$$
\begin{tikzcd}
 & \mathcal{D}_1 : [\boldsymbol{X}][\boldsymbol{Y}_1] \arrow[r] & h_1 : \mathcal{D}_1 \to \hat{\boldsymbol{Y}}_{\mathbf{1}} \\
 & \mathcal{D}_2 : [\boldsymbol{X}][\boldsymbol{Y}_2] \arrow[r] & h_2 : \mathcal{D}_2 \to \hat{\boldsymbol{Y}}_{\mathbf{2}} \\
\mathcal{D} : [\boldsymbol{X}][\boldsymbol{Y}] \arrow[uur, bend left] \arrow[ur, bend left] \arrow[dr, bend right] & \vdots & \\
 & \mathcal{D}_m : [\boldsymbol{X}][\boldsymbol{Y}_m] \arrow[r] & h_m : \mathcal{D}_m \to \hat{\boldsymbol{Y}}_{\boldsymbol{m}}
\end{tikzcd}
$$

Multi-Target Support Vector Regression (SVR)

Input: Training dataset $\mathcal{D}$
Output: ST models $h_j, j = 1, \ldots, m$

for $j = 1$ to m **do**
  $\mathcal{D}_j = \{\boldsymbol{X}, \boldsymbol{Y}_j\}$ ▷ Get ST data
  $h_j : \boldsymbol{X} \to \mathbb{R}$ ▷ Build ST model for the j^{th} target
end for

Build Chained Model

Input: Training dataset $\mathcal{D}$, random chain $\boldsymbol{C}$
Output: A chained model $h_j, j = \{1, \ldots, m\}$

$\mathcal{D}_1 = \{\boldsymbol{X}, \boldsymbol{Y}_{\boldsymbol{C}_1}\}$ ▷ Initialize first dataset
for $j = 1$ to m **do** ▷ For each target in chain $\boldsymbol{C}$
  $h_j : \mathcal{D}_j \to \mathbb{R}$ ▷ Train model on appended dataset
  if $j < m$ **then**
    $\mathcal{D}_{j+1} = \left\{\mathcal{D}_j, \boldsymbol{Y}_{\boldsymbol{C}_j}\right\}$ ▷ Append new target in chain to dataset
  end if
end for

$$
\begin{tikzcd}[column sep=small]
 & \mathcal{D} : [\boldsymbol{X}][\boldsymbol{Y}_1\boldsymbol{Y}_2\boldsymbol{Y}_3] \arrow[dl] \arrow[d] \arrow[drr] & & \\
{[1,2,3]} \arrow[d] & {[1,3,2]} \arrow[d] & \ldots & {[3,2,1]} \arrow[d] \\
h_1 : [\boldsymbol{X}] \to \hat{\boldsymbol{Y}}_{\mathbf{1}} \arrow[d] & h_1 : [\boldsymbol{X}] \to \hat{\boldsymbol{Y}}_{\mathbf{1}} \arrow[d] & \ldots & h_1 : [\boldsymbol{X}] \to \hat{\boldsymbol{Y}}_{\mathbf{1}} \arrow[d] \\
h_2 : [\boldsymbol{X}\boldsymbol{Y}_{\mathbf{1}}] \to \hat{\boldsymbol{Y}}_{\mathbf{2}} \arrow[d] & h_2 : [\boldsymbol{X}\boldsymbol{Y}_{\mathbf{1}}] \to \hat{\boldsymbol{Y}}_{\mathbf{3}} \arrow[d] & \ldots & h_2 : [\boldsymbol{X}\boldsymbol{Y}_{\mathbf{3}}] \to \hat{\boldsymbol{Y}}_{\mathbf{2}} \arrow[d] \\
h_3 : [\boldsymbol{X}\boldsymbol{Y}_{\mathbf{1}}\boldsymbol{Y}_{\mathbf{2}}] \to \hat{\boldsymbol{Y}}_{\mathbf{3}} & h_3 : [\boldsymbol{X}\boldsymbol{Y}_{\mathbf{1}}\boldsymbol{Y}_{\mathbf{3}}] \to \hat{\boldsymbol{Y}}_{\mathbf{2}} & \ldots & h_3 : [\boldsymbol{X}\boldsymbol{Y}_{\mathbf{3}}\boldsymbol{Y}_{\mathbf{2}}] \to \hat{\boldsymbol{Y}}_{\mathbf{1}}
\end{tikzcd}
$$

Multi-Target SVR with Random-Chains (SVRRC)

Input: Training dataset $\mathcal{D}$, c random chains $\mathcal{C}$
Output: An ensemble of chained models $h_{\mathcal{C}}$

for each $\boldsymbol{C} \in \mathcal{C}$ **do** ▷ For each random chain
  $h_{\boldsymbol{C}} \leftarrow$ `buildChainedModel`$(\mathcal{D}, \boldsymbol{C})$ ▷ Build a chained model for chain $\boldsymbol{C}$
end for

$$\mathcal{D} : [\boldsymbol{X}][\boldsymbol{Y}_1\boldsymbol{Y}_2\boldsymbol{Y}_3] \xrightarrow[\frac{\mathbf{E}\left[(Y_i-\mu_i)(Y_j-\mu_j)\right]}{\sqrt{\mathbf{E}\left[(Y_i-\mu_i)(Y_i-\mu_i)\right]\mathbf{E}\left[(Y_j-\mu_j)(Y_j-\mu_j)\right]}}]{\textit{generate maximum correlation chain}} [1, 2, 3]$$

$$h_1 : [\boldsymbol{X}] \rightarrow \boldsymbol{\hat{Y}_1} \longrightarrow h_2 : [\boldsymbol{X}\boldsymbol{Y_1}] \rightarrow \boldsymbol{\hat{Y}_2} \longrightarrow h_3 : [\boldsymbol{X}\boldsymbol{Y_1}\boldsymbol{Y_2}] \rightarrow \boldsymbol{\hat{Y}_3}$$

