Uncertainty Shocks and Macroeconomic Effects: Insights from Volatility Term Structures

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Abstract

This paper introduces a new approach to measuring uncertainty shocks by exploiting the term structure of VIX futures. We document that VIX futures display notable within-month dynamics, including frequent episodes of inversion. Harnessing these dynamics allows us to assess the causal effects of uncertainty shocks, which can be either expansionary or contractionary depending on how the term structure rotates. Furthermore, we show that correlations between existing uncertainty measures—such as the Economic Policy Uncertainty index—and VIX futures vary across horizons: at times stronger with short-term futures and at other times with longer maturities. This variation provides evidence consistent with the potential decoupling of uncertainty indices, a phenomenon frequently emphasized in the literature.

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1 Introduction

Uncertainty shocks are widely recognized as a key driver of macroeconomic fluctuations, influencing investment behavior, consumption patterns, and labor market dynamics. When uncertainty increases, firms often delay capital expenditures, households increase precautionary savings, and labor market hiring slows. Since uncertainty is not directly observable, the use of proxies to capture its causal effects becomes inevitable. Various measures have been proposed in the literature, including realized and implied stock market volatilities (Bloom, 2009), the dispersion of survey-based forecasts (Bachmann, Elstner and Sims, 2013), text-based economic policy uncertainty index derived from newspapers (Baker, Bloom and Davis, 2016), and statistical methods that extract the common anticipated variation from forecast errors in various economic indicators (Jurado, Ludvigson and Ng, 2015; Jo and Sekkel, 2019), among others. Cascaldi-Garcia et al. (2023) summarizes the various measures available in the literature.

In this paper, we propose a more general and not widely explored measure of uncertainty by relying on VIX futures data. We suggest a comprehensive measure of uncertainty that has a scope broader than the standard VIX, which reflects market-wide expectations of volatility over the next 30 days.[1] While the VIX is useful for gauging short-term market volatility, it fails to capture shifts in longer-term perceived risk. In this sense, the most closely related studies are Barrero, Bloom and Wright (2017) and Krogh and Pellegrino (2024), both of which extend the use of VIX data beyond its conventional 30-day horizon. The former analyzes VIX maturities from 30 days up to 10 years, while the latter employs the VIX term structure within a structural VAR framework to separate short-term from long-term uncertainty shocks.

We show that, in general, the VIX futures term structure can be summarized with an upward-sloping relationship, where longer-maturity contracts, on average, tend to reflect higher expected uncertainty relative to shorter-maturity ones. In addition, we document

[1] The standard 30-day VIX is estimated using the market prices of out-of-the-money S&P 500 index options with maturities close to 30 calendar days. It therefore captures the option market's volatility expectations. For further details, see CBOE Volatility Index Methodology: https://cdn.cboe.com/api/global/us_indices/governance/VIX_Methodology.pdf.

periods of inversion, i.e., situations where long-term expected uncertainty is below that of the short-term or when the gap between the two narrows. Our more comprehensive measure provides two-dimensional insights. First, it can speak to the changing correlation structure of VIX futures and other measures of uncertainty. For instance, we find that though Economic Policy Uncertainty (EPU), another widely used measure of uncertainty, and VIX futures of various maturities often move together, there are periods where the relationship between the EPU and long-run and short-run VIX futures behaves differently, highlighting the need to consider a more general market-based measure of uncertainty when running parallels between the various measures of uncertainty.[2] Second, we use the whole VIX futures curve to characterize the dynamic causal effects of uncertainty shocks. We showcase that when using the whole futures curve, we encounter interesting episodes of surprises, where at times the short-run uncertainty peaks, while the long-run decreases and vice versa and many other combinations in between. Depending how the VIX futures curve shifts, we could expect either recessionary or expansionary effects of uncertainty. This exercise also contributes to the overall understanding of whether the effects of uncertainty shocks are large or small (see Alessandri, Gazzani and Vicondoa, 2023 and references therein). Taken together, these findings highlight the importance of moving beyond conventional short-term metrics and considering the entire term structure to more accurately identify uncertainty shocks and assess their broader macroeconomic relevance.

Our methodology offers several advantages over conventional uncertainty measures. First, it enables a more precise distinction between short-run and long-run uncertainty shocks, allowing for a more accurate assessment of how these shocks unfold across different horizons.[3] Second, as discussed in Inoue and Rossi (2021), Seong and Seo (2024), among others, the functional framework offers greater flexibility by capturing medium- and long-term dynamics that are frequently overlooked in conventional empirical analysis. Third, by integrating this

[2] Białkowski, Dang and Wei (2022) find that the usual positive link between policy uncertainty and market volatility weakens when political signals are poor, investor opinions are divided, or markets are strongly bullish, especially under low signal quality, which can halve the relationship. Bae, Jo and Shim (2025) document time-varying effects of the various uncertainty measures, with emphasis on EPU.

[3] In this context, short-run uncertainty refers to changes in expectations about short-term maturity futures, whereas long-run uncertainty reflects shifts in expectations concerning more distant maturities.

richer measure of uncertainty into a macroeconomic analysis, we provide new insights into the transmission channels through which uncertainty influences key economic variables, such as output and employment.

Our findings show that the macroeconomic effects of uncertainty shocks are critically shaped by the evolution of uncertainty across the term structure. Shocks that appear similar using conventional short-term measures, such as the 30-day VIX, often have divergent impacts when their full horizon is taken into account. Among our selected episodes, the July 2010 and July 2020 shocks both exhibit short-term declines in implied volatility, yet lead to different output and employment responses due to distinct shapes in their term structures. In contrast, the April 2015 shock—marked by a spike in short-term uncertainty but declines at longer horizons—is associated with rising industrial production and employment, challenging the interpretation offered by standard metrics. Meanwhile, the broad-based increase in uncertainty observed in December 2018 leads to a temporary decline in activity, but not to a persistent downturn, as long-term risk perceptions remained stable. These patterns highlight the importance of accounting for the shape and persistence of uncertainty shocks when assessing their real effects. By leveraging the term structure of VIX futures, our approach offers a more complete view of how uncertainty transmits to key macroeconomic outcomes, including output, unemployment, and payroll employment.

In light of this analysis, our findings highlight a fundamental limitation of conventional approaches to identifying uncertain shocks: they reduce uncertainty to a scalar value observed at a single point in time, typically based on short-term measures like the 30-day VIX. This approach neglects how uncertainty unfolds over different horizons and fails to distinguish between short-run shocks and long-run shifts in expectations. As a result, it imposes a one-size-fits-all interpretation on all increases in uncertainty, regardless of their nature. This simplification risks missing key episodes where uncertainty dynamics differ substantially across maturities and may lead to misleading conclusions about their macroeconomic consequences.

The remainder of this paper is organized as follows. Section 2 describes the data and Section 3 presents the methodological framework. Section 4 reports empirical findings on the propagation of VIX-futures based uncertainty shock, and Section 5 concludes.

2 Market-based uncertainty measure

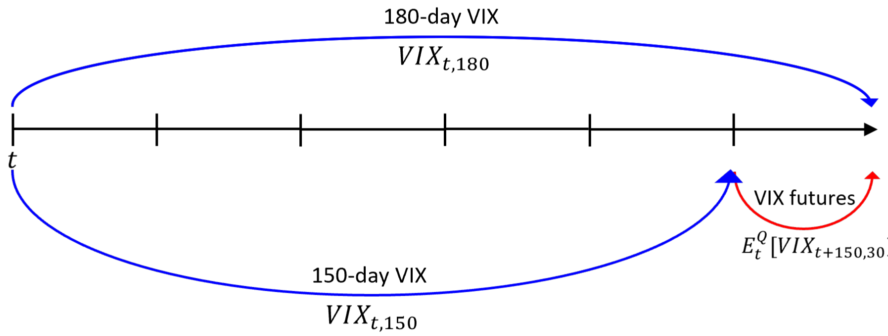
This section presents information about VIX, VIX futures, and their associated term structure. We begin by outlining the VIX as a market-implied measure of expected volatility and explain how VIX futures extend this measure to forward-looking horizons through standardized derivative contracts. We then examine the properties of the VIX futures term structure, focusing on its shape, dynamics, and the way it reflects market expectations regarding the persistence of volatility. Finally, we compare the VIX futures term structure with a widely used uncertainty measure, the EPU index. This comparison suggests that the VIX futures term structure embeds information on forward-looking uncertainty dynamics that may not be directly observable in existing indicators, making it a valuable tool for identifying uncertainty shocks while also complementing established approaches in the literature.

2.1 VIX and VIX futures

The Chicago Board Options Exchange (CBOE) Volatility Index, commonly referred to as the VIX, is a widely used market-implied measure of the expected volatility of the S&P 500 Index. The VIX is constructed from the implied volatility of S&P 500 index options and represents the consensus expectation of uncertainty of the market over the next 30 calendar days and longer-term VIX indices are available for maturities of up to ten contract months as longer-term S&P 500 index options exist in the market.[4] While providing a forward-looking measure of volatility for a fixed horizon from today, the VIX does not capture how these expectations evolve across different future periods. For instance, it does not directly convey today's expectation of equity market uncertainty during a 30-day window that lies further ahead, such as between 150 and 180 days from now. Figure 1 shows that, by construction, the VIX does not cover the future short-term periods (i.e. 30-day window between 150 and 180 days from today). The structure of VIX makes it difficult to compare expectations of future short-term uncertainty across different horizons. Moreover, VIX does not directly

[4] For more information about the VIX term structure and data, see https://www.cboe.com/tradable_products/vix/term_structure/

Figure 1: VIX and VIX futures



Notes: Figure 1 illustrates the relationship between VIX and VIX futures. $VIX_{t,150}$ and $VIX_{t,180}$ represent 150- and 180-day average volatility, respectively, and $\mathbb{E}_t^{\mathbb{Q}}[VIX_{t+150,30}]$ indicates 150 days ahead expected 30-day VIX. For theoretical relationship between VIX and VIX futures see Appendix A.

reflect the investors' expectation of future volatility as it is not a traded asset in the financial market.

In contrast, VIX futures extend the informational scope of the VIX by providing standardized derivative contracts. These contracts represent the market's forward-looking estimate of the 30-day VIX index value on specified future settlement dates. For example, in Figure 1, the price of 150-day VIX future reflects today's expectation of short-term, 30-day, stock market volatility (i.e. 30-day VIX in 150 days). Unlike the VIX, the value of VIX futures are determined in the futures market through trading. Hence, the price of a VIX futures contract embodies market expectations of future short-term volatility.

An increase in the current price of a VIX futures contract suggests that investors anticipate higher short-term volatility during the period from $t+150$ to $t+180$. On the settlement date, which is 150 days from today, the buyer of a 150-day VIX futures contract receives a payoff from the seller equal to the prevailing 30-day VIX index level multiplied by \$1,000 per contract[5]. The higher the VIX at maturity, the larger the payoff to the long position. Consequently, VIX futures are widely used both by hedgers, who seek protection against adverse volatility episodes, such as heightened uncertainty triggered by negative economic shocks, and by speculators, who take positions to profit from anticipated movements in volatility.

[5]For Mini VIX futures, the multiplier is \$100 per contract.

