BOGAZICI UNIVERSITY

IE440 Nonlinear Models in Operations Research

Final
Question 1

SOM for the Euclidean Travelling Salesman Problem

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Problem Definiton

My task in this assignment was to find the optimum soliton for the Euclidean Travelling Salesman Problem by using Kohonnen's Self-Organizing Map.

In my data file, I had the coordinates of 81 cities. I needed to find the best route for travelling all 81 cities with minimum distance traveled.

The First Approach: The Gaussian Kernel as the Neighbourhood Function

The first approach uses the Gaussian Kernel as the Neighbourhood Function. The gaussian kernel is defined as the following in the class:

$$\Lambda_{i,ip} = exp(-\frac{||w_i - w_{ip}||^2}{2\sigma^2})$$

where w_i is the current neuron and w_{ip} is the winner neuron. The source code for the implementation:

```python
while flag:
        # Shuffle the data (cities' locations) for each iteration
        np.random.shuffle(data_normalized)

        for sample in data_normalized:     # For each city,

            # Calculate distance between neurons and cities
            distances = neurons - sample
            euclideans = [np.linalg.norm(i) for i in distances]
            winner = float('inf')
            winner_index = None

            # Find the index of the winner neuron
            for i in range(len(euclideans)):
                if euclideans[i] < winner:
                    winner = euclideans[i]
                    winner_index = i

            # Update weights for all neurons
            for i in range(M):
                k = np.exp(-(np.linalg.norm(neurons[i]-neurons[winner_index]))**2 /
2*sigma**2)

                neurons[i] += lr * k * (sample - neurons[i])

        # Update the learning rate and sigma
        lr *= 0.95 , sigma *= 0.95

        # Stopping condition
        if lr < 0.00005:
            flag=False
```

The Gaussian Kernel neighbourhood function enables the neurons that are close to the winner neuron to get larger updates, and the neurons that are far from the winner neuron to get smaller updates.

To reach better results, I tried the following pairs of parameters:

| Run Number	Number of Neurons	Initial Learning Rate	Initial Sigma	Distance (Cost) (Normalized)
1.1	81	0.1	1	21.82
1.2		0.3	1.5	20.71
1.3	2*81	0.1	1	22.99
1.4		0.3	1.5	21.82
1.5	3*81	0.1	1	20.83
1.6		0.3	1.5	21.23
1.7	5*81	0.1	1	23.24
1.8		0.1	1.5	23.92
1.9	7*81	0.1	1	25.54
1.10		0.1	1.5	22.20

Table 1

As can be seen from the Table 1, the best results are obtained with distances 20.71 and 20.86 for runs 1.2 and 1.5 respectively. The resulting tour maps can be found below:
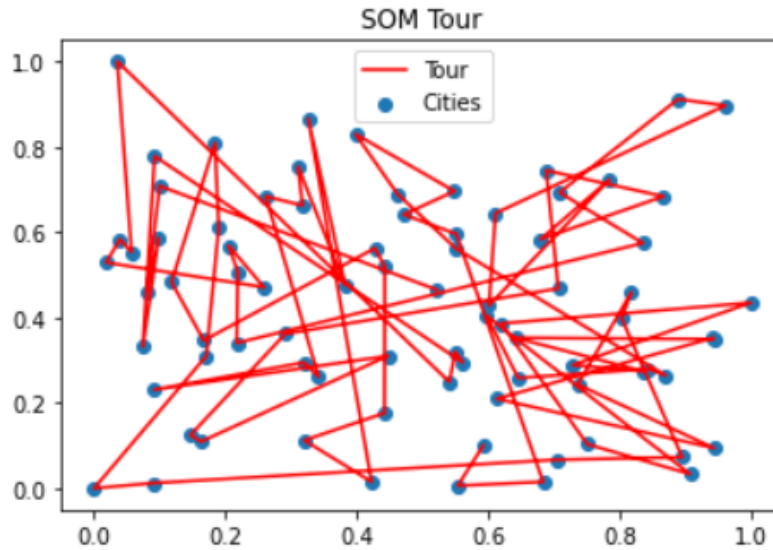
Map for run 1.2

Cities Visited in Order:
['Bayburt', 'Hatay', 'Osmaniye', 'Mersin', 'Yozgat', 'Samsun', 'Isparta', 'Eskisehir', 'Diyarbakir', 'Sinop', 'Corum', 'Amasya', 'Adana', 'Karaman', 'Cankiri', 'Gaziantep', 'Kirikkale', 'Kars', 'Istanbul', 'Kirklareli', 'Edirne', 'Kocaeli', 'Mardin', 'Ardahan', 'Yalova', 'Tekirdag', 'Artvin', 'Bursa', 'Ankara', 'Kirsehir', 'Aksaray', 'Erzincan', 'Nigde', 'Adiyaman', 'Konya', 'Ordu', 'Giresun', 'Karabuk', 'Sanliurfa', 'Bartin', 'Elazig', 'Malatya', 'Tunceli', 'Gumushane', 'Bolu', 'Kahramanmaras', 'Tokat', 'Kastamonu', 'Kilis', 'Sivas', 'Kayseri', 'Nevsehir', 'Mus', 'Van', 'Sirnak', 'Mugla', 'Siirt', 'Izmir', 'Kutahya', 'Igdir', 'Hakkari', 'Erzurum', 'Agri', 'Denizli', 'Canakkale', 'Burdur', 'Bitlis', 'Manisa', 'Bilecik', 'Batman', 'Balikesir', 'Aydin', 'Antalya', 'Trabzon', 'Zonguldak', 'Rize', 'Duzce', 'Sakarya', 'Usak', 'Afyonkarahisar', 'Bingol']
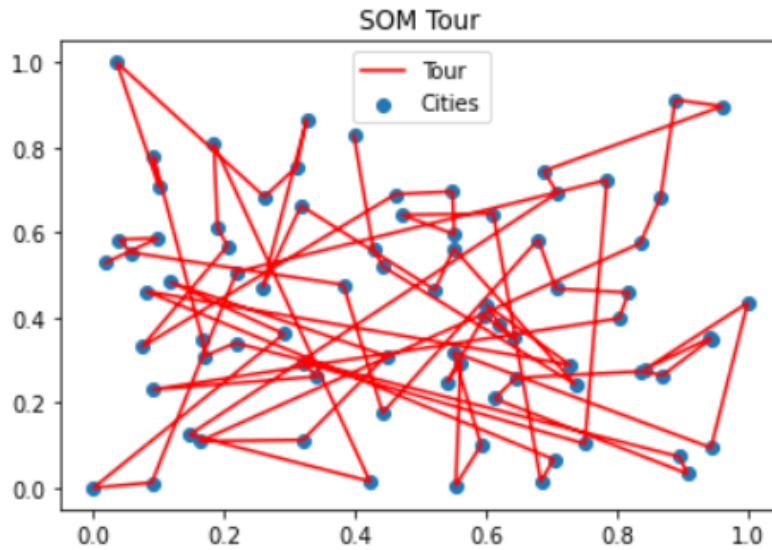
Map for run 1.2

