Adversarial Auto-Encoders (AAEs) and Wasserstein Auto-Encoders (WAEs)

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WAE Paper

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Wasserstein Auto-Encoders

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- The NLL cannot be optimized directly, so VAEs use an upper bound.

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- Take-Away: VAEs minimize an upper bound on KL divergence between the data and the model.

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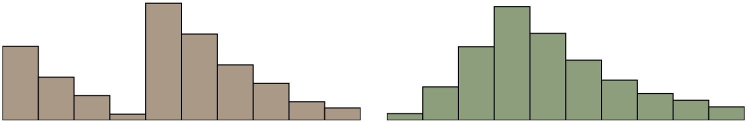
- Defined with respect to a cost function $c : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_+$ as

$$W_c(P_X, P_Y) = \inf_{\Gamma \in \mathcal{P}(X \sim P_X, Y \sim P_Y)} \mathbb{E}_{(X,Y)\sim\Gamma}[c(X, Y)]$$

where $\Gamma(x, y)$ is the "transport plan".



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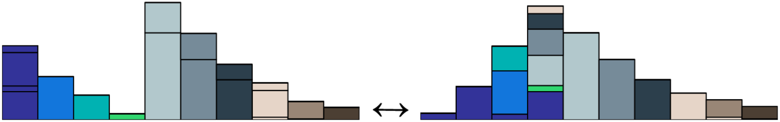

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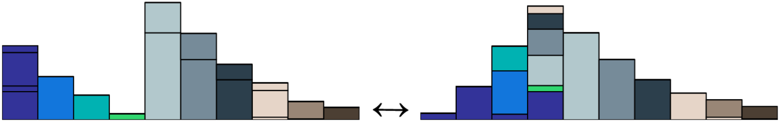

- Γ must be some joint distribution of X, Y because
  - $\int_x \Gamma(x, y)dx = P_X(x) \Rightarrow$ Total earth leaving x = total earth at x
  - $\int_y \Gamma(x, y)dy = P_Y(y) \Rightarrow$ Total earth entering y = total earth at y

Wasserstein Auto-Encoder

- Consider a latent space $\mathcal{Z}$ with prior P_Z. We want to find a model (i.e. decoder) $G : \mathcal{Z} \to \mathcal{X}$ that minimizes the Wasserstein distance

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- **[Bousquet et al. (2017)]** This is equivalent to minimizing

$$W_c(P_X, P_G) = \inf_{\underbrace{Q\,:\,Q_Z = P_Z}_{\text{min over encoders with marginal } P_Z}} \mathbb{E}_{P_X} \underbrace{\mathbb{E}_{Q(Z|X)}[c(X, G(Z))]}_{\text{reconstruction cost}}$$

where $Q_Z = \int Q(Z|X)P_X(X)dX$.

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- The WAE objective relaxes the constraint $Q_Z = P_Z$ by adding a penalty. It minimizes

$$D_{WAE}(P_X, P_G) = \inf_{Q(Z|X) \in \mathcal{Q}} \mathbb{E}_{P_X} \mathbb{E}_{Q(Z|X)}[c(X, G(Z)] + \lambda \cdot \mathcal{D}_Z(Q_Z, P_Z)$$

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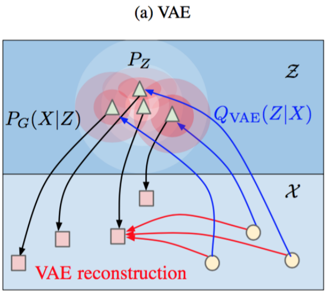
- Claims to fix blurriness issue of VAEs



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     - Theoretical justification of AAE as minimizing 2-Wasserstein distance

Experiments

- Test VAE, WAE-GAN, WAE-MMD on MNIST and CelebA
- Record Frechet Inception Distance (FID) to assess quality of images

Algorithm	**FID**
VAE	82
WAE-MMD	55
WAE-GAN	42

Table 1: FID scores for samples on CelebA (smaller is better).

Extensions

ADVERSARIALLY REGULARIZED AUTOENCODERS

Junbo (Jake) Zhao[1], Yoon Kim[2], Kelly Zhang[1], Alexander M. Rush[2], Yann LeCun[1,3]
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[2] School of Engineering and Applied Sciences, Harvard University
[3] Facebook AI Research
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WASSERSTEIN AUTO-ENCODERS:
LATENT DIMENSIONALITY AND RANDOM ENCODERS

Paul Rubenstein,* Bernhard Schölkopf, Ilya Tolstikhin
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LEARNING DISENTANGLED REPRESENTATIONS WITH
WASSERSTEIN AUTO-ENCODERS

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ORIGINAL TEXT	MODERN TEXT
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  - Formal English vs. Informal English

Informal:	*I'd say it is punk though.*
Formal:	*However, I do believe it to be punk.*
Informal:	*Gotta see both sides of the story.*
Formal:	*You have to consider both sides of the story.*