Multi-Target SVR with max-Correlation Chain (SVRCC)

$\mathbf{P} = corrcoef(\boldsymbol{Y})$ ▷ Find correlation coefficient matrix for target variables
$\boldsymbol{C} = \sum_{i=1}^{n} |\mathbf{P}_{ij}|, \forall j = 1, \ldots, m$ ▷ Sum rows of the correlation matrix
$\boldsymbol{C} = \texttt{sort}\,(\boldsymbol{C}, \textbf{decreasing})$ ▷ Sort sums in decreasing order
$h_{\boldsymbol{C}} = \texttt{buildChainedModel}(\mathcal{D}, \boldsymbol{C})$ ▷ Build a max-correlation chained model

---

Average Relative Root Mean Square Error (aRRMSE) for MT regressors

Datasets	MORF	ST	MTS	MTSC	RC	ERC	ERCC	SVR	SVRRC	SVRCC
Slump	0.6939	0.6886	0.6690	0.6938	0.7019	0.7022	0.6886	0.5765	**0.5545**	0.5560
Polymer	0.6159	0.5971	0.5778	0.6493	0.6270	0.6544	0.6131	0.5573	0.5253	**0.5116**
Andro	0.5097	0.5979	0.5155	0.5633	0.5924	0.5885	0.5666	0.4856	0.4651	**0.4455**
EDM	0.7337	0.7442	0.7413	0.7446	0.7449	0.7452	0.7443	0.7058	0.7070	**0.6978**
Solar Flare 1	1.3046	1.1357	1.1168	1.0758	0.9951	1.0457	1.0887	0.9917	0.9455	**0.9320**
Jura	0.5969	0.5874	0.5906	0.5892	0.5910	0.5896	0.5880	0.5952	**0.5764**	0.5885
Enb	0.1210	0.1165	0.1231	0.1211	0.1268	0.1250	0.1139	0.0977	0.0910	**0.0899**
Solar Flare 2	1.4167	1.1503	**0.9483**	1.0840	1.0092	1.0522	1.0928	1.0385	1.0253	1.0298
Wisconsin Cancer	0.9413	0.9314	0.9308	0.9336	**0.9305**	0.9313	0.9323	0.9555	0.9483	0.9427
California Housing	0.6611	0.6447	0.6974	0.6630	0.7131	0.6690	0.6146	0.6130	0.5945	**0.5852**
Stock	0.1653	0.1844	0.1787	0.1803	0.1802	0.1789	0.1752	0.1364	**0.1337**	0.1388
SCPF	0.8273	0.8348	0.8436	0.8308	0.8263	0.8105	0.8290	0.8164	0.8037	**0.8013**
Puma8NH	0.7858	0.8142	0.8118	0.8311	0.8199	0.8205	0.8207	**0.7655**	0.7744	0.7676
Friedman	0.9394	0.9214	0.9231	0.9210	0.9231	0.9209	0.9204	0.9218	0.9208	**0.9196**
Puma32H	0.9406	**0.8713**	0.8727	0.8791	0.8752	0.8729	0.8740	0.9364	0.9367	0.9319
Water Quality	**0.8994**	0.9085	0.9109	0.9093	0.9121	0.9097	0.9057	0.9343	0.9310	0.9045
M5SPEC	0.5910	0.5523	0.5974	0.5671	0.5552	0.5542	0.5558	0.2951	0.2935	**0.2925**
MP5SPEC	0.5522	0.5120	0.5683	0.5133	0.5145	0.5143	0.5119	0.2484	**0.2323**	0.2358
MP6SPEC	0.5553	0.5152	0.5686	0.5119	0.5198	0.5187	0.5109	0.2850	0.2669	**0.2623**
ATP7d	0.5563	0.5308	**0.5141**	0.5142	0.5558	0.5397	0.5182	0.5455	0.5371	0.5342
OES97	0.5490	0.5230	0.5229	0.5217	0.5239	0.5237	0.5222	0.4641	**0.4618**	0.4635
Osales	0.7596	0.7471	**0.7086**	0.7268	0.8318	0.7258	0.7101	0.7924	0.7924	0.7811
ATP1d	0.4173	0.3732	0.3733	0.3712	0.3790	**0.3696**	0.3721	0.3773	0.3707	0.3775
OES10	0.4518	0.4174	0.4176	0.4171	0.4178	0.4180	0.4166	0.3570	0.3555	**0.3538**
Average	0.6910	0.6625	0.6551	0.6589	0.6611	0.6575	0.6536	0.6039	0.5935	**0.5893**
Ranks	7.5000	5.7708	5.9375	6.1667	7.4375	6.3750	4.9792	4.7708	3.2708	**2.7917**

