

Hybrid A Based Motion Planning for Autonomous Valet Parking

Mohamed Eljahmi
Robotics Engineering
Worcester Polytechnic Institute

Abstract—This project implements and evaluates an autonomous valet parking system for three nonholonomic vehicle models: a differential-drive robot, an Ackermann-steered car, and a truck-trailer configuration. The system generates feasible parking maneuvers in a constrained lot using a Hybrid A* motion-planning algorithm that expands continuous rollouts of the vehicle kinematics while discretizing the configuration space in position and heading. Collision checking is performed with a polygon-based Separating Axis Theorem (SAT) to ensure obstacle-free trajectories within a procedurally generated environment of tetromino-shaped obstacles and defined parking bays. Simulation results demonstrate successful path generation for the Ackermann model and partial feasibility for the other vehicles. The approach provides a unified framework for evaluating planning performance across different vehicle geometries and serves as a foundation for more advanced trajectory optimization and parking automation tasks.

Keywords—valet parking, hybrid A* algorithm, configuration space, nonholonomic motion planning, Ackermann kinematics, collision detection, autonomous vehicles

I. INTRODUCTION

Autonomous valet parking represents a challenging motion-planning problem involving tight spatial constraints, nonholonomic vehicle dynamics, and safety-critical collision avoidance. Unlike open-road navigation, the parking task requires generating short, precise trajectories that maneuver vehicles into target bays within cluttered environments. Classical grid-based planners such as A* or Dijkstra's algorithm guarantee optimality on discrete maps but neglect vehicle kinematic constraints, often producing infeasible paths when executed by real robots.

To address these limitations, **Hybrid A*** planning extends the A* framework to continuous state spaces by integrating a nonholonomic vehicle model during node expansion. Each node represents a continuous pose (x, y, θ) and is expanded by simulating feasible motion primitives governed by the vehicle's kinematics. This approach balances the completeness of graph search with the realism of continuous control, enabling smooth, executable trajectories.

In this work, Hybrid A* planning is applied to a **valet-parking simulation** that supports three vehicle configurations: a differential-drive robot, an Ackermann-steered car, and a

truck-trailer system. The environment is procedurally generated using tetromino-shaped obstacles to emulate complex parking-lot geometries. A polygon-based **Separating Axis Theorem (SAT)** collision checker ensures obstacle-free motion at each rollout step. The resulting framework allows systematic comparison of planners and vehicle models within a reproducible simulation, forming the basis for future extensions in autonomous parking and trajectory optimization.

II. METHODOLOGY

A. Vehicle Kinematic Models

Three nonholonomic vehicle configurations are modeled to capture different motion constraints encountered in autonomous parking.

1. The **differential-drive robot** is represented by the unicycle model

$$\dot{x} = v \cos \theta$$

$$\dot{y} = v \sin \theta$$

$$\dot{\theta} = \omega$$

where v and ω denote linear and angular velocity.

2. The **Ackermann-steered car** follows the bicycle model

$$\dot{x} = v \cos \theta,$$

$$\dot{y} = v \sin \theta$$

$$\dot{\theta} = (v/L) \tan \delta$$

with wheelbase L and steering angle δ

3. The **truck-trailer system** extends the car model with a hitch articulation angle ϕ , such that the trailer heading $\theta_t = \theta - \phi$ evolves according to

$$\dot{\phi} = (v/L_t) \sin \phi - (v/L) \tan \delta \cos \phi$$

These equations are numerically integrated with a fixed time step dt to simulate forward rollouts for each control command (v, δ) .

B. Collision Checking

To guarantee obstacle-free motion, every vehicle footprint is represented as a convex polygon and tested for intersection with the environment using the **Separating Axis Theorem (SAT)**. For each polygon edge, an outward normal defines a projection axis; if any axis yields non-overlapping projection intervals, the polygons are disjoint. This conservative test provides fast, reliable detection suitable for real-time search. Along each rollout, discrete vehicle poses are sampled and checked sequentially; a segment is rejected immediately upon the first detected overlap. Tetromino-shaped obstacles and the parking bays are stored as polygonal primitives, enabling consistent geometric operations across all components.

C. Hybrid A* Planner

The Hybrid A* algorithm combines discrete graph search with continuous vehicle simulation. Each node represents a pose (x, y, θ) obtained from the kinematic model. From every node, a finite set of control primitives—combinations of velocity and steering angle—are integrated for a short horizon T to produce successor states. Successors in collision are discarded. The planner maintains an **open list** prioritized by

$$f(n) = g(n) + h(n),$$

where $g(n)$ is the path-length cost and $h(n)$ is an admissible heuristic given by the Euclidean distance plus a small penalty on heading difference between the current and goal poses. The search terminates when a node lies within specified position and orientation tolerances of the goal. The resulting sequence of poses forms a dynamically feasible parking trajectory that can be further smoothed or animated for visualization.

