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IE 202 OPERATIONS RESEARCH I
ASSIGNMENT REPORT

1) This problem can be solved by the TP Model (Transportation Problem). To start with, I calculated the total demand and supply, found out that total demand is less than total supply. Therefore, I added a Dummy demand node having 40 demand per month of 10^3 bottles. Now, the question is ready to be modeled as such:

c(i, j) = *Unit transportation cost between supply node i to demand node j per month*

x(i, j) = *Flow between supply node i to demand node j per month;*

i = 1, 2, 3, 4 & j =1, 2, 3, 4, 5, Dummy

a(i) = *Monthly production capacity for plant i (10^3 bottles)*

a(1) = 290, a(2) = 220, a(3) = 180, a(4) = 280

b(j) = *Monthly demand for customer j (10^3 bottles)*

b(1) = 180, b(2) = 200, b(3) = 160, b(4) = 140, b(5) = 250, b(Dummy) = 40

Min (z) = sum((i, j), c(i, j)*x(i, j))

Subject to; sum(j, x(i, j) = a(i)

sum(i, x(i, j) = b(j)

x(i, j) >= 0

i = 1, 2, 3, 4 & j =1, 2, 3, 4, 5, Dummy

1.a)

Plants\Customers	1	2	3	4	5	Dummy
1	8.5	7	8	6.5	9	0
2	7.5	8	7	10	8.5	0
3	11	6	6.5	8	7	0
4	9	7	12	6	7.5	0

Table c(i, j) unit transportation costs by train

If the company uses only railroad, the cost table will be as above. The optimal solution found by GAMS is as follows:

x(1,2) = 200, x(1,4) = 50, x(1,Dummy) = 40,
x(2,3) = 40, x(3,3) = 120, x(3,5) = 60,
x(4,4) = 90, x(4,5) = 190, x(i,j) = 0 otherwise.

Objective Value (z) is 6520.

1.b) If the company uses ships where available, and uses railroad where transportation by ship is not feasible; the costs are increased due to the investment costs for ships where ships are available. The new unit transportation costs will be : (unit transportation cost of ship + 1/20th of the investment cost of ship at that cell). The new cost table of the shipment is below:

Plants\Customers	1	2	3	4	5	Dummy
1	7.5	9	8	5.5	8	0
2	6	6.5	8	7.5	8	0
3	11	6	7	7	9.5	0
4	10	7.5	10	7	7.5	0

Table c(i, j) unit transportation costs by ship

The optimal solution found by GAMS with the same model as above is as follows:

x(1,3) = 140, **x(1,4) = 140,** **x(1,Dummy) = 10,**
x(2,1) = 180, **x(2,2) = 40,** **x(3,2) = 160,**
x(3,3) = 20 **x(4,5) = 250,** **x(4,Dummy) = 30,**
x(i,j) = 0 otherwise.

Objective Value (z) is 6205.

1.c) If the company can use either ships or trains, it will choose the cheaper one. By this assumption, the cost table of the shipment is below:

Plants\Customers	1	2	3	4	5	Dummy
1	7.5	7	8	5.5	8	0
2	6	6.5	7	7.5	8	0
3	11	6	6.5	7	7	0
4	9	7	10	6	7.5	0

Table c(i, j) min unit transportation cost

The optimal solution found by GAMS with the same model as above is as follows:

x(1,2) = 140, **x(1,4) = 120,** **x(1,Dummy) = 10,**
x(2,1) = 180, **x(2,2) = 40,** **x(3,2) = 20,**
x(3,3) = 160, **x(4,5) = 250,** **x(4,Dummy) = 30,**
x(i,j) = 0 otherwise.

Objective Value (z) is 6125.

In this solution, the routes where ship is used are: x(1,2), x(1,4), x(2,1), x(2,2).

the routes where train is used are: x(3,2), x(3,3), x(4,5)

2) The minimum cost is obtained in the question 1.c where the company uses either ships or train independently.