Run Time (seconds) for MT regressors

Datasets	MORF	ST	MTS	MTSC	RC	ERC	ERCC	SVR	SVRRC	SVRCC
Slump	38.1	2.6	9.9	15.9	1.8	11.1	50.5	**0.6**	1.9	0.7
Polymer	7.6	2.7	9.1	15.5	1.9	14.9	80.5	**0.5**	2.6	**0.5**
Andro	25.7	4.4	15.0	34.2	3.4	33.2	197.9	**1.1**	6.2	**1.1**
EDM	24.8	2.8	9.4	18.1	2.1	5.8	19.0	**0.9**	1.0	**0.9**
Solar Flare 1	34.1	3.5	13.6	26.7	2.7	17.7	86.9	**2.3**	9.3	2.6
Jura	64.3	7.9	31.8	74.3	6.4	43.5	254.2	**4.7**	18.7	5.3
Enb	71.4	6.6	26.1	63.6	**5.4**	15.6	69.6	11.3	17.7	15.9
Solar Flare 2	55.4	7.4	30.7	68.0	**6.3**	42.9	241.5	9.4	53.5	15.6
Wisconsin Cancer	51.4	6.1	21.9	53.7	4.9	14.8	61.6	**2.0**	2.4	**2.0**
California Housing	93.0	9.7	34.8	75.9	**8.2**	21.3	102.0	15.8	25.2	23.6
Stock	93.7	11.7	46.8	96.7	**11.0**	75.4	427.3	18.5	90.5	26.3
SCPF	66.3	19.3	65.9	176.3	**15.0**	104.2	734.2	32.8	162.8	48.8
Puma8NH	130.4	29.7	106.7	288.6	**27.9**	201.6	1227.7	94.1	516.6	177.1
Friedman	79.5	27.0	81.2	258.3	25.0	273.7	2871.6	**12.3**	322.3	18.8
Puma32H	93.9	68.1	181.0	635.0	87.7	667.9	6087.0	**32.2**	1018.7	53.1
Water Quality	108.4	**93.1**	262.1	912.3	127.2	925.4	10993.3	110.2	2567.9	189.5
M5SPEC	89.8	68.9	166.3	604.6	73.7	262.3	3132.1	**39.2**	546.7	45.1
MP5SPEC	84.5	94.6	221.2	888.3	91.5	557.0	6864.1	**49.3**	1132.1	58.4
MP6SPEC	90.3	93.4	212.6	871.0	89.1	557.6	6761.3	**47.2**	1227.1	58.5
ATP7d	**70.5**	262.6	452.1	2319.8	242.1	1779.2	24373.8	80.0	1897.4	136.5
OES97	**83.4**	485.3	1146.6	4928.9	499.8	5315.0	58072.1	148.2	3759.1	342.6
Osales	**92.0**	1094.8	2340.7	8322.2	986.5	11361.2	122265.3	437.0	4830.1	843.6
ATP1d	**70.7**	272.9	476.5	2568.9	261.9	2138.9	26768.9	95.0	2127.8	174.4
OES10	**90.0**	738.9	1633.6	6682.9	688.5	7150.8	83533.1	229.1	5419.4	577.1
Average	71.2	142.2	316.5	1250.0	136.2	1316.3	14803.2	**61.4**	1073.2	117.4
Ranks	5.5	3.71	6.0	8.29	3.0	7.08	9.92	**1.88**	6.71	2.92

$$\min_{\boldsymbol{w},b,\boldsymbol{\xi}} \frac{1}{2}||\boldsymbol{w}||^2 + C\sum_{I}\xi_I,$$
$$\text{s.t. } Y_I(\langle \boldsymbol{w}, \boldsymbol{x}_{s_I}\rangle + b) \geq 1 - \xi_I,\ \forall I \in \{1,\ldots,n\},$$
$$\xi_I \geq 0,\ \forall I \in \{1,\ldots,n\},$$
$$s_I = \underset{i\in I}{\operatorname{argmax}}(\langle \boldsymbol{w},\, \boldsymbol{x}_i\rangle + b),\ \forall I \in \{1,\ldots,n\}$$

$$\max_{\boldsymbol{\alpha}} \sum_I \alpha_I - \frac{1}{2}\sum_I \sum_{K\in I} \alpha_I \alpha_K Y_I Y_K \mathcal{K}\left(\boldsymbol{x}_{s_I}, \boldsymbol{x}_{s_K}\right)$$
$$\text{s.t. } \sum_I \alpha_I Y_I = 0,$$
$$0 \leq \alpha_I \leq C,\ \forall I \in \{1,\ldots,n\},$$
$$s_I = \underset{i\in I}{\operatorname{argmax}}(\boldsymbol{o}_I),\ \forall I \in \{1,\ldots,n\}$$

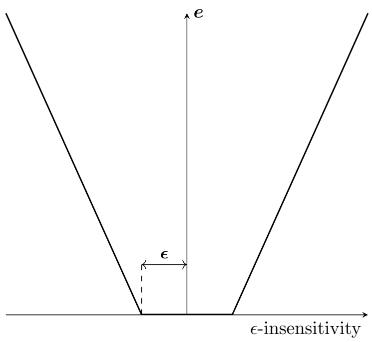


Figure 1: Vapnik's ϵ-insensitivity loss function.

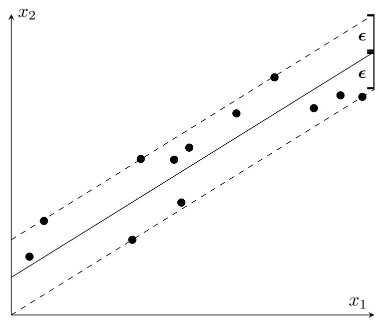
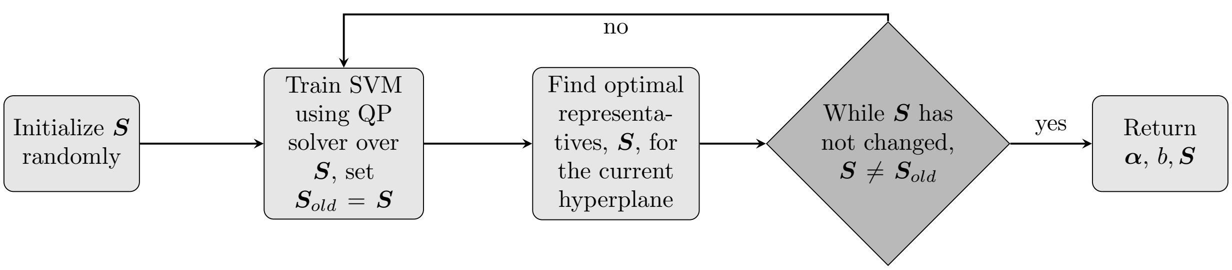


Figure 2: Linear support vector regression example solution on a toy 2D dataset.



