Target group: M2SAAS - Universite Evry Paris Saclay

Aerial Robot TP2 Lab Report

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Quadrotor modeling and control

Assumptions

- The quadrotor is a rigid body
- The propellers of the motors are also rigid
- The quadrotor is symmetrical along x and y axis in the body frame
- The center of the body coincides with its center of gravity. Therefore Iyz = Ixy = Ixz = 0
- The motor inertia is negligible.

Problem formulation

The relation between the linear velocity in the inertial frame W and the linear velocity in the body frame B or the translational kinematics can be given by:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix}\cos\theta\cos\psi & \cos\psi\sin\theta\sin\phi - \cos\phi\sin\psi & \cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi\\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\theta\sin\phi\sin\psi & -\cos\psi\sin\phi + \cos\phi\sin\theta\sin\psi\\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi\end{bmatrix}\begin{bmatrix}u\\ v\\ w\end{bmatrix}$$

While the rotational kinematics is the relation between the angular rates in body and fixed frames and given by:

$$\begin{bmatrix}p\\ q\\ r\end{bmatrix} = \begin{bmatrix}1 & 0 & -\mathrm{Sin}[\theta]\\ 0 & \mathrm{Cos}[\phi] & \mathrm{Sin}[\phi]\mathrm{Cos}[\theta]\\ 0 & -\mathrm{Sin}[\phi] & \mathrm{Cos}[\phi].\mathrm{Cos}[\theta]\end{bmatrix}\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}$$

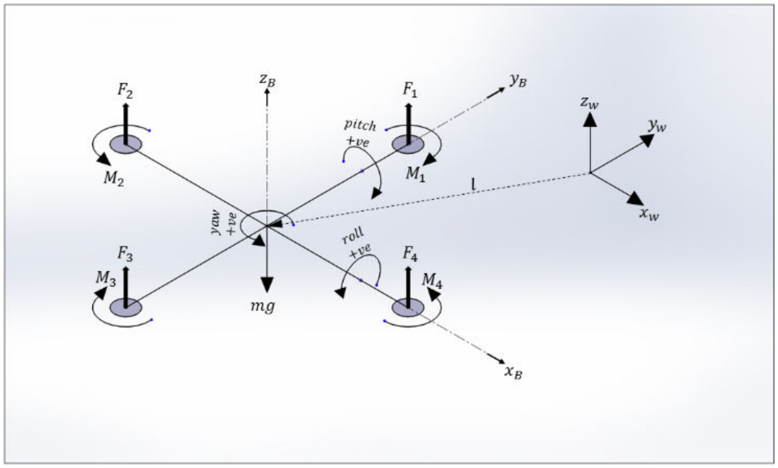

Figure 1: The Cartesian coordinate system of a quadrotor

In classical mechanics, the Newton Euler equations show the translational and rotational dynamics of a rigid body. These laws relate the motion of the center of gravity of a rigid body with the sum of external forces and moments applied on it. Without loss of generality, the origin of the body frame coincides with the center of gravity. Supposing there is a 6 DOF(Degrees of Freedom) rigid body which has a mass m and an inertia matrix I about the center of gravity. The linear velocity of the center of gravity is Vb, and the body angular velocity is Ωb in the body frame. The external force and the moment are Fb and Mb in the body frame. Therefore, the relation between the velocities and the external forces and moments in the body frame is given by:

$$\begin{bmatrix} mI & 0_{3*3} \\ 0_{3*3} & I \end{bmatrix} \begin{bmatrix} \dot{V}_b \\ \dot{\Omega}_b \end{bmatrix} + \begin{bmatrix} \Omega_b \times (mV_b) \\ \Omega_b \times (I\Omega_b) \end{bmatrix} = \begin{bmatrix} F_b \\ M_b \end{bmatrix}$$

Questions

1. show that from last equation we have

$$\begin{bmatrix} \ddot{x}_b \\ \ddot{y}_b \\ \ddot{z}_b \end{bmatrix} = \begin{bmatrix} r\dot{y}_b - q\dot{z}_b \\ p\dot{z}_b - r\dot{x}_b \\ q\dot{x}_b - p\dot{y}_b \end{bmatrix} + \frac{1}{m} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \ and \ \begin{bmatrix} I_{xx}\dot{p} \\ I_{yy}\dot{q} \\ I_{zz}\dot{r} \end{bmatrix} = \begin{bmatrix} (I_{yy} - I_{zz})qr \\ (I_{zz} - I_{xx})pr \\ (I_{xx} - I_{yy})pq \end{bmatrix} + \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

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Sol^{n}

$$V_b = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \quad \text{and} \quad \Omega_b = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad I_{inertia} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

From the equation of the following,

$$\begin{bmatrix} mI & 0_{3\times3} \\ 0_{3\times3} & I_{inertia} \end{bmatrix} \begin{bmatrix} \dot{V}_b \\ \dot{\Omega}_b \end{bmatrix} + \begin{bmatrix} \Omega_b \times (mV_b) \\ \Omega_b \times I\Omega_b \end{bmatrix} = \begin{bmatrix} F_b \\ M_b \end{bmatrix}$$

Take it separately

From row 1

$$mI\dot{V}_b + \Omega_b \times mV_b = F_b \quad \text{--- divide by } m$$

$$\dot{V}_b + \Omega_b \times V_b = \frac{1}{m} F_b$$

$$\boxed{\begin{bmatrix} \ddot{X}_b \\ \ddot{Y}_b \\ \ddot{Z}_b \end{bmatrix} = \begin{bmatrix} r\dot{y}_b - q\dot{z}_b \\ p\dot{z}_b - r\dot{x}_b \\ q\dot{x}_b - p\dot{y}_b \end{bmatrix} + \frac{1}{m} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}} \text{ Ans}$$

$$\begin{array}{cccc} & i & j & k \\ \Omega_b & p & q & r \\ V_b & \dot{x} & \dot{y} & \dot{z} \end{array}$$

$$\Omega_b \times V_b = (q\dot{z} - r\dot{y})i + (r\dot{x} - p\dot{z})j + (p\dot{y} - q\dot{x})k$$

