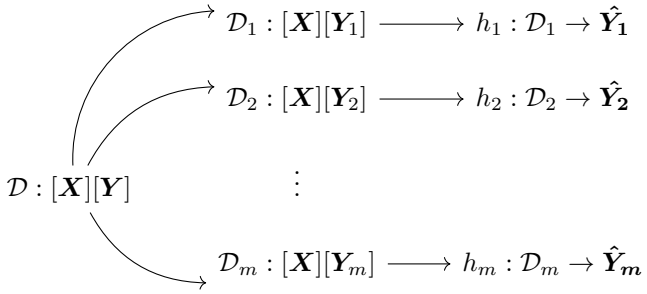


Multi-Target Support Vector Regression (SVR)

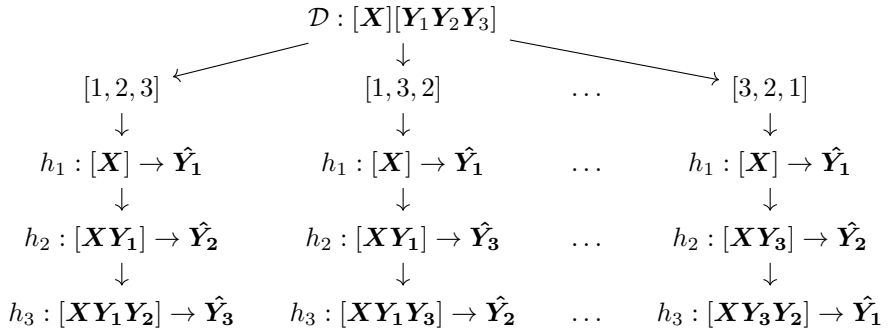
Input: Training dataset $\mathcal{D}$
Output: ST models $h_j, j = 1, \ldots, m$

for $j = 1$ to m **do**
  $\mathcal{D}_j = \{\boldsymbol{X}, \boldsymbol{Y}_j\}$ ▷ Get ST data
  $h_j : \boldsymbol{X} \rightarrow \mathbb{R}$ ▷ Build ST model for the j^{th} target
end for

Build Chained Model

Input: Training dataset $\mathcal{D}$, random chain $\boldsymbol{C}$
Output: A chained model $h_j, j = \{1, \ldots, m\}$

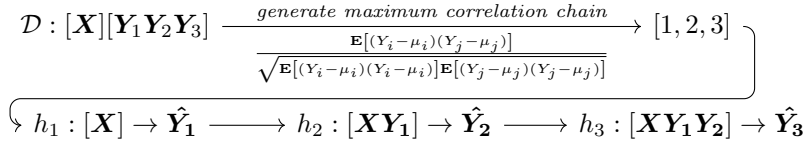
$\mathcal{D}_1 = \{\boldsymbol{X}, \boldsymbol{Y}_{\boldsymbol{C}_1}\}$ ▷ Initialize first dataset
for $j = 1$ to m **do** ▷ For each target in chain $\boldsymbol{C}$
  $h_j : \mathcal{D}_j \rightarrow \mathbb{R}$ ▷ Train model on appended dataset
  if $j < m$ **then**
    $\mathcal{D}_{j+1} = \{\mathcal{D}_j, \boldsymbol{Y}_{\boldsymbol{C}_j}\}$ ▷ Append new target in chain to dataset
  end if
end for



Multi-Target SVR with Random-Chains (SVRRC)

Input: Training dataset $\mathcal{D}$, c random chains $\mathcal{C}$
Output: An ensemble of chained models $h_\mathcal{C}$

for each $\boldsymbol{C} \in \mathcal{C}$ **do** ▷ For each random chain
  $h_{\boldsymbol{C}} \leftarrow$ `buildChainedModel`$(\mathcal{D}, \boldsymbol{C})$ ▷ Build a chained model for chain $\boldsymbol{C}$
end for



Multi-Target SVR with max-Correlation Chain (SVRCC)

1: $\mathbf{P} = corrcoef(\boldsymbol{Y})$ ▷ Find correlation coefficient matrix for target variables
2: $\boldsymbol{C} = \sum_{i=1}^{n} \mathbf{P}_{ij}, \forall j = 1, \ldots, m$ ▷ Sum rows of the correlation matrix
3: $\boldsymbol{C} = \texttt{sort}\,(\boldsymbol{C}, \textbf{decreasing})$ ▷ Sort sums in decreasing order
4: $h_{\boldsymbol{C}} = \texttt{buildChainedModel}(\mathcal{D}, \boldsymbol{C})$ ▷ Build a max-correlation chained model

Average Relative Root Mean Square Error (aRRMSE) for MT regressors

Datasets	MORF	ST	MTS	MTSC	RC	ERC	ERCC	SVR	SVRRC	SVRCC
Slump	0.6939	0.6886	0.6690	0.6938	0.7019	0.7022	0.6886	0.5765	**0.5545**	0.5560
Polymer	0.6159	0.5971	0.5778	0.6493	0.6270	0.6544	0.6131	0.5573	0.5253	**0.5116**
Andro	0.5097	0.5979	0.5155	0.5633	0.5924	0.5885	0.5666	0.4856	0.4651	**0.4455**
EDM	0.7337	0.7442	0.7413	0.7446	0.7449	0.7452	0.7443	0.7058	0.7070	**0.6978**
Solar Flare 1	1.3046	1.1357	1.1168	1.0758	0.9951	1.0457	1.0887	0.9917	0.9455	**0.9320**
Jura	0.5969	0.5874	0.5906	0.5892	0.5910	0.5896	0.5880	0.5952	**0.5764**	0.5885
Enb	0.1210	0.1165	0.1231	0.1211	0.1268	0.1250	0.1139	0.0977	0.0910	**0.0899**
Solar Flare 2	1.4167	1.1503	**0.9483**	1.0840	1.0092	1.0522	1.0928	1.0385	1.0253	1.0298
Wisconsin Cancer	0.9413	0.9314	0.9308	0.9336	**0.9305**	0.9313	0.9323	0.9555	0.9483	0.9427
California Housing	0.6611	0.6447	0.6974	0.6630	0.7131	0.6690	0.6146	0.6130	0.5945	**0.5852**
Stock	0.1653	0.1844	0.1787	0.1803	0.1802	0.1789	0.1752	0.1364	**0.1337**	0.1388
SCPF	0.8273	0.8348	0.8436	0.8308	0.8263	0.8105	0.8290	0.8164	0.8037	**0.8013**
Puma8NH	0.7858	0.8142	0.8118	0.8311	0.8199	0.8205	0.8207	**0.7655**	0.7744	0.7676
Friedman	0.9394	0.9214	0.9231	0.9210	0.9231	0.9209	0.9204	0.9218	0.9208	**0.9196**
Puma32H	0.9406	**0.8713**	0.8727	0.8791	0.8752	0.8729	0.8740	0.9364	0.9367	0.9319
Water Quality	**0.8994**	0.9085	0.9109	0.9093	0.9121	0.9097	0.9057	0.9343	0.9310	0.9045
M5SPEC	0.5910	0.5523	0.5974	0.5671	0.5552	0.5542	0.5558	0.2951	0.2935	**0.2925**
MP5SPEC	0.5522	0.5120	0.5683	0.5133	0.5145	0.5143	0.5119	0.2484	**0.2323**	0.2358
MP6SPEC	0.5553	0.5152	0.5686	0.5119	0.5198	0.5187	0.5109	0.2850	0.2669	**0.2623**
ATP7d	0.5563	0.5308	**0.5141**	0.5142	0.5558	0.5397	0.5182	0.5455	0.5371	0.5342
OES97	0.5490	0.5230	0.5229	0.5217	0.5239	0.5237	0.5222	0.4641	**0.4618**	0.4635
Osales	0.7596	0.7471	**0.7086**	0.7268	0.8318	0.7258	0.7101	0.7924	0.7924	0.7811
ATP1d	0.4173	0.3732	0.3733	0.3712	0.3790	**0.3696**	0.3721	0.3773	0.3707	0.3775
OES10	0.4518	0.4174	0.4176	0.4171	0.4178	0.4180	0.4166	0.3570	0.3555	**0.3538**
Average	0.6910	0.6625	0.6551	0.6589	0.6611	0.6575	0.6536	0.6039	0.5935	**0.5893**
Ranks	7.5000	5.7708	5.9375	6.1667	7.4375	6.3750	4.9792	4.7708	3.2708	**2.7917**