Multi-Instance Representative SVM (MIRSVM)

Input: Training dataset $\mathcal{D}$, SVM Parameters C and σ
Output: SVM model parameters $\boldsymbol{\alpha}$ and b, Bag Representative IDs $\boldsymbol{S}$

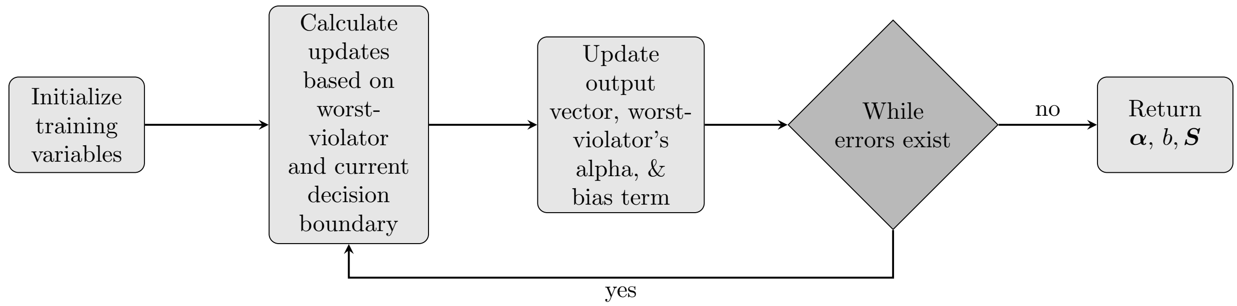
```
for I ∈ {1,...,n} do
    S_I ← rand(|B_I|, 1, 1)                          ▷ Assign each bag a random instance
end for
while S ≠ S_old do
    S_old ← S
    X_S ← X(S), Y_S ← Y(S)                           ▷ Initialize the representative dataset
    G ← (Y_S × Y_S) · K(X_S, X_S, σ)                 ▷ Build Gram matrix
    α ← quadprog(G, −1^n, Y_S, 0^n, 0^n, C^n)        ▷ Solve QP Problem
    sv ← find(0 < α ≤ C)                             ▷ Get the support vector indices
    n_sv ← count(0 < α ≤ C)                          ▷ Get the number of support vectors
    b ← (1/n_sv) Σ_{i=1}^{n_sv} (Y_sv − G_sv ∗ (α_sv · Y_sv))   ▷ Calculate the bias term
    for I ∈ {1,...,n} do
        G_I ← (Y_I × Y_S) · K(B_I, X_S, σ)
        S_I ← argmax_{i∈I}(G_I ∗ α + b)              ▷ Select optimal bag-representatives
    end for
end while
```

$$\min_{(\boldsymbol{w},b)\in\mathcal{H}_o\times\mathbb{R}} R \;=\; \frac{1}{2}||\boldsymbol{w}||^2 + C\sum_{i=1}^{n} L(y_i,\, o_{(w,b)}(\boldsymbol{x}_i)) \tag{1}$$

$$L(y_i,\, o_{(w,b)}(\boldsymbol{x}_i)) = \max\left\{0, 1 - y_i o_{(w,b)}(\boldsymbol{x}_i)\right\} \tag{2}$$

$$\min_{(\boldsymbol{w},b)\in\mathcal{H}_o\times\mathbb{R}} R \;=\; \frac{1}{2}||\boldsymbol{w}||^2 + C\sum_{i=1}^{n}(|y_i - o_{(w,b)}(\boldsymbol{x_i})|_\epsilon) \tag{3}$$

$$L(y_i,\, o_{(w,b)}(\boldsymbol{x}_i)) = \begin{cases} 0 & if|y_i - o_{(w,b)}(\boldsymbol{x_i})| \leq \epsilon \\ |y_i - o_{(w,b)}(\boldsymbol{x_i})| - \epsilon & \text{otherwise.} \end{cases} \tag{4}$$



OnLine Learning Algorithm using Worst-Violators (OLLAWV)

Input: $\mathcal{D}$, C, γ, β, M
Output: $\boldsymbol{\alpha}$, b, $\boldsymbol{S}$

$\boldsymbol{\alpha} \leftarrow \mathbf{0}$, $b \leftarrow 0, \boldsymbol{S} \leftarrow \mathbf{0}$ ▷ Initialize OLLAWV model parameters
$\boldsymbol{o} \leftarrow \mathbf{0}$, $t \leftarrow 0$ ▷ Initialize the output vector and iteration counter
$wv \leftarrow 0$, $yo \leftarrow y_{wv} * \boldsymbol{o}_{wv}$ ▷ Initialize hinge loss error and worst-violator index
while $yo < M$ **do**
  $t \leftarrow t + 1$
  $\eta \leftarrow 2/\sqrt{t}$ ▷ Learning rate

  $\Lambda \leftarrow \eta * C * y_{wv}$ ▷ Calculate hinge loss update
  $B \leftarrow (\Lambda * \beta)/n$ ▷ Calculate bias update
  $\boldsymbol{o} \leftarrow \boldsymbol{o} + \Lambda * \mathcal{K}(\boldsymbol{x}_{\neg \boldsymbol{S}}, \boldsymbol{x}_{wv}, \gamma) + B$ ▷ Update output vector
  $\boldsymbol{\alpha}_{wv} \leftarrow \boldsymbol{\alpha}_{wv} + \Lambda$ ▷ Update worst-violator's alpha value
  $b \leftarrow b + B$ ▷ Update bias term

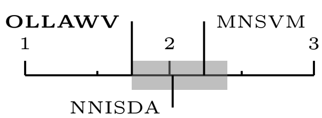
  $\boldsymbol{S}_t \leftarrow wv$ ▷ Save index of worst-violator
  $[yo, wv] \leftarrow \min\limits_{wv \in \{\neg \boldsymbol{S}\}} \{y_{wv} \cdot o_{wv}\}$ ▷ Find the worst-violator
end while