Cities Visited in Order:
['Adana', 'Mersin', 'Amasya', 'Corum', 'Sanliurfa', 'Tunceli', 'Yalova', 'Bursa', 'Mugla', 'Sirnak', 'Nigde', 'Konya', 'Adiyaman', 'Bitlis', 'Ardahan', 'Gaziantep', 'Nevsehir', 'Kayseri', 'Ankara', 'Tokat', 'Erzincan', 'Malatya', 'Aksaray', 'Kars', 'Tekirdag', 'Kirklareli', 'Edirne', 'Artvin', 'Istanbul', 'Kirsehir', 'Elazig', 'Kastamonu', 'Kilis', 'Cankiri', 'Yozgat', 'Trabzon', 'Rize', 'Sakarya', 'Duzce', 'Kocaeli', 'Usak', 'Mus', 'Osmaniye', 'Denizli', 'Mardin', 'Burdur', 'Erzurum', 'Bilecik', 'Diyarbakir', 'Isparta', 'Afyonkarahisar', 'Bingol', 'Eskisehir', 'Izmir', 'Canakkale', 'Balikesir', 'Igdir', 'Hakkari', 'Aydin', 'Agri', 'Batman', 'Kutahya', 'Manisa', 'Van', 'Kirikkale', 'Sivas', 'Kahramanmaras', 'Samsun', 'Hatay', 'Sinop', 'Ordu', 'Karabuk', 'Bartin', 'Karaman', 'Giresun', 'Bayburt', 'Antalya', 'Siirt', 'Gumushane', 'Bolu', 'Zonguldak']

The resulting maps and cities visited in order can be found in Appendix. It is important to note that the optimum value for cost and the order of cities visited may differ from run to run since we randomly locate the neurons and shuffle the cities at each iteration.

Even though the resulting map did not satify me much, this is the best I can obtain. I also tried various different values for number of neurons, initial learning rate and sigma, multipliers for decreasing learning rate and sigma, and even stopping conditions, but none of them worked well as the above table of runs.

The Second Approach: Neighbourhood Function as defined in the Elastic Band

In this approach, the neighbourhood function is defined as follows:

$$d_{iip} = \min \{ |i - i_p|, \quad I - |i - i_p| \}$$

$$\Lambda_{i,ip} = exp(-\frac{d_{iip}^2}{\sigma^2})$$

where i is the index of the current neuron and i_p is the index of the winner neuron. The source code for the implementation:

```python
while flag:

        # Shuffle the data (cities' locations) for each iteration
        np.random.shuffle(data_normalized)

        # For each city,
        for sample in data_normalized:

                # Calculate distance between neurons and cities
                distances = neurons - sample

                euclideans = [np.linalg.norm(i) for i in distances]

                winner = float('inf')
                winner_index = None

                for i in range(len(euclideans)):
                    if euclideans[i] < winner:
                        winner = euclideans[i]
                        winner_index = i

                # Update weights for all neurons
                for i in range(M):
                        distance_to_bmu = min(np.abs(i - winner_index), M - np.abs(i -
winner_index))
                        k = np.exp(-distance_to_bmu**2 / sigma**2)
                        neurons[i] += lr * k * (sample - neurons[i])

        # Update the learning rate and sigma
        lr *= 0.95
        sigma *= 0.95

        # Stopping condition
        if lr < 0.001:
            flag=False
```

I tried the same set of parameters and obtained the following results:

Run Number	Number of Neurons	Initial Learning Rate	Initial Sigma	Distance (Cost) (Normalized)
2.1	81	0.1	1	35.30
2.2		0.3	1.5	32.20
2.3	2*81	0.1	1	33.16
2.4		0.3	1.5	32.11
2.5	3*81	0.1	1	35.22
2.6		0.3	1.5	34.93
2.7	5*81	0.1	1	38.21
2.8		0.1	1.5	39.05
2.9	7*81	0.1	1	40.24
2.10		0.1	1.5	38.37

Table 2

As can be seen from the Table 2, the best results are obtained with distances 32.20 and 32.11 for runs 2.2 and 2.4 respectively. These results are clearly worse than the results of the first approach. Therefore I also tried the following set of parameters for this approach:

| Run Number	Number of Neurons	Initial Learning Rate	Initial Sigma	Distance (Cost) (Normalized)
2.11	81	0.5	4	20.99
2.12		0.8	3.5	20.62
2.13	2*81	0.5	4	32.28
2.14		0.8	3.5	32.80
2.15	3*81	0.5	4	36.45
2.16		0.8	3.5	36.45