Run Time (seconds) for MT regressors

Datasets	MORF	ST	MTS	MTSC	RC	ERC	ERCC	SVR	SVRRC	SVRCC
Slump	38.1	2.6	9.9	15.9	1.8	11.1	50.5	**0.6**	1.9	0.7
Polymer	7.6	2.7	9.1	15.5	1.9	14.9	80.5	**0.5**	2.6	**0.5**
Andro	25.7	4.4	15.0	34.2	3.4	33.2	197.9	**1.1**	6.2	**1.1**
EDM	24.8	2.8	9.4	18.1	2.1	5.8	19.0	**0.9**	1.0	**0.9**
Solar Flare 1	34.1	3.5	13.6	26.7	2.7	17.7	86.9	**2.3**	9.3	2.6
Jura	64.3	7.9	31.8	74.3	6.4	43.5	254.2	**4.7**	18.7	5.3
Enb	71.4	6.6	26.1	63.6	**5.4**	15.6	69.6	11.3	17.7	15.9
Solar Flare 2	55.4	7.4	30.7	68.0	**6.3**	42.9	241.5	9.4	53.5	15.6
Wisconsin Cancer	51.4	6.1	21.9	53.7	4.9	14.8	61.6	**2.0**	2.4	**2.0**
California Housing	93.0	9.7	34.8	75.9	**8.2**	21.3	102.0	15.8	25.2	23.6
Stock	93.7	11.7	46.8	96.7	**11.0**	75.4	427.3	18.5	90.5	26.3
SCPF	66.3	19.3	65.9	176.3	**15.0**	104.2	734.2	32.8	162.8	48.8
Puma8NH	130.4	29.7	106.7	288.6	**27.9**	201.6	1227.7	94.1	516.6	177.1
Friedman	79.5	27.0	81.2	258.3	25.0	273.7	2871.6	**12.3**	322.3	18.8
Puma32H	93.9	68.1	181.0	635.0	87.7	667.9	6087.0	**32.2**	1018.7	53.1
Water Quality	108.4	**93.1**	262.1	912.3	127.2	925.4	10993.3	110.2	2567.9	189.5
M5SPEC	89.8	68.9	166.3	604.6	73.7	262.3	3132.1	**39.2**	546.7	45.1
MP5SPEC	84.5	94.6	221.2	888.3	91.5	557.0	6864.1	**49.3**	1132.1	58.4
MP6SPEC	90.3	93.4	212.6	871.0	89.1	557.6	6761.3	**47.2**	1227.1	58.5
ATP7d	**70.5**	262.6	452.1	2319.8	242.1	1779.2	24373.8	80.0	1897.4	136.5
OES97	**83.4**	485.3	1146.6	4928.9	499.8	5315.0	58072.1	148.2	3759.1	342.6
Osales	**92.0**	1094.8	2340.7	8322.2	986.5	11361.2	122265.3	437.0	4830.1	843.6
ATP1d	**70.7**	272.9	476.5	2568.9	261.9	2138.9	26768.9	95.0	2127.8	174.4
OES10	**90.0**	738.9	1633.6	6682.9	688.5	7150.8	83533.1	229.1	5419.4	577.1
Average	71.2	142.2	316.5	1250.0	136.2	1316.3	14803.2	**61.4**	1073.2	117.4
Ranks	5.5	3.71	6.0	8.29	3.0	7.08	9.92	**1.88**	6.71	2.92

$$\min_{\boldsymbol{w},b,\boldsymbol{\xi}} \frac{1}{2}||\boldsymbol{w}||^2 + C\sum_I \xi_I, \qquad \max_{\boldsymbol{\alpha}} \sum_I \alpha_I - \frac{1}{2}\sum_I \sum_{K\in I} \alpha_I \alpha_K Y_I Y_K \mathcal{K}\left(\boldsymbol{x}_{s_I}, \boldsymbol{x}_{s_K}\right)$$

$$\text{s.t. } Y_I(\langle \boldsymbol{w}, \boldsymbol{x}_{s_I}\rangle + b) \geq 1 - \xi_I,\ \forall I \in \{1,\ldots,n\}, \qquad \text{s.t.} \sum_I \alpha_I Y_I = 0,$$

$$\xi_I \geq 0,\ \forall I \in \{1,\ldots,n\}, \qquad 0 \leq \alpha_I \leq C,\ \forall I \in \{1,\ldots,n\},$$

$$s_I = \underset{i\in I}{\text{argmax}}(\langle \boldsymbol{w}, \boldsymbol{x}_i\rangle + b),\ \forall I \in \{1,\ldots,n\}. \qquad s_I = \underset{i\in I}{\text{argmax}}(\boldsymbol{o}_I),\ \forall I \in \{1,\ldots,n\}.$$

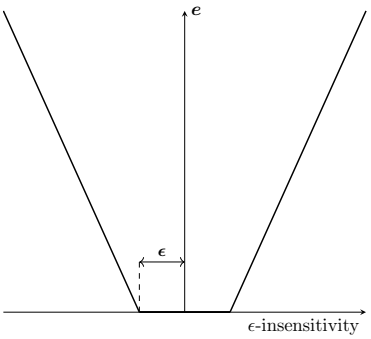


Figure 1: Vapnik's ϵ-insensitivity loss function.

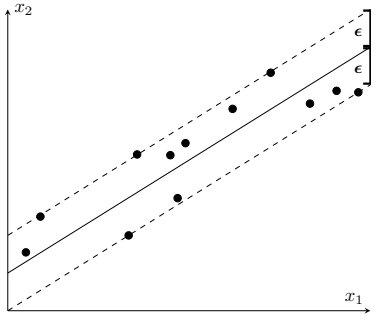
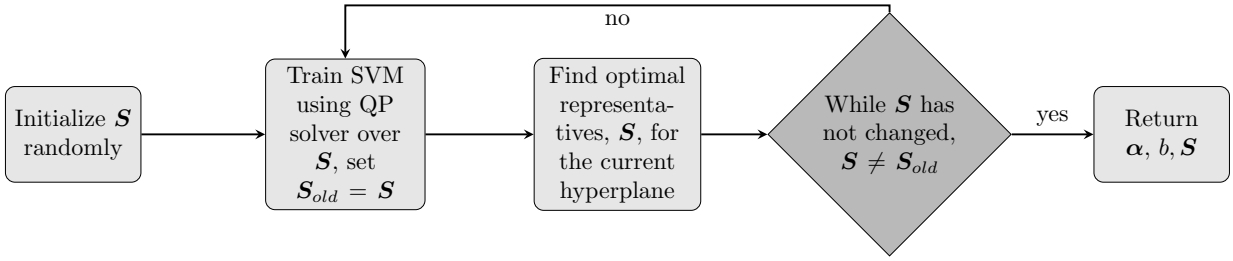


Figure 2: Linear support vector regression example solution on a toy 2D dataset.