For the empirical analysis, we use historical daily closing prices of VIX futures obtained from the Bloomberg Terminal. The sample spans the period from March 2006 to June 2024 and includes contracts with expirations ranging from one to eight months. To identify the relevant tickers, we extract the `FUT_CUR_GEN_TICKER` series using Bloomberg indices "UX1 Index" through "UX8 Index," which represent the first through eighth nearest-maturity VIX futures contracts. Using these tickers, we retrieve the individual contracts and construct the VIX futures term structure. Since exact maturities are not always available in the market, we apply a B-spline approach to estimate the constant-maturity VIX futures at horizons of 30, 60, 90, 120, 150, 180, 210, and 240 days, thereby creating a balanced panel for analysis.[6]

2.2 VIX futures term structure and its dynamics

Based on the constant maturity VIX futures framework described in the previous section, we plot the term structure of VIX futures in Figure 2. The term structure depicts the relationship between futures prices and their time to maturity, providing insight into how market expectations of short-term volatility evolve across different horizons.[7] Observing its evolution over an extended sample period allows us to identify patterns in its shape, such as periods of contango and backwardation, and to examine how these configurations respond to shifts in market conditions.

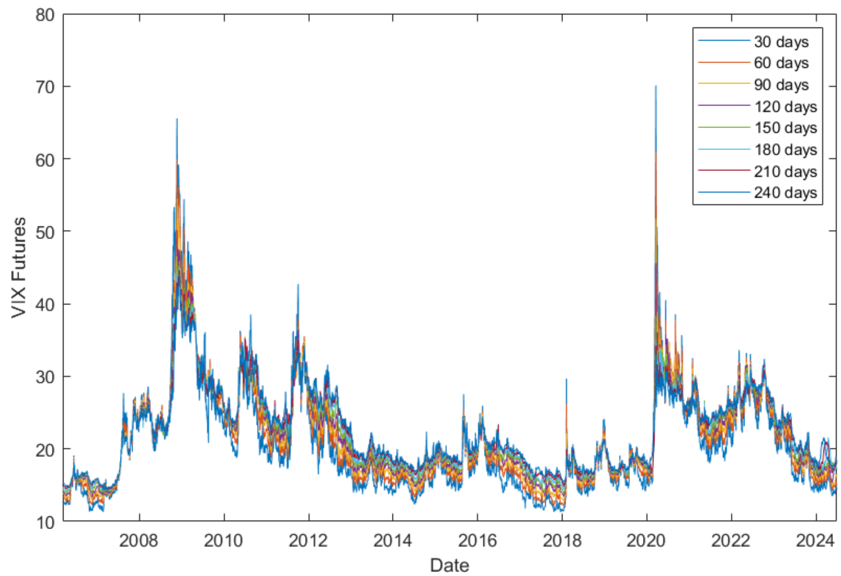
In Figure 2, Panel (a) shows daily observations of VIX futures across multiple maturities from March 2006 to June 2024. The contracts generally move together, with shorter maturities exhibiting more frequent short-term volatility spikes. Panel (b) provides a three-dimensional view of the term structure with the horizontal axes representing time and contract maturity, and the vertical axis indicating the value of the futures. This figure illustrates that there is heterogeneity in the slope of the curve over time, usually changing with economic conditions (more on this in Figure 3). Over the full sample, the term structure is

[6]For more details on the construction of constant-maturity VIX futures, refer to Appendix B.

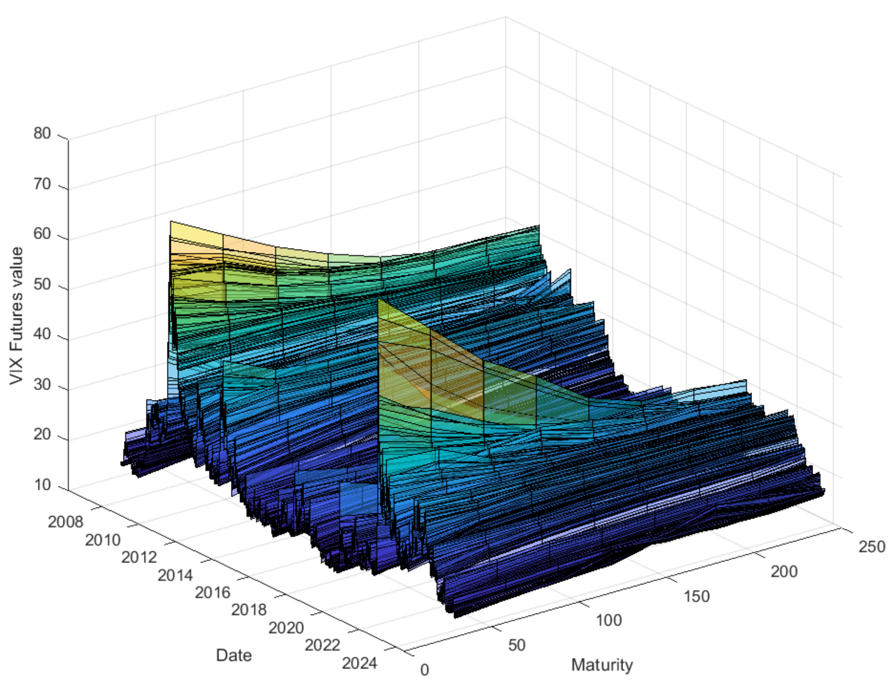
[7]To compare the VIX futures term structure with the VIX term structure, it is crucial to recognize their conceptual differences. The VIX futures term structure reflects market-implied pricing of expected 30-day volatility at different maturities, whereas the VIX term structure represents option-implied volatility across horizons, with its longer end corresponding to the average expected volatility over the entire horizon.

Figure 2: The term structure of volatility index

(a) VIX Futures over time



(b) VIX Futures term structure



Notes: Figure 2, Panel (a) presents daily VIX futures prices for various maturities over time. Panel (b) shows the three-dimensional term structure for eight maturities (30, 60, 90, 120, 150, 180, 210, and 240 days) from March 2006 to June 2024.

Figure 3: VIX futures term structure

Notes: Figure 3, top panel, shows VIX futures prices for four selected maturities (30, 90, 150, and 240 days) from March 2006 to June 2024, with shaded areas denoting NBER recessions. The bottom panel plots the slope of the term structure, defined as the difference between the 240-day and 30-day VIX futures prices, where shaded areas indicate periods when the slope is negative.

upward sloping in 81% of daily observations and downward sloping in 19%.[8]

To examine these inversions more closely, Figure 3 plots the level of VIX futures for selected maturities (30, 90, 150, and 240 days) alongside the slope defined as the difference between the 240-day and 30-day contracts. The top panel shows how longer-maturity contracts generally trade above shorter-maturity ones outside of stress periods, while the bottom panel captures episodes when the slope turns negative, marking backwardation.[9] The shaded regions correspond to NBER-dated recessions, highlighting the tendency for inversions to cluster around major economic downturns, such as the 2008–2009 financial crisis

[8]Out of 4,526 daily observations, 3,644 exhibit an upward slope and 882 a downward slope.

[9]In Appendix C, we plot the VIX term structure for selected maturities and observe similar inversions during recessions over the sample period from October 2010 to June 2024, although such inversions occur less frequently.

and the COVID-19 shock in early 2020, though there are also some other periods of curve inversions, not necessarily associated with recessions. Outside of the recessions, there are also interesting movements in the cross-sectional dynamics of the VIX futures. For instance, before the recessions, the various VIX futures tend to converge, indicating a flattened term structure.

2.3 Connection to policy uncertainty

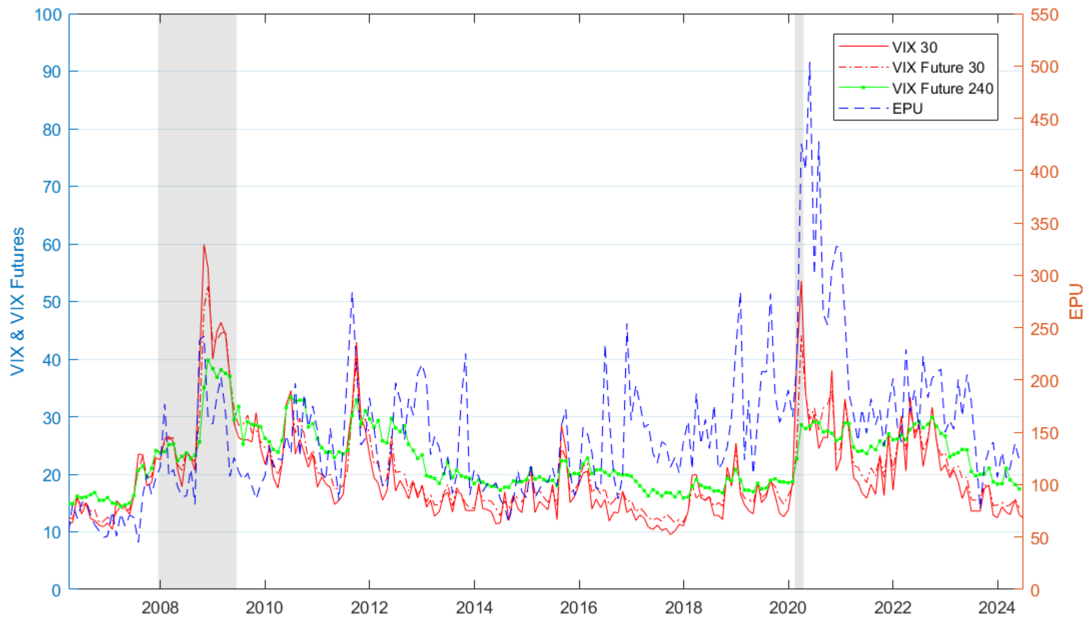
Among the various proxies for economic uncertainty, VIX and the EPU index have been considered as two of the most widely used measures. Despite originating from different sources, with VIX reflecting market-implied volatility and the EPU derived from news-based content, both aim to quantify underlying economic uncertainty. As such, it is natural to expect them to co-move, particularly during periods of elevated risk or macroeconomic turmoil. In this section, we analyze the relationship between market-based measures, including the VIX and its futures, and the EPU index.

The EPU, introduced by Baker, Bloom and Davis (2016), is a widely used measure of policy-related economic uncertainty in the United States, constructed from news coverage in ten major newspapers and three key components.[10] The first identifies policy-related articles based on specific keywords, the second tracks scheduled expirations of federal tax provisions, and the third captures forecast uncertainty through disagreement among professional forecasters. Together, these elements provide a comprehensive index of policy uncertainty, shedding light on how shifts in policy expectations influence economic behavior.

Having established the definitions and key features of each measure, we now examine the interrelationships among our key variables: the VIX, VIX futures (30-day and 240-day horizons), and the EPU index. Figure 4 shows that short-term market volatility, proxied by the 30-day VIX, has historically moved closely with the EPU index, with episodes of elevated policy uncertainty, often linked to geopolitical shocks or unexpected regulatory

[10]The EPU index measures policy-related economic uncertainty by analyzing coverage in ten major newspapers, including *USA Today, Miami Herald, Chicago Tribune, The Washington Post, Los Angeles Times, The Boston Globe, San Francisco Chronicle, The Dallas Morning News, Houston Chronicle*, and *The Wall Street Journal*.

Figure 4: VIX, VIX future, and EPU (monthly)



Notes: Figure 4 illustrates the co-movement between the VIX, VIX futures (30-day and 240-day maturity), and EPU over the sample period from March 2006 to June 2024. The shaded area indicates NBER recession periods.

changes, coinciding with sharp spikes in volatility. The figure also illustrates that longer-horizon VIX futures, such as the 240-day contract, are less volatile and smoother, reflecting the persistence of uncertainty than its immediate fluctuations. Together, these dynamics highlight how different segments of the term structure convey complementary information about market expectations.

Prior to the pandemic, the literature raised concerns about the divergence between the EPU and VIX, as discussed in Białkowski, Dang and Wei (2022). In light of these concerns, we assess the stability of our broader measure of uncertainty– constructed from the entire term structure of VIX futures– in relation to EPU. To do so, we consider the linear projection below:

$$ln(EPU_t) = \alpha_m + \gamma_m ln(V_{m,t}) + \epsilon_{m,t}, \tag{1}$$

where $V_{m,t}$ denotes the values of VIX futures at various maturities m at time t.