Table 3

The best results now are obtained with distances 20.99 and 20.62 for runs 2.11 and 2.12 respectively. These results made me discover that as the number of neurons increase, the result get worse for this approach. The resulting tour maps can be found below:
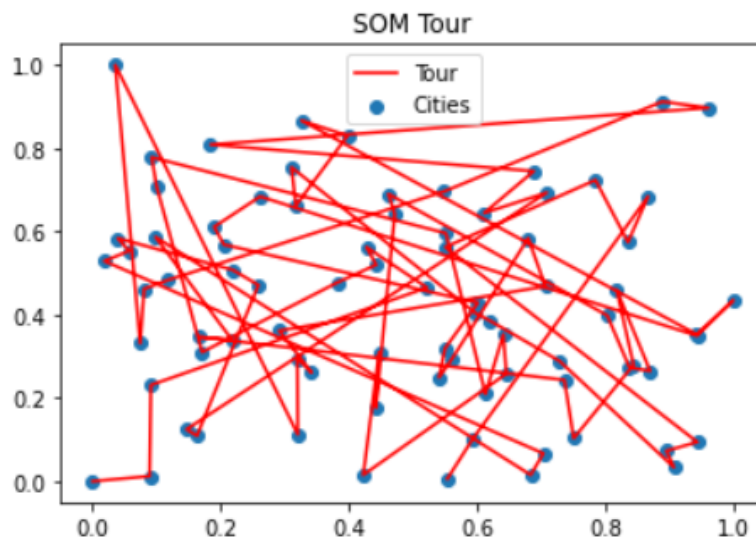
Map for run 2.11

Cities Visited in Order:
['Afyonkarahisar', 'Bingol', 'Tekirdag', 'Kars', 'Ardahan', 'Antalya', 'Izmir', 'Van', 'Manisa', 'Denizli', 'Burdur', 'Bilecik', 'Karabuk', 'Giresun', 'Zonguldak', 'Bartin', 'Igdir', 'Canakkale', 'Hakkari', 'Ordu', 'Kilis', 'Cankiri', 'Mardin', 'Eskisehir', 'Diyarbakir', 'Isparta', 'Yalova', 'Bursa', 'Ankara', 'Konya', 'Adiyaman', 'Malatya', 'Sinop', 'Yozgat', 'Osmaniye', 'Tokat', 'Rize', 'Kirklareli', 'Edirne', 'Kastamonu', 'Artvin', 'Kocaeli', 'Istanbul', 'Kirsehir', 'Aksaray', 'Karaman', 'Gaziantep', 'Amasya', 'Corum', 'Adana', 'Mus', 'Usak', 'Mersin', 'Kirikkale', 'Kahramanmaras', 'Siirt', 'Bitlis', 'Balikesir', 'Mugla', 'Aydin', 'Agri', 'Sirnak', 'Trabzon', 'Duzce', 'Tunceli', 'Erzincan', 'Elazig', 'Batman', 'Erzurum', 'Kutahya', 'Sakarya', 'Samsun', 'Hatay', 'Sivas', 'Sanliurfa', 'Gumushane', 'Bolu', 'Bayburt', 'Nigde', 'Nevsehir', 'Kayseri']
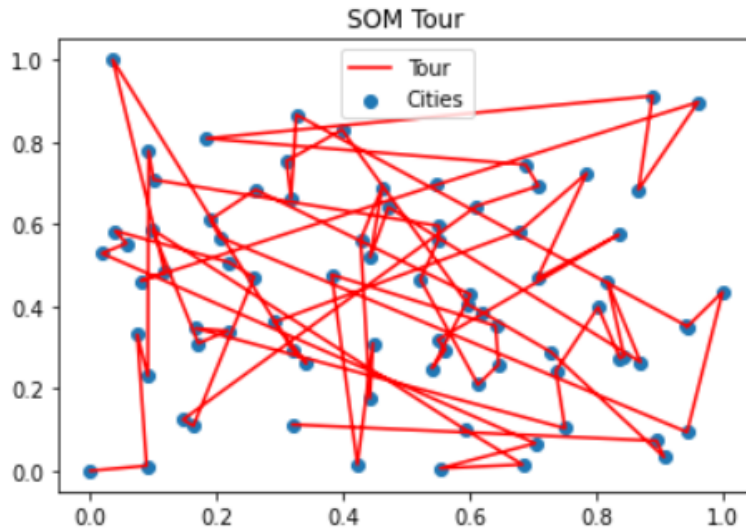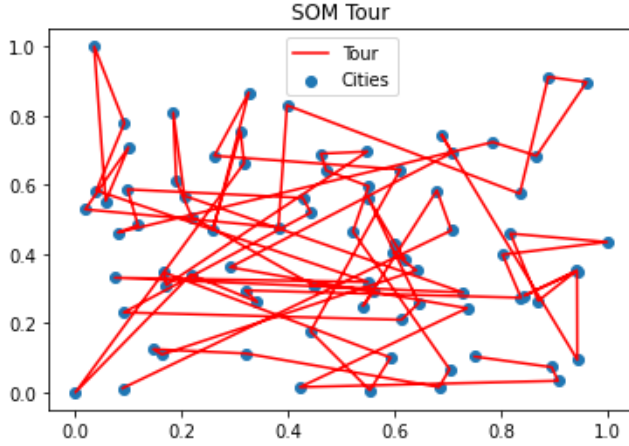

Map for run 2.12

Cities Visited in Order:
['Malatya', 'Van', 'Manisa', 'Denizli', 'Burdur', 'Bilecik', 'Zonguldak', 'Karabuk', 'Giresun', 'Bartin', 'Igdir', 'Hakkari', 'Canakkale', 'Izmir', 'Cankiri', 'Kilis', 'Ordu', 'Mardin', 'Diyarbakir', 'Isparta', 'Eskisehir', 'Kars', 'Bursa', 'Ardahan', 'Yalova', 'Ankara', 'Konya', 'Adiyaman', 'Sinop', 'Tokat', 'Osmaniye', 'Rize', 'Edirne', 'Tekirdag', 'Kirklareli', 'Kastamonu', 'Artvin', 'Istanbul', 'Kocaeli', 'Kirsehir', 'Aksaray', 'Karaman', 'Gaziantep', 'Amasya', 'Corum', 'Adana', 'Mus', 'Afyonkarahisar', 'Usak', 'Bingol', 'Mersin', 'Sivas', 'Kirikkale', 'Kahramanmaras', 'Bitlis', 'Siirt', 'Sirnak', 'Agri', 'Balikesir', 'Mugla', 'Aydin', 'Duzce', 'Gumushane', 'Trabzon', 'Bolu', 'Tunceli', 'Erzincan', 'Elazig', 'Sanliurfa', 'Erzurum', 'Batman', 'Kutahya', 'Bayburt', 'Antalya', 'Sakarya', 'Hatay', 'Samsun', 'Nigde', 'Yozgat', 'Nevsehir', 'Kayseri']

