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Answer 1

According to Fermat's Little Theorem:
We begin by considering the first p - 1 positive multiples of a; that is, the integers a, 2a, 3a, ..., (p - 1)a. None of these numbers are congruent modulo p to any other, nor is any congruent to zero. Indeed if it happened that
$r * a \equiv s * a (modp), 1 \leq r < s \leq p - 1$ then, r ≡ s (mod p), which is impossible because r and s are both between 1 and p - 1. Hence, the previous set of integers must be congruent modulo p to 1, 2, ... , p - 1. Multiplying these together gives us
$a * 2a * 3a * ... * (p - 1) * a \equiv 1 * 2 * 3 * ... * (p - 1)(modp)$
meaning
$a^{p-1} * (p - 1)! \equiv (p - 1)!(modp)$.
Cancelling (p - 1)! from both sides we obtain
$a^{p-1} \equiv 1(modp)$,(end of the proof of theorem).
When we put x instead of a, we obtain that y=(p-1) and also y|(p-1).

Answer 2

For this statement to be true, right-hand side expression must be equal one of the divisors of 169. Divisors of 169 are 13,1 and 169.
Now, we should check equality:
1.$2n^2 + 10n - 7 = 13 \rightarrow 2n^2 + 10n - 20 = 0 \rightarrow n^2 + 5n - 10 = 0$
2.$2n^2 + 10n - 7 = 1 \rightarrow 2n^2 + 10n - 8 = 0 \rightarrow n^2 + 5n - 4 = 0$
3.$2n^2 + 10n - 7 = 169 \rightarrow 2n^2 + 10n - 176 = 0 \rightarrow n^2 + 5n - 88 = 0$
None of this equations has a root that is positive integer, so that we can say that any positive integer for n for the equation cannot divide 169.

Answer 3

Let's say a ≡ b(mod m*n), then $a = k * m * n + b$ for some k.
When we write $a - b = k * m * n$, then we can obtain that $a - b \equiv 0mod(m)$ since m is the divisor of the (a-b).
Also, we can obtain $a - b \equiv 0mod(n)$ with the same idea.
All in all we prove that $a \equiv bmod(m)$ and $a \equiv bmod(n)$.

Answer 4

Let's say j.(j+1).(j+2)....(j+k-1) is the function that is P(j).
Base Step:
Take n=1 $\rightarrow$ $\sum_{j=1}^{1} P(j) = 1.2.3.4...k = (?)$
$\dfrac{n.(n+1)..(n+k)}{k+1} = \dfrac{1.2..(1+k)}{k+1}$ (Cancel out (k+1)'s)
= 1.2.3.4...k They're equal $\checkmark$
Inductive Step:

Let's say $\sum_{j=1}^{p} P(j) = \dfrac{p.(p+1)..(p+k)}{k+1}$ *is true*. We need to prove this for p+1:
$\sum_{j=1}^{p+1} P(j) = \sum_{j=1}^{p} P(j) + (p+1).(p+2).(p+3)....(p+k)$

$= \dfrac{p.(p+1)..(p+k)}{k+1} + (p+1).(p+2).(p+3)....(p+k)$ From the assumption above.
When we do the rational addition, we obtain $\dfrac{(p+k+1).(p+1)..(p+k)}{k+1}$ and this equals:
$\dfrac{(p+1).(p+2)..(p+k).(p+k+1)}{k+1}$ $\checkmark$
End of the mathematical induction proof.

Answer 5

Base Step:
n=3 : $H_3 = 5H_2 + 5H_1 + 63H_0$
$= 5.5 + 5.3 + 63.1 = 103 < 7^3 \checkmark$

Inductive Step:
Let's say $H_3, H_4, H_5....., H_k$ is true, then we need to prove H_{k+1} is true :

$$\begin{array}{llll} H_{k+1} = & 5H_k & + 5H_{k-1} & + 63H_{k-2} \\ & \downarrow & \downarrow & \downarrow \\ & \leq 5.7^k & \leq 5.7^{k-1} & \leq 63.7^{k-2} \text{ (From the assumption above)} \\ & \downarrow & \downarrow & \downarrow \\ & \leq 35.7^{k-1}+ & \leq 5.7^{k-1}+ & \leq 9.7^{k-1} \end{array}$$

So,
$H_{k+1} \leq 49.7^{k-1}$
$H_{k+1} \leq 7^{k+1} \checkmark$