Multi-Instance Representative SVM (MIRSVM)

Input: Training dataset $\mathcal{D}$, SVM Parameters C and σ
Output: SVM model parameters $\boldsymbol{\alpha}$ and b, Bag Representative IDs $\boldsymbol{S}$

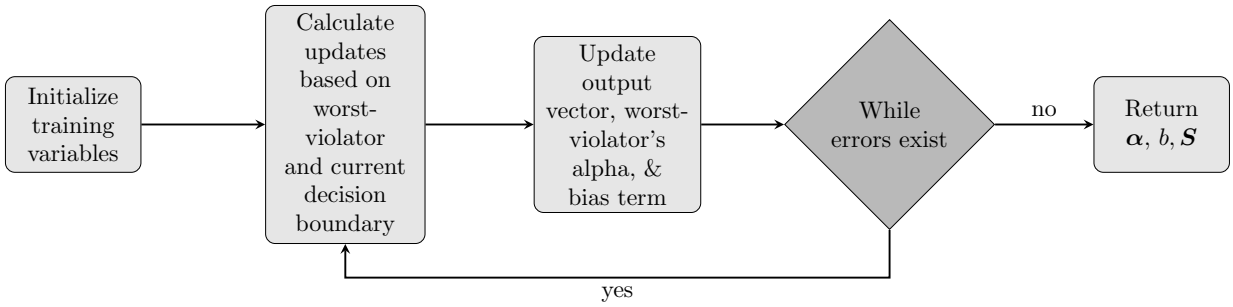
for $I \in \{1,\ldots,n\}$ **do**
  $\boldsymbol{S}_I \leftarrow$ rand $(|\mathcal{B}_I|, 1, 1)$ ▷ Assign each bag a random instance
end for
while $\boldsymbol{S} \neq \boldsymbol{S}_{old}$ **do**
  $\boldsymbol{S}_{old} \leftarrow \boldsymbol{S}$
  $\boldsymbol{X}_S \leftarrow \boldsymbol{X}(\boldsymbol{S})$, $\boldsymbol{Y}_S \leftarrow \boldsymbol{Y}(\boldsymbol{S})$ ▷ Initialize the representative dataset
  $\boldsymbol{G} \leftarrow (\boldsymbol{Y}_S \times \boldsymbol{Y}_S) \cdot \mathcal{K}\left(\boldsymbol{X}_S, \boldsymbol{X}_S, \sigma\right)$ ▷ Build Gram matrix
  $\boldsymbol{\alpha} \leftarrow$ quadprog $(\boldsymbol{G}, -\boldsymbol{1}^n, \boldsymbol{Y}_S, \boldsymbol{0}^n, \boldsymbol{0}^n, \boldsymbol{C}^n)$ ▷ Solve QP Problem
  $\boldsymbol{sv} \leftarrow$ find $(0 < \boldsymbol{\alpha} \leq C)$ ▷ Get the support vector indices
  $n_{\boldsymbol{sv}} \leftarrow$ count $(0 < \boldsymbol{\alpha} \leq C)$ ▷ Get the number of support vectors
  $b \leftarrow \frac{1}{n_{\boldsymbol{sv}}} \sum_{i=1}^{n_{\boldsymbol{sv}}} (\boldsymbol{Y}_{\boldsymbol{sv}} - \boldsymbol{G}_{\boldsymbol{sv}} * (\boldsymbol{\alpha}_{\boldsymbol{sv}} \cdot \boldsymbol{Y}_{\boldsymbol{sv}}))$ ▷ Calculate the bias term
  for $I \in \{1,\ldots,n\}$ **do**
    $\boldsymbol{G}_I \leftarrow (Y_I \times \boldsymbol{Y}_S) \cdot \mathcal{K}\left(\mathcal{B}_I, \boldsymbol{X}_S, \sigma\right)$
    $\boldsymbol{S}_I \leftarrow \text{argmax}_{i\in I}\left(\boldsymbol{G}_I * \boldsymbol{\alpha} + b\right)$ ▷ Select optimal bag-representatives
  end for
end while

$$\min_{(\boldsymbol{w},b)\in\mathcal{H}_o\times\mathbb{R}} R \;=\; \frac{1}{2}||\boldsymbol{w}||^2 + C\sum_{i=1}^{n} L(y_i,\, o_{(w,b)}(\boldsymbol{x}_i)) \tag{1}$$

$$L(y_i,\, o_{(w,b)}(\boldsymbol{x}_i)) = \max\left\{0, 1 - y_i o_{(w,b)}(\boldsymbol{x}_i)\right\} \tag{2}$$

$$\min_{(\boldsymbol{w},b)\in\mathcal{H}_o\times\mathbb{R}} R \;=\; \frac{1}{2}||\boldsymbol{w}||^2 + C\sum_{i=1}^{n} (|y_i - o_{(w,b)}(\boldsymbol{x_i})|_\epsilon) \tag{3}$$

$$L(y_i,\, o_{(w,b)}(\boldsymbol{x}_i)) = \begin{cases} 0 & if|y_i - o_{(w,b)}(\boldsymbol{x_i})| \leq \epsilon \\ |y_i - o_{(w,b)}(\boldsymbol{x_i})| - \epsilon & \text{otherwise.} \end{cases} \tag{4}$$



OnLine Learning Algorithm using Worst-Violators (OLLAWV)

Input: $\mathcal{D}$, C, γ, β, M
Output: $\boldsymbol{\alpha}$, b, $\boldsymbol{S}$

$\boldsymbol{\alpha} \leftarrow \mathbf{0}$, $b \leftarrow 0$, $\boldsymbol{S} \leftarrow \mathbf{0}$ ▷ Initialize OLLAWV model parameters
$\boldsymbol{o} \leftarrow \mathbf{0}$, $t \leftarrow 0$ ▷ Initialize the output vector and iteration counter
$wv \leftarrow 0$, $yo \leftarrow y_{wv} * \boldsymbol{o}_{wv}$ ▷ Initialize hinge loss error and worst-violator index
while $yo < M$ **do**
    $t \leftarrow t + 1$
    $\eta \leftarrow 2/\sqrt{t}$ ▷ Learning rate

    $\Lambda \leftarrow \eta * C * y_{wv}$ ▷ Calculate hinge loss update
    $B \leftarrow (\Lambda * \beta)/n$ ▷ Calculate bias update
    $\boldsymbol{o} \leftarrow \boldsymbol{o} + \Lambda * \mathcal{K}(\boldsymbol{x}_{\neg \boldsymbol{S}}, \boldsymbol{x}_{wv}, \gamma) + B$ ▷ Update output vector
    $\boldsymbol{\alpha}_{wv} \leftarrow \boldsymbol{\alpha}_{wv} + \Lambda$ ▷ Update worst-violator's alpha value
    $b \leftarrow b + B$ ▷ Update bias term