From the 2nd row

$$I_{inertia}\dot{\Omega}_b + \Omega_b \times I\Omega_b = M_b$$

$$\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} pI_{xx} \\ qI_{yy} \\ rI_{zz} \end{bmatrix} = M_b$$

$$\begin{array}{cccc} & i & j & k \\ \Omega_b & p & q & r \\ I\Omega_b & pI_{xx} & qI_{yy} & rI_{zz} \end{array}$$

$$(qrI_{zz} - rqI_{yy})i + (rpI_{xx} - prI_{zz})j + (pqI_{yy} - pqI_{xx})k$$

$$\boxed{\begin{bmatrix} I_{xx}\dot{p} \\ I_{yy}\dot{q} \\ I_{zz}\dot{r} \end{bmatrix} = \begin{bmatrix} (I_{yy} - I_{zz})qr \\ (I_{zz} - I_{xx})pr \\ (I_{xx} - I_{yy})pq \end{bmatrix} + \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}} \text{ -- Ans}$$

```
% inertia matrix because of symmetry
syms Ixx Iyy Izz
I = diag([Ixx,Iyy, Izz])
```

I =

$$\begin{pmatrix} \mathrm{Ixx} & 0 & 0 \\ 0 & \mathrm{Iyy} & 0 \\ 0 & 0 & \mathrm{Izz} \end{pmatrix}$$

```
% If we use element wise multiplication like the above one, we can take I as vector
of
I = [Ixx; Iyy; Izz]
```

I =

$$\begin{pmatrix} \mathrm{Ixx} \\ \mathrm{Iyy} \\ \mathrm{Izz} \end{pmatrix}$$

2. Give the gravity force in the body-fixed frame and in the inertial axes.

```
% Define the variables (adjust these as needed)
syms phi theta psi
% Transposed transformation matrix
R = [
    cos(theta)*cos(psi), cos(psi)*sin(theta)*sin(phi) - cos(phi)*sin(psi),
cos(phi)*cos(psi)*sin(theta) + sin(phi)*sin(psi);
    cos(theta)*sin(psi), cos(psi)*cos(phi) + sin(theta)*sin(psi)*sin(phi),
-cos(psi)*sin(phi) + cos(phi)*sin(theta)*sin(psi);
    -sin(theta),           sin(phi)*cos(theta),
cos(theta)*cos(phi)
];

disp('Transposed Transformation Matrix (Explicit):');
```

Transposed Transformation Matrix (Explicit):

```
disp(R);
```

$$\begin{pmatrix} \cos(\psi)\cos(\theta) & \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\phi)\sin(\psi) & \sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\sin(\theta) \\ \cos(\theta)\sin(\psi) & \cos(\phi)\cos(\psi) + \sin(\phi)\sin(\psi)\sin(\theta) & \cos(\phi)\sin(\psi)\sin(\theta) - \cos(\psi)\sin(\phi) \\ -\sin(\theta) & \cos(\theta)\sin(\phi) & \cos(\phi)\cos(\theta) \end{pmatrix}$$

```
syms mg
F_ig = [0; 0; -mg];
disp(F_ig);
```

$$\begin{pmatrix} 0 \\ 0 \\ -\mathrm{mg} \end{pmatrix}$$

```
F_bg = R'*F_ig;
disp(F_bg);
```

$$\begin{pmatrix} \mathrm{mg}\sin(\overline{\theta}) \\ -\mathrm{mg}\cos(\overline{\theta})\sin(\overline{\phi}) \\ -\mathrm{mg}\cos(\overline{\phi})\cos(\overline{\theta}) \end{pmatrix}$$

3. Detail the pitching, rolling and yawing moments.

Lets start from defining the control inputs:

$$u = [u_1; u_2; u_3; u_4] \text{ control inputs}$$

$$F_i = K_F \omega_i^2 \text{ forces in i direction}$$

$$M_i = K_m \omega_i^2 \text{ moments in i rotation}$$

- �1 is the resulting upwards force of the four rotors which is responsible for the altitude of the quad-rotor and its rate of change(�, dz).
- u2 is the moment about �b axis resulting from the difference in thrust between rotors 2 and 4 which is responsible for the **roll rotation** and its rate of change(phi, dphi).
- �3 on the other hand represents the moment about �b axis resulting from the difference in thrust between rotors 1 and 3 thus generating the **pitch rotation** and its rate of change(theta, dtheta).
- Finally �4 is the difference in torque between the two clockwise turning rotors and the two counter clockwise turning rotors generating the yaw rotation and ultimately its rate of change (psi, dpsi).

$$\begin{aligned} F_z &= u_1 = \sum F_i \\ M_x &= u_2 = L(F_2 - F_4) \\ M_y &= u_3 = L(F_3 - F_1) \\ M_z &= u_4 = M_1 - M_2 + M_3 - M_4 \end{aligned}$$

4. Using small disturbance theory deduce a linear model.

The quadrotor's complete mathematical model can be expressed as a nonlinear state-space representation by combining the translation and rotation equations.

$$
f(x,u) = \begin{bmatrix} x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ (\cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi)\dfrac{u_1}{m} \\ (-\cos\psi\sin\phi + \cos\phi\sin\theta\sin\psi)\dfrac{u_1}{m} \\ \cos\theta\cos\phi\dfrac{u_1}{m} - g \\ \dfrac{qrI_{yy} - qrI_{zz} + u_2}{I_{xx}} \\ \dfrac{-prI_{xx} + prI_{zz} + u_3}{I_{yy}} \\ \dfrac{pqI_{xx} - pqI_{yy} + u_4}{I_{zz}} \end{bmatrix}
$$

- Small disturbance theory used to deduce a linear model from nonlinear equations.

