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Answer 1

a)

Let P_n be the string with n 1's.
$P_0 \rightarrow 000000000 \rightarrow 1way$
$P_1 \rightarrow \underline{10}\ 0000000 \rightarrow \dfrac{8!}{7!} = 8way$
$P_2 \rightarrow \underline{10}\ \underline{10}\ 00000 \rightarrow \dfrac{7!}{5!.2!} = 21way$
$P_3 \rightarrow \underline{10}\ \underline{10}\ \underline{10}\ 000 \rightarrow \dfrac{6!}{3!.3!} = 20ways$
$P_4 \rightarrow \underline{10}\ \underline{10}\ \underline{10}\ \ \underline{10}\ 0 \rightarrow \dfrac{5!}{4!} = 5ways$
There cannot be more than 4 1's.Hence the result is 1+8+21+20+5=55.

b)

Let B_n be the string with n 1's.
$B_8 \rightarrow 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0 \rightarrow \dfrac{10!}{8!.2!} = 45$
$B_9 \rightarrow 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0 \rightarrow \dfrac{10!}{9!.1!} = 10$
$B_10 \rightarrow 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \rightarrow \dfrac{10!}{10!} = 1$
Hence the result is 45 + 10 +1 =56.

c)

From the Theorem 1 from the textbook(8.6) :
$n^m - C(n,1)(n-1)^m + ... + (-1)^{n-1}C(n,n-1).1^m$
$=3^4 - C(3,1)2^4 + C(3,2)1^3$=36.

d)

Answer 2

a)

Let's say a subset A of a_n that does not contain consecutive numbers.We can divide A into two groups.
Group 1 contains the element n,Group 2 does not contain n.
We first get an expression for the number of Group 1,where n≥2.Such a subset cannot contain n-1 due to constraint.So, any Group 1 subset can be obtained by adding n to a non-consecutive subset of a_{n-2}.
Now we obtain an expression for the number of subset of Group 2 of a_n.Such a subset is a subset of a_{n-1}.
A non-consecutive subset of a_n is either of Group 1 or of Group 2.So the number of appropriate subsets of a_n is

$$a_n = a_{n-2} + a_{n-1} \text{ for } n \geq 2.$$

The initial conditions are $a_1 = 2$,because all of the subsets of this are non-consecutive and it is 2^1 from 2^n; and $a_2 = 3$ because it has just 1 consecutive subset which is itself so it is $2^2 - 1$ from $2^n - 1$.

b)

$a_1 = 2$
$a_2 = 3$
$a_n = a_{n-1} + a_{n-2}$
Let f(x) = $\sum_{n=0}^{\infty} a_n x^n$

$$\sum_{n=2}^{\infty} a_n x^n = \sum_{n=2}^{\infty} (a_{n-1} + a_{n-2}) x^n$$

$$\sum_{n=2}^{\infty} a_n x^n = \sum_{n=2}^{\infty} a_{n-1} x^n + \sum_{n=2}^{\infty} a_{n-2} x^n$$

$$\underbrace{\sum_{n=3}^{\infty} a_n x^n}_{f(x) - a_1 x - a_2 x^2} = \underbrace{\sum_{n=3}^{\infty} a_{n-1} x^n}_{x\sum_{n=3}^{\infty} a_{n-1} x^{n-1}} + \underbrace{\sum_{n=3}^{\infty} a_{n-2} x^n}_{x^2 \sum_{n=3}^{\infty} a_{n-2} x^{n-2}}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$f(x) - 2x - 3x^2 = x(f(x) - 2x) + x^2 f(x)$$
$$= -x^2 - 2x = f(x)(x^2 + x - 1)$$
$$= f(x) = \frac{-x^2 - 2x}{(x^2 + x - 1)}$$

Answer 3

Theorem 1 from the textbook(8.2) can be used to solve this problem.The characteristic equation of the recurrence relation is $r^3 = 4r^2 + r - 4$.It's root's are r=4,r=1,r=-1.Hence, the sequence a_n is a solution to the recurrence relation if and only if

$a_n = \alpha_1 4^n + \alpha_2 1^n + \alpha_3(-1)^n$ for some constants $\alpha_1, \alpha_2, \alpha_3$.

From the initial conditions,it follows that

$$a_0 = 4 = \alpha_1 + \alpha_2 + \alpha_3$$
$$a_1 = 8 = 4\alpha_1 + \alpha_2 - \alpha_3$$
$$a_3 = 34 = 16\alpha_1 + \alpha_2 + \alpha_3$$

Solving these equations shows that $\alpha_1 = 2, \alpha_2 = 1$ and $\alpha_3 = 1$.Hence we conculude that

$$a_n = 2.4^n + 1^n + (-1)^n$$

Answer 4

Definition:An equivalence relation on a set X is a binary relation on X which is reflexive, symmetric and transitive.

We check whether R is equivalence relation as follows:

(a) (*Reflexivity*): (x_1, y_1)R(x_1, y_1) hence $3x_1 - 2y_1 = 3x_1 - 2y_1$

(a) (*Symmetry*): If $3x_1 - 2y_1 = 3x_2 - 2y_2$ then $3x_2 - 2y_2 = 3x_1 - 2y_1$.Hence (x_2, y_2)R(x_1, y_1)

(a) (*Transitivity*):If $(x_1, y_1)R(x_2, y_2)$ and $(x_2, y_2)R(x_3, y_3)$ then $3x_1 - 2y_1 = 3x_2 - 2y_2$ and $3x_2 - 2y_2 = 3x_3 - 2y_3$.Since both of them equal $3x_2 - 2y_2$, $3x_1 - 2y_1 = 3x_3 - 2y_3$.Hence $(x_1, y_1)R(x_3, y_3)$.

Consequently R is equivalence.