

1) Interpretation of the results of the linear regression

The results of the linear regression based on the Capital Asset Pricing Model (CAPM) provide insights for the relationship between the asset's returns. We can evaluate the asset's performance by analyzing the alpha, beta, expected return and idiosyncratic Variance.

Beta measures the sensitivity of the asset's returns to the overall market. A higher beta (>1) means the stock is more volatile than the market and a beta less than 1.0 indicates a stock with lower volatility. The low beta stocks are good for investors who are looking for stable, long-term investments. But, the high beta stocks have the potential to provide investors with a higher rate of return.

By looking at our results (figure 1) we can conclude below results;

Most of our beta values are greater than 1, meaning most of our stocks are more volatile than the market (**S&P 500**).

NVDA has the highest beta (1.72). This means it moves 1.7 times more than the market. This could be interpreted as high risk investment but the reward could be also high.

IBM, the lowest beta with 0.8, could be seen as a safe stock compared to other stocks. But it will provide smaller returns than riskier investments.

The stock's beta that close to 1 means it moves closely to the market for example **AMAZN**.

	A	B	C	D	E
1		alpha	beta	expected_return	idiosyncratic_variance
2	<u>AAPL</u>	0.00055788671958	1.1970842545313	0.00070211328063	0.000143237797756
3	<u>MSFT</u>	0.00055737757478	1.18135904941551	0.00069299439689	0.000115580839079
4	<u>GOOGL</u>	0.00027231547103	1.13350817801897	0.00066524617169	0.000175603862203
5	<u>AMZN</u>	0.00017629005571	1.06411395597433	0.00062500518037	0.000285283834912
6	<u>TSLA</u>	0.00176234125098	1.51251855573394	0.00088503036802	0.001247523334583
7	<u>META</u>	0.00052047140652	1.30629120507288	0.00076544126126	0.000473575678787
8	<u>NFLX</u>	0.00025610079891	1.03102372727108	0.00060581649914	0.000645264308589
9	<u>NVDA</u>	0.00196895885776	1.72938941213726	0.00101079153364	0.000525475460628
10	<u>IBM</u>	0.0001420175653	0.80709554557974	0.00047596286361	0.000160487745664
11	<u>INTC</u>	-0.00046570182783	1.2092127746728	0.00070914648396	0.00035034535975

Figure 1: CAPM Results for 10 Assets, with Highest Values in Red and Lowest Values in Green

Alpha tells us how much a stock outperforms or underperforms the market-adjusted expectations. Higher alpha (>0) means stock outperforms the market expectation and is a strong performer(good investment). Lower alpha (negative <0) means stock underperforms and not a good investment relative to its risk.

Except **INTC**, all of our stock's outperforms the market expectation. **TSLA** has a higher alpha value, it generates a high excess return even after controlling for market risk.

Idiosyncratic Variance represents the portion of an asset's risk that is not explained by market movements.

As can be seen on figure 1, **TSLA** has higher idiosyncratic variance (0.0012) (thanks to Elon Musk) which means stock's price fluctuations are highly sensitive to company-specific news and events. For example; by mid-March 2025, the stock had fallen approximately 50% from its peak due to Elon Musk's political involvement [1].

MSFT (0.00011) and **AAPL** (0.00014) have the lowest idiosyncratic variance, showing that their risks are more influenced by market factors rather than company-specific factors.

Expected Return is the average return expected from the asset over a given period. Higher expected returns indicate higher potential profits.

NVDA has the highest expected return (0.001) while **IBM** is the lowest(0.0004)

2) Analysis of the optimal portfolio weights and their sensitivity to changes in expected returns and risks.

I determined the target return variable based on the result of the efficient frontier model. I chose 0.000892 as the target return (as explained in the forth section) and calculated the optimal objective using this target value. The optimal objective is 2.92916362e-04, which represents the minimized variance (risk).

	Stock	Weight
0	AAPL	1.010194e-01
1	MSFT	6.055573e-02
2	GOOGL	6.388668e-07
3	AMZN	1.137075e-07
4	TSLA	1.317309e-01
5	META	1.401167e-01
6	NFLX	6.677485e-08
7	NVDA	5.088262e-01
8	IBM	1.382662e-08
9	INTC	5.775023e-02

Figure 2: Portfolio Optimization Weights for Each Asset Using Gurobi

The weights represent the percentage of the portfolio that should be invested in each stock. Using these weights, we can distribute our investment for each asset. For example, if we have \$100,000 to invest, we would buy 50% of it **NVDA** (based on the weights) given the results of the figure 3.