The model that captures the nonlinear dynamics of the quadrotor has been developed using multiple inputs, specifically the rotational speeds (RPM) of the four motors. To facilitate stability and control analysis, these complex equations have been linearized around the hover condition, which serves as our trim point.The control system generates four distinct command signals. These commands are then translated into specific angular velocities for each of the four motors through control allocation.

Assumption we took:

- The system is operating near an equilibrium or or trim condition.
- Using small disturbance theory we converted the nonlinear terms into linear terms by assuming that:

$x = x_0 + \Delta x$

Where: x is the variable

x_0 is the equilibrium value

Δx is the small perturbation

Forces and moments:

The total thrust force is the sum of u_{1o}(the equilibrium thrust), and Δu_1 is a small change in thrust.

$$u_1 = u_{1o} + \Delta u_1$$

A small change in thrust is equal to the sum of small changes in individual rotor forces.

$$\Delta u_1 = \Delta \sum F_i$$

The rolling moment is the product of L(distance from the center of mass to the rotor), and $(F_2 - F_4)$(the difference in forces between opposing rotors).

$$u_2 = L(F_2 - F_4)$$

The pitching moment, which is the product of L(distance from the center of mass to the rotor), and $(F_3 - F_1)$(the difference in forces between the other pair of opposing rotors).

$$u_3 = L(F_3 - F_1)$$

The yawing moment, where M_i are the individual torques produced by each rotor.

$$u_4 = M_1 - M_2 + M_3 - M_4$$

The hovering conditions describe the equilibrium state of a quadrotor UAV: the quadrotor has no linear motion in any direction (x, y, or z), is not rotating around any of its axes, and the quadrotor maintains a stable position in the air without any translational or rotational motion.

From the above explanation, we assumed that all the initial condition at hovering state will be:

$$u_0 = v_0 = w_0 = 0$$

$$p_0 = q_0 = r_0 = 0$$

$$\varphi_0 = \theta_0 = \psi_0 = 0$$

The thrust force produced by the rotors exactly counteracts the weight of the vehicle.

$$w = m \cdot g = u_{10}$$

$$\frac{u_{10}}{m} = g$$

For small angles, the higher-order terms become negligible.

$$\sin(\theta) \approx \theta$$

$$\cos(\theta) \approx 1$$

$$\sin(\varphi) \approx \varphi$$

$$\cos(\varphi) \approx 1$$

$$\tan(\theta) \approx \theta$$

$$\tan(\varphi) \approx \varphi$$

<u>Translational motion matrix</u>

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} (\cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi)\dfrac{u_1}{m} \\ (-\cos\psi\sin\phi + \cos\phi\sin\theta\sin\psi)\dfrac{u_1}{m} \\ \cos\theta\cos\phi\dfrac{u_1}{m} - g \end{bmatrix}$$

- From the equation 1, the **linear acceleration in the x** direction:

$$\ddot{x} = (\cos\phi \cos\psi \sin\theta + \sin\phi \sin\psi)\frac{u_1}{m}$$

Apply small disturbance theory, i.e $x = x_0 + \Delta x$

$$\ddot{x}_0 + \Delta\ddot{x} = (\sin(\theta_0 + \Delta\theta)\cos(\psi_0 + \Delta\psi)\cos(\phi_0 + \Delta\phi) + \sin(\phi_0 + \Delta\phi)\sin(\psi_0 + \Delta\psi)) \cdot \frac{u1_0 + \Delta u_1}{m}$$

Small perturbations Δx assumed small compared to their respective baseline values x

- Nonlinear terms (e.g., products of small perturbations like Δx^2) are neglected.

$$\Delta\ddot{x} = (\Delta\theta \cos(\Delta\psi) + \Delta\phi \sin(\Delta\psi)) \cdot \frac{u1_0 + \Delta u1}{m}$$

- From the equation 2, **linear acceleration in the y** direction:

$$\ddot{y} = (-\cos(\psi)\sin(\phi) + \cos(\phi)\sin(\theta)\sin(\psi)) \cdot \frac{u_1}{m}$$

$$\ddot{y}_0 + \Delta\ddot{y} = (-\sin(\phi_0 + \Delta\phi)\cos(\psi_0 + \Delta\psi) + \sin(\theta_0 + \Delta\theta)\cos(\phi_0 + \Delta\phi)\sin(\psi_0 + \Delta\psi)) \cdot \frac{u1_0 + \Delta u_1}{m}$$

$$\Delta\ddot{y} = (-\Delta\phi \cos(\Delta\psi) + \Delta\theta \sin(\Delta\psi)) \cdot \frac{u1_0 + \Delta u_1}{m}$$

- From the equation 3, **linear acceleration in the z** direction:

$$\ddot{z} = \frac{\cos(\theta)\cos(\phi)u_1}{m} - g$$

$$\ddot{z}_0 + \Delta\ddot{z} = (\cos(\phi_0 + \Delta\phi)\cos(\theta_0 + \Delta\theta)) \cdot \frac{u1_0 + \Delta u_1}{m} - g$$

$$\Delta\ddot{z} = \frac{u1_0 + \Delta u_1}{m} - g$$

The angular velocity matrix

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} qr\frac{(I_{yy} - I_{zz})}{I_{xx}} + \frac{u_2}{I_{xx}} \\ pr\frac{(I_{zz} - I_{xx})}{I_{yy}} + \frac{u_3}{I_{yy}} \\ pq\frac{(I_{xx} - I_{yy})}{I_{zz}} + \frac{u_4}{I_{zz}} \end{bmatrix}$$

