]:

```
print(lmbdas)
# plotting
fig, ax = plt.subplots(1)
ax.scatter(lmbdas, mean_train, c = 'blue')
ax.scatter(lmbdas, mean_test, c = 'red')
plt.xlabel('True Value')
ax.set_xscale('log')
ax.set_xlabel('Lambda (log scale)')
ax.set_ylabel('Mean Cross-Validated RMSE')
ax.set_title('RMSE vs. Regularization Strength')
plt.show()
```

```
[ 0.1         0.10985411  0.12067926  0.13257114  0.14563485  0.15998587
  0.17575106  0.19306977  0.21209509  0.23299518  0.25595479  0.28117687
  0.30888436  0.33932218  0.37275937  0.40949151  0.44984327  0.49417134
  0.54286754  0.59636233  0.65512856  0.71968567  0.79060432  0.86851137
  0.95409548  1.04811313  1.1513954   1.26485522  1.38949549  1.52641797
  1.67683294  1.84206997  2.02358965  2.22299648  2.44205309  2.6826958
  2.9470517   3.23745754  3.55648031  3.90693994  4.29193426  4.71486636
  5.17947468  5.68986603  6.25055193  6.86648845  7.54312006  8.28642773
  9.10298178 10.        ]
```



What value of lambda minimizes your cross-validated (test) RMSE? Fix that value of lambda, and train a new model using all of your training data with that value of lambda (i.e., use the entire 80% of the data that you set aside in 4.1). Calcuate the RMSE for this model on the 20% of "true" test data. How does your test RMSE compare to the RMSE from 4.3, 4.2, 2.3, and to the RMSE from nearest neighbors? What do you make of these results?

In [49]:

```
minval = np.argmin(mean_test)
print(mean_test)
print(minval)
```

```
minlambda = lmbdas[minval]
print(minlambda)
```

```
[3.6506756425642597, 3.645008564272085, 3.6384866564923053, 3.639841422559659,
3.6350855264740636, 3.629851857574663, 3.620299271938399, 3.6287767055735713, 3.
6182028647146116, 3.6165792515639703, 3.623273585546612, 3.6142014333487746, 3.6
14427142078098, 3.607469370416297, 3.6033965807340147, 3.606954895498968, 3.5961
1511445351, 3.5961014244698895, 3.587612823180593, 3.592319198593114, 3.58063696
39698503, 3.5787942074353856, 3.5769442879708606, 3.5775980532961635, 3.58575971
73956863, 3.580702449170745, 3.5769955497882227, 3.5769892570864377, 3.585006700
54247, 3.5883919370368296, 3.58379277113, 3.578437904701202, 3.577886692579158,
3.5873768381600324, 3.586929098728261, 3.5894993515087883, 3.589742067422252, 3.
5948471021851915, 3.60749998871727, 3.611760202839292, 3.6180901379512194, 3.627
571503596237, 3.645078346462796, 3.653180714695336, 3.6653955105269054, 3.684250
0539584893, 3.6988345193462293, 3.717969618068782, 3.731808121342249, 3.75219594
24196347]
22
0.7906043210907697
```

In [53]:

```
a, b = reg_multivariate_ols(X_train, y_train, R=0.01, MaxIterations=100000, lam=
predictions = model(a, b, X_test)
rmse_final = compute_rmse(predictions,y_test)
print(rmse_final)
```

```
Time taken: 1.47 seconds
Iteration number: 14087
4.346925334933998
```

How does your test RMSE compare to the RMSE from 4.3, 4.2, 3.2, and to the RMSE from nearest neighbors? What do you make of these results?

4.4 results: 4.346925334933998 4.3 results: Training RMSE: 2.791560637154338; Testing RMSE: 4.329249888142999 4.2 results: Training RMSE: 2.719573979435247; Testing RMSE: 4.3016431756475875 3.2 results: 6.153580529876927

Using cross validation and optimizing our lambda, we find that our final RMSE is 4.34. This is worse than our results from 4.3 and 4.2, which makes sense as these RMSEs were overfitted due to the absense of cross validation. Comparing our final RMSE of 4.34 to our 3.2 RMSE of 6.15, we can see that our model using all the variables established in 4.1 is a better model even with cross validation and penalizing for complexity. The graph demonstrates that as the penalty for complexity (lambda) increases, our RMSE will get higher.

Extra Credit 2: AdaGrad

AdaGrad is a method to implement gradient descent with different learning rates for each feature. Adaptive algorithms like this one are being extensively used especially in neural network training. Implement AdaGrad on 2.3 but now use CRIM, RM and DIS as independent variables. Standardize these variables before inputting them to the gradient descent algorithm. Tune the algorithm until you estimate the regression coefficients within a tolerance of 1e-1. Use mini-batch gradient descent in this implementation. In summary for each parameter (in our case one intercept and three slopes) the update step of the gradient (in this example β_j) at iteration k of the GD algorithm becomes:

$$\beta_j = \beta_j - \frac{R}{\sqrt{G_j^{(k)}}} \frac{\partial J(\alpha, \beta_1, \ldots)}{\partial \beta_j}$$

where $G_j^{(k)} = \sum_{i=1}^{k} (\frac{\partial J^{(i)}(\alpha,\beta_1,\ldots)}{\partial \beta_j})^2$ and R is your learning rate. The notation $\frac{\partial J^{(i)}(\alpha,\beta_1,\ldots)}{\partial \beta_j}$ corresponds to the value of the gradient at iteration (i). Essentially we are "storing" information about previous iteration gradients. Doing that we effectively decrease the learning rate slower when a feature x_i is sparse (i.e. has many zero values which would lead to zero gradients). Although this method is not necessary for our regression problem, it is good to be familiar with these methods as they are widely used in neural network training.

In [38]:

```
# Your code here
```

Discuss your results here