Based on the results given in figure 3, we can interfere that;

NVDA has the highest weight (50%). This means optimizer considers **NVDA** is the best risk-return tradeoff. This suggests **NVDA** has a favorable expected return in the portfolio. **META** (14%) and **TSLA** (13.1%) are the other assets that has moderate weights.

IBM, **NFLX**, **AMAZN**, **GOOGL** have weights close to zero. This could be happens if;

- Their risk-adjusted return is not optimal for the portfolio.
- They may have high covariance with other assets (increasing overall risk)
- They may not contribute to achieving the target return.

3) The results and the shape of the efficient frontier and its implications for portfolio diversification.

The Efficient Frontier Calculation is needed to analyze the best possible portfolios at different levels of return and risk.

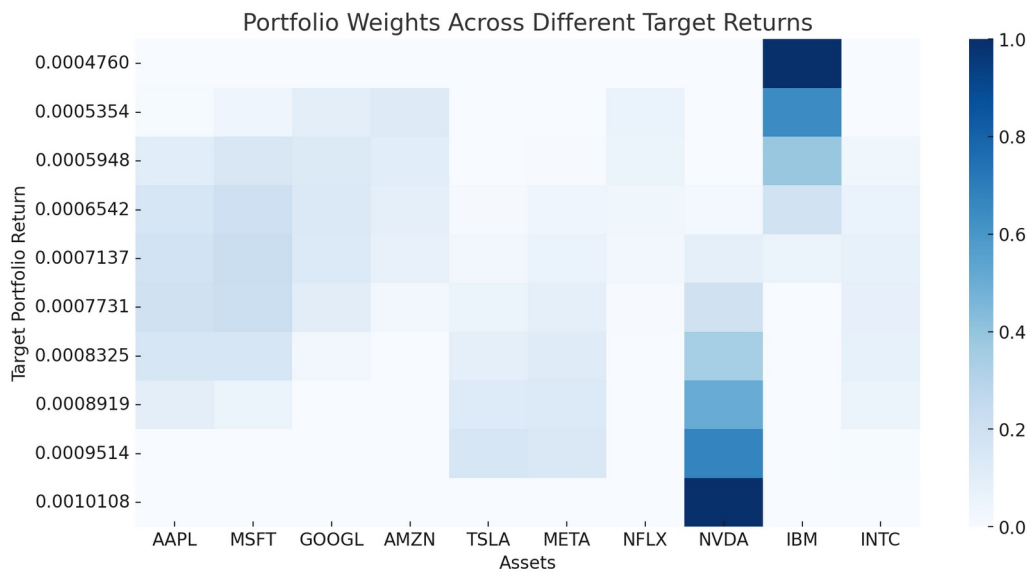


Figure 3: Efficient Frontier Weights

From the weight table (figure 3), low risk portfolios (left side of the curve of the figure 4) are mostly the assets like **IBM** (safe asset). As we increase return targets, weight shifts to the volatile stocks like **TSLA, NVDA**.



Figure 4: Efficient Frontier for Portfolio Optimization

As can be seen in the figure 4, portfolios with higher returns also have higher risk.

The lower-left part of the curve contains portfolios which are inefficient (they have higher risk for lower returns). These portfolios are below the efficient frontier since they are dominated by better choices. We need to avoid them. The upper convex region is the efficient frontier which means these portfolios provide the best risk-return trade-offs.

For the conservative portfolio, it would be a good choice to select a point on the left side of the upper convex region (low risk, moderate return).

4) Coding choices

On the portfolio optimization part I took target return as 0.000892. Because I am willing to take more risk for higher return. Besides this portfolio is more diversified and contains higher-growth stocks. But if I go with the much lower target return like 0.000535, my portfolio will be heavily weighted in IBM with little exposure to high-volatility stocks like TSLA or NVDA.

Reference

- [1] Investopedia. (2025, March 12). Dow Jones Today: Stock Market News and Trends. Investopedia. Retrieved from <https://www.investopedia.com/dow-jones-today-03122025-11695293>