- From the equation 1 of the angular acceleration:

$$p = qr\frac{(I_{yy} - I_{zz})}{I_{xx}} + \frac{u_2}{I_{xx}}$$

$$\dot{p}_0 + \Delta\dot{p} = (q_0 + \Delta q)\cdot(r_0 + \Delta r)\cdot\frac{(I_{yy} - I_{zz})}{I_{xx}} + \frac{u_2}{I_{xx}}$$

After we neglet the higher order terms

$$\Delta\dot{p} = \frac{u_2}{I_{xx}}$$

- From the equation 2 of the angular acceleration:

$$q = \frac{pr(I_{zz} - I_{xx})}{I_{yy}} + \frac{u_3}{I_{yy}}$$

$$\dot{q}_0 + \Delta\dot{q} = (p_0 + \Delta p)\cdot(r_0 + \Delta r)\cdot\frac{(I_{zz} - I_{xx})}{I_{yy}} + \frac{u_3}{I_{yy}}$$

- After we neglecting higher order terms

$$\Delta\dot{q} = \frac{u_3}{I_y}$$

From the equation 3 of the angular acceleration:

$$\dot{r} = \frac{pq(I_{xx} - I_{yy})}{I_{zz}} + \frac{u_4}{I_{zz}}$$

$$\dot{r}_0 + \Delta\dot{r} = (p_0 + \Delta p)\cdot(q_0 + \Delta q)\cdot\frac{(I_{xx} - I_{yy})}{I_{zz}} + \frac{u_4}{I_z}$$

After we neglecting higher order terms

$$\Delta\dot{r} = \frac{u_4}{I_{zz}}$$

<u>Euler angle matrix</u>

$$\begin{bmatrix}\dot{\phi}\\ \dot{\psi}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix} p & \sin(\phi)\tan(\theta)*q & \cos(\phi)\tan(\theta)*r\\ 0 & \cos(\phi)*q & -\sin(\phi)*r\\ 0 & \frac{\sin(\phi)}{\cos(\phi)}*q & \frac{\cos(\phi)}{\cos(\theta)}*r \end{bmatrix}$$

From Equation 1 of the euler angle matrix

$$\dot{\phi} = p + q\sin(\phi)\tan(\theta) + r\cos(\phi)\tan(\theta)$$

$$\dot{\phi}_0 + \Delta\dot{\phi} = (p_0 + \Delta p) + (q_0 + \Delta q)\sin(\phi_0 + \Delta\phi)\tan(\theta_0 + \Delta\theta) + (r_0 + \Delta r)\cos(\phi_0 + \Delta\phi)\tan(\theta_0 + \Delta\theta)$$

At hovering condition

$$\Delta\dot{\phi} = \Delta p + \Delta q \Delta\phi \Delta\theta + \Delta r \Delta\theta$$

After we neglet the higher order terms

$$\Delta\dot{\phi} = \Delta p$$

From Equation 2 of the euler angle matrix

$$\dot{\theta} = q\cos(\phi) - r\sin(\phi)$$
$$\dot{\theta}_0 + \Delta\dot{\theta} = (q_0 + \Delta q)\cos(\phi_0 + \Delta\phi) - (r_0 + \Delta r)\sin(\phi_0 + \Delta\phi)$$

At hovering condition and after neglecting higher order terms

$$\Delta\dot{\theta} = \Delta q$$

From Equation 3 of the euler angle matrix

$$\dot{\psi} = \frac{q\sin(\phi)}{\cos(\theta)} + \frac{r\cos(\phi)}{\cos(\theta)}$$

$$\dot{\psi}_0 + \Delta\dot{\psi} = \frac{(q_0 + \Delta q)\sin(\phi_0 + \Delta\phi)}{\cos(\theta_0 + \Delta\theta)} + \frac{(r_0 + \Delta r)\cos(\phi_0 + \Delta\phi)}{\cos(\theta_0 + \Delta\theta)}$$

At hovering condition and after neglecting higher order terms

$$\Delta\dot{\psi} = \Delta r$$

After the assumption of small angle and near hover assumption. The euler rate matrix will be

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\psi} \\ \dot{\phi} \\ \dot{\theta} \end{bmatrix}$$

after small angle approximation, the non linear dynamics becomes

$$
f(x,u) = \begin{bmatrix} x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ (\cos(\psi)\theta + \phi\sin(\psi))\dfrac{u_1}{m} \\ (-\cos(\psi)\phi + \theta\sin(\psi))\dfrac{u_1}{m} \\ \dfrac{u_1}{m} - g \\ \dfrac{u_2}{I_{xx}} \\ \dfrac{u_3}{I_{yy}} \\ \dfrac{u_4}{I_{zz}} \end{bmatrix}
$$