APPENDIX

Map for run 1.2

Cities Visited in Order:
['Nevsehir', 'Bursa', 'Yalova', 'Diyarbakir', 'Isparta', 'Erzincan', 'Gaziantep', 'Kilis', 'Kastamonu', 'Cankiri', 'Denizli', 'Yozgat', 'Eskisehir', 'Amasya', 'Samsun', 'Sinop', 'Corum', 'Hatay', 'Adana', 'Sanliurfa', 'Zonguldak', 'Kars', 'Kirklareli', 'Edirne', 'Tekirdag', 'Ardahan', 'Osmaniye', 'Tokat', 'Mersin', 'Bolu', 'Gumushane', 'Nigde', 'Kutahya', 'Batman', 'Antalya', 'Mardin', 'Burdur', 'Bilecik', 'Erzurum', 'Aksaray', 'Kirsehir', 'Malatya', 'Usak', 'Mus', 'Bayburt', 'Sakarya', 'Duzce', 'Trabzon', 'Rize', 'Sivas', 'Kahramanmaras', 'Bingol', 'Afyonkarahisar', 'Tunceli', 'Kocaeli', 'Ordu', 'Bartin', 'Karaman', 'Karabuk', 'Giresun', 'Kayseri', 'Kirikkale', 'Konya', 'Adiyaman', 'Agri', 'Sirnak', 'Canakkale', 'Mugla', 'Aydin', 'Igdir', 'Balikesir', 'Hakkari', 'Izmir', 'Artvin', 'Istanbul', 'Ankara', 'Siirt', 'Elazig', 'Manisa', 'Van', 'Bitlis']
Cost for M=81: 21.82
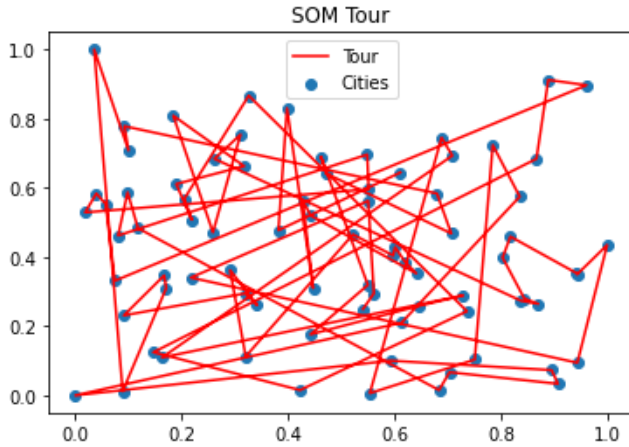
Map for run 1.3

Cities Visited in Order:
['Kocaeli', 'Sanliurfa', 'Zonguldak', 'Erzincan', 'Bolu', 'Gumushane', 'Erzurum', 'Burdur', 'Mardin', 'Bilecik', 'Bayburt', 'Malatya', 'Ankara', 'Konya', 'Adiyaman', 'Nigde', 'Kahramanmaras', 'Sivas', 'Nevsehir', 'Sinop', 'Hatay', 'Samsun', 'Yalova', 'Bursa', 'Duzce', 'Trabzon', 'Eskisehir', 'Tunceli', 'Batman', 'Denizli', 'Aksaray', 'Istanbul', 'Artvin', 'Afyonkarahisar', 'Bitlis', 'Ardahan', 'Kars', 'Kutahya', 'Kayseri', 'Bingol', 'Van', 'Manisa', 'Mus', 'Usak', 'Tokat', 'Mersin', 'Osmaniye', 'Rize', 'Sakarya', 'Adana', 'Amasya', 'Corum', 'Yozgat', 'Edirne', 'Kirklareli', 'Tekirdag', 'Kirsehir', 'Elazig', 'Siirt', 'Bartin', 'Cankiri', 'Gaziantep', 'Kilis', 'Giresun', 'Kastamonu', 'Karaman', 'Karabuk', 'Ordu', 'Agri', 'Aydin', 'Balikesir', 'Sirnak', 'Mugla', 'Igdir', 'Hakkari', 'Canakkale', 'Izmir', 'Kirikkale', 'Antalya', 'Isparta', 'Diyarbakir']
Cost for M=162: 22.99
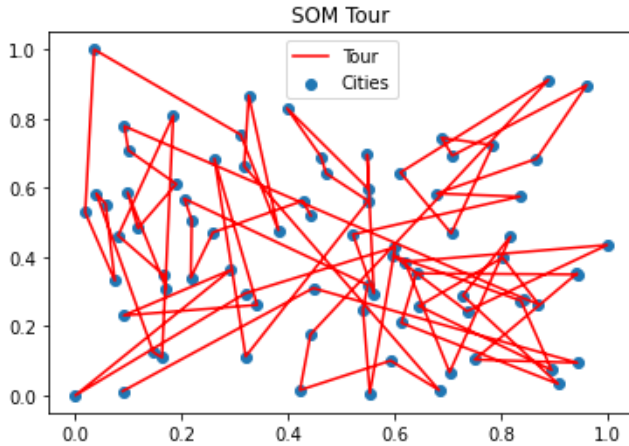
Map for run 1.4