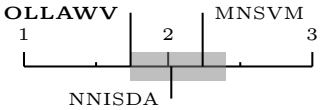
    $\boldsymbol{S}_t \leftarrow wv$ ▷ Save index of worst-violator
    $[yo, wv] \leftarrow \min_{wv \in \{\neg \boldsymbol{S}\}} \{y_{wv} \cdot o_{wv}\}$ ▷ Find the worst-violator
end while

Classification Datasets

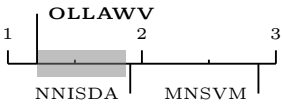
Dataset	# Samples	# Attributes	# Classes
small datasets			
iris	150	4	3
teach	151	5	3
wine	178	13	3
cancer	198	32	2
sonar	208	60	2
glass	214	9	6
vote	232	16	2
heart	270	13	2
dermatology	366	33	6
prokaryotic	997	20	3
eukaryotic	2,427	20	4
medium datasets			
optdigits	5,620	64	10
satimage	6,435	36	6
usps	9,298	256	10
pendigits	10,992	16	10
reuters	11,069	8,315	2
letter	20,000	16	26
large datasets			
adult	48,842	123	2
w3a	49,749	300	2
shuttle	58,000	7	7
web (w8a)	64,700	300	2
ijcnn1	141,691	22	2
intrusion	5,209,460	127	2

Comparison of OLLAWV vs. NNISDA and MNSVM

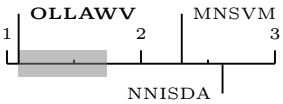
Dataset	Accuracy (%)			Run Time (s)			Support Vectors (%)		
	OLLAWV	NNISDA	MNSVM	OLLAWV	NNISDA	MNSVM	OLLAWV	NNISDA	MNSVM
small datasets									
iris	**97.33**	94.00	96.67	**0.05**	0.27	3.57	**13.50**	40.20	29.80
teach	52.32	52.31	**52.95**	**0.12**	0.44	8.85	**69.19**	99.80	87.40
wine	**98.87**	96.60	96.60	**0.28**	0.43	4.84	**15.02**	44.40	48.60
cancer	80.36	**81.86**	81.38	**0.49**	0.85	4.46	**42.79**	83.80	89.60
sonar	**92.32**	89.48	87.57	**0.59**	0.98	3.03	**31.26**	73.00	66.00
glass	**72.41**	67.81	69.30	**0.46**	1.01	11.94	**62.84**	90.80	87.60
vote	**96.54**	96.11	93.99	**0.26**	0.46	1.49	**13.36**	33.20	34.00
heart	82.22	**83.33**	**83.33**	**0.50**	0.91	6.45	**37.69**	73.00	82.00
dermatology	97.82	**98.36**	**98.36**	**1.62**	2.47	11.68	**36.94**	59.00	59.80
prokaryotic	88.96	88.86	**88.97**	**6.09**	10.64	50.86	**29.01**	51.20	49.00
eukaryotic	77.38	79.56	**81.21**	61.95	**49.16**	342.76	**54.11**	76.40	72.60
medium datasets									
optdigits	99.11	99.29	**99.31**	**411**	528	787	**28.64**	31.60	30.60
satimage	91.66	**92.39**	92.35	1,334	**687**	1,094	**20.72**	45.00	44.80
usps	97.49	98.05	**98.24**	10,214	**5,245**	7,777	**11.22**	29.40	28.00
pendigits	99.56	**99.62**	99.61	**723**	909	1,500	**10.27**	17.60	16.60
reuters	98.03	**98.08**	97.99	**954**	1,368	1,657	**8.770**	18.20	18.60
letter	96.99	99.11	**99.13**	**5,259**	12,009	26,551	**43.56**	57.60	56.60
large datasets									
adult	84.75	85.07	**85.13**	**21,025**	72,552	123,067	**34.66**	56.00	56.60
w3a	**98.86**	98.82	98.82	**6,532**	15,951	24,562	**3.270**	14.60	12.40
shuttle	99.77	99.83	**99.87**	**2,833**	7,420	45,062	**2.010**	6.00	16.40
web	98.94	**99.00**	99.00	**12,067**	30,583	38,040	**4.320**	13.20	10.80
ijcnn1	98.31	99.34	**99.41**	**162,587**	296,917	370,144	16.36	11.00	**7.600**
intrusion	**99.77**	99.67	99.66	**2,402,804**	4,646,810	3,772,113	**0.780**	2.000	1.700
Average	**91.29**	91.15	91.25	**114,209**	221,350	191,861	**25.66**	44.65	43.79
Ranks	**1.739**	2.022	2.239	**1.217**	1.913	2.869	**1.087**	2.609	2.304



Bonferroni-Dunn test for Accuracy



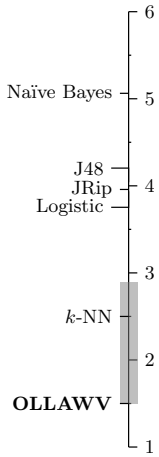
Bonferroni-Dunn test for Run Time

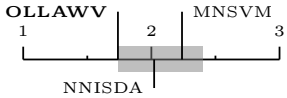


Bonferroni-Dunn test for % Support Vectors

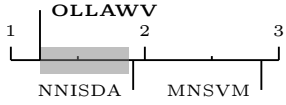
Accuracy (%) for Non-SVM Methods vs. OLLAWV

Dataset	OLLAWV	k-NN	J48	JRip	Naïve Bayes	Logistic
small datasets						
iris	**97.33 ± 1.49**	96.00 ± 3.65	94.00 ± 2.79	90.67 ± 4.35	96.00 ± 2.79	97.33 ± 2.79
teach	52.32 ± 3.46	**59.64 ± 2.89**	49.72 ± 7.58	56.75 ± 9.60	53.75 ± 6.46	51.77 ± 6.68
wine	**98.87 ± 1.54**	97.73 ± 3.72	90.43 ± 5.83	93.24 ± 3.27	96.60 ± 3.14	96.05 ± 2.58
cancer	**80.36 ± 5.80**	77.32 ± 6.93	73.81 ± 8.57	73.78 ± 5.81	67.73 ± 5.07	77.32 ± 7.78
sonar	**92.32 ± 3.11**	88.99 ± 4.59	76.16 ± 10.6	75.18 ± 6.77	73.69 ± 7.65	75.18 ± 7.31
glass	**72.41 ± 2.28**	67.73 ± 5.91	65.06 ± 5.51	65.59 ± 9.66	49.46 ± 5.19	62.04 ± 5.75
vote	**96.54 ± 1.87**	92.26 ± 3.19	95.70 ± 2.12	96.54 ± 2.45	92.24 ± 3.24	93.54 ± 2.59
heart	82.22 ± 2.93	79.63 ± 5.71	78.52 ± 2.81	80.74 ± 4.06	**84.44 ± 4.46**	83.33 ± 3.93
dermatology	**97.82 ± 0.05**	96.18 ± 1.78	94.52 ± 2.21	91.27 ± 5.08	97.28 ± 1.64	96.98 ± 2.28
prokaryotic	**88.96 ± 2.14**	87.96 ± 3.01	78.54 ± 1.62	79.13 ± 2.78	62.38 ± 3.54	87.57 ± 2.56
eukaryotic	77.38 ± 1.96	**81.42 ± 2.06**	65.27 ± 2.92	66.42 ± 3.47	39.27 ± 3.43	69.55 ± 1.34
medium datasets						
optdigits	**99.11 ± 0.38**	98.74 ± 0.39	90.87 ± 1.09	91.28 ± 0.40	92.42 ± 0.75	95.05 ± 0.91
satimage	**91.66 ± 0.80**	90.38 ± 0.72	85.64 ± 1.21	85.33 ± 0.77	85.41 ± 0.92	88.14 ± 1.11
usps	**97.49 ± 0.22**	97.04 ± 0.47	88.73 ± 0.46	89.20 ± 1.00	79.45 ± 0.59	91.88 ± 0.65
pendigits	**99.56 ± 0.12**	99.33 ± 0.17	96.24 ± 0.31	96.34 ± 0.41	88.34 ± 0.65	95.59 ± 0.18
reuters	**98.03 ± 0.22**	97.15 ± 0.43	96.90 ± 0.32	97.18 ± 0.44	93.52 ± 0.02	69.54 ± 0.28
letter	**96.99 ± 0.21**	95.71 ± 0.19	87.34 ± 0.68	87.02 ± 0.66	74.12 ± 0.97	77.45 ± 0.16
large datasets						
adult	**84.75 ± 0.26**	83.85 ± 0.28	84.38 ± 0.28	83.73 ± 0.17	80.57 ± 0.09	82.46 ± 0.14
w3a	**98.86 ± 0.04**	98.60 ± 0.06	98.71 ± 0.05	98.41 ± 0.10	96.71 ± 0.20	98.61 ± 0.12
shuttle	99.77 ± 0.03	99.93 ± 0.03	**99.97 ± 0.02**	99.96 ± 0.02	98.57 ± 0.24	96.83 ± 0.12
web	**98.94 ± 0.05**	98.89 ± 0.06	98.79 ± 0.09	98.50 ± 0.13	96.71 ± 0.21	98.70 ± 0.08
ijcnn1	98.31 ± 0.07	**98.48 ± 0.04**	98.40 ± 0.09	98.11 ± 0.10	90.69 ± 0.26	92.29 ± 0.16
intrusion	**99.77 ± 0.02**	88.20 ± 1.06	58.01 ± 26.6	87.66 ± 3.79	49.75 ± 30.7	65.15 ± 15.7
Average	**91.29 ± 1.26**	90.05 ± 2.06	84.60 ± 3.64	86.18 ± 2.84	79.96 ± 3.58	84.45 ± 2.83
Ranks	**1.500**	2.500	4.041	3.958	5.063	3.938