5. Design a control law to stabilize and control the deduced model

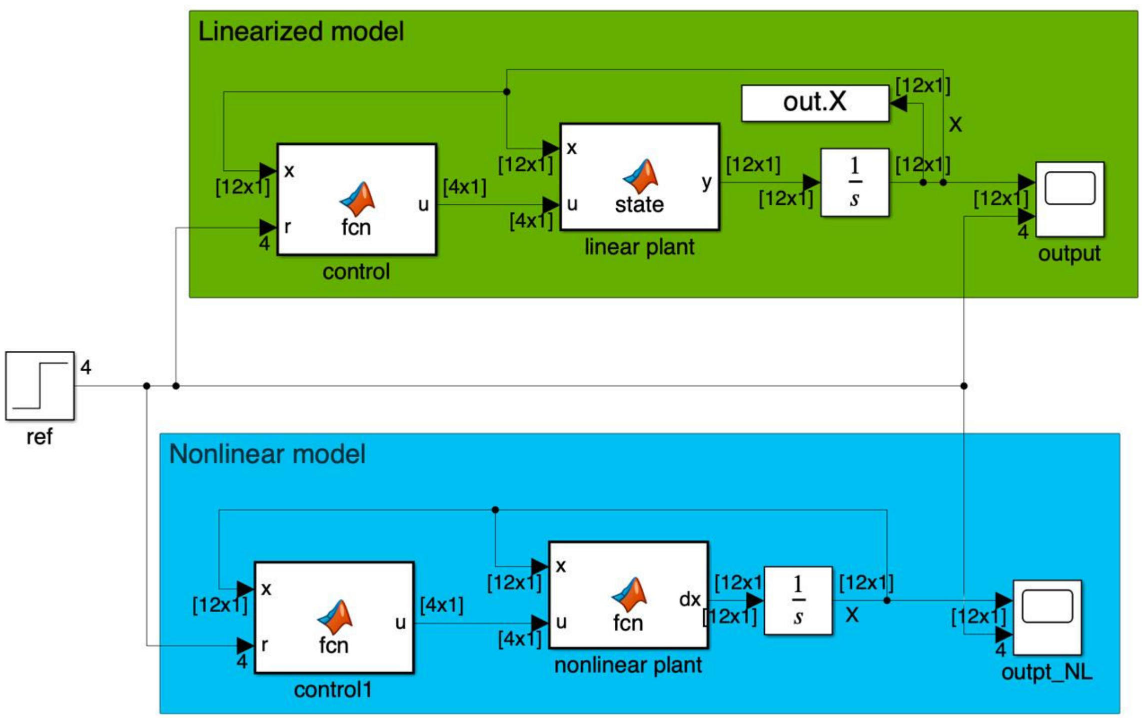

Figure 2. The non linear and linear plant dynamics of the plant

we use a state feedback linearization method to calculate the controller input

$$\dot{x} = Ax + Bu; \ \ where \ \ ; u = -Kx + v$$
$$\dot{x} = (A - BK)x \ + \ Bv$$

A = [0 1; 0 0]; B = [0; 1];

p = [-5, -6];

K = acker(A, B, p)

We used the above method when we design the controller u1

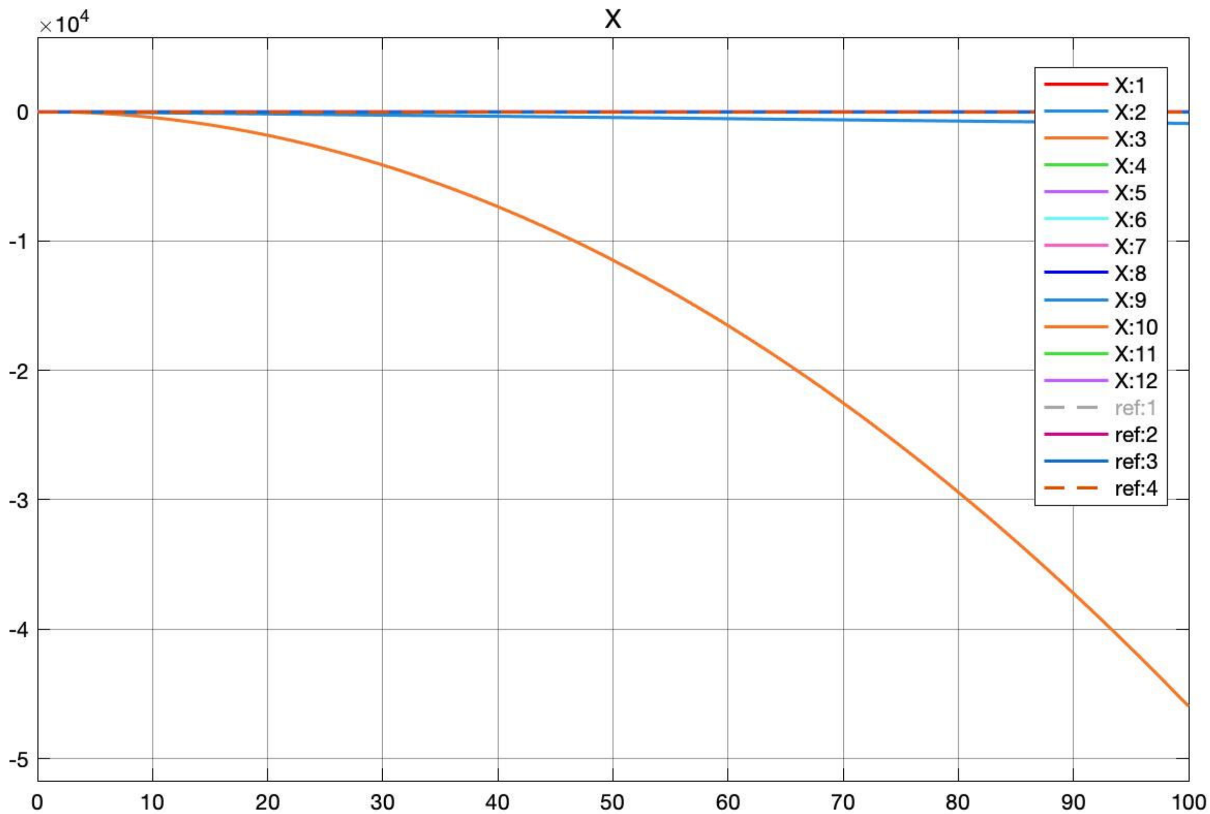

Figure 3. The open loop result of the linear system

From the observation of the figure 3, the linear system in the z-direction shows instability or divergence, that means it need for a controllertp ensure the system maintains or reaches a desired trajectory or equilibrium point.