Classification Datasets

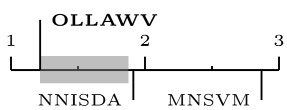
Dataset	# Samples	# Attributes	# Classes
small datasets			
iris	150	4	3
teach	151	5	3
wine	178	13	3
cancer	198	32	2
sonar	208	60	2
glass	214	9	6
vote	232	16	2
heart	270	13	2
dermatology	366	33	6
prokaryotic	997	20	3
eukaryotic	2,427	20	4
medium datasets			
optdigits	5,620	64	10
satimage	6,435	36	6
usps	9,298	256	10
pendigits	10,992	16	10
reuters	11,069	8,315	2
letter	20,000	16	26
large datasets			
adult	48,842	123	2
w3a	49,749	300	2
shuttle	58,000	7	7
web (w8a)	64,700	300	2
ijcnn1	141,691	22	2
intrusion	5,209,460	127	2

Comparison of OLLAWV vs. NNISDA and MNSVM

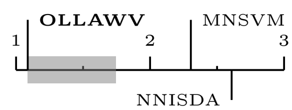
Dataset	Accuracy (%)			Run Time (s)			Support Vectors (%)		
	OLLAWV	NNISDA	MNSVM	OLLAWV	NNISDA	MNSVM	OLLAWV	NNISDA	MNSVM
small datasets									
iris	**97.33**	94.00	96.67	**0.05**	0.27	3.57	**13.50**	40.20	29.80
teach	52.32	52.31	**52.95**	**0.12**	0.44	8.85	**69.19**	99.80	87.40
wine	**98.87**	96.60	96.60	**0.28**	0.43	4.84	**15.02**	44.40	48.60
cancer	80.36	**81.86**	81.38	**0.49**	0.85	4.46	**42.79**	83.80	89.60
sonar	**92.32**	89.48	87.57	**0.59**	0.98	3.03	**31.26**	73.00	66.00
glass	**72.41**	67.81	69.30	**0.46**	1.01	11.94	**62.84**	90.80	87.60
vote	**96.54**	96.11	93.99	**0.26**	0.46	1.49	**13.36**	33.20	34.00
heart	82.22	**83.33**	**83.33**	**0.50**	0.91	6.45	**37.69**	73.00	82.00
dermatology	97.82	**98.36**	**98.36**	**1.62**	2.47	11.68	**36.94**	59.00	59.80
prokaryotic	88.96	88.86	**88.97**	**6.09**	10.64	50.86	**29.01**	51.20	49.00
eukaryotic	77.38	79.56	**81.21**	61.95	**49.16**	342.76	**54.11**	76.40	72.60
medium datasets									
optdigits	99.11	99.29	**99.31**	**411**	528	787	**28.64**	31.60	30.60
satimage	91.66	**92.39**	92.35	1,334	**687**	1,094	**20.72**	45.00	44.80
usps	97.49	98.05	**98.24**	10,214	**5,245**	7,777	**11.22**	29.40	28.00
pendigits	99.56	**99.62**	99.61	**723**	909	1,500	**10.27**	17.60	16.60
reuters	98.03	**98.08**	97.99	**954**	1,368	1,657	**8.770**	18.20	18.60
letter	96.99	99.11	**99.13**	**5,259**	12,009	26,551	**43.56**	57.60	56.60
large datasets									
adult	84.75	85.07	**85.13**	**21,025**	72,552	123,067	**34.66**	56.00	56.60
w3a	**98.86**	98.82	98.82	**6,532**	15,951	24,562	**3.270**	14.60	12.40
shuttle	99.77	99.83	**99.87**	**2,833**	7,420	45,062	**2.010**	6.00	16.40
web	98.94	**99.00**	99.00	**12,067**	30,583	38,040	**4.320**	13.20	10.80
ijcnn1	98.31	99.34	**99.41**	**162,587**	296,917	370,144	16.36	11.00	**7.600**
intrusion	**99.77**	99.67	99.66	**2,402,804**	4,646,810	3,772,113	**0.780**	2.000	1.700
Average	**91.29**	91.15	91.25	**114,209**	221,350	191,861	**25.66**	44.65	43.79
Ranks	**1.739**	2.022	2.239	**1.217**	1.913	2.869	**1.087**	2.609	2.304



Bonferroni-Dunn test for Accuracy



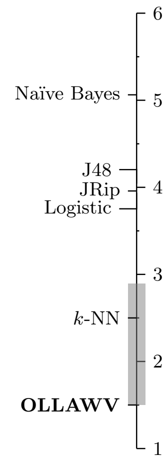
Bonferroni-Dunn test for Run Time

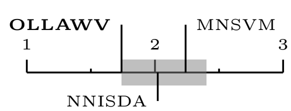


Bonferroni-Dunn test for % Support Vectors

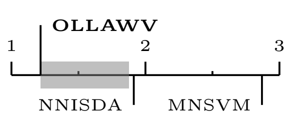
Accuracy (%) for Non-SVM Methods vs. OLLAWV