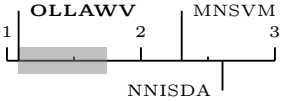




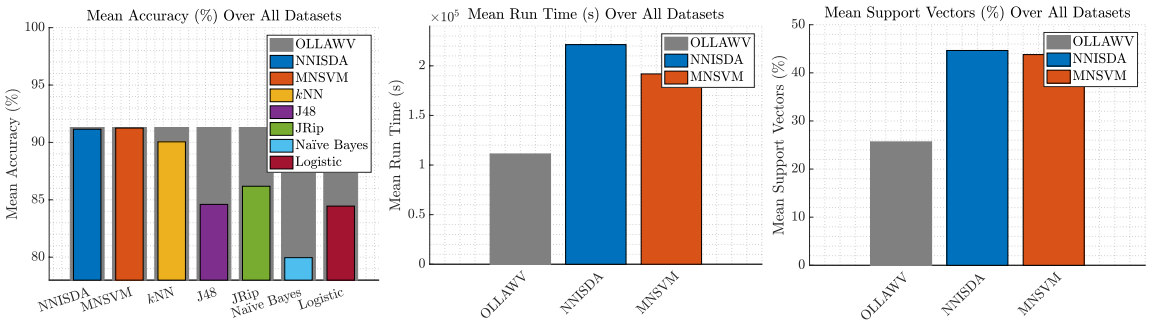
Accuracy (%)



Run Time (s)



Support Vectors (%)



$$C \in \{4^n\}, \quad n = \{-2, \ldots, 5\} \tag{5a}$$

$$\gamma \in \{4^n\}, \quad n = \{-5, \ldots, 2\} \tag{5b}$$

Algorithm Hyperparameters

Algorithm	Parameters
SVM	Penalty: $C \in \{4^n\}, n = \{-2, \ldots, 5\}$ RBF Kernel: $\gamma \in \{4^n\}, n = \{-5, \ldots, 2\}$
k-NN	Number of neighbors: $k \in \{1, 3, 5, 7\}$
J48	Pruning: {True, False} Pruning Confidence: {0.1, 0.25, 0.5}
JRip	Pruning: {True, False}
Naïve Bayes	Use kernel estimation: {True, False}
Logistic	Log-likelihood: $\{1e^{-7}, 1e^{-8}, 1e^{-9}\}$

$$\min_{(\boldsymbol{w},b)\in\mathcal{H}_o\times\mathbb{R}} R \;=\; \frac{1}{2}||\boldsymbol{w}||^2 + C\sum_{i=1}^{n} L(y_i,\, o_{(w,b)}(\boldsymbol{x}_i)),$$

$$\min_{(\boldsymbol{w},b)\in\mathcal{H}_o\times\mathbb{R}} R \;=\; \frac{1}{2}||\boldsymbol{w}||^2 + CL(y_i,\, o_{(w,b)}(\boldsymbol{x}_i)).$$

$$\min_{\boldsymbol{w}\in\mathcal{H}_o\times\mathbb{R}} R \;=\; \frac{1}{2}||\boldsymbol{w}||^2 + C\sum_{i=1}^{n} \max\left\{0, 1 - y_i o_{(w)}(\boldsymbol{x}_i)\right\}$$

$$\boldsymbol{w} \leftarrow \boldsymbol{w} - \eta\frac{\partial R}{\partial \boldsymbol{w}}$$
$$\boldsymbol{w} \leftarrow \boldsymbol{w} + \eta\begin{cases} Cy_i\boldsymbol{x_i} - \boldsymbol{w} & y_i o_i < 1 \\ -\boldsymbol{w} & \text{otherwise} \end{cases}$$

$$\sum_{i=1}^{n}\alpha_i\phi(\boldsymbol{x}_i) \leftarrow \sum_{i=1}^{n}\alpha_i\phi(\boldsymbol{x}_i) + \eta\begin{cases} Cy_i\phi(\boldsymbol{x_i}) - \sum_{i=1}^{n}\alpha_i\phi(\boldsymbol{x}_i) & y_i o_i < 1 \\ -\sum_{i=1}^{n}\alpha_i\phi(\boldsymbol{x}_i) & \text{otherwise} \end{cases}$$

$$\forall i : \alpha_i\phi(\boldsymbol{x}_i) \leftarrow \alpha_i\phi(\boldsymbol{x}_i) + \eta\begin{cases} (Cy_i\phi(\boldsymbol{x_i}) - \alpha_i\phi(\boldsymbol{x}_i)) & y_i o_i < 1 \\ (-\alpha_i\phi(\boldsymbol{x}_i)) & \text{otherwise} \end{cases}$$
$$\forall i : \alpha_i \leftarrow \alpha_i + \eta\begin{cases} (Cy_i - \alpha_i) & y_i o_i < 1 \\ (-\alpha_i) & \text{otherwise} \end{cases}$$