Cities Visited in Order:
['Gumushane', 'Bolu', 'Karaman', 'Kirikkale', 'Gaziantep', 'Cankiri', 'Eskisehir', 'Diyarbakir', 'Afyonkarahisar', 'Mardin', 'Adiyaman', 'Kayseri', 'Ankara', 'Nigde', 'Konya', 'Ordu', 'Malatya', 'Antalya', 'Duzce', 'Trabzon', 'Zonguldak', 'Sakarya', 'Rize', 'Isparta', 'Bayburt', 'Kars', 'Yalova', 'Tekirdag', 'Edirne', 'Istanbul', 'Artvin', 'Ardahan', 'Bursa', 'Kocaeli', 'Kirklareli', 'Tunceli', 'Elazig', 'Bingol', 'Usak', 'Giresun', 'Bartin', 'Sanliurfa', 'Karabuk', 'Sinop', 'Adana', 'Yozgat', 'Corum', 'Amasya', 'Kirsehir', 'Aksaray', 'Kastamonu', 'Osmaniye', 'Kahramanmaras', 'Sivas', 'Mersin', 'Tokat', 'Kilis', 'Hatay', 'Samsun', 'Balikesir', 'Aydin', 'Sirnak', 'Batman', 'Van', 'Agri', 'Bilecik', 'Siirt', 'Canakkale', 'Burdur', 'Mus', 'Mugla', 'Denizli', 'Manisa', 'Kutahya', 'Erzurum', 'Hakkari', 'Igdir', 'Bitlis', 'Izmir', 'Erzincan', 'Nevsehir']
Cost for M=162: 21.82
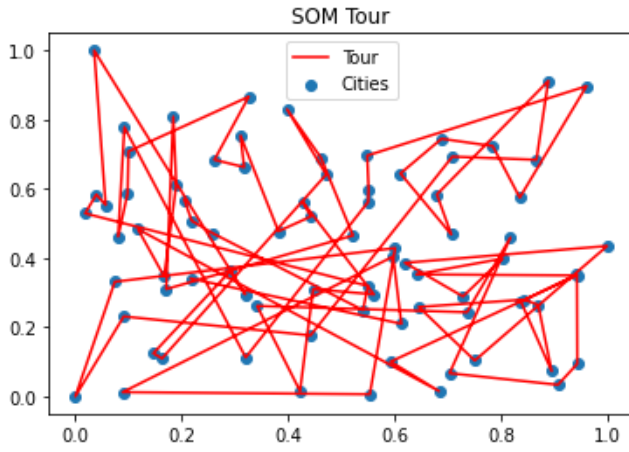
Map for run 1.6

Cities Visited in Order:
['Kirikkale', 'Kutahya', 'Bilecik', 'Nevsehir', 'Afyonkarahisar', 'Mardin', 'Yozgat', 'Kayseri', 'Nigde', 'Tunceli', 'Istanbul', 'Tekirdag', 'Kirklareli', 'Yalova', 'Bursa', 'Kocaeli', 'Artvin', 'Ardahan', 'Kars', 'Edirne', 'Rize', 'Sakarya', 'Antalya', 'Malatya', 'Samsun', 'Osmaniye', 'Mersin', 'Hatay', 'Ordu', 'Giresun', 'Karabuk', 'Sanliurfa', 'Gumushane', 'Bolu', 'Isparta', 'Erzincan', 'Elazig', 'Konya', 'Siirt', 'Mugla', 'Mus', 'Manisa', 'Izmir', 'Igdir', 'Agri', 'Van', 'Aydin', 'Balikesir', 'Batman', 'Bitlis', 'Canakkale', 'Burdur', 'Denizli', 'Sirnak', 'Erzurum', 'Hakkari', 'Bingol', 'Usak', 'Tokat', 'Kahramanmaras', 'Kilis', 'Kastamonu', 'Sivas', 'Bayburt', 'Trabzon', 'Zonguldak', 'Duzce', 'Aksaray', 'Kirsehir', 'Ankara', 'Adiyaman', 'Sinop', 'Corum', 'Amasya', 'Adana', 'Eskisehir', 'Diyarbakir', 'Karaman', 'Gaziantep', 'Cankiri']
Cost for M=243: 21.23
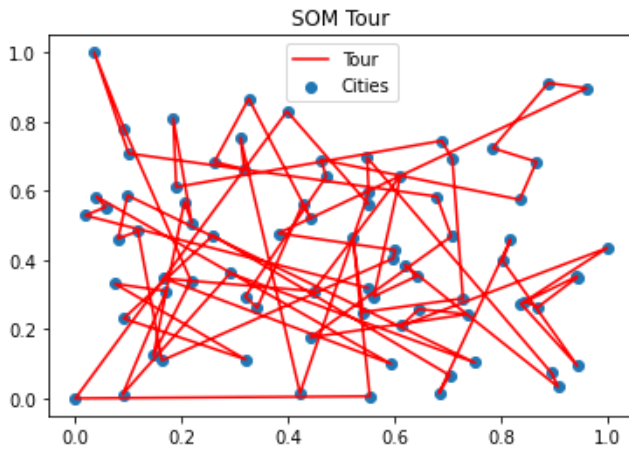
Map for run 1.7

Cities Visited in Order:
['Eskisehir', 'Adana', 'Corum', 'Amasya', 'Bingol', 'Kahramanmaras', 'Erzincan', 'Kocaeli', 'Isparta', 'Erzurum', 'Burdur', 'Manisa', 'Van', 'Rize', 'Sakarya', 'Elazig', 'Karabuk', 'Giresun', 'Ordu', 'Bartin', 'Gumushane', 'Bolu', 'Konya', 'Adiyaman', 'Duzce', 'Trabzon', 'Kars', 'Tekirdag', 'Ardahan', 'Kirklareli', 'Edirne', 'Sanliurfa', 'Mardin', 'Bilecik', 'Aksaray', 'Tokat', 'Osmaniye', 'Mersin', 'Bitlis', 'Ankara', 'Nevsehir', 'Sivas', 'Yozgat', 'Malatya', 'Nigde', 'Karaman', 'Mus', 'Usak', 'Mugla', 'Sirnak', 'Izmir', 'Agri', 'Igdir', 'Hakkari', 'Aydin', 'Balikesir', 'Canakkale', 'Kutahya', 'Batman', 'Siirt', 'Tunceli', 'Bursa', 'Yalova', 'Hatay', 'Sinop', 'Samsun', 'Kirikkale', 'Kirsehir', 'Cankiri', 'Gaziantep', 'Kastamonu', 'Kilis', 'Artvin', 'Istanbul', 'Denizli', 'Diyarbakir', 'Bayburt', 'Afyonkarahisar', 'Kayseri', 'Zonguldak', 'Antalya']
Cost for M=405: 23.24
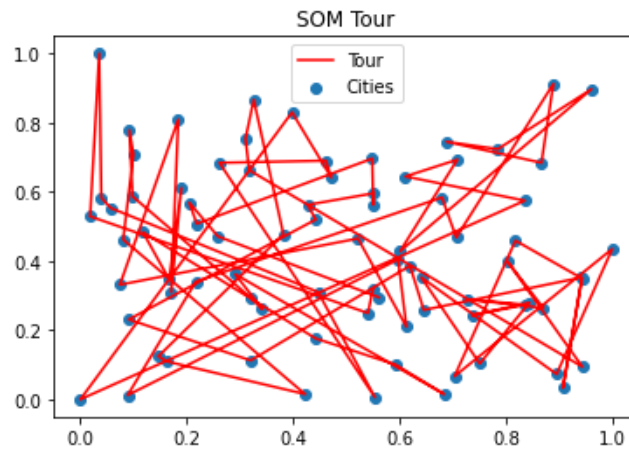
Map for run 1.8

