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CECS 457 Applied ML
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Probability & Statistics

Exercise 1. Give a real-world example of a joint distribution P(x,y) where x is discrete and y is continuous. Do not use examples involving coins and dice.
Answer 1.
If X represents the discrete number of people in a room and Y represents the maximum temperature recorded in that room. The joint distribution P(x, y) would describe the probability of how hot a room gets as more people enter it.

Exercise 2. What remains if I marginalize a joint distribution P(v,w,x,y,z) over five variables with respect to variables w and y ? What remains if I marginalize the resulting distribution with respect to v?
Answer 2.
If you marginalize a joint distribution P(v,w,x,y,z) with respect to variables w and y, then you sum over all possibilities of variables w and y from the joint distribution, which will leave P(v,x,z), a distribution involving variables v, x, and z. If you marginalize the resulting distribution with respect to v then you will end up with P(x,z), a distribution involving just the variables x and z.

Exercise 3. If variables x and y are independent and variables x and z are independent, does it follow that variables y and z are independent?
Answer 3.
No, the independence of x and y and the independence of x and z doesn't inherently say anything about the relationship between y and z so you cannot conclude that variables y and z are independent with that information.

Exercise 4. Show that the following relation is true: P(w,x,y,z)=P(x,y)P(z|w,x,y)P(w|x,y)
Answer 4.
P(w,x,y,z) = P(z|w,x,y) * P(w,x,y) *Expand P(w,x,y,z) using product rule*
= P(z|w,x,y) * P(w|x,y) * P(x,y) *Expand P(w,x,y) using product rule*

Product rule: P(A,B) = P(A|B)P(B)

Exercise 5. In my pocket there are two coins. Coin 1 is a fair coin, so the probability of getting heads is 0.5 and the likelihood of getting tails is also 0.5. Coin 2 is biased, so the probability of getting heads is 0.8 and the probability of getting tails is 0.2. I reach into my pocket and draw one of the coins at random. I assume there is an equal chance I might have picked either coin. Then I flip that coin and observe a head.
Think about the Bayesian framework and describe what is the prior, what is the likelihood in this case.
Use Bayes' rule to compute the posterior probability that I chose coin 2.
Answer 5.
1. **Prior:** The prior is the probability of choosing each coin, P(C=1) & P(C=2), before any new evidence is introduced since both coins have an equal chance of getting chosen (P(C=1) = P(C=2) = 0.5), the prior is 0.5.
2. **Likelihood:** The likelihood in this case is how likely it is for the coin to be heads if coin 2 was chosen, P(H=1|C=2). This information was given so the likelihood is 0.8.
3. Marginal = P(H=1) = 0.5 * P(H=1|C=1) + 0.5 P(H=1|C=2) = 0.5*0.5 + 0.5*0.8 = 0.65
4. **Posterior Probability:**

$P(C=2| H=1) = (Prior * Likelihood) / Marginal$
$= (P(C=2) * P(H=1|C=2)) / P(H=1)$
$= (0.5 * 0.8) / 0.65 \approx 0.62$

Exercise 6. Consider a biased die where the probabilities of rolling sides {1,2,3,4,5,6} are {1/12,1/12,1/12,1/12,1/6,1/2}, respectively. What is the expected value of the outcome? If I roll the die twice, what is the expected value of the sum of the two rolls?

Answer 6.

The expected values, E(x), can be found by adding the products of each rolling side value, x, and its probability, P(x).

$E(x) = \Sigma\, xP(x) = (1 * (1/12)) + (2 * (1/12)) + (3 * (1/12)) + (4 * (1/12)) + (5 * (1/6)) + (6 * (1/2))$
$= 1/12 + 2/12 + 3/12 + 4/12 + 10/12 + 36/12$
$= 56/12 \approx 4.66$

Since each roll is independent, we can assume the expected value will be the same for each roll. Therefore, the expected value of the sum of the two rolls, E(y), is double the expected value of one roll.

$E(y) = E(x) * 2 = 56/12 * 2 = 112/12 \approx 9.33$