

Student Information

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Answer 1

a)

p	q	$\neg p$	$q \rightarrow \neg p$	$p \leftrightarrow q$	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

b)

p	q	r	$p \vee q$	$p \leftrightarrow r$	$q \rightarrow r$	$(p \vee q) \wedge (p \leftrightarrow r) \wedge (q \rightarrow r)$	$((p \vee q) \wedge (p \leftrightarrow r) \wedge (q \rightarrow r)) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Since at the end we reach all True's, then this means that we found tautology.

Answer 2

We need to prove $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$:

$$\neg p \rightarrow (q \rightarrow r)$$

$$\equiv p \vee (q \rightarrow r) \text{ logical eqv. implication elim.}$$

$$\equiv p \vee (\neg q \vee r) \text{ logical eqv. implication elim.}$$

$$\equiv \neg q \vee (p \vee r) \text{ assosiative law}$$

$$\equiv q \rightarrow (p \vee r) \text{ logical egv. implicaton introduction}$$

Answer 3

a) $\forall x L(x, \text{Burak})$

b) $\forall y L(\text{Hazal}, y)$

c) $\forall x \exists y L(x, y)$

d) $\neg \exists x (\forall y L(x,y))$

e)

f) $\neg \exists x (L(x,Burak) \wedge L(x,Mustafa))$

g) $\exists x \exists y (L(Ceren,x) \wedge L(Ceren,y) \wedge (x \neq y)) \wedge \forall z (L(Ceren,z) \rightarrow ((z=x) \vee (z=y)))$

h) $\forall x ((\exists y L(x,y)) \wedge \forall z (L(x,z) \rightarrow (z=y)))$

i) $\forall x \neg L(x,x)$

j) $\exists x \exists y (L(x,y) \wedge y \neg x) \wedge \forall z (L(x,z) \wedge (z=y) \wedge (z \neq x))$

Answer 4

1	$p \rightarrow (r \rightarrow q)$	<i>premise</i>
2	$q \rightarrow s$	<i>premise</i>
3	p	<i>premise</i>
4	$r \rightarrow q$	$\rightarrow e$ 1,3
5	q	<i>Assumption</i>
6	r	<i>Assumption</i>
7	q	$\rightarrow e$, 4
8	\perp	$\neg e$, 2, 5
9	$\neg r$	$\neg i$, 6 – 8
10	$\neg r \vee s$	$\vee i$, 9
11	$\neg q \rightarrow (s \vee \neg r)$	$\rightarrow i$, 5-10

Answer 5

1	$\forall x(p(x) \rightarrow q(x))$	<i>premise</i>
2	$\exists x \neg r(x)$	<i>premise</i>
3	$\exists x(p(x) \wedge q(x))$	<i>premise</i>
4	$\neg r(a)$	<i>Assumption</i>
5	c $p(c) \vee r(a)$	<i>Assumption</i>
6	$p(c)$	<i>Assumption</i>
7	$p(c)$	copy
8	$r(a)$	<i>Assumption</i>
9	\perp	$\neg e, 4, 8$
10	$p(c)$	$\perp e, 9$
11	$p(c)$	$\vee e, 5, 6 - 7, 8 - 10$
12	$\forall y(p(y) \rightarrow q(y))$	$\forall xe, 1$
13	$p(c) \rightarrow q(c)$	$\forall xe, 12$
14	$q(c)$	$\rightarrow e, 11, 13$
15	$\exists z q(z)$	$\exists zi, 5 - 14$
16	$\exists z q(z)$	$\exists ze, 4 - 15$