Cities Visited in Order:
['Eskisehir', 'Diyarbakir', 'Adana', 'Sinop', 'Amasya', 'Corum', 'Erzincan', 'Afyonkarahisar', 'Ordu', 'Trabzon', 'Duzce', 'Zonguldak', 'Kayseri', 'Kars', 'Kocaeli', 'Istanbul', 'Mardin', 'Edirne', 'Ardahan', 'Artvin', 'Tekirdag', 'Kirklareli', 'Bursa', 'Yalova', 'Kirikkale', 'Nevsehir', 'Bilecik', 'Kutahya', 'Bayburt', 'Yozgat', 'Kastamonu', 'Sivas', 'Kilis', 'Kahramanmaras', 'Tokat', 'Tunceli', 'Bingol', 'Usak', 'Ankara', 'Konya', 'Adiyaman', 'Mersin', 'Samsun', 'Hatay', 'Osmaniye', 'Elazig', 'Aksaray', 'Kirsehir', 'Sanliurfa', 'Bartin', 'Karabuk', 'Giresun', 'Izmir', 'Sirnak', 'Aydin', 'Denizli', 'Agri', 'Siirt', 'Balikesir', 'Bitlis', 'Erzurum', 'Van', 'Canakkale', 'Mus', 'Mugla', 'Hakkari', 'Manisa', 'Igdir', 'Batman', 'Burdur', 'Malatya', 'Nigde', 'Gumushane', 'Bolu', 'Sakarya', 'Antalya', 'Rize', 'Gaziantep', 'Cankiri', 'Karaman', 'Isparta']
Cost for M=405: 23.92
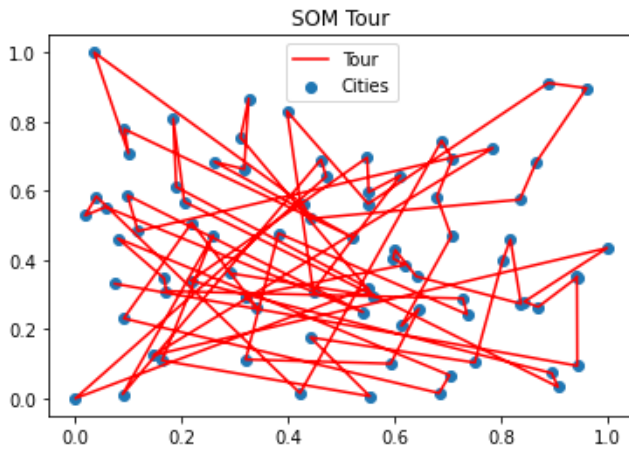
Map for run 1.9

Cities Visited in Order:
['Yozgat', 'Izmir', 'Igdir', 'Hakkari', 'Aydin', 'Balikesir', 'Canakkale', 'Kirsehir', 'Ardahan', 'Tokat', 'Mersin', 'Diyarbakir', 'Corum', 'Adana', 'Amasya', 'Elazig', 'Artvin', 'Istanbul', 'Yalova', 'Bursa', 'Kutahya', 'Batman', 'Bingol', 'Malatya', 'Sanliurfa', 'Siirt', 'Denizli', 'Sivas', 'Kahramanmaras', 'Osmaniye', 'Mus', 'Usak', 'Nigde', 'Gaziantep', 'Konya', 'Adiyaman', 'Burdur', 'Bilecik', 'Mardin', 'Erzurum', 'Agri', 'Mugla', 'Sirnak', 'Bitlis', 'Tunceli', 'Afyonkarahisar', 'Aksaray', 'Karaman', 'Nevsehir', 'Trabzon', 'Duzce', 'Kayseri', 'Eskisehir', 'Kilis', 'Kastamonu', 'Cankiri', 'Manisa', 'Van', 'Kirikkale', 'Rize', 'Sakarya', 'Kocaeli', 'Erzincan', 'Zonguldak', 'Antalya', 'Kirklareli', 'Edirne', 'Tekirdag', 'Kars', 'Gumushane', 'Bolu', 'Sinop', 'Hatay', 'Samsun', 'Bayburt', 'Ordu', 'Giresun', 'Bartin', 'Karabuk', 'Isparta', 'Ankara']
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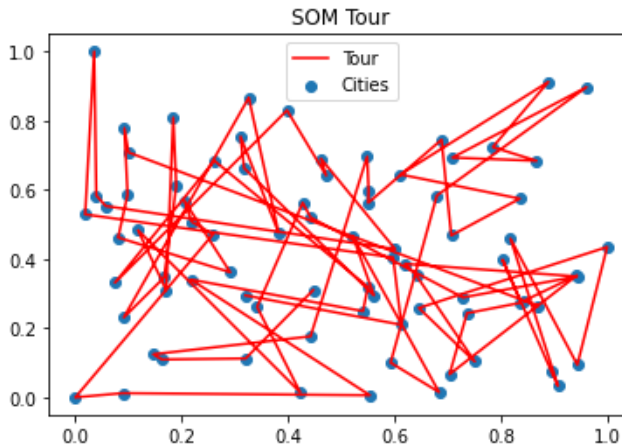
Map for run 1.10