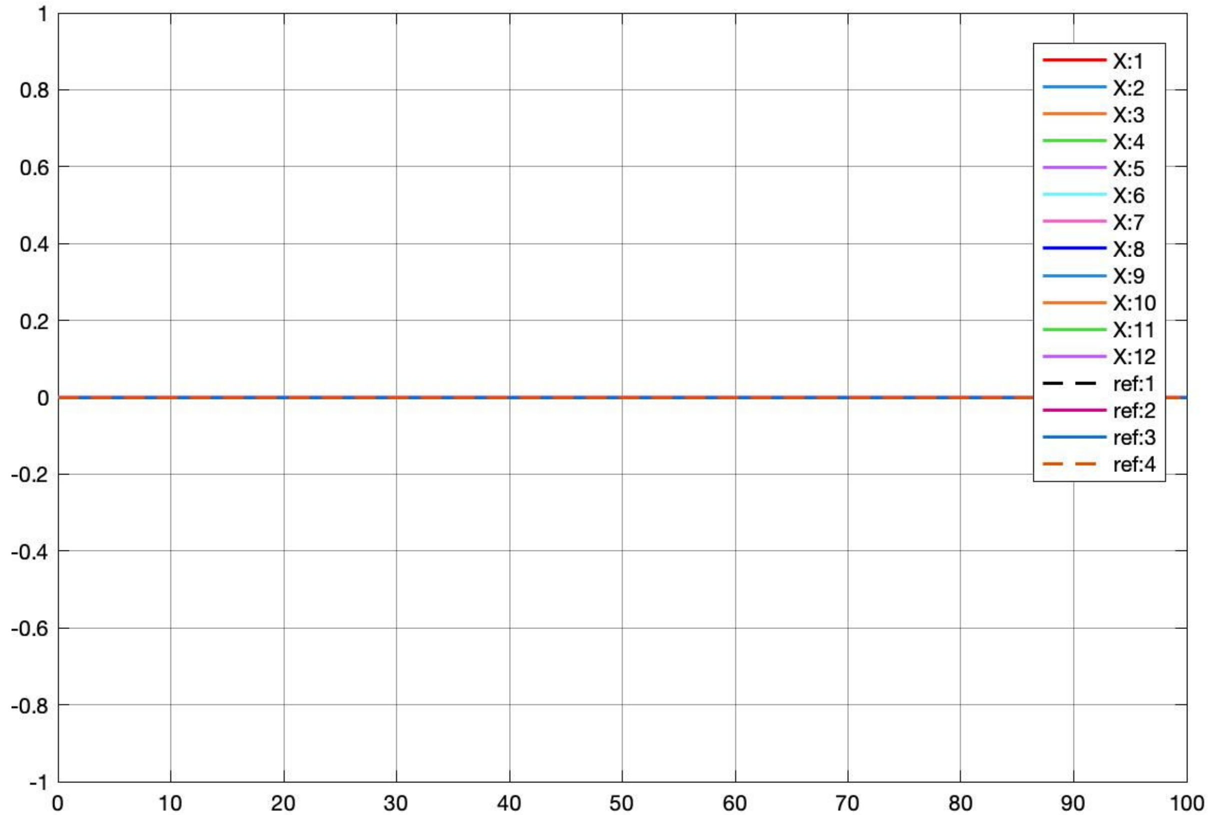

Figure 4. The linearized model at equilibrium point

The linearized model at the equilibrium point, designed using small angle approximations, shows stability with negligible deviations from the reference.