Dataset	OLLAWV	k-NN	J48	JRip	Naïve Bayes	Logistic
small datasets						
iris	**97.33 ± 1.49**	96.00 ± 3.65	94.00 ± 2.79	90.67 ± 4.35	96.00 ± 2.79	97.33 ± 2.79
teach	52.32 ± 3.46	**59.64 ± 2.89**	49.72 ± 7.58	56.75 ± 9.60	53.75 ± 6.46	51.77 ± 6.68
wine	**98.87 ± 1.54**	97.73 ± 3.72	90.43 ± 5.83	93.24 ± 3.27	96.60 ± 3.14	96.05 ± 2.58
cancer	**80.36 ± 5.80**	77.32 ± 6.93	73.81 ± 8.57	73.78 ± 5.81	67.73 ± 5.07	77.32 ± 7.78
sonar	**92.32 ± 3.11**	88.99 ± 4.59	76.16 ± 10.6	75.18 ± 6.77	73.69 ± 7.65	75.18 ± 7.31
glass	**72.41 ± 2.28**	67.73 ± 5.91	65.06 ± 5.51	65.59 ± 9.66	49.46 ± 5.19	62.04 ± 5.75
vote	**96.54 ± 1.87**	92.26 ± 3.19	95.70 ± 2.12	96.54 ± 2.45	92.24 ± 3.24	93.54 ± 2.59
heart	82.22 ± 2.93	79.63 ± 5.71	78.52 ± 2.81	80.74 ± 4.06	**84.44 ± 4.46**	83.33 ± 3.93
dermatology	**97.82 ± 0.05**	96.18 ± 1.78	94.52 ± 2.21	91.27 ± 5.08	97.28 ± 1.64	96.98 ± 2.28
prokaryotic	**88.96 ± 2.14**	87.96 ± 3.01	78.54 ± 1.62	79.13 ± 2.78	62.38 ± 3.54	87.57 ± 2.56
eukaryotic	77.38 ± 1.96	**81.42 ± 2.06**	65.27 ± 2.92	66.42 ± 3.47	39.27 ± 3.43	69.55 ± 1.34
medium datasets						
optdigits	**99.11 ± 0.38**	98.74 ± 0.39	90.87 ± 1.09	91.28 ± 0.40	92.42 ± 0.75	95.05 ± 0.91
satimage	**91.66 ± 0.80**	90.38 ± 0.72	85.64 ± 1.21	85.33 ± 0.77	85.41 ± 0.92	88.14 ± 1.11
usps	**97.49 ± 0.22**	97.04 ± 0.47	88.73 ± 0.46	89.20 ± 1.00	79.45 ± 0.59	91.88 ± 0.65
pendigits	**99.56 ± 0.12**	99.33 ± 0.17	96.24 ± 0.31	96.34 ± 0.41	88.34 ± 0.65	95.59 ± 0.18
reuters	**98.03 ± 0.22**	97.15 ± 0.43	96.90 ± 0.32	97.18 ± 0.44	93.52 ± 0.02	69.54 ± 0.28
letter	**96.99 ± 0.21**	95.71 ± 0.19	87.34 ± 0.68	87.02 ± 0.66	74.12 ± 0.97	77.45 ± 0.16
large datasets						
adult	**84.75 ± 0.26**	83.85 ± 0.28	84.38 ± 0.28	83.73 ± 0.17	80.57 ± 0.09	82.46 ± 0.14
w3a	**98.86 ± 0.04**	98.60 ± 0.06	98.71 ± 0.05	98.41 ± 0.10	96.71 ± 0.20	98.61 ± 0.12
shuttle	99.77 ± 0.03	99.93 ± 0.03	**99.97 ± 0.02**	99.96 ± 0.02	98.57 ± 0.24	96.83 ± 0.12
web	**98.94 ± 0.05**	98.89 ± 0.06	98.79 ± 0.09	98.50 ± 0.13	96.71 ± 0.21	98.70 ± 0.08
ijcnn1	98.31 ± 0.07	**98.48 ± 0.04**	98.40 ± 0.09	98.11 ± 0.10	90.69 ± 0.26	92.29 ± 0.16
intrusion	**99.77 ± 0.02**	88.20 ± 1.06	58.01 ± 26.6	87.66 ± 3.79	49.75 ± 30.7	65.15 ± 15.7
Average	**91.29 ± 1.26**	90.05 ± 2.06	84.60 ± 3.64	86.18 ± 2.84	79.96 ± 3.58	84.45 ± 2.83
Ranks	**1.500**	2.500	4.041	3.958	5.063	3.938

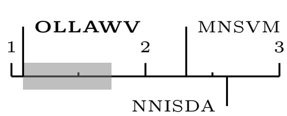




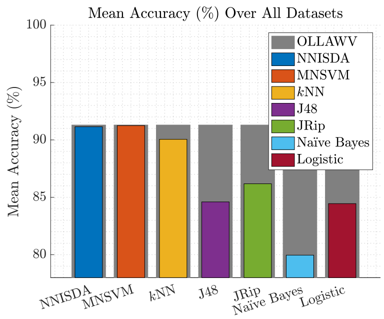
Accuracy (%)

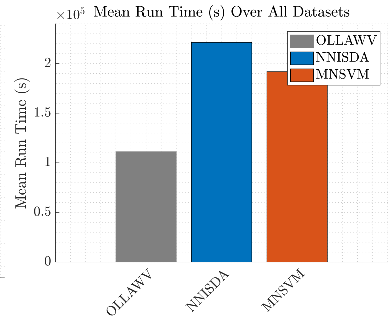


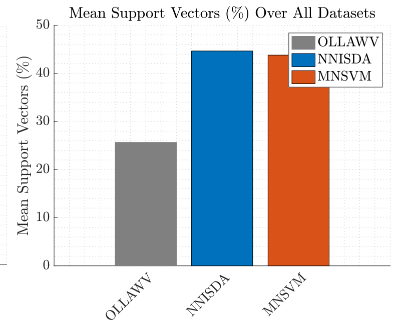
Run Time (s)



Support Vectors (%)







$$C \in \{4^n\}, \quad n = \{-2, \ldots, 5\} \tag{5a}$$
$$\gamma \in \{4^n\}, \quad n = \{-5, \ldots, 2\} \tag{5b}$$