Since we are interested in the time variation in coefficients γ_m in the above projection, we investigate whether γ_m is different from zero using a test robust to instabilities as described in Rossi and Sekhposyan (2016). Specifically, we estimate the regressions in equation (1) in rolling windows of R observations, where $R = 30$ in our empirical application. Let $\widehat{\gamma}_{m,t}$ denote the parameter estimated sequentially in regression (1) for $t = R/2, ..., T - R/2$ using observations centered around time t – that is, the most recent $R/2$ observations and the following $R/2$ ones. We then construct a t-statistic at each point in time t:

$$\tau_{m,t} = \widehat{\gamma}_{m,t} / \sqrt{\widehat{\sigma}_m^2 / R} \tag{2}$$

where $\widehat{\sigma}_m^2$ is the Newey and West (1987) HAC estimator of the asymptotic variance of the parameter estimate in the rolling window centered at time t.[11] The Fluctuation test statistic is:

$$\mathcal{F}_m = max_t \left| \tau_{m,t} \right|, \tag{3}$$

which we use to test the null hypothesis that $\gamma_{m,t} = 0$ at every point in time t against the alternative that $\gamma_{m,t} \neq 0$ at some point in time t.

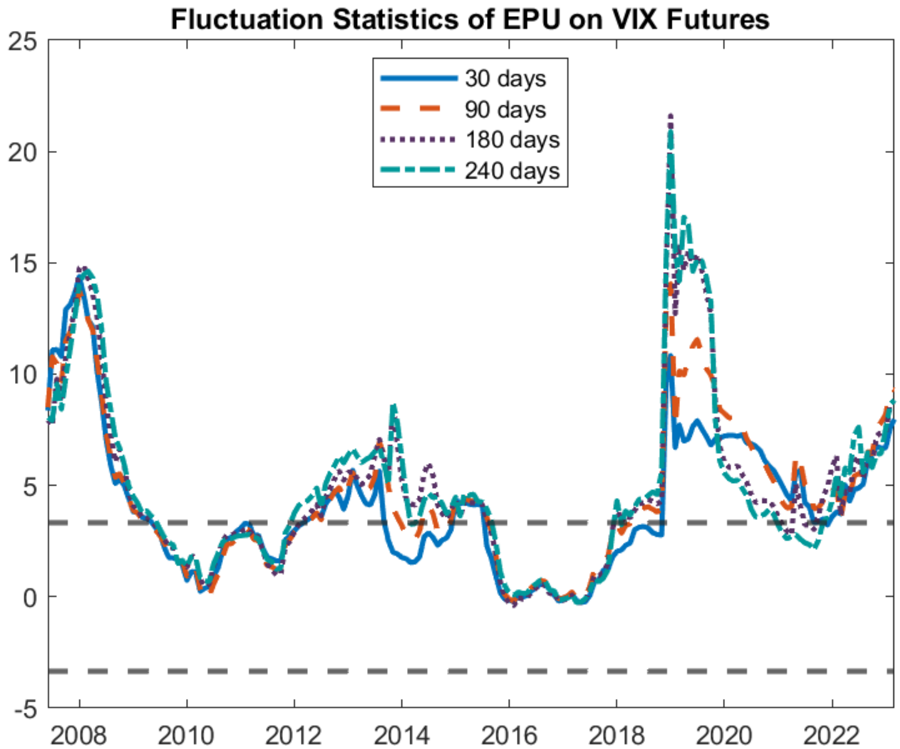
Figure 5 depicts $\tau_{m,t}$ over time. The largest (absolute) value in the sequence of $\tau_{m,t}$ is the Fluctuation test statistic, $\mathcal{F}_m$. The dashed horizontal lines denote the corresponding (two-sided) five percent critical value.[12] When $\mathcal{F}_m$ is outside the critical value lines, the test rejects the null hypothesis that $\gamma_{m,t} = 0$ for every t, and we conclude that the EPU and VIX futures of maturity m had a significant relationship at some point in time. Importantly, the critical value properly controls size and guards against sequential testing bias. Though $\mathcal{F}_m$ is the test statistic guiding the statistical decisions, the path of $\tau_{m,t}$ contains valuable information on evolving dynamics of the EPU and VIX futures.

Figure 5 shows that the correlation of EPU with VIX futures of various maturities is not constant over time. In fact, the correlation has declined in the early part of 2010s and

[11]The variance estimate is based on a Newey and West (1987) HAC estimator using a truncation lag equal to $R^{1/4}$. For details on the variance estimator, see Rossi and Sekhposyan (2016).

[12]The relevant critical value is the t-statistic analog to the Wald test critical values for the survey and model-free forecasts reported in Table II Panel C of Rossi and Sekhposyan (2016), re-simulated to closely match our sample size and rolling window size.

Figure 5: Fluctuation Test: EPU and VIX Futures



Notes: Figure 5 shows $\tau_{m,t}$ from eq. (1) based on $R = 30$ meetings rolling windows using a Newey-West covariance estimator with a truncation lag of $R^{1/4}$. Horizontal axes correspond to mid-window dates. Dashed lines denote 5% critical value lines based on Rossi and Sekhposyan (2016)'s two-sided Fluctuation test.

then again between 2016 to 2019. The figure also shows that though the significance of the correlations across various VIX futures usually moves together, there are periods where that is not the case. For instance, between 2013 and 2016, the correlation of EPU with VIX appears to be statistically insignificant, but the correlation between EPU and longer maturity VIX futures appears to be statistically significant. A similar type of episode occurs right before the pandemic. On the other hand, after the pandemic the reverse seems to be the case in that the correlation with the longer maturity VIX futures becomes statistically insignificant, while that with shorter maturities is significant. This exercise suggests the importance of the maturity structure of the VIX futures and the usefulness of the whole VIX futures term structure for a more complete characterization of the uncertainty dynamics and the analysis of its relationship with other measures.

3 Measuring uncertainty

3.1 Identification of the uncertainty shock

Quantifying economic uncertainty in empirical research has traditionally relied on indirect measurement through observable proxies. These include indicators such as the implied stock market volatility (Bloom, 2009), the EPU index based on newspaper frequency counts (Baker, Bloom and Davis, 2016), and other measures like forecast disagreement and macro surprise-based uncertainty indexes (Jurado, Ludvigson and Ng, 2015; Scotti, 2016; Kozeniauskas, Orlik and Veldkamp, 2018). While these indicators often spike around recessions and tend to co-move to some extent, they capture distinct dimensions of uncertainty, such as financial, policy-related, and macroeconomic factors, which may obscure important differences between them.

A widely adopted strategy for identifying uncertainty shocks is the standard recursive identification approach used in VAR models, which assumes that unexpected variation in the uncertainty measure, i.e., uncertainty innovations as captured by the VAR, are orthogonal to other sources of variation in the system contemporaneously. Despite its popularity, this approach has notable limitations in settings where uncertainty and macroeconomic variables are jointly determined, which can mislead the identification of causal effects (see Kilian, Plante and Richter, 2025 for a discussion).

We propose an alternative approach for identifying uncertainty shocks, which we refer to as *functional uncertainty shocks*. The shift to a higher frequency, i.e. biweekly, in the construction of uncertainty shocks mitigates (though does not completely resolve) the exogeneity concerns associated with recursive identification.[13] Our methodology is inspired by the concept of functional monetary policy shocks proposed by Inoue and Rossi (2021), but departs in a key respect: while their framework focuses on responses to discrete monetary policy announcements, we instead observe the continuous evolution of the VIX futures curve

[13]At the moment, our VIX futures data is at a daily frequency. In order to approximate the futures curve with reasonable accuracy, we use two weeks' worth of data. Higher frequency observations of the VIX futures could help improve the identification strategy utilized in this paper. In fact, Alessandri, Gazzani and Vicondoa (2023) argue for identification of causal effects in a monthly frequency, which could then be used as an instrument for characterizing the dynamic effects on low frequency economic variables.

over the course of a month. This perspective introduces identification-related challenges, which we attempt to resolve with high-frequency identification. Our method, however, can be refined if we were concerned with uncertainty about particular narratives and isolated events.

Formally, our functional uncertainty shock at time t is defined as follows:

$$\varepsilon_{f,t}(.) := V_{t_+}(.) - V_{t_-}(.), \tag{4}$$

where $V_{t_-}(.)$ and $V_{t_+}(.)$ represent the VIX futures curves in the first and second halves of month t, respectively. That is, our measure of uncertainty shock $\varepsilon_{f,t}(.)$ captures intra-month shifts in investor expectations about future volatility across different maturities.

To model the VIX futures curves in each half of the month, we adopt the widely used Nelson and Siegel (1987) factor model, follwoing Inoue and Rossi (2021). For each time t and maturity $m \in [0, 240]$, we let

$$V_{t_-}(m) = \beta_{1,t}^{-} + \beta_{2,t}^{-}\left(\frac{1-e^{-\lambda m}}{\lambda m}\right) + \beta_{3,t}^{-}\left(\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right), \tag{5}$$

and

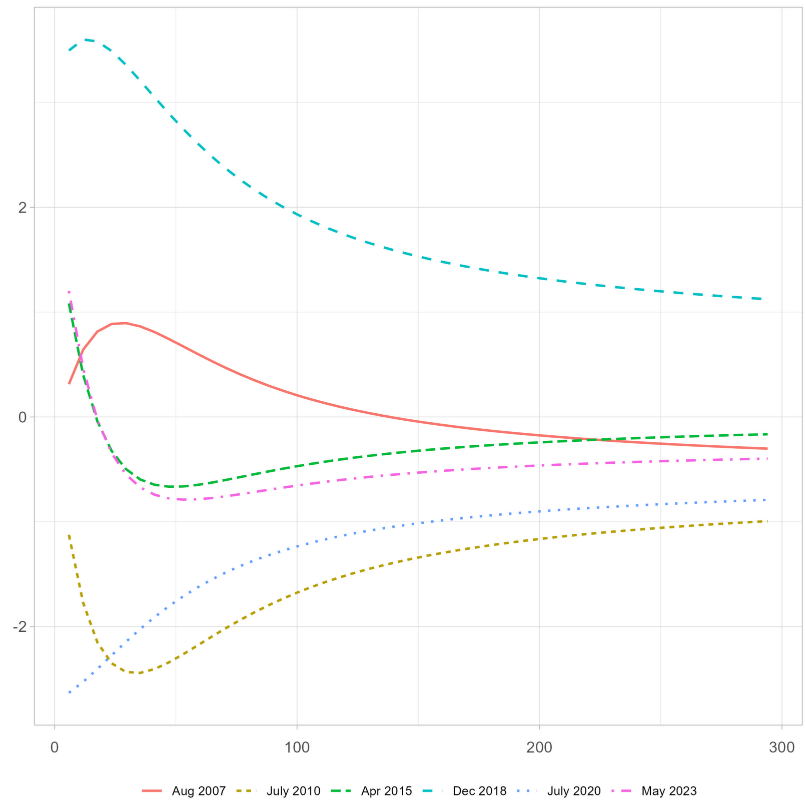
$$V_{t_+}(m) = \beta_{1,t}^{+} + \beta_{2,t}^{+}\left(\frac{1-e^{-\lambda m}}{\lambda m}\right) + \beta_{3,t}^{+}\left(\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right). \tag{6}$$

The three factors in $V_{t_-}(.)$ and $V_{t_+}(.)$ are estimated by using observations in the first and second half of month t, respectively. The shape parameter λ in $V_{\tau_-}(.)$ and $V_{\tau_+}(.)$ is associated with the decay rate of the slope and curvature components, and thus that of $\varepsilon_{f,t}(.)$. As λ increases, the uncertainty curve tends to have a faster decay rate, whereas it is likely to decay slowly when λ is small.[14] The parameter λ in our empirical application is set to maximize the curvature component at 30 days maturity as in Diebold and Li (2006).