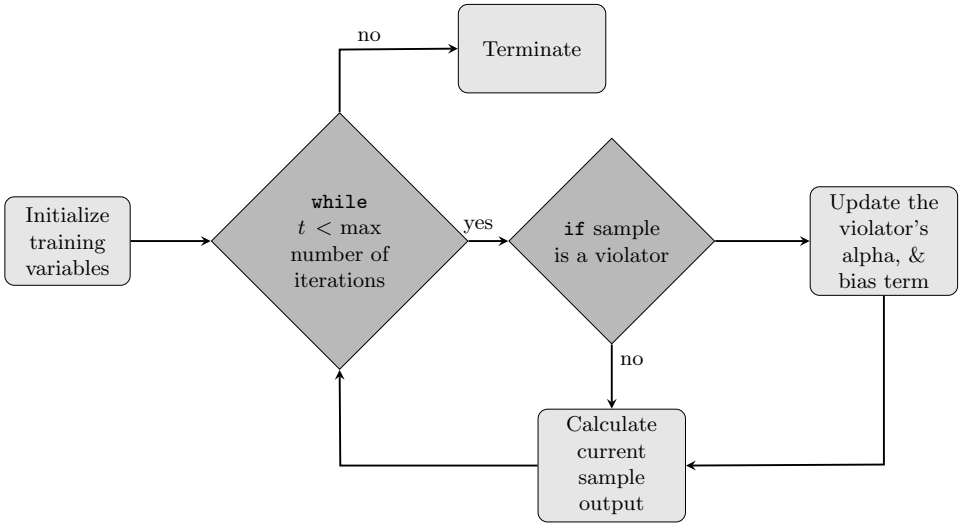
$$\Lambda \leftarrow Cy_i$$
$$\alpha_i \leftarrow \alpha_i + \eta\Lambda$$
$$b \leftarrow b + \eta\Lambda$$

$$\forall i : b \leftarrow b + \eta\frac{Cy_i}{n}$$

$$\boldsymbol{o} \leftarrow \boldsymbol{o} + \Lambda * \mathcal{K}\left(\boldsymbol{x}_{\neg \boldsymbol{S}},\, \boldsymbol{x}_{wv}, \gamma\right) + B \tag{6}$$

Comparison of OLLAWV vs. OLLA-L2

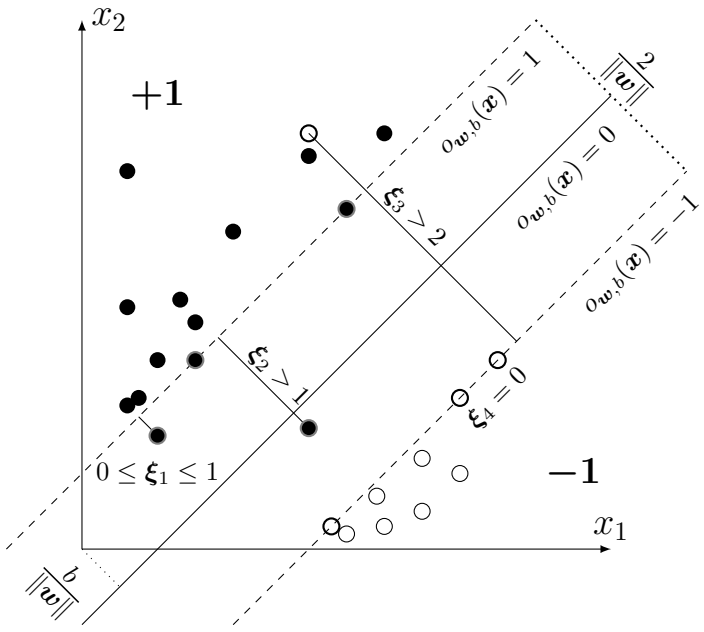
Dataset	Accuracy (%)		Run Time (s)	
	OLLA-L2	OLLAWV	OLLA-L2	OLLAWV
RBFNoDrift	93.07	**94.21**	**0.0238**	0.0329
HyperplaneSlow	87.40	**90.09**	**0.0261**	0.0353
HyperplaneFaster	87.40	**89.51**	**0.0256**	0.0263
STAGGERGeneratorF1	**100.0**	**100.0**	0.0034	**0.0021**
HyperplaneFaster02	87.41	**89.49**	**0.0257**	0.0268
MixedGeneratorBT	92.45	**98.00**	**0.0108**	0.0205
MixedGeneratorBF	92.55	**98.03**	**0.0107**	0.0299
SineGeneratorF1BF	97.37	**97.79**	**0.0091**	0.0122
SineGeneratorF2BF	97.37	**97.79**	**0.0091**	0.0121
STAGGERGeneratorF1BF	**100.0**	**100.0**	0.0035	**0.0021**
STAGGERGeneratorF2BF	**100.0**	**100.0**	0.0039	**0.0022**
HyperplaneFasterAN0	87.40	**89.51**	**0.0255**	0.0263
HyperplaneFasterAN5	87.29	**89.29**	**0.0258**	0.0264
SEASuddenAN0	84.01	**87.80**	0.0494	**0.0208**
SEASuddenAN05	83.69	**87.53**	0.0494	**0.0284**
Average	91.83	**93.94**	**0.0201**	0.0203
Rank	1.900	**1.100**	**1.3333**	1.6667



OnLine Learning Algorithm - List 2 (OLLA-L2)

Input: $\boldsymbol{X}, \boldsymbol{Y}, \beta, n, e$
Output: $\boldsymbol{\alpha}$, b, $\boldsymbol{S}$

$\boldsymbol{\alpha} \leftarrow \mathbf{0}, b \leftarrow 0, \boldsymbol{S} \leftarrow \mathbf{0}$, $\boldsymbol{o} \leftarrow \mathbf{0}$, $i \leftarrow 0$ ▷ Initialize model and algorithm parameters
for $t = 1, \ldots, n * e$ **do**
    $\eta \leftarrow 2/\sqrt{t}$ ▷ Learning rate in function of time
    if $y_i\, o_i \leq 1$ **then** ▷ Check if current sample is a violator
        Calculate Λ *and* P ▷ Calculate update parameters
        $\boldsymbol{S} \leftarrow [\boldsymbol{S} \cup i]$ ▷ Save index of current violator
        $\alpha_i \leftarrow \alpha_i + (\Lambda - \mathrm{P})$ ▷ Update violator's alpha value
        $b \leftarrow b + (\Lambda - \mathrm{P})\beta$ ▷ Update bias term
    end if
    $i \leftarrow i + 1$ ▷ Get new sample
    if $i = n$ **then** ▷ If the sample index exceeds the number of samples
        $i = 0$ ▷ Reset sample index
    end if
    $o_i \leftarrow \boldsymbol{K}(\boldsymbol{x}_i, \boldsymbol{x}_{\boldsymbol{S}})\, \boldsymbol{\alpha}_{\boldsymbol{S}} + b$ ▷ Calculate the new sample's output value
end for


x_2
$+1$
$o_{\boldsymbol{w},b}(\boldsymbol{x}) = 1$
$\frac{2}{\|\boldsymbol{w}\|}$
$o_{\boldsymbol{w},b}(\boldsymbol{x}) = 0$
$o_{\boldsymbol{w},b}(\boldsymbol{x}) = -1$
$\boldsymbol{\xi}_3 > 2$
$\boldsymbol{\xi}_2 > 1$
$\boldsymbol{\xi}_4 = 0$
$0 \leq \boldsymbol{\xi}_1 \leq 1$
-1
x_1
$\frac{b}{\|\boldsymbol{w}\|}$

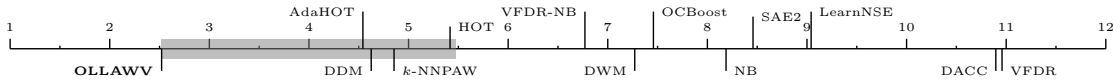
OnLine Learning Algorithm - List 2 (OLLA-L2)

