Université d'Évry-Val-d'Essonne, UFR Sciences et Technologies, Master2 E3A: Smart Aerospace & Autonomous Systems

Aerial Vehicles Lab

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Problem: Quadrotor modeling and control

assumptions

- the quadrotor is a rigid body
- the propellers of the motors are also rigid
- the quadrotor is symmetrical along x and y axis in the body frame. The center of the body coincides with its center of gravity. There fore $I_{yz} = I_{xy} = I_{xz} = 0$
- the motor inertia is negligible.

problem formulation

The relation between the linear velocity in the inertial frame W and the linear velocity in the body frame B or the translational kinematics can be given by:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \cos\psi\sin\theta\sin\phi - \cos\phi\sin\psi & \cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\theta\sin\phi\sin\psi & -\cos\psi\sin\phi + \cos\phi\sin\theta\sin\psi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

While the rotational kinematics is the relation between the angular rates in body and fixed frames and given by:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\text{Sin}[\theta] \\ 0 & \text{Cos}[\phi] & \text{Sin}[\phi]\text{Cos}[\theta] \\ 0 & -\text{Sin}[\phi] & \text{Cos}[\phi].\text{Cos}[\theta] \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

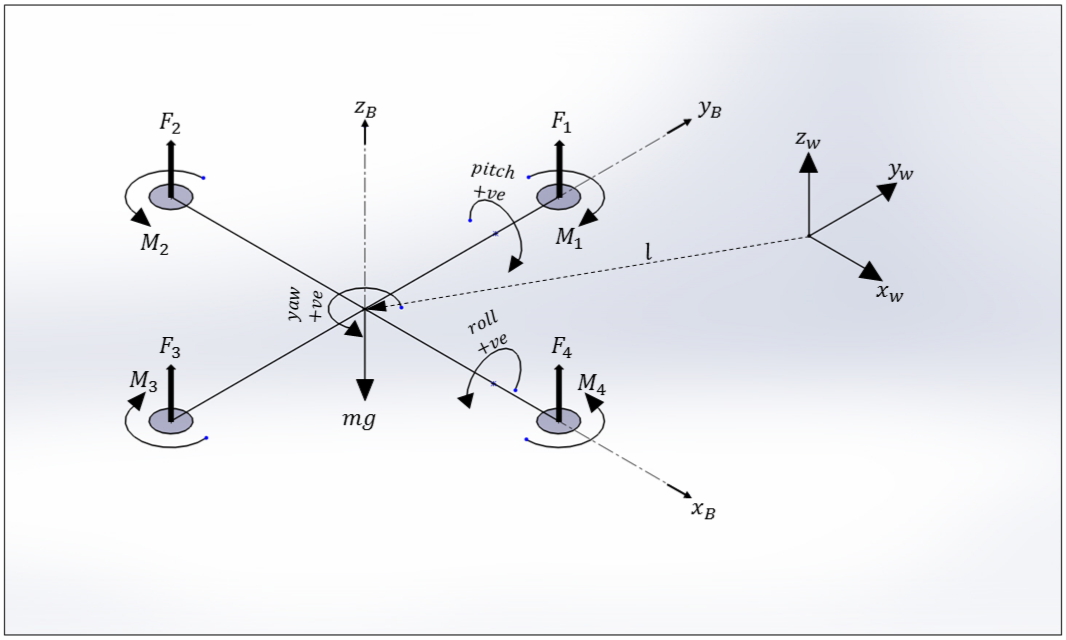

Figure 1: The Cartesian coordinate system of a quadrotor

In classical mechanics, the Newton Euler equations show the translational and rotational dynamics of a rigid body. These laws relate the motion of the center of gravity of a rigid body with the sum of external forces and moments applied on it. Without loss of generality, the origin of the body frame coincides with the center of gravity. Supposing there is a 6 DOF(Degrees of Freedom) rigid body which has a mass m and an inertia matrix I about the center of gravity. The linear velocity of the center of gravity is V_b, and the body angular velocity is Ω_b in the body frame. The external force and the moment are F_b and M_b in the body frame. Therefore, the relation between the velocities and the external forces and moments in the body frame is given by:

$$\begin{bmatrix} mI & 0_{3*3} \\ 0_{3*3} & I \end{bmatrix} \begin{bmatrix} \dot{V}_b \\ \dot{\Omega}_b \end{bmatrix} + \begin{bmatrix} \Omega_b \times (mV_b) \\ \Omega_b \times (I\Omega_b) \end{bmatrix} = \begin{bmatrix} F_b \\ M_b \end{bmatrix}$$

Questions

1. show that from last equation we have

$$\begin{bmatrix} \ddot{x}_b \\ \ddot{y}_b \\ \ddot{z}_b \end{bmatrix} = \begin{bmatrix} r\dot{y}_b - q\dot{z}_b \\ p\dot{z}_b - r\dot{x}_b \\ q\dot{x}_b - p\dot{y}_b \end{bmatrix} + \frac{1}{m} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \; and \; \begin{bmatrix} I_{xx}\dot{p} \\ I_{yy}\dot{q} \\ I_{zz}\dot{r} \end{bmatrix} = \begin{bmatrix} (I_{yy} - I_{zz})qr \\ (I_{zz} - I_{xx})pr \\ (I_{xx} - I_{yy})pq \end{bmatrix} + \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

2. Give the gravity force in the body-fixed frame and in the inertial axes.
3. Detail the pitching, rolling and yawing moments.
4. using small disturbance theory deduce a linear model.
5. design a control law to stabilize and control the deduced model.