The Nelson-Siegel factor model, originally developed for interest rates, provides a practical framework for characterizing the VIX curve despite its theoretical limitations in this

[14] In Appendix D, Figure D.1 presents the functional uncertainty shocks with $\lambda = 90$. The figure shows that the uncertainty curve exhibits lower persistence relative to the baseline case (i.e., $\lambda = 30$), while maintaining a broadly similar pattern in terms of the baseline uncertainty shape across the selected periods.

Figure 6: Representative examples of uncertainty shocks



Notes: Figure 6 presents the collected examples of uncertainty shocks over our sample periods spanning from March 2006 to June 2024.

context. Nevertheless, considering (i) the scarcity of literature on the VIX future dynamics and (ii) the association of the factors with Hermite polynomials, commonly used in the econometrics literature on nonparametric analysis to approximate a curve from discrete realizations, the three-factor model proposed by Nelson and Siegel (1987) would provide a practical insight into understanding the complicated nature of VIX curve dynamics.

While a nonparametric approach—such as local kernel regression—could be used to estimate $V_{t_-}(.)$ and $V_{t_+}(.)$, such methods pose challenges in subsequent impulse response analysis due to infinite-dimensional nature of function-valued predictors.[15] The Nelson-Siegel frame-

[15] As detailed in the literature on the functional regression model, estimation involving function-valued regressors is complicated due to the infinite dimensionality of the explanatory variable and the associated inverse problem, see, e.g., Imaizumi and Kato (2018); Florens and Van Bellegem (2015); Seong and Seo (2024) and references therein. Therefore, any form of dimension reduction is necessary to effectively utilize the information from a function-valued variable.

work, therefore, strikes a balance between tractability and flexibility, allowing us to extract meaningful features from the VIX future curve while avoiding high-dimensional complexity.

Figure 6 presents representative examples of functional uncertainty shocks (which are essentially the bi-monthly differences in the factor loadings) across several notable episodes from March 2006 to June 2024. Each curve reflects the intra-month change in the VIX futures term structure, providing a snapshot of how investor expectations about volatility evolved over time and across maturities within a given month. The shape and level of each curve highlight the heterogeneity of uncertainty shocks both in magnitude and in how they impact different segments of the term structure.

For instance, the August 2007 shock shows a sharp upward shift concentrated at short to medium maturities, reflecting a surge in near-term uncertainty at the onset of the global financial crisis. In contrast, the July 2010 and July 2020 shocks exhibit negative slopes with increasing values at longer horizons, pointing to declines in short- to medium-term uncertainty while long-term uncertainty remained relatively stable. Other episodes highlight distinct patterns: the April 2015 and May 2023 shocks reveal modest upward shifts concentrated at short maturities with limited impact on longer horizons, suggesting short-lived or localized increases in uncertainty that did not significantly alter expectations further out the term structure. The December 2018 shock is distinctly positive and persistent across maturities, suggesting a broad-based surge in uncertainty likely tied to tightening financial conditions.

Overall, these patterns highlight the richness and flexibility of our functional approach. By modeling uncertainty shocks as shifts in entire curves, we capture not just the direction and magnitude of changes in perceived risk and uncertainty, but also how these changes vary across time horizons–providing a more comprehensive picture of market sentiment that a single measure alone can offer.

3.2 Estimation methodology

To examine how uncertainty shocks influence macroeconomic outcomes, we employ the *functional local projection* approach as in Inoue and Rossi (2021). This method enables us to estimate impulse responses that vary not only in magnitude but also in shape, depending on the nature of the shock. Specifically, we estimate the following regression model:

$$\begin{aligned} y_{t+h} &= \mu_h + \Theta_h(L) \int_{\mathcal{T}} \omega(m) \varepsilon_{f,t}(m) dm + u_{h,t+h} \\ &= \mu_h + \Theta_h^{(1)}(L)\beta_{1,t} + \Theta_h^{(2)}(L)\beta_{2,t} + \Theta_h^{(3)}(L)\beta_{3,t} + \varphi_1' X_{t-1} + \varphi_2' X_{t-2} + u_{h,t+h}, \end{aligned} \tag{7}$$

for some weight function $\omega(.) : \mathcal{T} \to \mathbb{R}$ as in Inoue and Rossi (2021, eq. (2)). The coefficient $\beta_{j,t}$ is equal to $\beta_{j,t}^{+} - \beta_{j,t}^{-}$ for $\{\beta_{j,t}^{+}, \beta_{j,t}^{-}\}_{j=1,2,3}$ defined as the coefficients of the constant, level, and slope components in equations (5) and (6). Essentially, we are capturing the shifts in the factor loadings within the month and treating them as the perturbation of interest. Under the assumption that $\beta_{1,t}, \beta_{2,t}, \beta_{3,t}$ are indeed shocks, in that they are linearly unpredictable using the controls ($Cov(\beta_{i,t}, F_{t-1}) = 0,\ i = 1, 2, 3,\ F_{t-1} = [X_{t-1}, X_{t-2}]$), then our procedure will uncover dynamic causal effects of uncertainty shocks.[16] Though the identification strategy for uncertainty shocks is far from being clear (see Kilian, Plante and Richter, 2025 for a discussion), our approach extends and refines the existing strategies that use recursive assumptions for the identification of the shocks. In addition, it is also possible to extend the analysis to instrumental variable local projections, which would correspond to a further refinement of the methodology.

We consider three different target variables of y_{t+h}, each of which represents output growth (ind_{t+h}), unemployment rate (u_{t+h}), and payroll employment growth (e_{t+h}). The output (resp. payroll employment) growth is measured by the annual percentage change in the industrial production index (resp. total nonfarm payroll employment index). The variable X_t is a vector of finite dimensional covariates consisting of lagged output and payroll growths (i.e., $(ind_t, u_t)'$), except when we study the impact to the unemployment rate. To avoid multicollinearity in that case, we set X_t as a vector consisting of output growth and the lagged unemployment rate.

[16]Implicitly, this corresponds to a recursive ordering in a VAR. It is possible to extend this identification strategy to identification with controls, where we also control for contemporaneous values of other variables that could potentially affect the revision of our factor loadings contemporaneously.

Each lag polynomial $\Theta_h^{(j)}(L)$ denotes the response of y_{t+h} to a shock $\beta_{j,t}$, and is given by:

$$\Theta_h^{(j)}(L) = I - \Theta_{1,h}^{(j)}L - \Theta_{2,h}^{(j)}L^2 - \cdots - \Theta_{p,h}^{(j)}L^p. \tag{8}$$

The lag order p is set to 2 throughout the analysis. We assume that an unexpected uncertainty shock jointly shifts the entire term structure, i.e., all $\beta_{j,t}$ as in Inoue and Rossi (2021). Using the chain rule from their eqn. (27), the impulse response of macroeconomic variables to a functional uncertainty shock is similarly identified as follows:

$$\frac{\partial y_{t+h}}{\partial \varepsilon_{f,t}(.)} = \frac{\partial y_{t+h}}{\partial \beta_{j,t}} \frac{\partial \beta_{j,t}^d}{\partial \varepsilon_{f,t}(.)} = \sum_{j=1}^{p} \Theta_h^{(j)} \beta_{j,t}. \tag{9}$$

The impulse response derived from equations (7) and (9) is shaped by both the magnitude and the profile of the uncertainty shock, which is captured by a functional representation. This distinctive feature offers deeper insights into how uncertainty shocks influence macroeconomic variables.

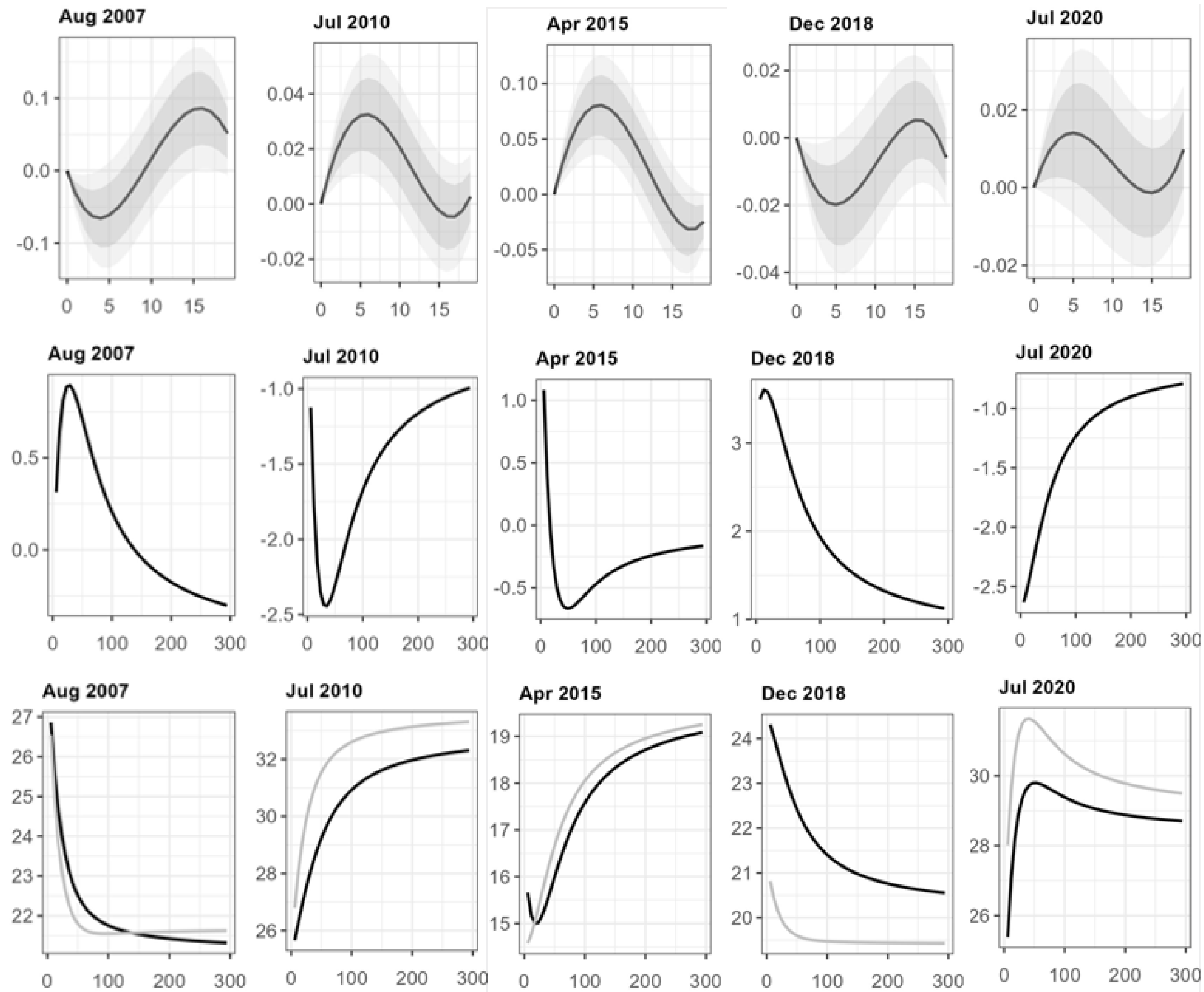
4 The effects of uncertainty shocks

We next present the results from functional uncertainy shocks and contrast them with the traditional uncertainty shocks obtained using the VIX with recursive identification. The following section presents the estimated responses of macroeconomic variables, such as industrial production and employment, to selected functional uncertainty shocks.[17]

Figure 7 illustrates the effects of selected functional uncertainty shocks on industrial production, highlighting how the shape of the identified shock (as shown Figure 6) translates into distinct economic outcomes. The August 2007 shock reflects an early rise in short-run uncertainty, marked by a sharp increase in medium-term uncertainty peaking around the 60-

[17]As noted in Section 3.1, we construct intra-month shifts in the VIX futures term structure over the period from March 2006 to June 2024. By design, the resulting shock series captures changes at a monthly frequency, yielding a total of 219 observations. In this section, we highlight a selection of representative functional uncertainty shocks—each exhibiting distinct shapes across the maturity spectrum—and present the corresponding results.