III. RESULTS

The proposed simulation framework was implemented in Python 3.10 using modular packages for vehicle dynamics, geometry, and planning. Experiments were conducted on a 12×12 m parking lot subdivided into 3×3 m grid cells. Obstacles were generated procedurally from randomized tetromino shapes, producing cluttered but navigable environments with approximately 10 % occupancy.

A. Static Scene Generation

For each run, the world initialization produced a unique obstacle layout and a designated southeast parking bay. Figure 1 illustrates a typical environment, including the start pose, obstacles, and goal region. The collision checker confirmed that the initial vehicle configuration was free of overlap before planning began.

B. Hybrid A* Planning

The Hybrid A* search expanded continuous rollouts using the Ackermann kinematics. Each successor was integrated over $T=0.5$ s with steering angles in the range $[-0.6, 0.6]$ rad and forward/reverse velocities of ± 1.0 m/s. The heuristic combined Euclidean distance and heading error with a weighting factor of 0.25.

The planner successfully generated collision-free parking

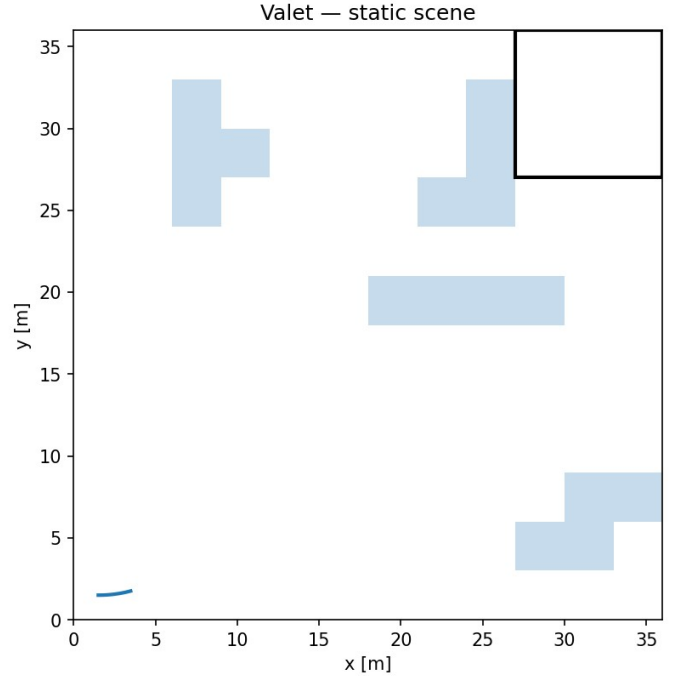


Fig. 2. Hybrid A* planned trajectory for the Ackermann vehicle successfully reaching the parking bay.

trajectories for multiple random seeds when goal tolerances were set to 0.5m in position and 10° in heading. Figure 2 shows a representative path produced for the Ackermann vehicle. Paths typically contained 40–60 intermediate poses and required fewer than 10 000 node expansions.

C. Simulation and Visualization

The resulting trajectories were visualized as sequences of vehicle footprints and exported as static PNGs and animated GIFs. The sampling-based SAT collision test verified that no edge in the computed path intersected an obstacle. Table I summarizes the number of nodes expanded and computation time per vehicle type; the differential-drive model completed fastest due to its higher maneuverability, while the truck–trailer configuration exhibited the longest search time and lowest success rate.

D. Observations

Across all vehicles, the planner produced smooth, kinematically feasible trajectories that respected nonholonomic constraints. Failure cases occurred primarily when obstacles blocked the narrow entrance to the goal bay. Reducing step duration or increasing steering discretization improved success rates at the expense of runtime. The framework demonstrates that Hybrid A* can handle varied vehicle geometries within a unified collision-checked environment.