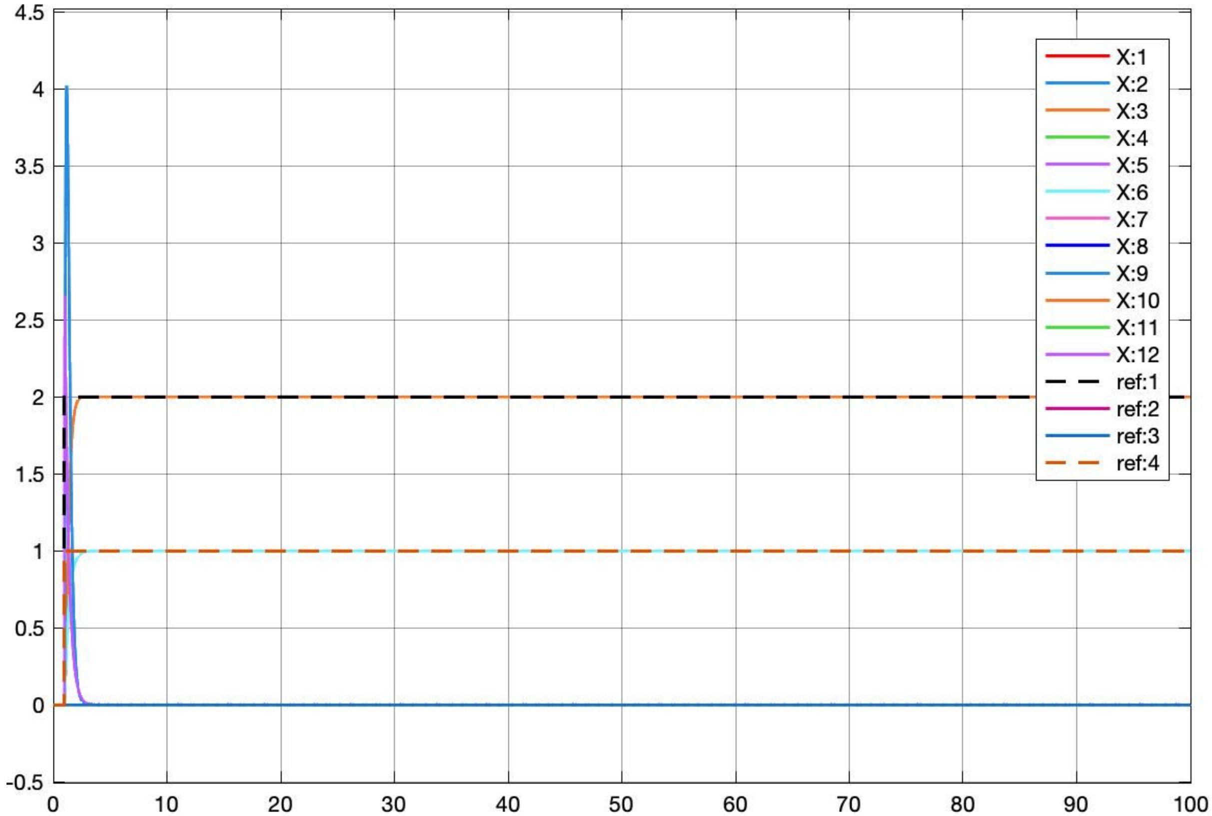

Figure 5. The linearized model with a given reference

The linearized model with a given reference as shown in Figure 5showed, the system quickly converges to the reference values with minimal overshoot and steady-state error.

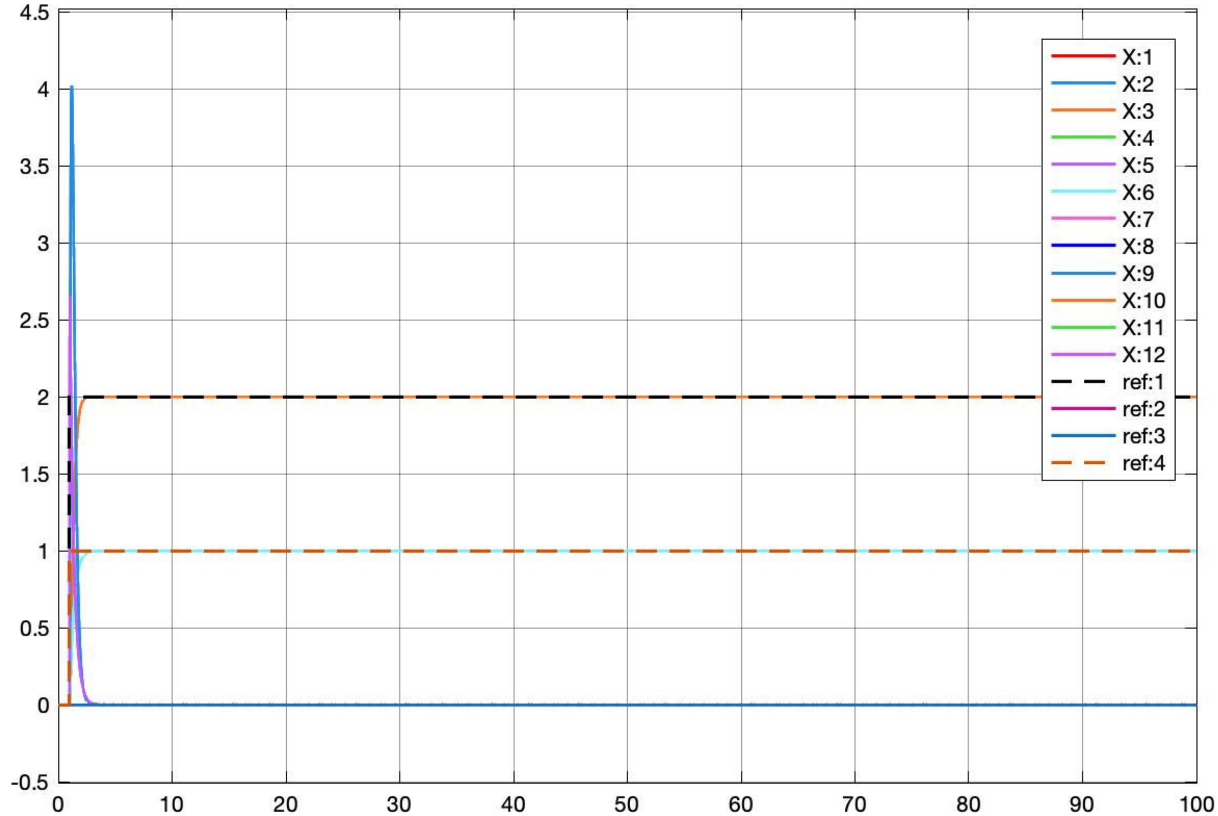

Figure 6. The non linear model with a given reference at lower disturbance

Based on the figure 6, we concluded that, the small disturbance approximation control method effectively controls the non linear plant around a small disturbance from the equilibrium point.However, the some states have internal dynamics and when the disturbance increase, they became divergent.