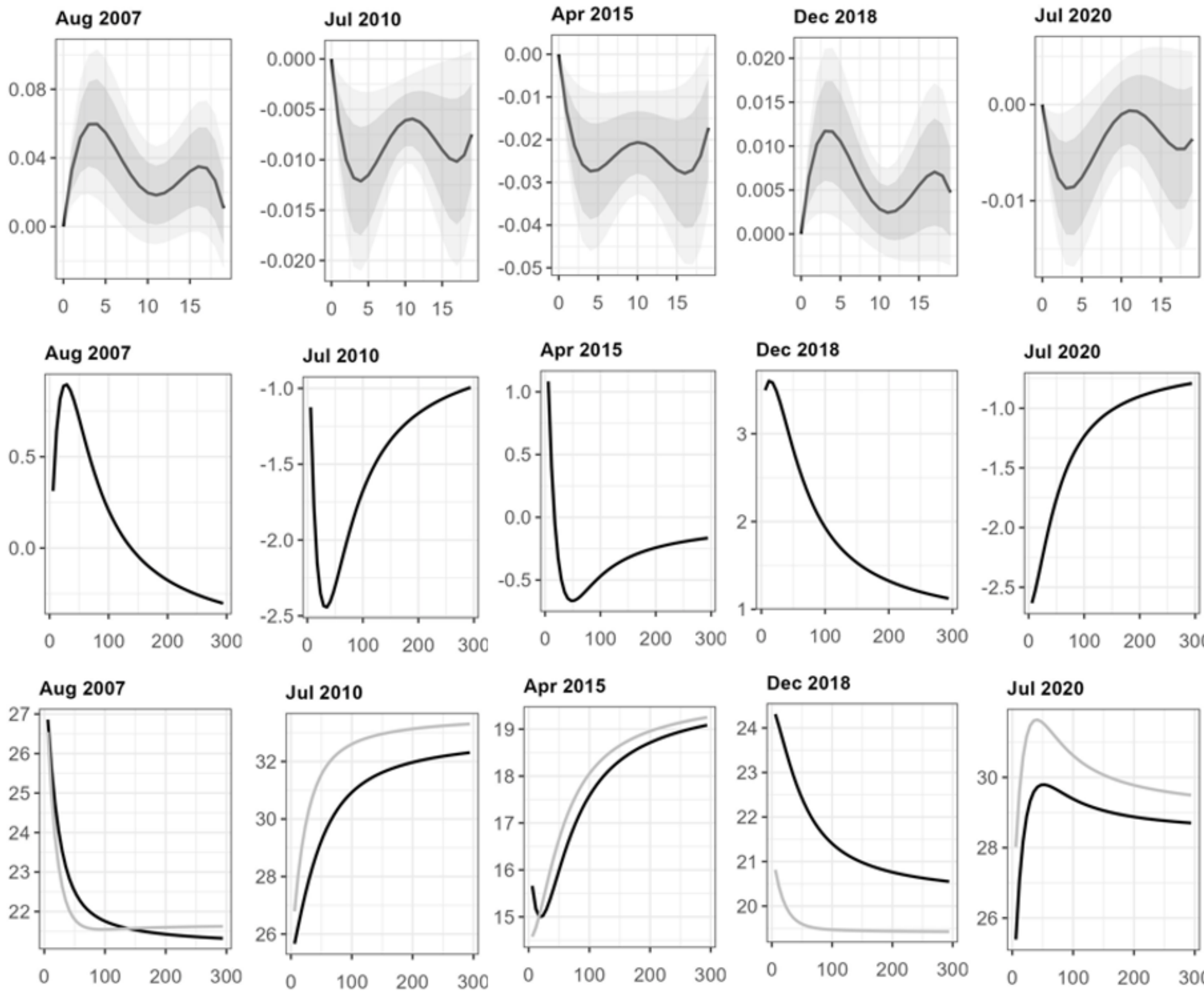
Figure 7: Industrial production responses to uncertainty shocks



Notes: Figure 7 presents the effects of uncertainty shocks on industrial production across different episodes. The top row reports the impulse responses with 68% and 90% confidence intervals. The second row depicts the functional shocks, capturing intra-month differences in VIX futures, and the third row contrasts their profiles before (grey line) and after (black line).

day horizon. Instead, the long-run uncertainty (120 and more days into the future) declines. In that sense, this shock is similar to the one occurring in June 2020, though the shape and the magnitude of the shock are slightly different. The shock occurring in June 2020 implies a shift of the whole VIX futures curve, where the long-run uncertainty is still elevated. Consequently, the impact of uncertainty is recessionary across all forecast horizons, with the highest impact around one year. The impact of the August 2007 shock is also recessionary

Figure 8: Unemployment rate responses to uncertainty shocks

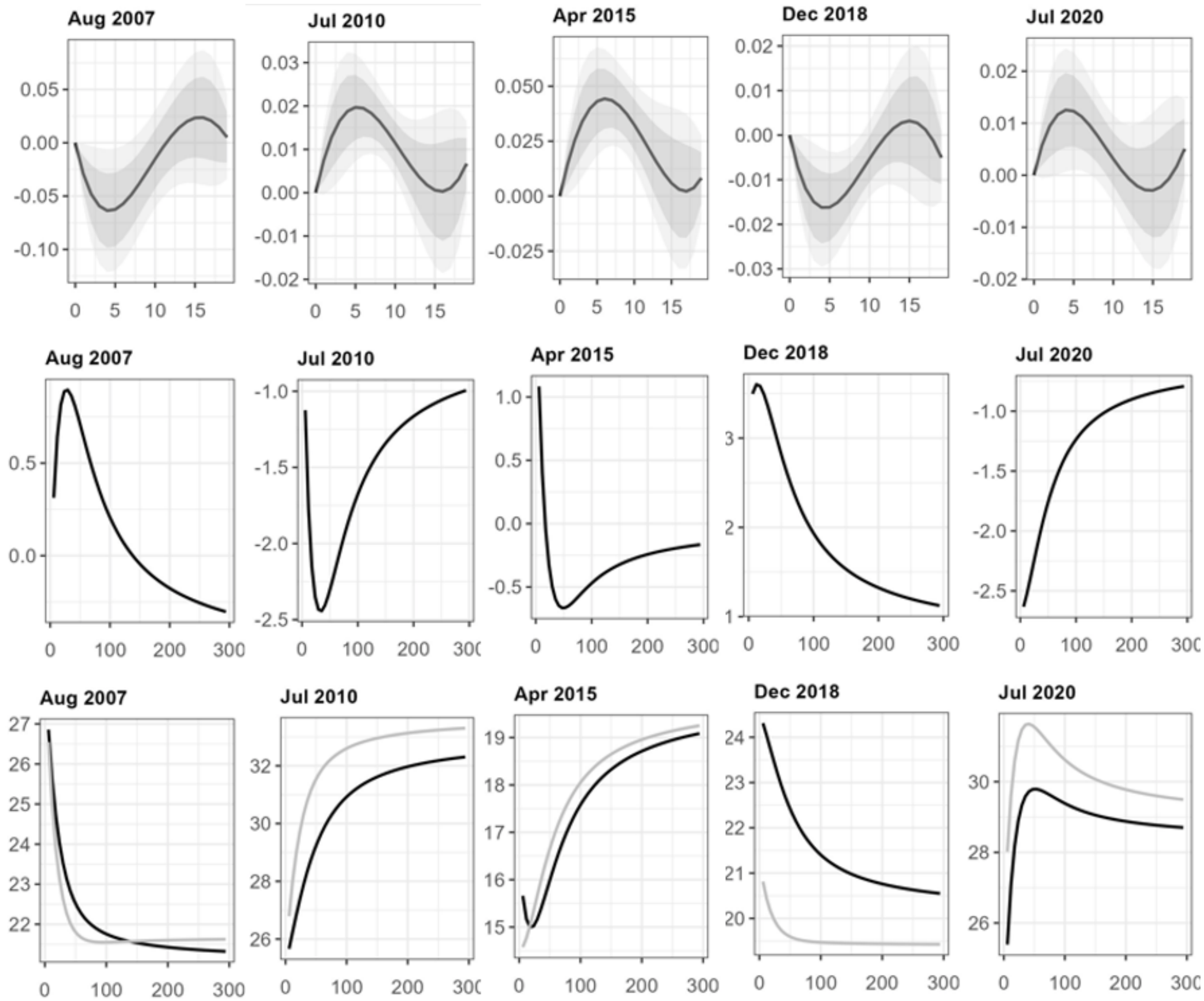


Notes: Figure 8 presents the effects of uncertainty shocks on the unemployment rate across different episodes. The top row reports the impulse responses with 68% and 90% confidence intervals. The second row depicts the functional shocks, capturing intra-month differences in VIX futures, and the third row contrasts their profiles before (grey line) and after (black line).

up to 10 months into the future and turns positive thereafter. This example highlights the importance of controlling for various dimensions of uncertainty while assessing the dynamic causal effects.

Figure 8 and 9 present the labor market responses to selected uncertainty shocks, focusing on the unemployment rate and payroll employment, respectively. These results complement the earlier findings on industrial production by capturing how uncertainty shocks affect both the intensive (employment levels) and extensive (unemployment) margins of the labor mar-

Figure 9: Payroll employment responses to uncertainty shocks



Notes: Figure 9 presents the effects of uncertainty shocks on the payroll employment across different episodes. The top row reports the impulse responses with 68% and 90% confidence intervals. The second row depicts the functional shocks, capturing intra-month differences in VIX futures, and the third row contrasts their profiles before (grey line) and after (black line).

ket. In Figure 8, the August 2007 shock, marked by a sharp rise in medium-term uncertainty, is associated with a modest and short-lived increase in the unemployment rate, consistent with temporary labor market disruptions as firms held back hiring amid heightened perceived risk. By contrast, the July 2010 and July 2020 shocks, which involve steep declines in short-term uncertainty, are followed by a gradual decline in unemployment, reflecting improved expectations and a resumption of labor demand. The April 2015 shock, characterized by a localized increase in short-term uncertainty, shows even negative effects on unemployment, while the December 2018 episode induces only a modest and short-lived uptick, suggesting that the absence of elevated long-term concerns prevented a more persistent weakening of the labor market.