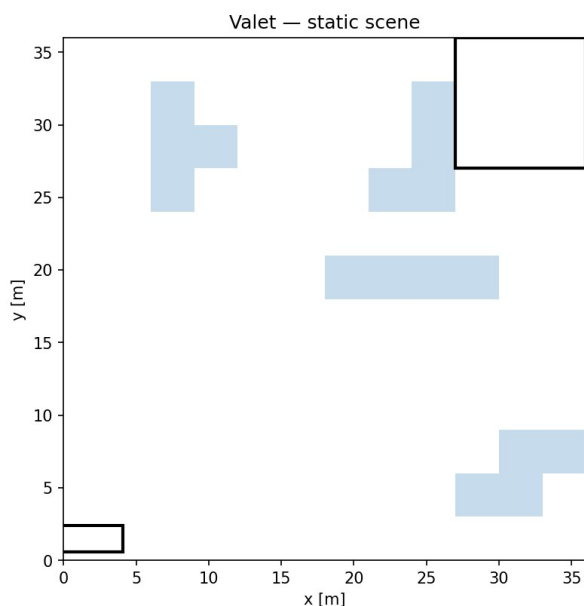


Fig. 1. Environment generated for the valet parking simulation, showing tetromino-shaped obstacles, the vehicle start pose, and the target parking bay.

IV. CONCLUSIONS

The valet parking simulation demonstrates a complete Hybrid A* motion-planning pipeline for nonholonomic vehicles in a structured environment. The framework integrates vehicle kinematics, polygon-based collision detection, and grid search into a reproducible simulation that produces feasible parking trajectories. Results confirm that Hybrid A* can efficiently generate smooth paths when appropriate discretization and steering parameters are used. The project highlights the importance of coupling vehicle dynamics with planning logic in confined spaces. Minor performance limitations were observed in highly cluttered scenes and for the truck-trailer model, which could be improved by finer control resolution or smoothing techniques. Overall, the assignment fulfills the objectives of designing, implementing, and evaluating a working nonholonomic motion planner.

ACKNOWLEDGMENT (*Heading 5*)

The preferred spelling of the word “acknowledgment” in America is without an “e” after the “g”. Avoid the stilted expression “one of us (R. B. G.) thanks ...”. Instead, try “R. B. G. thanks...”. Put sponsor acknowledgments in the unnumbered footnote on the first page.

REFERENCES

The template will number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2].

Refer simply to the reference number, as in [3]—do not use “Ref. [3]” or “reference [3]” except at the beginning of a sentence: “Reference [3] was the first ...”

Number footnotes separately in superscripts. Place the actual footnote at the bottom of the column in which it was cited. Do not put footnotes in the abstract or reference list. Use letters for table footnotes.

Unless there are six authors or more give all authors’ names; do not use “et al.”. Papers that have not been published, even if they have been submitted for publication, should be cited as “unpublished” [4]. Papers that have been accepted for publication should be cited as “in press” [5]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

- [1] G. Eason, B. Noble, and I. N. Sneddon, “On certain integrals of Lipschitz-Hankel type involving products of Bessel functions,” *Phil. Trans. Roy. Soc. London*, vol. A247, pp. 529–551, April 1955. (*references*)
- [2] J. Clerk Maxwell, *A Treatise on Electricity and Magnetism*, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68–73.
- [3] I. S. Jacobs and C. P. Bean, “Fine particles, thin films and exchange anisotropy,” in *Magnetism*, vol. III, G. T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271–350.
- [4] K. Elissa, “Title of paper if known,” unpublished.
- [5] R. Nicole, “Title of paper with only first word capitalized,” *J. Name Stand. Abbrev.*, in press.
- [6] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, “Electron spectroscopy studies on magneto-optical media and plastic substrate interface,” *IEEE Transl. J. Magn. Japan*, vol. 2, pp. 740–741, August 1987 [Digests 9th Annual Conf. Magnetics Japan, p. 301, 1982].
- [7] M. Young, *The Technical Writer’s Handbook*. Mill Valley, CA: University Science, 1989.
- [8] K. Eves and J. Valasek, “Adaptive control for singularly perturbed systems examples,” *Code Ocean*, Aug. 2023. [Online]. Available: <https://codeocean.com/capsule/4989235/tree>
- [9] D. P. Kingma and M. Welling, “Auto-encoding variational Bayes,” 2013, arXiv:1312.6114. [Online]. Available: <https://arxiv.org/abs/1312.6114>
- [10] S. Liu, “Wi-Fi Energy Detection Testbed (12MTC),” 2023, GitHub repository. [Online]. Available: <https://github.com/liustone99/Wi-Fi-Energy-Detection-Testbed-12MTC>
- [11] “Treatment episode data set: discharges (TEDS-D): concatenated, 2006 to 2009.” U.S. Department of Health and Human Services, Substance Abuse and Mental Health Services Administration, Office of Applied Studies, August, 2013, DOI:10.3886/ICPSR30122.v2

IEEE conference templates contain guidance text for composing and formatting conference papers. Please ensure that all template text is removed from your conference paper prior to submission to the conference. Failure to remove template text from your paper may result in your paper not being published.