Input: $\boldsymbol{X}, \boldsymbol{Y}, \beta, n, e$
Output: $\boldsymbol{\alpha}$, b, $\boldsymbol{S}$

$\boldsymbol{\alpha} \leftarrow \mathbf{0}, b \leftarrow 0, \boldsymbol{S} \leftarrow \mathbf{0}$, $\boldsymbol{o} \leftarrow \mathbf{0}$, $i \leftarrow 0$ ▷ Initialize model and algorithm parameters
for $t = 1, \ldots, n * e$ **do**
    $\eta \leftarrow 2/\sqrt{t}$ ▷ Learning rate in function of time
    if $y_i\, o_i \leq 1$ **then** ▷ Check if current sample is a violator
        Calculate Λ *and* P ▷ Calculate update parameters
        $\boldsymbol{S} \leftarrow [\boldsymbol{S} \cup i]$ ▷ Save index of current violator
        $\alpha_i \leftarrow \alpha_i + (\Lambda - \mathrm{P})$ ▷ Update violator's alpha value
        $b \leftarrow b + (\Lambda - \mathrm{P})\beta$ ▷ Update bias term
    end if
    $i \leftarrow i + 1$ ▷ Get new sample
    if $i = n$ **then** ▷ If the sample index exceeds the number of samples
        $i = 0$ ▷ Reset sample index
    end if
    $o_i \leftarrow \boldsymbol{K}(\boldsymbol{x}_i, \boldsymbol{x}_{\boldsymbol{S}})\, \boldsymbol{\alpha}_{\boldsymbol{S}} + b$ ▷ Calculate the new sample's output value
end for

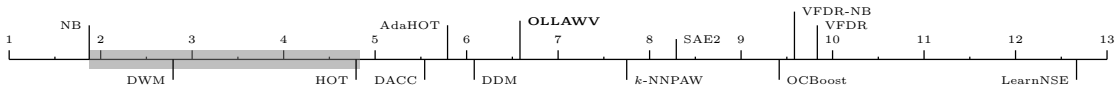
Accuracy (%) for Data Stream Classifiers

Dataset	OLLAWV	HOT	AdaHOT	NB	k-NNPAW	DDM	VFDR	VFDR-NB	SAE2	LearnNSE	DWM	DACC	OCBoost
CovType	**90.28**	85.34	86.22	60.04	87.89	59.56	60.32	75.58	76.21	69.97	71.96	61.70	71.29
Census	93.76	94.70	**94.74**	87.02	93.65	91.85	93.65	84.06	90.13	84.14	91.40	90.37	93.47
Shuttle	**99.67**	98.18	98.52	90.02	99.26	98.49	88.40	96.06	90.24	93.79	89.91	92.03	74.21
RBFNoDrift	**94.21**	92.94	92.96	71.99	93.75	92.48	77.53	81.71	89.16	70.28	70.43	65.01	92.08
LEDNoDrift	73.83	73.85	73.84	**73.94**	65.83	73.64	41.16	73.75	67.60	67.84	71.15	48.27	17.44
HyperplaneSlow	**90.09**	82.10	82.42	77.69	84.03	81.57	68.88	85.19	82.67	86.20	88.06	80.66	85.78
HyperplaneFaster	**89.51**	82.72	85.34	77.23	84.27	84.33	78.63	85.18	83.01	86.50	86.76	81.15	87.80
RBFGradualRecurring	98.41	94.63	94.44	58.33	**98.43**	93.47	60.08	86.16	88.48	72.87	74.96	61.91	49.62
RBFBlips	**99.07**	95.67	95.60	60.83	98.94	94.92	66.90	88.35	89.04	77.53	79.98	68.27	47.46
WaveformGenerator	83.94	82.99	**84.14**	80.41	80.13	83.61	63.88	75.84	80.18	80.19	78.39	73.59	55.05
STAGGERGeneratorF1	**100.0**	99.99	99.99	**100.0**	**100.0**	**100.0**	99.91	**100.0**	95.04	89.82	**100.0**	99.96	**100.0**
HyperplaneFaster02	**89.49**	82.77	85.36	77.25	84.27	87.64	78.89	85.11	83.04	86.51	86.76	81.23	87.59
RBFGradualRecurringv2	**97.18**	93.29	93.00	57.47	95.73	93.19	57.96	80.71	84.45	62.41	63.79	49.42	48.88
MixedGeneratorBT	98.00	99.11	**99.32**	91.93	97.67	99.11	83.12	93.28	93.16	90.91	91.19	89.16	98.98
MixedGeneratorBF	98.03	99.18	**99.36**	92.04	97.59	99.20	89.96	94.30	93.41	90.76	91.46	88.61	98.94
RandomRBFGeneratorC4A25	**99.12**	97.43	97.14	81.59	98.69	96.89	72.33	89.03	90.61	78.23	79.67	63.02	52.16
RandomRBFGeneratorC4A50	**99.74**	99.16	99.14	91.90	99.17	98.99	80.63	95.63	92.00	86.03	90.45	73.78	50.66
SineGeneratorF1BF	97.79	**99.75**	99.73	93.55	95.54	99.66	94.83	95.90	94.51	92.55	93.30	92.20	99.51
SineGeneratoF2BF	97.79	**99.75**	99.73	93.55	95.41	99.66	95.26	96.10	94.57	92.55	93.34	92.20	99.48
STAGGERGeneratorF1BF	**100.0**	99.99	99.99	**100.0**	**100.0**	**100.0**	99.91	**100.0**	95.04	89.82	**100.0**	99.96	**100.0**
STAGGERGeneratorF2BF	**100.0**	99.98	99.98	**100.0**	**100.0**	99.98	99.87	**100.0**	95.02	44.41	**100.0**	**100.0**	0.61
HyperplaneFasterAN5	**89.29**	82.69	85.24	87.38	84.19	88.74	79.14	84.89	82.92	86.41	86.67	81.12	87.38
SEASuddenAN0	87.80	84.92	85.18	88.23	87.22	**88.97**	81.56	85.17	85.11	85.77	86.93	83.73	88.23
SEASuddenAN05	87.53	84.57	84.82	87.53	86.96	**88.28**	81.55	85.11	84.57	85.63	86.54	83.53	87.53
Average	**93.94**	91.90	92.34	82.50	92.03	91.43	78.93	88.21	87.51	81.30	85.55	79.20	73.92
Rank	**2.52**	5.42	4.54	8.19	4.8542	4.63	10.96	6.77	8.46	9.04	7.27	10.90	7.46