Algorithm Hyperparameters

Algorithm	Parameters
SVM	Penalty: $C \in \{4^n\}, n = \{-2, \ldots, 5\}$ RBF Kernel: $\gamma \in \{4^n\}, n = \{-5, \ldots, 2\}$
k-NN	Number of neighbors: $k \in \{1, 3, 5, 7\}$
J48	Pruning: {True, False} Pruning Confidence: {0.1, 0.25, 0.5}
JRip	Pruning: {True, False}
Naïve Bayes	Use kernel estimation: {True, False}
Logistic	Log-likelihood: $\{1\mathrm{e}^{-7}, 1\mathrm{e}^{-8}, 1\mathrm{e}^{-9}\}$

$$\min_{(\boldsymbol{w},b)\in\mathcal{H}_o\times\mathbb{R}} R \;=\; \frac{1}{2}||\boldsymbol{w}||^2 + C\sum_{i=1}^{n} L(y_i,\, o_{(w,b)}(\boldsymbol{x}_i)),$$

$$\min_{(\boldsymbol{w},b)\in\mathcal{H}_o\times\mathbb{R}} R \;=\; \frac{1}{2}||\boldsymbol{w}||^2 + CL(y_j,\, o_{(w,b)}(\boldsymbol{x}_j)).$$

$$\min_{\boldsymbol{w}\in\mathcal{H}_o\times\mathbb{R}} R \;=\; \frac{1}{2}||\boldsymbol{w}||^2 + C\sum_{i=1}^{n} \max\left\{0, 1 - y_i o_{(w)}(\boldsymbol{x}_i)\right\}$$

$$\boldsymbol{w} \leftarrow \boldsymbol{w} - \eta\frac{\partial R}{\partial \boldsymbol{w}}$$

$$\boldsymbol{w} \leftarrow \boldsymbol{w} + \eta \begin{cases} Cy_i\boldsymbol{x_i} - \boldsymbol{w} & y_i o_i < 1 \\ -\boldsymbol{w} & \text{otherwise} \end{cases}$$

$$\sum_{i=1}^{n} \alpha_i\phi(\boldsymbol{x}_i) \leftarrow \sum_{i=1}^{n} \alpha_i\phi(\boldsymbol{x}_i) + \eta \begin{cases} Cy_i\phi(\boldsymbol{x_i}) - \sum_{i=1}^{n}\alpha_i\phi(\boldsymbol{x}_i) & y_i o_i < 1 \\ -\sum_{i=1}^{n}\alpha_i\phi(\boldsymbol{x}_i) & \text{otherwise} \end{cases}$$

$$\forall i : \alpha_i\phi(\boldsymbol{x}_i) \leftarrow \alpha_i\phi(\boldsymbol{x}_i) + \eta \begin{cases} (Cy_i\phi(\boldsymbol{x_i}) - \alpha_i\phi(\boldsymbol{x}_i)) & y_i o_i < 1 \\ (-\alpha_i\phi(\boldsymbol{x}_i)) & \text{otherwise} \end{cases}$$

$$\forall i : \alpha_i \leftarrow \alpha_i + \eta \begin{cases} (Cy_i - \alpha_i) & y_i o_i < 1 \\ (-\alpha_i) & \text{otherwise} \end{cases}$$

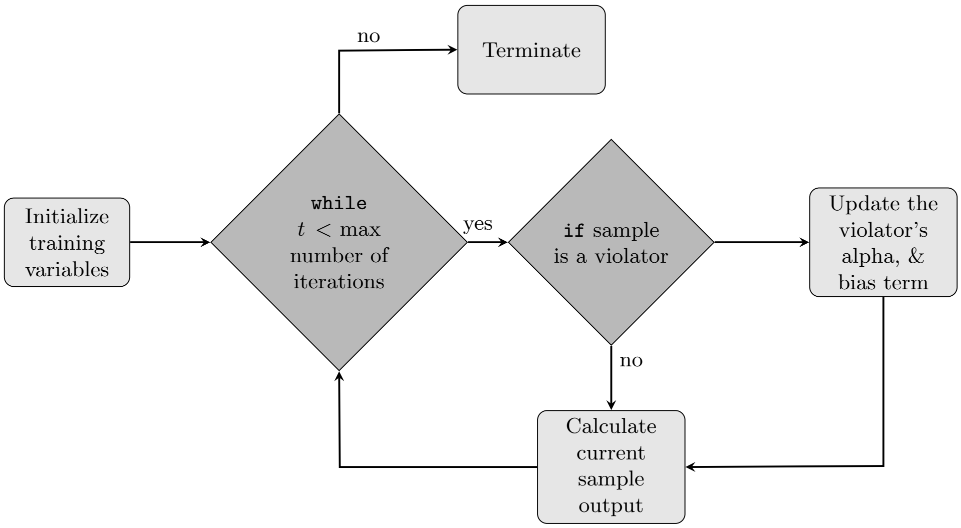
$$\Lambda \leftarrow Cy_i$$
$$\alpha_i \leftarrow \alpha_i + \eta\Lambda$$
$$b \leftarrow b + \eta\Lambda$$

$$\forall i : b \leftarrow b + \eta\frac{Cy_i}{n}$$

$$\boldsymbol{o} \leftarrow \boldsymbol{o} + \Lambda * \mathcal{K}\left(\boldsymbol{x}_{\neg \boldsymbol{S}},\, \boldsymbol{x}_{wv}, \gamma\right) + B \tag{6}$$