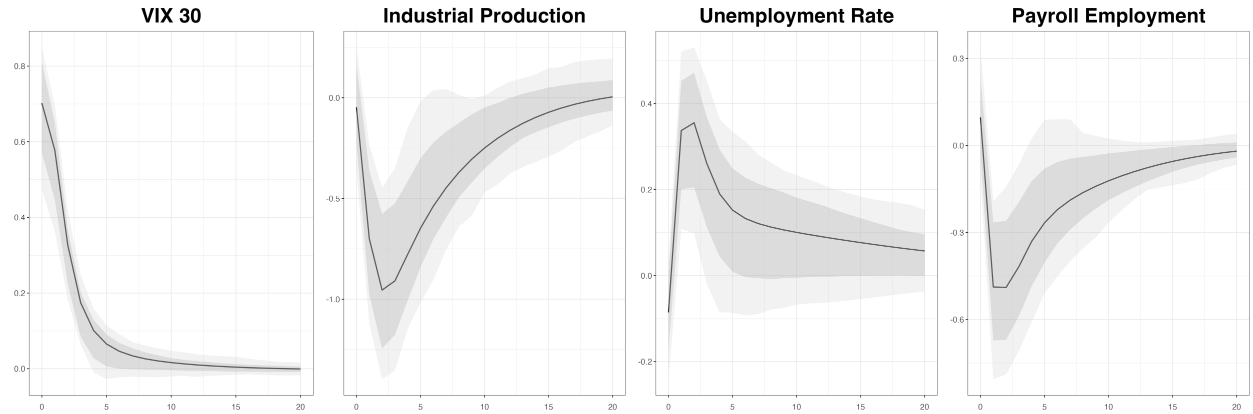
Figure 9 shows corresponding payroll employment responses, capturing changes in the overall level of total nonfarm employment, which is a broad proxy for output growth. The August 2007 shock leads to a brief decline in employment, which stabilizes relatively quickly, mirroring the short-lived rise in unemployment. In contrast, the July 2010 and July 2020 shocks produce strong and sustained increases in payroll employment, reflecting a rebound in labor demand as uncertainty receded. The April 2015 and December 2018 shocks generate only limited effects, again reinforcing the idea that more localized or non-persistent uncertainty shocks tend to have muted labor market impacts.

These results highlight the importance of the shape and persistence of uncertainty shocks in driving output and labor market dynamics. Conventional measures like the 30-day VIX treat shocks with similar short-term movements as equivalent, while our functional approach reveals meaningful differences across maturities. For instance, the July 2010 and July 2020 shocks both show short-term declines in uncertainty, yet their distinct term structure shapes lead to different macroeconomic responses. Similarly, the April 2015 shock, despite rising short-term uncertainty, coincides with improving economic conditions due to declines at longer horizons. These patterns emphasize the value of incorporating horizon-specific information when assessing the macroeconomic effects of uncertainty.

4.1 Comparison to conventional uncertainty shock

We next showcase the impulse responses from the conventional approach to identifying uncertainty shocks, which captures only the magnitude of a shock at a specific point in time, offering no information about how uncertainty is expected to evolve across different horizons. This restricts our ability to interpret the economic consequences of an uncertainty shock, particularly when short- and long-term components move in different directions. Such simplification can lead to two critical issues: the omission of important sources of variation

Figure 10: Effects of conventional uncertainty shocks



Notes: Figure 10 presents the impulse responses of industrial production, the unemployment rate, and total non-farm payroll employment to a one-unit increase in the uncertainty shock, as measured by the VIX30 index. The shaded areas represent 68% and 90% confidence intervals, respectively.

and the failure to differentiate between shocks with distinct temporal profiles.

These limitations have direct consequences for empirical analysis. When responses to uncertainty shocks are estimated using a scalar representation, the underlying heterogeneity in the term structure is muted. As a result, the estimated macroeconomic responses often reflect an average over very different types of uncertainty events, making interpretation difficult and potentially misleading. Figure 10 illustrates this concern: it shows the impulse responses of industrial production, the unemployment rate, and payroll employment to a conventional uncertainty shock.[18] The results are broadly consistent with standard findings that industrial production and payroll employment decline while the unemployment rate increases.[19] However, this "average" response fails to reveal which component of the uncertainty term structure drives these dynamics or whether they would look different under an alternative configuration.

This is the gap that out functional uncertainty approach aims to fill. By modeling uncertainty as a function over different horizons, we can recover the entire shape of the

[18]We conduct a standard vector autoregression (VAR) analysis in which the shock variable, VIX 30, is ordered first, followed by the variables of interest: industrial production and the unemployment rate. When analyzing labor market outcomes, the VAR includes VIX, industrial production, and payroll employment. The lag length is set to 2, and confidence bands are generated using 10,000 bootstrap replications.

[19]See, for example, Bloom (2009); Jurado, Ludvigson and Ng (2015); Basu and Bundick (2017).

uncertainty shock, rather than collapsing it into a single point. This allows us to identify episodes where uncertainty increases in the short term but falls in the medium to long run—as was the case in April 2015—and understand how such shifts influence economic outcomes in ways that the conventional approach cannot detect. The functional method thus enables a much richer characterization of shocks, distinguishing between transitory noise and deeper, more persistent uncertainty. In turn, it offers a more accurate and flexible framework for interpreting the macroeconomic effects of uncertainty. Rather than assuming all shocks are alike, we can now ask more refined questions: Is uncertainty expected to persist? Is it front-loaded or back-loaded? Does it signal a fundamental shift in outlook, or merely reflect near-term volatility? These distinctions matter not only for empirical analysis, but also for policy responses, which may need to be tailored depending on the nature and expected duration of the uncertainty shock.

5 Conclusions

This paper explores the importance of using the VIX term structure to more accurately capture economic uncertainty and its macroeconomic implications. We show that increases in short-term uncertainty can have both expansionary and recessionary effects, depending on whether it is accompanied by an increase or a decrease in long-term uncertainty. We further show that accounting for market-based uncertainty at different maturities can help us understand the time-varying correlation with other uncertainty measures. More specifically, we show that though EPU and VIX (at various future maturities) are usually strongly related, there are episodes of breakdown, though these episodes do not co-occur; at times, the EPU correlation is higher with short maturity VIX futures, and at times with longer horizon ones.

In addition to improving the measurement of uncertainty, our findings also contribute to the debate on how uncertainty shocks should be identified in empirical models. We make an effort towards high-frequency identification and integration of that to low-frequency models, though extensions into functional IV estimation could be investigated further.

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Appendix

A Conceptual foundations of the VIX and VIX Futures

Assume that the instantaneous stock market return, dS_t/S_t, follows a diffusion process in a continuous time model, $dS_t/S_t = \mu_t dt + \sigma_t dZ_t$, where S_t is level of S&P500 index at time t, μ_t and σ_t are instantaneous mean and volatility, respectively, and Z_t denotes the standard Brownian motion. Realized variance (RV^2) over T period from time t is defined as a time-series average of instantaneous variances (σ^2) over the period as follows:

$$RV_{t,T}^2 \equiv \frac{1}{T}\mathbb{E}_t^{\mathbb{P}}\left[\int_t^{t+T}\sigma_s^2 ds\right]. \tag{10}$$

Reflecting subjective probabilities of risk-averse investors, the expectation under the physical probability measure ($\mathbb{P}$) incorporates all information available to investors at time t. Since the realized volatility is an ex-post measure, it is not measurable at time t but at time T.

Instead, the VIX is widely used as a market-based estimate of expected volatility computed using prices of S&P500 index options.[20] Similar to the realized variance in (10), the squared VIX in (11) reflects investors' average expected short-term variance over a given period under the risk-neutral probability ($\mathbb{Q}$) at time t:

$$VIX_{t,T}^2 \equiv \mathbb{E}_t^{\mathbb{Q}}\left[RV_{t,T}^2\right] = \frac{1}{T}\mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{t+T}\sigma_s^2 ds\right]. \tag{11}$$

The risk neutral measure ($\mathbb{Q}$) is used in the equation (11) as the risk preference of investors does not affect the pricing in the derivative market (i.e., individual's subjective probability is not incorporated in the pricing).

Following Johnson (2017), future instantaneous variances can be decomposed into separate time intervals. Let k denote the number of days ahead from today (i.e., from time t), so that $t + k$ represents a future point k days from now, and $t + k + 30$ represents a point

[20] See Carr and Madan (1998) for details.

30 days beyond that. Using this notation, the squared VIX over $k+30$ period at time t is given by:

$$
\begin{aligned}
VIX^2_{t,k+30} &\equiv \frac{1}{k+30}\mathbb{E}^{\mathbb{Q}}_t\left[\int_t^{t+k+30}\sigma^2_s ds\right] \\
&= \frac{1}{k+30}\mathbb{E}^{\mathbb{Q}}_t\left[\int_t^{t+k}\sigma^2_s ds + \int_{t+k}^{t+k+30}\sigma^2_s ds\right] \\
&= \frac{k}{k+30}VIX^2_{t,k} + \frac{1}{k+30}\mathbb{E}^{\mathbb{Q}}_t\left[\mathbb{E}^{\mathbb{Q}}_{t+k}\left[\int_{t+k}^{t+k+30}\sigma^2_s ds\right]\right] \\
&= \frac{k}{k+30}VIX^2_{t,k} + \frac{30}{k+30}\mathbb{E}^{\mathbb{Q}}_t\left[VIX^2_{t+k,30}\right].
\end{aligned}
\tag{12}
$$

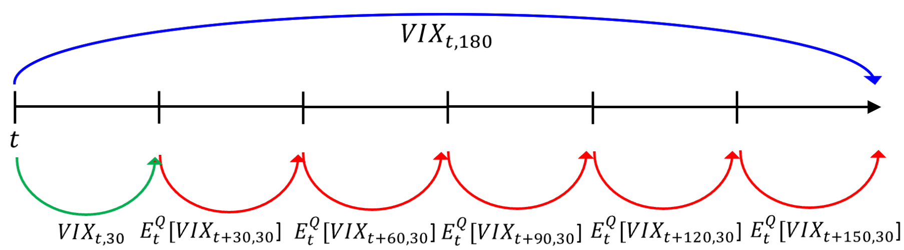
This decomposition implies that the squared VIX for the $k+30$ days horizon, $VIX^2_{t,k+30}$, is a term-weighted average of the near-term squared VIX, $VIX^2_{t,k}$, and the expected future squared VIX, $\mathbb{E}^{\mathbb{Q}}_t[VIX^2_{t+k,30}]$, under the risk-neutral measure. The expected future VIX, denoted $\mathbb{E}^{\mathbb{Q}}_t\left[VIX_{t+k,30}\right]$, represents the market's current expectation of average volatility over the 30-day interval from $t+k$ to $t+k+30$. Institutional investors, who account for the majority of participants in the volatility derivatives market, commonly trade futures on this expected future VIX (i.e., $\mathbb{E}^{\mathbb{Q}}_t\left[VIX_{t+k,30}\right]$), known as VIX futures. These contracts serve both speculative and hedging purposes—particularly as protection against adverse volatility events, such as increased market uncertainty following negative economic shocks.

Moreover, the above decomposition can be extended to incorporate the full term structure of VIX futures, offering further insight into the market's evolving expectations of future volatility. For example, the 180-day VIX example in Figure 1 can be further decomposed similarly to include the term structure of VIX futures as follows.

$$
\begin{aligned}
VIX^2_{t,180} \equiv &\frac{30}{180}\left[VIX^2_{t,30} + \mathbb{E}^{\mathbb{Q}}_t\left[VIX^2_{t+30,30}\right] + \mathbb{E}^{\mathbb{Q}}_t\left[VIX^2_{t+60,30}\right]\right. \\
&\left. + \mathbb{E}^{\mathbb{Q}}_t\left[VIX^2_{t+90,30}\right] + \mathbb{E}^{\mathbb{Q}}_t\left[VIX^2_{t+120,30}\right] + \mathbb{E}^{\mathbb{Q}}_t\left[VIX^2_{t+150,30}\right]\right]
\end{aligned}
\tag{13}
$$

The equation shows that the 180-day long-term VIX is decomposed into the short-term VIX and VIX futures and this relationship is illustrated in the Figure A.1.