Cities Visited in Order:
['Adiyaman', 'Kutahya', 'Bilecik', 'Adana', 'Sinop', 'Amasya', 'Corum', 'Mardin', 'Trabzon', 'Duzce', 'Zonguldak', 'Yozgat', 'Ordu', 'Eskisehir', 'Diyarbakir', 'Kirikkale', 'Afyonkarahisar', 'Nevsehir', 'Kayseri', 'Karaman', 'Gaziantep', 'Cankiri', 'Ankara', 'Osmaniye', 'Mersin', 'Samsun', 'Hatay', 'Aydin', 'Balikesir', 'Igdir', 'Mus', 'Siirt', 'Agri', 'Sirnak', 'Van', 'Manisa', 'Mugla', 'Izmir', 'Canakkale', 'Batman', 'Bitlis', 'Erzurum', 'Burdur', 'Hakkari', 'Denizli', 'Gumushane', 'Bolu', 'Konya', 'Elazig', 'Tokat', 'Sivas', 'Kastamonu', 'Kilis', 'Kahramanmaras', 'Nigde', 'Bartin', 'Sanliurfa', 'Karabuk', 'Giresun', 'Isparta', 'Bayburt', 'Usak', 'Bingol', 'Yalova', 'Edirne', 'Istanbul', 'Tekirdag', 'Ardahan', 'Kirklareli', 'Kocaeli', 'Kars', 'Bursa', 'Artvin', 'Antalya', 'Sakarya', 'Rize', 'Tunceli', 'Kirsehir', 'Aksaray', 'Malatya', 'Erzincan']
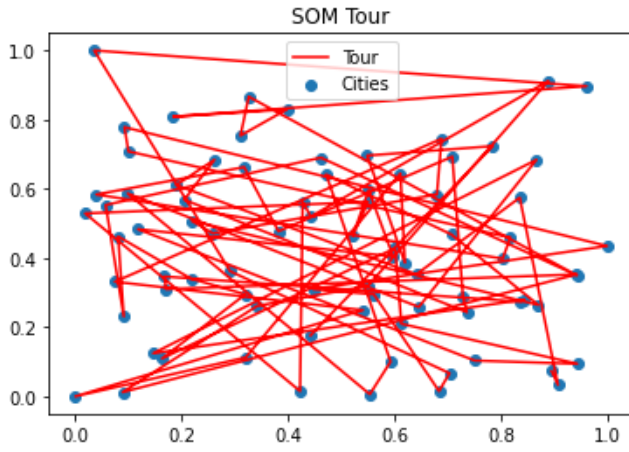Cost for M=567: 22.20
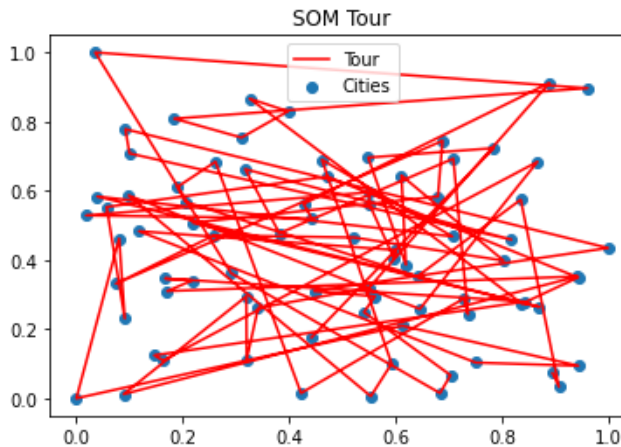
Map for run 2.13

Cities Visited in Order:
['Tunceli', 'Ardahan', 'Rize', 'Burdur', 'Kocaeli', 'Sirnak', 'Bayburt', 'Karaman', 'Yozgat', 'Artvin', 'Yalova', 'Gaziantep', 'Aydin', 'Amasya', 'Bursa', 'Trabzon', 'Bingol', 'Afyonkarahisar', 'Mersin', 'Batman', 'Tekirdag', 'Aksaray', 'Kirsehir', 'Balikesir', 'Kahramanmaras', 'Kirikkale', 'Sivas', 'Igdir', 'Nevsehir', 'Adiyaman', 'Malatya', 'Konya', 'Eskisehir', 'Kirklareli', 'Bolu', 'Gumushane', 'Adana', 'Sakarya', 'Antalya', 'Bilecik', 'Mardin', 'Tokat', 'Bitlis', 'Izmir', 'Kutahya', 'Kayseri', 'Osmaniye', 'Nigde', 'Corum', 'Duzce', 'Hakkari', 'Agri', 'Bartin', 'Zonguldak', 'Karabuk', 'Kastamonu', 'Edirne', 'Sinop', 'Ankara', 'Usak', 'Mus', 'Diyarbakir', 'Isparta', 'Erzincan', 'Canakkale', 'Samsun', 'Hatay', 'Mugla', 'Giresun', 'Sanliurfa', 'Istanbul', 'Siirt', 'Denizli', 'Elazig', 'Ordu', 'Kilis', 'Cankiri', 'Erzurum', 'Kars', 'Manisa', 'Van']
Cost for M=162: 32.28