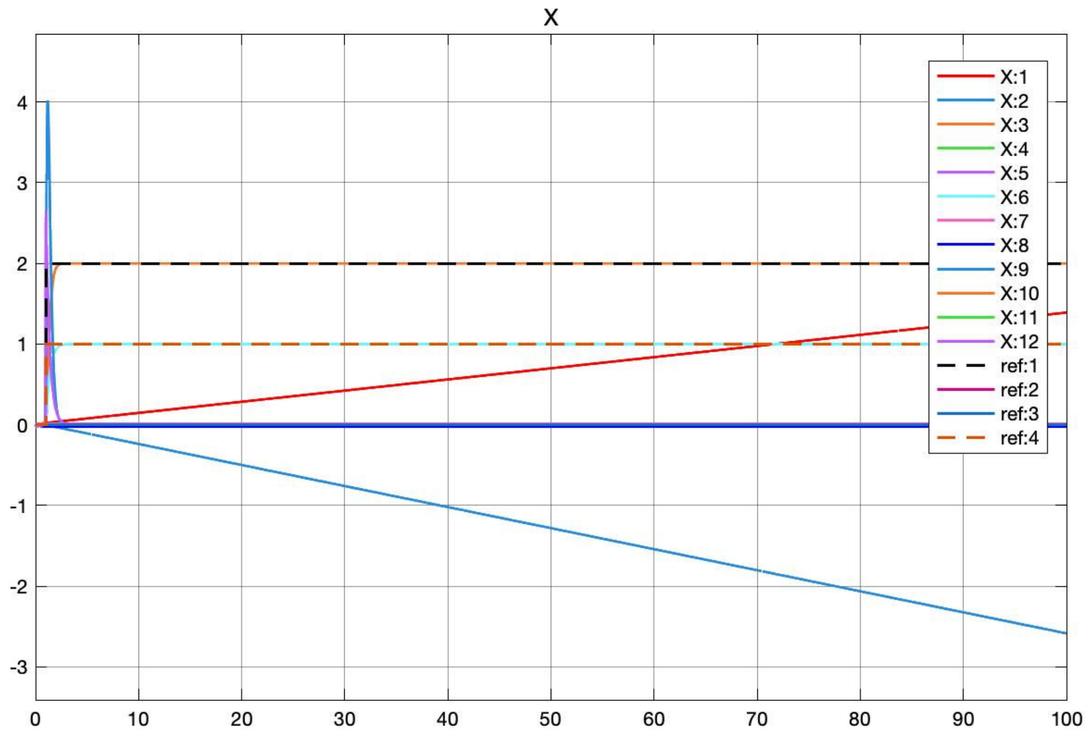

Figure 7. The non linear model with a given reference at higher disturbance

Conclusion

The lab report provided the analysis of quadrotor modeling and control, focusing on the derivation of kinematic and dynamic equations, simplification using small disturbance theory, and the design of a control law to stabilize the system.

The state feedback controller effectively stabilized the linearized system and performed well for small disturbances in the nonlinear system. Performance degradation for moderate disturbances highlights the limitations of linearization and the need to consider nonlinearity for larger deviations. Control inputs remained within operational limits, suggesting the feasibility of implementation in practical systems.

Reference

Eslam Abousalima graduation project

Appendix

nonlinear model

```
function dx = fcn(x,u)
g  = 9.81;
m = 1.62;
Ixx = 0.01184;
Iyy = 0.01667;
Izz = 0.026;
```

```
dx = zeros(12,1);
dx(1) = x(7);
dx(2) = x(8);
dx(3) = x(9);
dx(4) = x(10);
dx(5) = x(11);
dx(6) = x(12);
dx(7) = (cos(x(4))cos(x(6))*sin(x(5)) + sin(x(4)*sin(x(5)))(u(1)/m));
dx(8) = (-cos(x(6))sin(x(4)) + cos(x(4))*sin(x(6))*sin(x(5)))(u(1)/m);
dx(9) = -g + cos(x(4))cos(x(5))(u(1)/m);
dx(10) = 1/Ixx*(x(11)x(12)(Iyy-Izz) + u(2));
dx(11) = 1/Iyy*(x(10)x(12)(-Ixx + Izz) + u(3));
dx(12) = 1/Izz*(x(10)x(11)(Ixx - Izz) + u(4));
end
```

controller function

```
function u = fcn(x, r, K)
% r - reference input for z, phi, and theta
k1 = 5;
k2 = 4;
m = 1.62;
g = 9.81;
u = zeros(4,1);
% u(1) = 1;
u(1) = (K(1)*(r(1)-x(3)) - K(2)*x(9) + g)*m;
u(2) = K(1)*(r(2)-x(4)) - K(2)*x(10);
u(3) = K(1)*(r(3)-x(5)) - K(2)*x(11);
u(4) = K(1)*(r(4)-x(6)) - K(2)*x(12);
end
```

Linearized model

```
function y = state(x, u)
% Define the state derivatives
% x(1) = x7, x(2) = x8, ..., x(6) = x12

% Constants
m = 1.62;
Izz = 0.026;
Iyy = 0.01667;
Ixx = 0.0184;
g = 9.81;

% Inputs
u1 = u(1);
u2 = u(2);
u3 = u(3);
u4 = u(4);
```

```
% State variables
phi = x(4);  % Roll angle
theta = x(5); % Pitch angle
psi = x(6);   % Yaw angle

% Dynamics equations
dx = zeros(12, 1);
dx(1) = x(7);
dx(2) = x(8);
dx(3) = x(9);
dx(4) = x(10);
dx(5) = x(11);
dx(6) = x(12);
dx(7) = (cos(x(6)) * x(5) + x(4) * sin(x(6))) * u1 / m;
dx(8) = (-cos(x(6)) * x(4) + x(5) * sin(x(6))) * u1 / m;
dx(9) = u1/m - g;
dx(10) = u2 / Ixx;
dx(11) = u3 / Iyy;
dx(12) = u4 / Izz;
y = dx;
end
```