Figure A.1: VIX and term structure of VIX futures

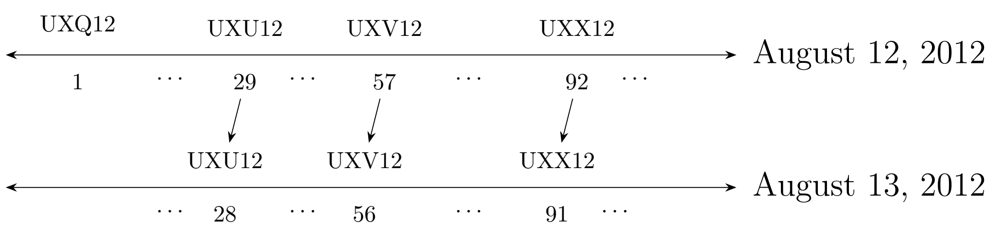


Notes: Figure A.1 illustrates the conceptual relationship between VIX and VIX futures. $VIX_{t,180}$ represents 180-day average volatility and $\mathbb{E}_t^{\mathbb{Q}}[VIX_{t+k,30}]$ indicates k days ahead expected 30-day VIX.

B Construction of constant maturity VIX futures

This section explains how constant maturity VIX futures are constructed. Constant maturity VIX futures refer to modified VIX futures that maintain a fixed time to maturity throughout the sample period. In actual VIX futures markets, as well as in other markets trading financial instruments with varying maturities, the maturities in the term structure decline over time. Consequently, it is not possible to observe a balanced panel of fixed, or constant, maturities over time, meaning that only an unbalanced panel can be obtained without approximation.[21]

Figure B.1: Observed data points on 2012/08/12 and 2012/08/13



Notes: Figure B.1 presents observed data points on two consecutive days, 2012/08/12 and 2012/08/13. Each point represents remaining maturity associated with each VIX future.

For instance, Figure B.1 shows that, on August 12, 2012, VIX futures are quoted at nine different maturity points: 1, 29, 57, 92, 120, 148, 176, 211, and 239 days. On the following day, only eight maturity points are observed: 28, 56, 91, 119, 147, 175, 210, and 238 days as the future with a one-day maturity on August 13 has expired.[22] Consequently, the number of available cross-sectional observations decreases until a new VIX futures contract is issued, while the maturity structure simultaneously changes due to time decay, resulting in an unbalanced panel.

[21]The expiration date of VIX futures is always 30 days before the maturity of S&P500 options. These options typically expire on a Wednesday, although exceptions exist when public holidays alter the schedule. See https://www.macroption.com/vix-expiration-calendar/ for historical VIX futures and S&P500 options expiration dates.

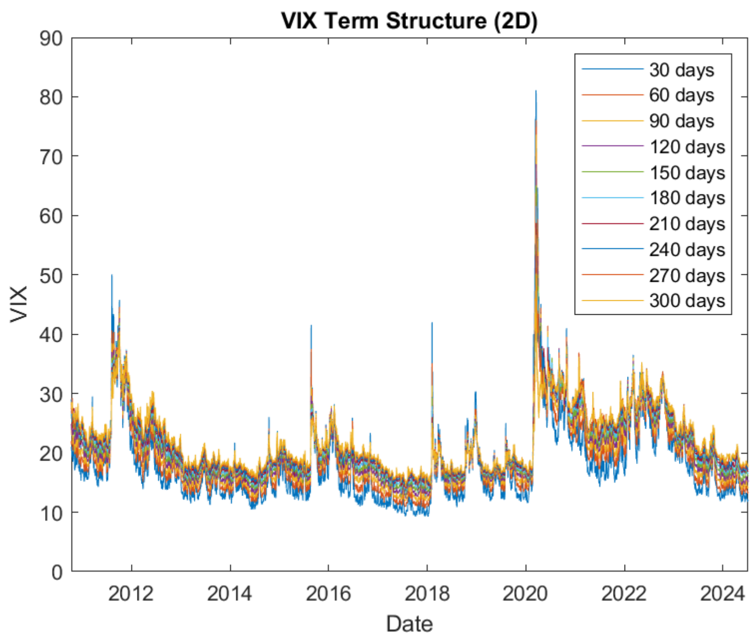
[22]In this example, no new contract was generated, resulting in only eight available observations. The creation of new contracts depends on the date and follows the schedule published on the CBOE website. Due to this schedule, the number of observations within our sample periods varies. For further details, see CBOE contract specifications: https://www.cboe.com/tradable_products/vix/vix_futures/specifications/.

The term structure of VIX futures is assumed to follow a smooth functional form, which allows for the construction of a balanced panel of maturities. Similar to the methodology of Barrero, Bloom and Wright (2017) and related studies including Ramsay and Silverman (2005), we construct a balanced panel of constant-maturity VIX futures with $k \in \{30, 60, 90, 120, 150, 180, 210, 240\}$ as described in Section 2.1 using B-spline approaches. Although the original sample includes VIX futures with maturities beyond 240 days, our construction of constant maturity VIX futures is restricted to contracts with maturities of 240 days or less to avoid extrapolation, since longer-maturity VIX futures are issued infrequently throughout the sample period. Note that from Section 3 onward, the raw VIX futures data are used instead of the constant-maturity VIX futures, since the models take maturity, m, as an input to reflect the time-varying maturity structure.

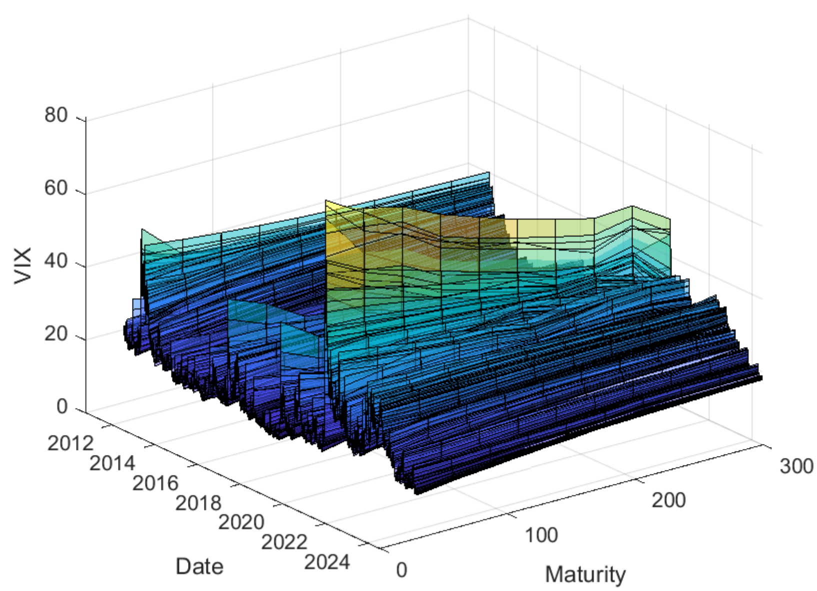
C Volatility Index (VIX) term structure

Figure C.1: The term structure of volatility index

(a) VIX index over time

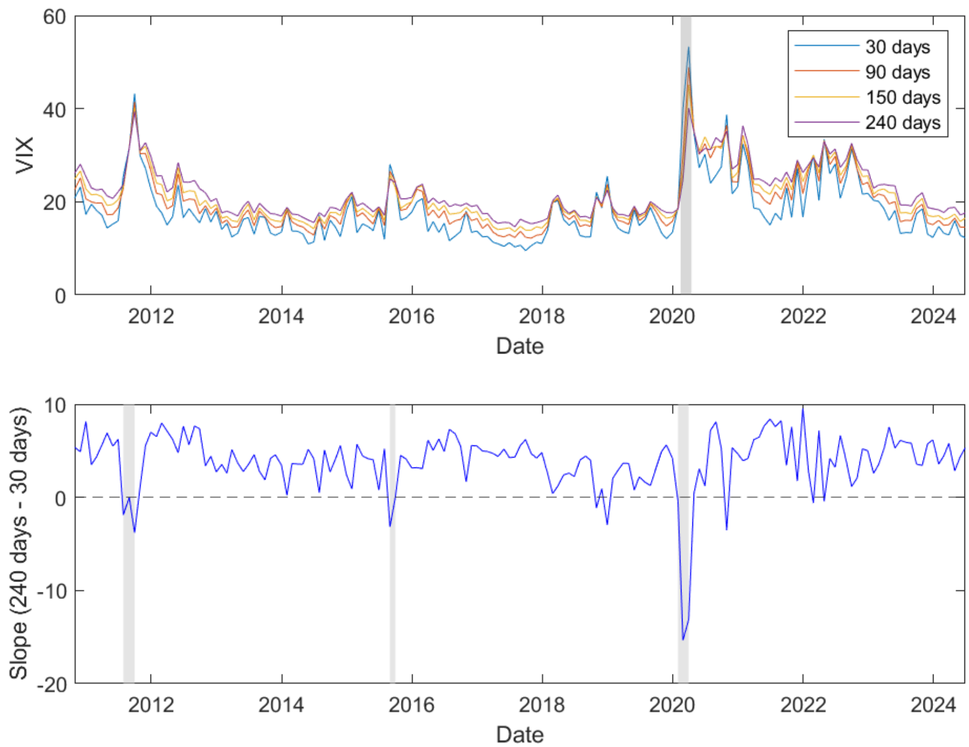


(b) VIX term structure



Notes: Figure C.1 Panel (a) presents daily VIX index with various maturities over time. Panel (b) illustrates the three-dimensional term structure of VIX across different maturities spanning from 30 to 300 days from October 2010 to June 2024.

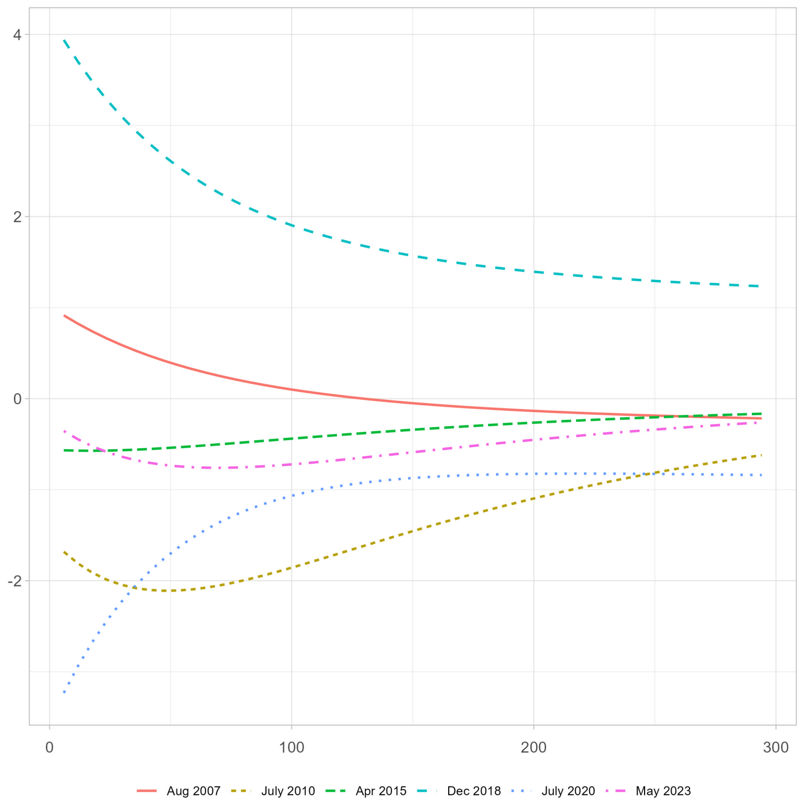
Figure C.2: VIX term structure



Notes: Figure C.2, top panel, shows VIX levels for four selected maturities (30, 90, 150, and 240 days) from October 2010 to June 2024, with shaded areas denoting NBER recessions. The bottom panel plots the slope of the term structure, defined as the difference between the 240-day and 30-day VIX levels, where shaded areas indicate periods when the slope is negative.