2.a) In order to estimate the new total cost when the second plant's capacity is increased, one should check the marginal cost of supply constraint 2. In the solution of 1.c, where the second plant's capacity is 220, the marginal cost of second supply equation is found as -0.5 by GAMS. That is to say, if we were to increase the right-hand side of the second supply equation by 1, which is the capacity of second plant, the objective value would decrease by 0.5. Therefore, I estimate that if we increase the capacity from 220 to 230, the objective value will decrease by 0.5 * 10 = 5.

I made the GAMS resolve the problem with a capacity of 230 for the second plant and a demand of 50 for Dummy customer. The new objective value turned out to be 6120, which is 5 less than the old objective value of 6125 meaning that my estimation is correct.

2.b) In order to estimate the new total cost when the second customer's demand is decreased, one should check the marginal cost of demand constraint 2. In the solution of 1.c, where the second customer's demand is 200, the marginal cost of second demand equation is found as 7 by GAMS. That means, if we were to increase the right-hand side of the second demand equation by 1, which is the demand of second customer, the objective value would increase by 7. So, my estimation is that if we decrease the demand from 200 to 195, the objective value will decrease by 5 * 7 = 35.

When I resolve the problem by GAMS with a demand of 195 for the second plant and a demand of 45 for Dummy customer, the new objective value turned out to be 6090, which is 35 less than the old objective value of 6125 meaning that my estimation is correct.

2.c) As I explained in the question 2.a, the marginal cost of the second supply equation is -0.5. So it appears that if we increase the right hand side of the second supply equation by 180, the objective value will decrease by 0.5 * 180 = 90. However, when we make a relatively big change like increasing the capacity from 220 to 400, the basis of the first solution may also change. Therefore, most probably this estimation is wrong.

When I resolve the problem with GAMS, the new objective value is 6055, which is 70 less than 6125, the old value. So, it is not possible to calculate the new value without resolving the problem since the change we make is not in the range of sensitivity.

2.d) We should check the bounds for x(1, 4) under the objective ranging part in the solution file of the question 1.c. The current unit cost for that flow is 5.5. When I check the objective rankings, I see that the lower bound for c(1, 4) is 0, and the upper bound for c(1, 4) is 6. So, the range of the unit cost at route 1-4 is (0, 6).

2.e) We should check the bounds for x(3, 1) under the objective ranging part in the solution file of the question 1.c. The current unit cost for that flow is 11 and x(3, 1) is not in the basis. When I check the objective ranges, I see that the lower bound for x(3, 1) is 5.5. That is to say, we should decrease the unit cost of that route by at least 5.5 to reach a maximum of 5.5, to make x(3, 1) enter the basis.

3) Now, we assume that instead of investment costs, there are monthly rent costs for ships. Since the monthly rent is $350,000, we should divide it by 1000 to be consistent in the units (we are using a cost of $1000 per 1000 units). So, the monthly rent cost will be $350 for the routes where ships are feasible. If transportation by ships is not available, the rent cost of that route will be 0. I denoted this monthly rent cost with "f", and constructed the table below:

Plants\Customers	1	2	3	4	5	Dummy
1	350	350	0	350	350	0
2	350	350	350	350	350	0
3	0	0	350	350	350	0
4	350	350	350	350	0	0

Table f(i,j) monthly rent cost of ship

Next, we should construct the unit cost table of transportations. As stated in the question, we are considering the conditions of question 1-b which only allows transportation by ships if available. So, the unit cost table will be same as the unit cost table of transportation by ship, with unit costs of train written in the cells where ships are not available.

Plants\Customers	1	2	3	4	5	Dummy
1	5.5	6	8	3.5	4	0
2	3	4.5	4	6.5	6	0
3	11	6	3	4	4.5	0
4	5	4.5	7	3	7.5	0

Table c(i, j) transportation costs by ship

3.a) The most important point in this question is, we should add the monthly rent cost of ships if shipment on that route is used. To do that, we should introduce a binary variable y(i,j); which takes the value of 1 if the route from plant i to customer j is used, and the value of 0 otherwise. In addition to this, x(i,j) (the flow from plant i to customer j) should get the value 0 if that route is not used (if y(i,j) = 0).

