# M2 E3A SAAS -
# Flight modeling and control
# Université d'Évry - Paris Saclay

## Objectives

The objectives of this lab are

- Familiarizaion with modeling and simulation with Matlab/Simulink
- Linear control systems : linearization and stabilisation
- Design an autopilot for a drone using the "exact linearization" approach and other nonlinear approaches.
- Stabilization and trajectory tracking control.

Consider a drone (rocket) having airships at angles in two perpendicular directions. The nonlinear model of the drone is expressed as a state representation form with 12 variables that correspond to positions, velocities, angular positions and angular velocities, which will be controlled via 4 control inputs. The 4 controls are respectively : the thrust $T$, the 2 angles of rotation $\alpha$ and $\beta$, the roll control input$M_{\psi}$.

By choosing the following state variables :

$$x = \begin{pmatrix} x \\ u \\ y \\ v \\ z \\ \omega \\ \phi \\ p \\ \theta \\ q \\ \psi \\ r \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{pmatrix} \Rightarrow \dot{x} = \begin{pmatrix} \dot{x} \\ \dot{u} \\ \dot{y} \\ \dot{v} \\ \dot{z} \\ \dot{\omega} \\ \dot{\phi} \\ \dot{p} \\ \dot{\theta} \\ \dot{q} \\ \dot{\psi} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \\ \dot{x}_{11} \\ \dot{x}_{12} \end{pmatrix} = \begin{pmatrix} x_2 \\ \dot{x}_2 \\ x_4 \\ \dot{x}_4 \\ x_6 \\ \dot{x}_6 \\ x_8 \\ \dot{x}_8 \\ x_{10} \\ \dot{x}_{10} \\ x_{12} \\ \dot{x}_{12} \end{pmatrix}$$

Then we obtain the state representation of our system, we replace these state variables in the equations of motion we get :

$$
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = x_{12}x_4 - x_{10}x_6 - g\sin x_9 - \frac{T}{m}\cos\beta\sin\alpha \\
\dot{x}_3 = x_4 \\
\dot{x}_4 = x_8x_6 - x_{12}x_2 + g\sin x_7\cos x_9 + \frac{T}{m}\sin\beta \\
\dot{x}_5 = x_6 \\
\dot{x}_6 = x_{10}x_2 - x_8x_4 + g\cos x_7\cos x_9 - \frac{T}{m}\cos\beta\cos\alpha \\
\dot{x}_7 = x_8 \\
\dot{x}_8 = \frac{I_{yy}-I_{zz}}{I_{xx}}x_{12}x_{10} + \frac{1}{I_{xx}}(-T\sin\beta(AB\cos\alpha + GA)) \\
\dot{x}_9 = x_{10} \\
\dot{x}_{10} = \frac{I_{zz}-I_{xx}}{I_{yy}}x_{12}x_8 + \frac{1}{I_{yy}}(-GA.T\cos\beta\sin\alpha) \\
\dot{x}_{11} = x_{12} \\
\dot{x}_{12} = \frac{I_{xx}-I_{yy}}{I_{zz}}x_8x_{10} + \frac{1}{I_{zz}}(AB.T\sin\beta\sin\alpha) + \frac{1}{I_{zz}}M_\psi
\end{cases} \tag{1}
$$

From the equations(1), we define the control input vector $u$ by :

$$
\begin{cases}
u_1 = -\frac{T}{m}\cos\beta\sin\alpha \\
u_2 = \frac{T}{m}\sin\beta \\
u_3 = -\frac{T}{m}\cos\beta\cos\alpha \\
u_4 = \frac{1}{I_{zz}}(AB.T\sin\beta\sin\alpha) \\
u_5 = \frac{1}{I_{zz}}M_\psi
\end{cases} \tag{2}
$$

For small rotation angles :

$$-10^\circ \leq \alpha \leq 10^\circ \tag{3}$$

et

$$-10^\circ \leq \beta \leq 10^\circ \tag{4}$$

it is possible to use the approximations :

$$
\begin{aligned}
&\cos\alpha = \cos\beta = 1 \\
&\sin\alpha = \alpha \\
&\sin\beta = \beta
\end{aligned}
$$

The, the control inputs $u_i, i = 1, ..., 5$ are simplified as follows :

$$
\begin{cases}
u_1 = -\frac{T}{m}\alpha \\
u_2 = \frac{T}{m}\beta \\
u_3 = -\frac{T}{m} \\
u_4 = \frac{T.AB}{I_{zz}}\alpha\beta \\
u_5 = \frac{1}{I_{zz}}M_\psi
\end{cases} \tag{5}
$$

which gives the following state space representation :