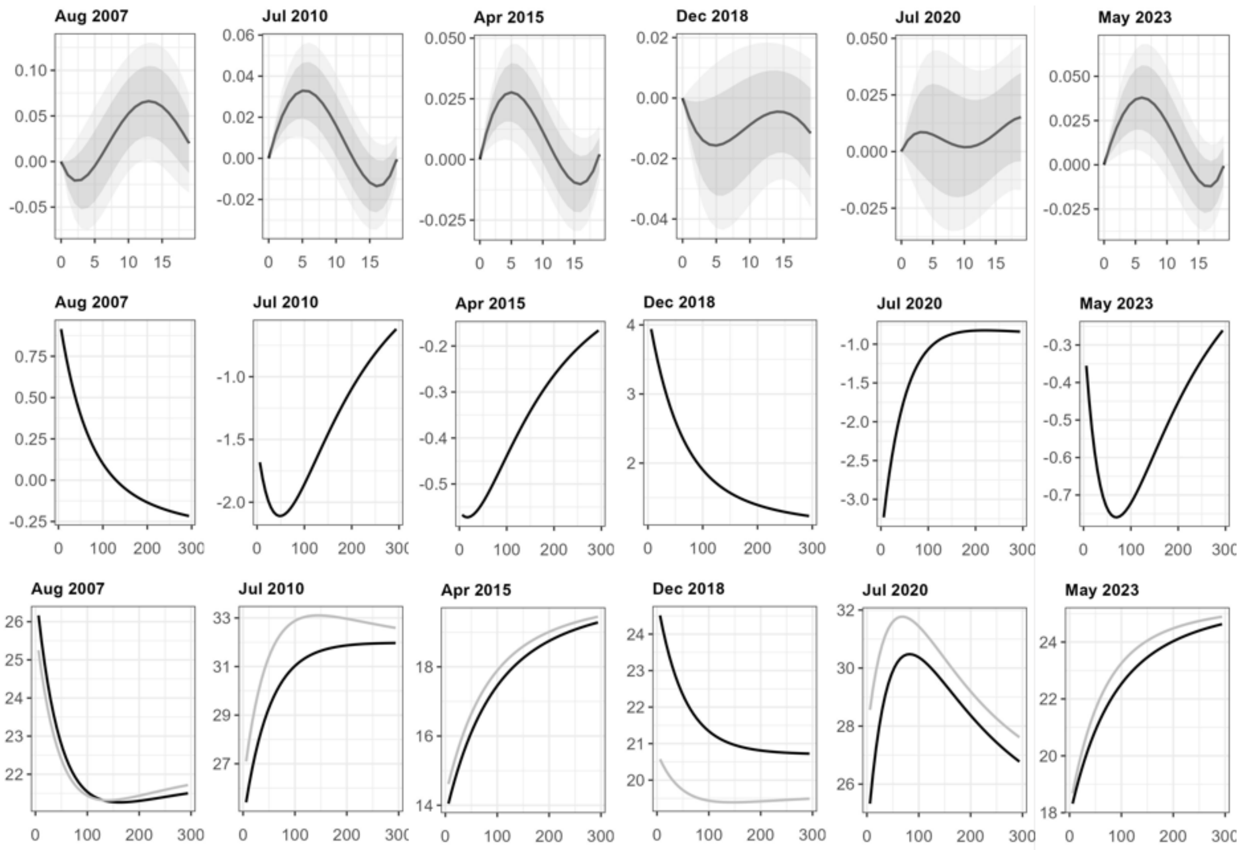
D Alternative shape parameter and functional uncertainty shocks

Figure D.1: Representative examples of uncertainty shocks



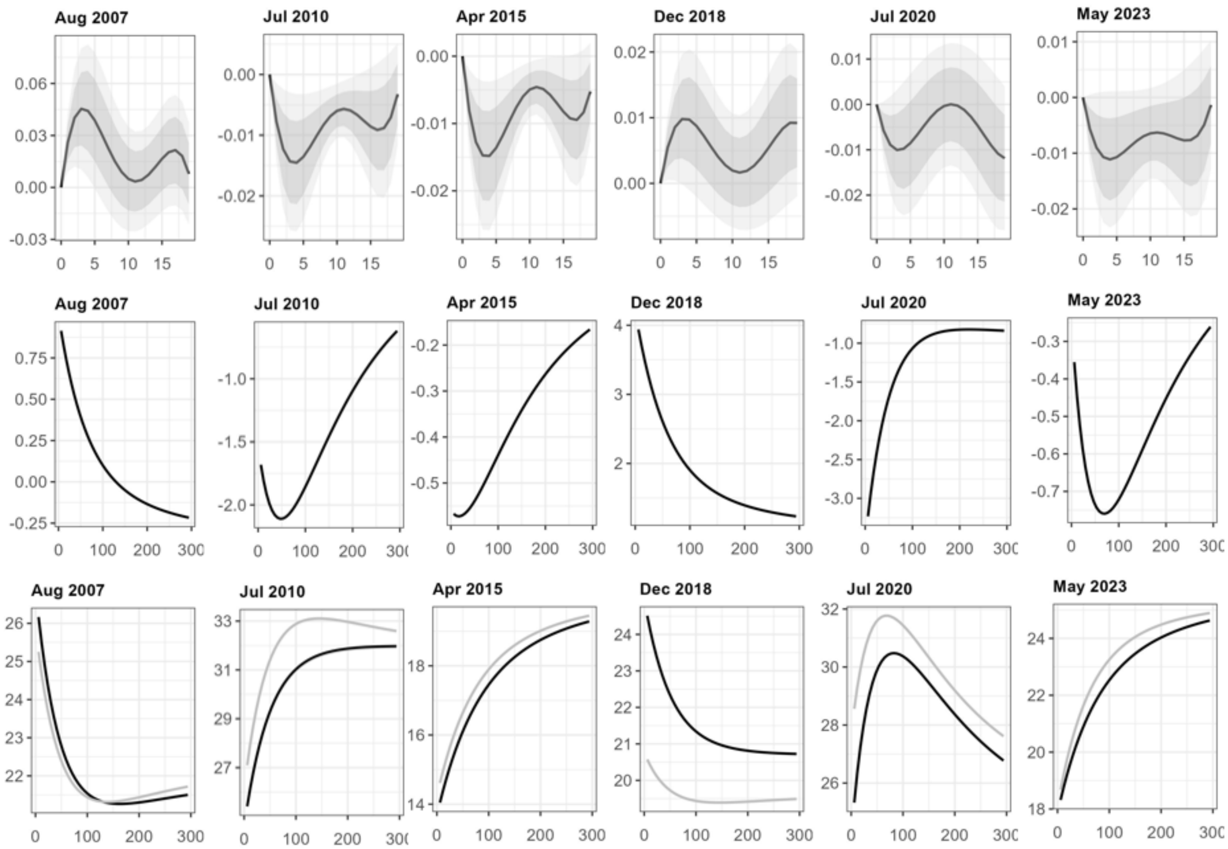
Notes: Figure D.1 presents examples of uncertainty shocks with $\lambda = 90$, our shape parameter, over the sample period from March 2006 to June 2024.

Figure D.2: Industrial production responses to uncertainty shocks



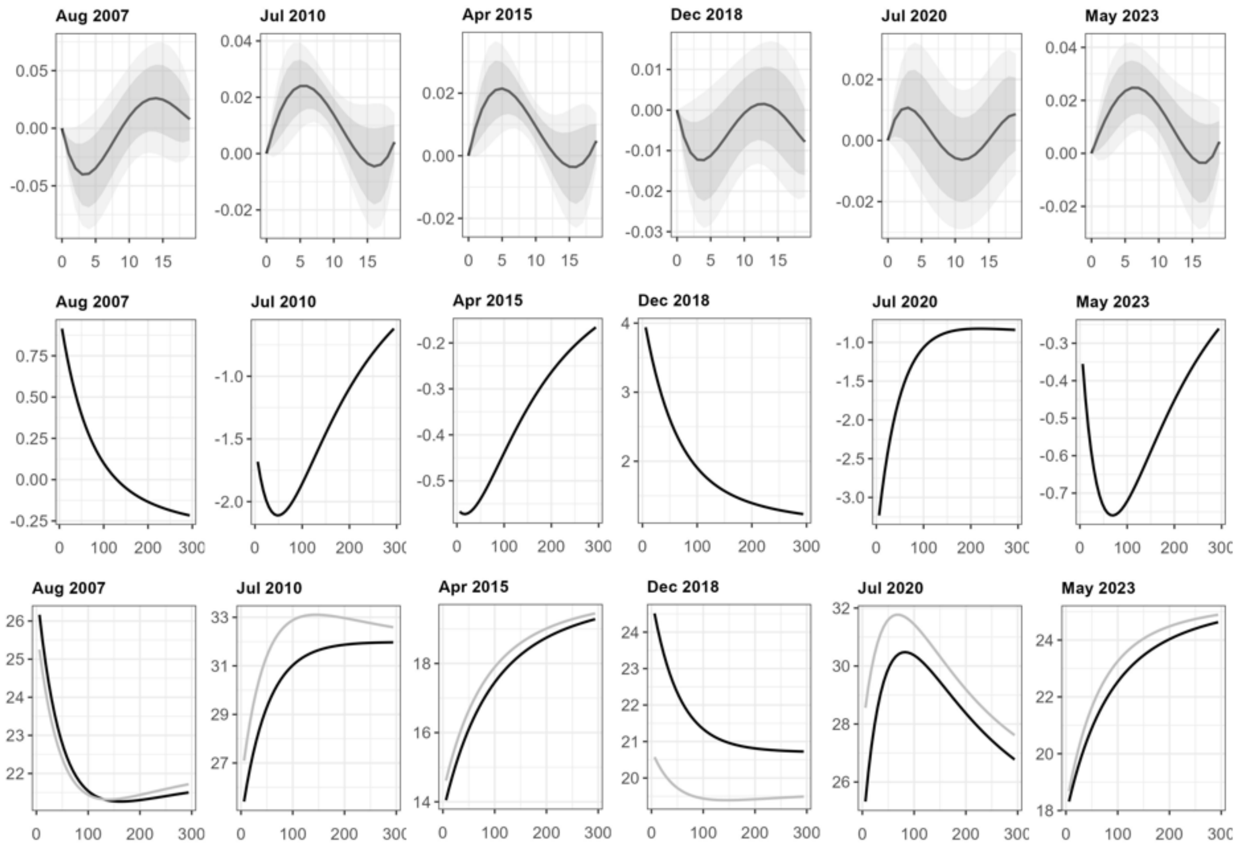
Notes: Figure D.2 presents the effects of uncertainty shocks on industrial production across different episodes when $\lambda = 90$. The top row reports the impulse responses with 68% and 90% confidence intervals. The second row depicts the functional shocks, capturing intra-month differences in VIX futures, and the third row contrasts their profiles before (grey line) and after (black line).

Figure D.3: Unemployment rate responses to uncertainty shocks



Notes: Figure D.3 presents the effects of uncertainty shocks on the unemployment rate across different episodes when $\lambda = 90$. The top row shows the impulse responses of industrial production to identified uncertainty shocks, with 68% and 90% confidence intervals. The second row displays the functional shocks, capturing the intra-month differences in VIX futures shown before (grey line) and after (black line) in the third row.

Figure D.4: Payroll employment responses to uncertainty shocks



Notes: Figure D.4 presents the effects of uncertainty shocks on the payroll employment across different episodes when $\lambda = 90$. The top row reports the impulse responses with 68% and 90% confidence intervals. The second row depicts the functional shocks, capturing intra-month differences in VIX futures, and the third row contrasts their profiles before (grey line) and after (black line).