Training Time (seconds) for Data Stream Classifiers

Dataset	OLLAWV	HOT	AdaHOT	NB	k-NNPAW	DDM	VFDR	VFDR-NB	SAE2	LearnNSE	DWM	DACC	OCBoost
CovType	0.0451	0.0765	0.0909	**0.0009**	0.0387	0.0577	0.0371	0.0445	0.1121	6.2806	0.0419	0.0357	0.0984
Census	0.0168	0.1040	0.1162	**0.0007**	0.0386	0.0345	0.0735	0.0458	0.0363	0.9368	0.0156	0.0175	0.0543
Shuttle	0.0103	0.0276	0.0298	**0.0004**	0.0386	0.0069	0.0108	0.0069	0.0217	0.2692	0.0104	0.0186	0.0581
RBFNoDrift	0.0329	0.0174	0.0247	**0.0002**	0.0383	0.0828	0.4267	0.3172	0.0491	3.4029	0.0104	0.0137	0.0402
LEDNoDrift	0.0697	0.0373	0.0649	**0.0003**	0.0389	0.0245	0.0097	0.0082	0.0694	5.4734	0.0175	0.0290	0.0547
HyperplaneSlow	0.0353	0.0094	0.0142	**0.0002**	0.0389	0.1405	0.1871	0.3006	0.0473	3.3734	0.0091	0.0142	0.0389
HyperplaneFaster	0.0263	0.0295	0.0416	**0.0002**	0.0384	0.1231	0.3016	0.3784	0.0424	3.2991	0.0098	0.0143	0.0376
RBFGradualRecurring	0.0456	0.0316	0.0369	**0.0003**	0.0389	0.0391	1.1284	0.9295	0.1004	11.7980	0.0335	0.0474	0.1070
RBFBlips	0.0396	0.0243	0.0299	**0.0003**	0.0384	0.0278	0.7587	0.8342	0.0948	11.9934	0.0342	0.0473	0.1050
WaveformGenerator	0.0394	0.0751	0.0965	**0.0006**	0.0390	0.1847	0.9670	0.9271	0.1715	17.9675	0.0511	0.0743	0.1318
STAGGERGeneratorF1	0.0021	0.0006	0.0007	**0.0001**	0.0389	0.0005	0.0013	0.0007	0.0018	0.5517	0.0003	0.0020	0.0113
HyperplaneFaster02	0.0268	0.0302	0.0465	**0.0002**	0.0388	0.0260	0.3192	0.3010	0.0456	3.2893	0.0095	0.0141	0.0372
RBFGradualRecurringv2	0.0545	0.0169	0.0203	**0.0003**	0.0383	0.0859	0.7622	0.7634	0.0552	12.0529	0.0367	0.0464	0.1053
MixedGeneratorBT	0.0205	0.0027	0.0026	**0.0001**	0.0336	0.0071	1.7640	1.5534	0.0086	0.9761	0.0024	0.0052	0.0204
MixedGeneratorBF	0.0299	0.0024	0.0024	**0.0001**	0.0334	0.0065	1.1035	1.0924	0.0093	0.8971	0.0024	0.0058	0.0209
RandomRBFGeneratorC4A25	0.0200	0.0441	0.0407	**0.0003**	0.0358	0.0730	1.7831	1.6927	0.1245	13.9876	0.0395	0.0719	0.1580
RandomRBFGeneratorC4A50	0.0157	0.0333	0.0337	**0.0007**	0.0350	0.0909	2.8802	2.5559	0.1917	27.8456	0.0752	0.1432	0.2974
SineGeneratorF1BF	0.0122	0.0056	0.0056	**0.0001**	0.0334	0.0073	1.9301	2.9169	0.0127	1.3768	0.0034	0.0071	0.0235
SineGeneratoF2BF	0.0121	0.0051	0.0053	**0.0001**	0.0314	0.0077	5.0598	5.9477	0.0117	1.3996	0.0027	0.0074	0.0236
STAGGERGeneratorF1BF	0.0021	0.0006	0.0006	**0.0001**	0.0323	0.0005	0.0006	0.0005	0.0015	0.6175	0.0003	0.0022	0.0143
STAGGERGeneratorF2BF	0.0022	0.0010	0.0009	**0.0001**	0.0340	0.0006	0.0005	0.0005	0.0017	0.6752	0.0003	0.0020	0.0139
HyperplaneFasterAN5	0.0264	0.0200	0.0200	0.0440	0.0269	0.0115	0.2298	0.2417	0.0312	2.5014	**0.0080**	0.0147	0.0740
SEASuddenAN0	0.0208	0.0051	0.0052	0.0268	0.0312	0.0099	0.0370	0.0373	0.0176	1.3545	**0.0032**	0.0059	0.0335
SEASuddenAN05	0.0284	0.0055	0.0055	0.0257	0.0309	0.0095	0.0321	0.0334	0.0167	1.3365	**0.0032**	0.0061	0.0326
Average	0.0264	0.0252	0.0307	**0.0043**	0.0359	0.0441	0.8252	0.8721	0.0531	5.5690	0.0175	0.0269	0.0663
Rank	6.5833	4.7917	5.7917	**1.8750**	7.7500	6.0833	9.8333	9.5833	8.2917	12.6667	2.7917	5.5417	9.4167

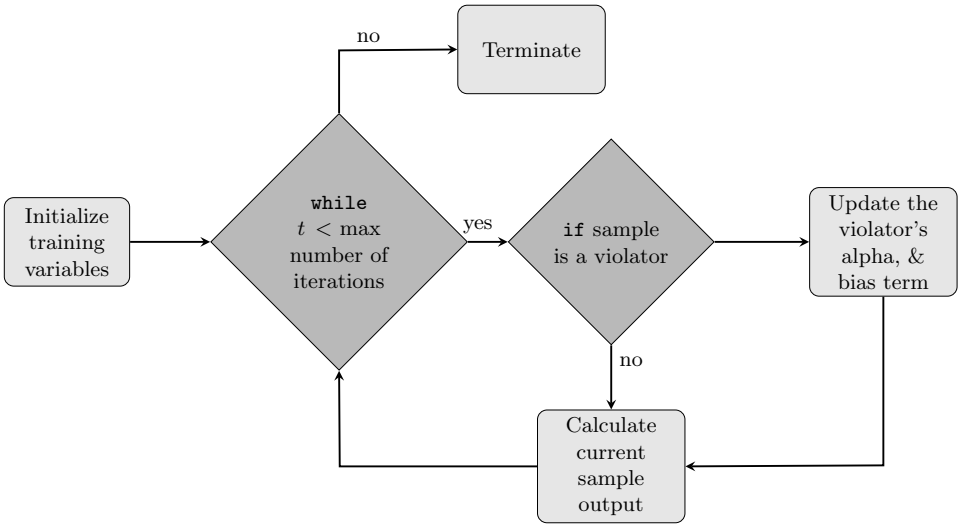


Algorithms

Algorithm	Description
HOT	Hoeffding Option Tree
AdaHOT	Adaptive Hoeffding Option Tree
NB	Naïve Bayes
k-NNPAW	k-NN with Probabilistic Adaptive Windows
DDM	Drift Detection Method with HOT
VFDR	Very Fast Decision Rules
VFDR-NB	VFDR with Naïve Bayes
SAE2	Social Adaptive Ensemble 2
Learn.NSE	Learn++ for Non-Stationary Environments
DWM	Dynamic Weighted Majority
DACC	Dynamic Adaptation to Concept Changes
OCBoost	Online Coordinate Boosting

Base Streamed Datasets & Generators

Dataset	# Samples	# Attributes	# Classes
Static			
Shuttle	57,999	10	7
Census	299,284	42	2
CovType	581,012	55	7
Generators			
RandomRBFGenerator	1,000,000	10	2
LEDGenerator	1,000,000	2	10
HyperplaneGenerator	1,000,000	10	2
WaveformGenerator	1,000,000	40	3
STAGGERGenerator	1,000,000	3	2
MixedGenerator	1,000,000	4	2
SineGenerator	1,000,000	2	2
SEAGenerator	1,000,000	2	2


Initialize training variables
while $t <$ max number of iterations
no
Terminate
yes
if sample is a violator
Update the violator's alpha, & bias term
no
Calculate current sample output