Map for run 2.14

Cities Visited in Order:
['Tunceli', 'Ardahan', 'Rize', 'Burdur', 'Kocaeli', 'Malatya', 'Kayseri', 'Diyarbakir', 'Yozgat', 'Artvin', 'Yalova', 'Karaman', 'Gaziantep', 'Aydin', 'Bursa', 'Duzce', 'Bingol', 'Afyonkarahisar', 'Mersin', 'Batman', 'Tekirdag', 'Kirsehir', 'Balikesir', 'Agri', 'Kahramanmaras', 'Sivas', 'Igdir', 'Nevsehir', 'Konya', 'Eskisehir', 'Kirklareli', 'Gumushane', 'Bolu', 'Elazig', 'Adana', 'Antalya', 'Bayburt', 'Sakarya', 'Bilecik', 'Tokat', 'Bitlis', 'Izmir', 'Kutahya', 'Mardin', 'Aksaray', 'Adiyaman', 'Kirikkale', 'Osmaniye', 'Nigde', 'Corum', 'Trabzon', 'Hakkari', 'Bartin', 'Karabuk', 'Zonguldak', 'Kastamonu', 'Edirne', 'Sinop', 'Ankara', 'Mus', 'Usak', 'Isparta', 'Erzincan', 'Canakkale', 'Samsun', 'Hatay', 'Mugla', 'Sirnak', 'Amasya', 'Giresun', 'Sanliurfa', 'Istanbul', 'Denizli', 'Siirt', 'Cankiri', 'Ordu', 'Kilis', 'Erzurum', 'Kars', 'Manisa', 'Van']
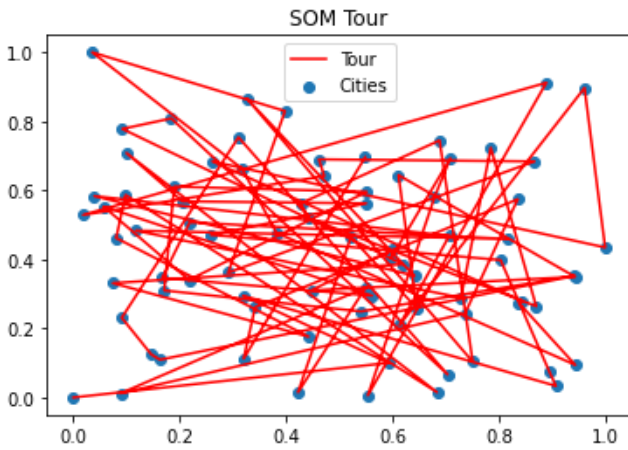Cost for M=162: 32.80
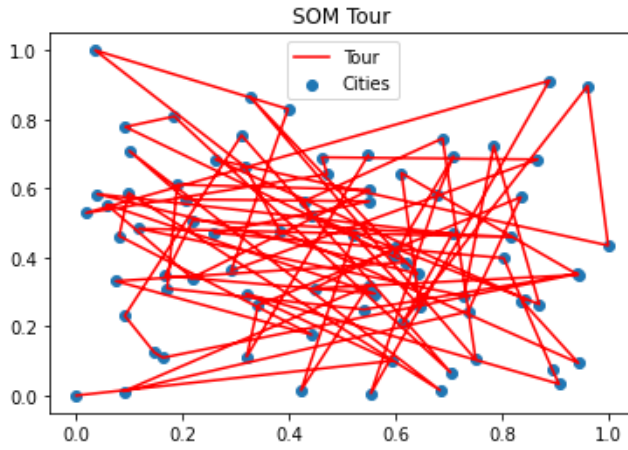
Map for run 2.15

Cities Visited in Order:
['Kayseri', 'Bingol', 'Adiyaman', 'Manisa', 'Van', 'Sirnak', 'Karaman', 'Antalya', 'Cankiri', 'Hatay', 'Usak', 'Mersin', 'Osmaniye', 'Tunceli', 'Yozgat', 'Burdur', 'Samsun', 'Kastamonu', 'Mus', 'Karabuk', 'Nigde', 'Kirsehir', 'Aksaray', 'Igdir', 'Diyarbakir', 'Sivas', 'Kilis', 'Sakarya', 'Kirikkale', 'Corum', 'Isparta', 'Kahramanmaras', 'Gaziantep', 'Mugla', 'Tokat', 'Denizli', 'Ardahan', 'Aydin', 'Gumushane', 'Bolu', 'Artvin', 'Bitlis', 'Bartin', 'Erzurum', 'Amasya', 'Bursa', 'Kutahya', 'Edirne', 'Canakkale', 'Ordu', 'Balikesir', 'Agri', 'Izmir', 'Sanliurfa', 'Giresun', 'Sinop', 'Zonguldak', 'Ankara', 'Tekirdag', 'Trabzon', 'Duzce', 'Adana', 'Kirklareli', 'Afyonkarahisar', 'Yalova', 'Istanbul', 'Bilecik', 'Mardin', 'Erzincan', 'Hakkari', 'Kocaeli', 'Batman', 'Bayburt', 'Elazig', 'Eskisehir', 'Nevsehir', 'Siirt', 'Kars', 'Konya', 'Malatya', 'Rize']
Cost for M=243: 36.45

Map for run 2.16

Cities Visited in Order:
['Kayseri', 'Bingol', 'Adiyaman', 'Manisa', 'Van', 'Sirnak', 'Karaman', 'Antalya', 'Cankiri', 'Hatay', 'Usak', 'Mersin', 'Osmaniye', 'Tunceli', 'Yozgat', 'Burdur', 'Samsun', 'Kastamonu', 'Mus', 'Karabuk', 'Nigde', 'Kirsehir', 'Aksaray', 'Igdir', 'Diyarbakir', 'Sivas', 'Kilis', 'Sakarya', 'Kirikkale', 'Corum', 'Isparta', 'Kahramanmaras', 'Gaziantep', 'Mugla', 'Tokat', 'Denizli', 'Ardahan', 'Aydin', 'Gumushane', 'Bolu', 'Artvin', 'Bitlis', 'Bartin', 'Erzurum', 'Amasya', 'Bursa', 'Kutahya', 'Edirne', 'Canakkale', 'Ordu', 'Balikesir', 'Agri', 'Izmir', 'Sanliurfa', 'Giresun', 'Sinop', 'Zonguldak', 'Ankara', 'Tekirdag', 'Trabzon', 'Duzce', 'Adana', 'Kirklareli', 'Afyonkarahisar', 'Yalova', 'Istanbul', 'Bilecik', 'Mardin', 'Erzincan', 'Hakkari', 'Kocaeli', 'Batman', 'Bayburt', 'Elazig', 'Eskisehir', 'Nevsehir', 'Siirt', 'Kars', 'Konya', 'Malatya', 'Rize']
Cost for M=243: 36.45