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_{12}x_4 - x_{10}x_6 - g\sin x_9 + u_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = x_8x_6 - x_{12}x_2 + g\sin x_7 \cos x_9 + u_2 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = x_{10}x_2 - x_8x_4 + g\cos x_7 \cos x_9 + u_3 \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = a_1x_{12}x_{10} + b_1u_2 \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = a_2x_{12}x_8 + b_2u_1 \\ \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = a_3x_8x_{10} + u_4 + u_5 \end{cases} \tag{6}$$

with

$$\begin{cases} a_1 = \frac{I_{yy}-I_{zz}}{I_{xx}} \\ \\ a_2 = \frac{I_{zz}-I_{xx}}{I_{yy}} \\ \\ a_3 = \frac{I_{xx}-I_{yy}}{I_{zz}} \end{cases} et \begin{cases} b_1 = \frac{m}{I_{xx}}(AB + GA) \\ \\ b_2 = -\frac{m}{I_{yy}}GA \end{cases} \tag{7}$$

1. Analyze the provided Simulink model
2. Compute the equilibrium points of the system and show that the origin is an equilibrium point.

# 1 Linear longitudinal and lateral control

After decoupling (under some conditions which will be explained), the decoupled, lateral and longitudinal motionss of the drone are given by the dynamical models :

$$Long \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -g\sin(x_9) + u_1 \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = b_2u_1 \end{cases}$$

where $x_1$ et $x_2$ represent the position and the velocity with respect to the $x-$axis and $x_9$ and $x_10$ the roll angle and its rate. $u_1$ is the trust generated by the wings with respect to $x-$axis.

$$Lat \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = g\sin(x_7) + u_2 \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = b_1u_2 \end{cases}$$

where $x_3$ and $x_4$ rrepresent the position and the velocity with respect to the $y-$axis and $x_7$ and $x_8$ the pitch angle and its rate. $u_2$ is the trust generated by the wings with respect to $y-$axis.

1. Compute the linearized models around the origin of each model ?
2. For each motion, compute a linear controller by state feedback control and pole placement technique.
3. Apply the proposed control techniques directly on the nonlinear model. Discuss the results.
4. Modify the control input in order to track :
   (a) a constant trajectory
   (b) a time varying trajectory

# 2 Nonlinear longitudinal and lateral control

1. For each motion, compute anonlinear controller by state feedback control with exact linearization approach.
2. Apply the proposed control techniques directly on the nonlinear model. Discuss the results.
3. Modify the control input in order to track :
   (a) a constant trajectory
   (b) a time varying trajectory
4. Compare the results with the results obtained by the linear controller

# 3 Nonlinear Vertical control

The decoupled vertical dynamics is described by

$$\begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = x_2 x_{10} - x_4 x_8 + g\cos(x_7)\cos(x_9) + u_3 \\ \dot{x}_{11} = x_8 \\ \dot{x}_{12} = a_3 x_8 x_{10} + \frac{1}{I_{zz}} M_3 + u_4 \end{cases} \tag{8}$$

This dynamic can be decoupled into two sub-systems with dedicated controllers, the vertical displacement of the drone along the $z-$axis and its rotation around this axis, which is why the method of exact linearization is considered more than the other calculation methods.

Consider the two controllers :

$$\begin{array}{l} u_3 = -x_2 x_{10} + x_4 x_8 - g\cos(x_7)\cos(x_9) - k_1 x_5 - k_2 x_6 \\ u_4 = -a_3 x_8 x_{10} - \frac{1}{I_{zz}} M_3 - k_3 x_{11} - k_4 x_{12} \end{array} \tag{9}$$

1. Prove that the following control inputs allows to obtain a linear closed-loop system and compute the values of $k_1$, $k_2$, $k_3$ and $k_4$ allonwing to stabilize asymptotically the system.
2. Simulate the vertical model with a 0-reference $z$ position and a non-zero constant and positive position (Hovering flight).

# 4 Autonomous control of the complete system

In this section, the previous controllers will be applied to the complete model.

1. Simulate the closed loop system with a setpoint in zero position.
2. Simulate the closed loop system with a reference trajectory :
   (a) Vertical and hovering flights at $1m$
   (b) At $10s$ longitudinal flight $1m$
   (c) At $20s$ latéral flight $1m$
   (d) At $40s$ landing.
3. Simulate the closed loop system with :
   (a) Vertical and hovering flights at $1m$
   (b) Circle flight in the $x - y$ plan