We can construct an MIP with these conditions and above tables as follows:

c(i, j) = *Unit transportation cost between plant i to customer j by ship per month*

x(i, j) = *Flow between plant i customer j per month;*

i = 1, 2, 3, 4 & j =1, 2, 3, 4, 5, Dummy

f(i,j) = *Monthly rent cost of ship*

f(i,j) = $350 where ship is feasible, 0 otherwise.

a(i) = *Monthly production capacity for plant i (10^3 bottles)*

a(1) = 290, a(2) = 220, a(3) = 180, a(4) = 280

b(j) = *Monthly demand for customer j (10^3 bottles)*

b(1) = 180, b(2) = 200, b(3) = 160, b(4) = 140, b(5) = 250, b(Dummy) = 40

Min (z) = sum((i,j), c(i,j)*x(i,j)) + sum((i,j), y(i,j)*f(i,j))

Subject to; sum(j, x(i, j) = a(i)

sum(i, x(i, j) = b(j)

x(i,j) <= M*y(i,j)

y(i,j) <= X(i,j)

x(i, j) >= 0, integer y(i,j) binary

i = 1, 2, 3, 4 & j =1, 2, 3, 4, 5, Dummy

With this model, the optimal solution found by GAMS is as follows ;

x(1,3) = 40,	**x(1,5) = 250,**	**x(2,1) = 180,**
x(2,Dummy) = 40,	**x(3,2) = 60,**	**x(3,3) = 120,**
x(4,2) = 140,	**x(4,4) = 140,**	**x(i,j) = 0 otherwise.**

y(1,3) = y(1,5) = y(2,1) = y(2,Dummy) = y(3,2) = y(3,3) = y(4,2) = y(4,4) = 1,
y(i,j) = 0 otherwise.

Objective Value (z) is 5380.

3.b) In this question, we should add 2 constraints to the above model. Since only transportation by ship is available for both the routes 3-3 and 1-4, we can say that either the route 3-3 or 1-4 can be used. This restriction can be described as follows :

$$y(3,3) + y(1,4) = 1$$

For the second restriction, we can say that sum of the binary variables of the routes, where transportation by ships is feasible, cannot exceed 4. This restriction can be described as follows :

$$y(1,1) + y(1,2) + y(1,4) + y(1,5) + y(2,1) + y(2,2) + y(2,3) + y(2,4) + y(2,5) + y(3,3) + y(3,4) + y(3,5) + y(4,1) + y(4,2) + y(4,3) + y(4,4) <= 4$$

The final MIP model will be like :

Min (z) = sum((i,j), c(i,j)*x(i,j)) + sum((i,j), y(i,j)*f(i,j))

Subject to; sum(j, x(i, j) = a(i)

sum(i, x(i, j) = b(j)

x(i,j) <= M*y(i,j)

y(i,j) <= X(i,j)

y(3,3) + y(1,4) = 1

y(1,1) + y(1,2) + y(1,4) + y(1,5) + y(2,1) + y(2,2) + y(2,3) + y(2,4) + y(2,5) + y(3,3) + y(3,4) + y(3,5) + y(4,1) + y(4,2) + y(4,3) + y(4,4) <= 4

x(i, j) >= 0, integer y(i,j) binary

i = 1, 2, 3, 4 & j =1, 2, 3, 4, 5, Dummy

With the additional constraints, the optimum solution found by GAMS is as follows:

x(1,3) = 120,	**x(1,4) = 140,**	**x(1,Dummy) = 30,**
x(2,1) = 180,	**x(2,3) = 40,**	**x(3,2) = 170,**
x(3,Dummy) = 10,	**x(4,2) = 30,**	**x(4,5) = 250**
	x(i,j) = 0 otherwise.	

**y(1,3) = y(1,4) = y(1,Dummy) = y(2,1) = y(2,3) = y(3,2) = y(3,Dummy) = y(4,2) = y(4,5) = 1,
y(i,j) = 0 otherwise.**

Objective Value (z) is 6580.

In this solution, the routes where ship is used are: x(1,4), x(2,1), x(2,3), x(4,2).

the routes where train is used are: x(1,3), x(3,2), x(4,5).

Note that the routes 3-3 and 1-4 are not used at the same time. Instead, only the route 1-4 is used in the solution. Also, the number of routes where ship is used is 4, agreeing with the second restriction.