M2 E3A-SAAS

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FLIGHT PLANNING
TD 5 : Trim trajectories

Exercice 1 : TRIM trajectories

Prove using the kinematic model of translation and orientation that equilibrium paths or trim trajectories are

represented by helices. The translational kinematic model is given by:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = R \begin{pmatrix} u \\ v \\ w \end{pmatrix} \text{ where } R = \begin{pmatrix} c\theta c\psi & s\theta c\psi s\phi - s\psi c\phi & s\theta c\psi c\phi + s\psi s\phi \\ c\theta s\psi & s\theta s\psi s\phi + c\psi c\phi & s\theta s\psi c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\psi c\phi \end{pmatrix}$$

and the rotational kinematic model is :

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = J \begin{pmatrix} p \\ q \\ r \end{pmatrix} \text{ where } J = \begin{pmatrix} 1 & s\phi \tan\theta & c\phi \tan\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi / c\theta & c\theta / c\phi \end{pmatrix}$$

or $$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = J^{-1} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & s\phi c\theta \\ 0 & -s\phi & c\theta c\phi \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

Exercice 2 : Quad-rotor trim trajectories characterization

The dynamic model of a quad-rotor is given by the following equations:

$$\dot{X} = f(X, Y) = \begin{pmatrix} \dot{\phi} \\ a_1 \dot{\theta}\dot{\psi} + a_2 \dot{\theta}\Omega_r + b_1 U_1 \\ \dot{\theta} \\ a_3 \dot{\phi}\dot{\psi} + a_4 \dot{\phi}\Omega_r + b_2 U_2 \\ \dot{\psi} \\ a_5 \dot{\theta}\dot{\phi} + b_3 U_3 \\ \dot{x} \\ \frac{\sin\psi \sin\phi + \cos\psi \sin\theta \cos\phi}{m} U_4 \\ \dot{y} \\ -\frac{-\cos\psi \sin\phi + \sin\psi \sin\theta \cos\phi}{m} U_4 \\ \dot{z} \\ g - \frac{\cos\psi \cos\phi}{m} U_4 \end{pmatrix}$$

with $X = \left(\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, x, \dot{x}, y, \dot{y}, z, \dot{z}\right)^T$ and

$$\begin{array}{rcl} U_1 & = & b\left(-\Omega_2^2 + \Omega_4^2\right) \\ U_2 & = & b\left(\Omega_1^2 - \Omega_3^2\right) \\ U_3 & = & d\left(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2\right) \\ U_4 & = & b\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right) \\ \Omega_r & = & -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \end{array}$$

$$\begin{array}{rcl} a_1 & = & \frac{I_{yy} - I_{zz}}{I_{xx}}, a_2 = \frac{J_r}{I_{xx}}, a_3 = \frac{I_{zz} - I_{xx}}{I_{yy}}, a_4 = \frac{J_r}{I_{yy},}, a_5 = \frac{I_{xx} - I_{yy}}{I_{zz}} \\ b_1 & = & \frac{l}{I_{xx}}, b_2 = \frac{l}{I_{yy}}, b_3 = \frac{l}{I_{zz}} \end{array}$$

Characterize the trim trajectories taking into account effective limitations on thrust and angles.