Comparison of OLLAWV vs. OLLA-L2

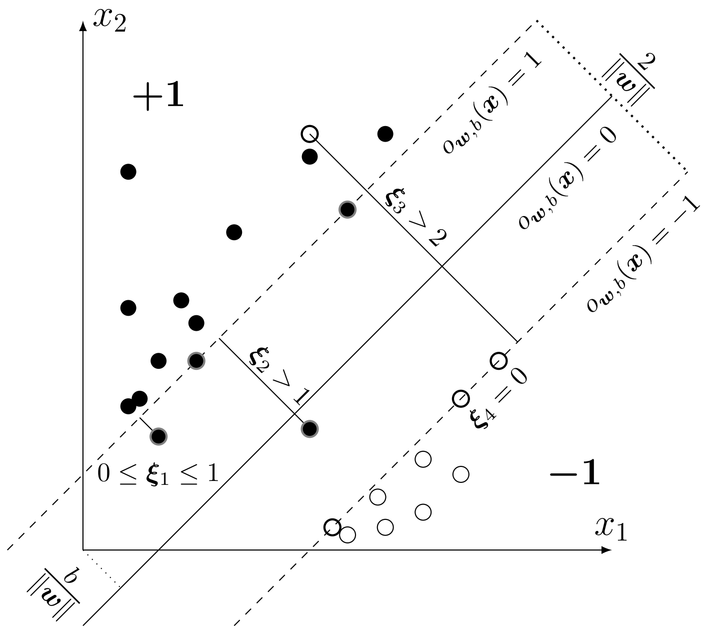
Dataset	Accuracy (%)		Run Time (s)	
	OLLA-L2	OLLAWV	OLLA-L2	OLLAWV
RBFNoDrift	93.07	**94.21**	**0.0238**	0.0329
HyperplaneSlow	87.40	**90.09**	**0.0261**	0.0353
HyperplaneFaster	87.40	**89.51**	**0.0256**	0.0263
STAGGERGeneratorF1	**100.0**	**100.0**	0.0034	**0.0021**
HyperplaneFaster02	87.41	**89.49**	**0.0257**	0.0268
MixedGeneratorBT	92.45	**98.00**	**0.0108**	0.0205
MixedGeneratorBF	92.55	**98.03**	**0.0107**	0.0299
SineGeneratorF1BF	97.37	**97.79**	**0.0091**	0.0122
SineGeneratorF2BF	97.37	**97.79**	**0.0091**	0.0121
STAGGERGeneratorF1BF	**100.0**	**100.0**	0.0035	**0.0021**
STAGGERGeneratorF2BF	**100.0**	**100.0**	0.0039	**0.0022**
HyperplaneFasterAN0	87.40	**89.51**	**0.0255**	0.0263
HyperplaneFasterAN5	87.29	**89.29**	**0.0258**	0.0264
SEASuddenAN0	84.01	**87.80**	0.0494	**0.0208**
SEASuddenAN05	83.69	**87.53**	0.0494	**0.0284**
Average	91.83	**93.94**	**0.0201**	0.0203
Rank	1.900	**1.100**	**1.3333**	1.6667



OnLine Learning Algorithm - List 2 (OLLA-L2)

Input: $\boldsymbol{X}, \boldsymbol{Y}, \beta, n, e$
Output: $\boldsymbol{\alpha}$, b, $\boldsymbol{S}$

$\boldsymbol{\alpha} \leftarrow \mathbf{0}, b \leftarrow 0, \boldsymbol{S} \leftarrow \mathbf{0}$, $\boldsymbol{o} \leftarrow \mathbf{0}$, $i \leftarrow 0$ ▷ Initialize model and algorithm parameters
for $t = 1, \ldots, n * e$ **do**
  $\eta \leftarrow 2/\sqrt{t}$ ▷ Learning rate in function of time
  if $y_i \, o_i \leq 1$ **then** ▷ Check if current sample is a violator
    Calculate Λ *and* P ▷ Calculate update parameters
    $\boldsymbol{S} \leftarrow [\boldsymbol{S} \cup i]$ ▷ Save index of current violator
    $\alpha_i \leftarrow \alpha_i + (\Lambda - \mathrm{P})$ ▷ Update violator's alpha value
    $b \leftarrow b + (\Lambda - \mathrm{P})\beta$ ▷ Update bias term
  end if
  $i \leftarrow i + 1$ ▷ Get new sample
  if $i = n$ **then** ▷ If the sample index exceeds the number of samples
    $i = 0$ ▷ Reset sample index
  end if
  $o_i \leftarrow \boldsymbol{K}(\boldsymbol{x}_i, \, \boldsymbol{x}_{\boldsymbol{S}}) \, \boldsymbol{\alpha}_{\boldsymbol{S}} + b$ ▷ Calculate the new sample's output value
end for


x_2
$+1$
$o_{w,b}(x) = 1$
$\frac{2}{\|w\|}$
$o_{w,b}(x) = 0$
$o_{w,b}(x) = -1$
$\xi_3 > 2$
$\xi_2 > 1$
$\xi_4 = 0$
$0 \leq \xi_1 \leq 1$
-1
x_1
$\frac{b}{\|w\|}$

OnLine Learning Algorithm - List 2 (OLLA-L2)

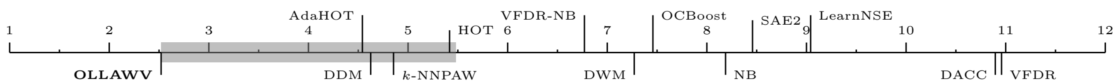
Input: $\boldsymbol{X}, \boldsymbol{Y}, \beta, n, e$

Output: $\boldsymbol{\alpha}$, b, $\boldsymbol{S}$

$\boldsymbol{\alpha} \leftarrow \mathbf{0}, b \leftarrow 0, \boldsymbol{S} \leftarrow \mathbf{0}$, $\boldsymbol{o} \leftarrow \mathbf{0}$, $i \leftarrow 0$ ▷ Initialize model and algorithm parameters
for $t = 1, \ldots, n * e$ **do**
  $\eta \leftarrow 2/\sqrt{t}$ ▷ Learning rate in function of time
  if $y_i\, o_i \leq 1$ **then** ▷ Check if current sample is a violator
    Calculate Λ *and* P ▷ Calculate update parameters
    $\boldsymbol{S} \leftarrow [\boldsymbol{S} \cup i]$ ▷ Save index of current violator
    $\alpha_i \leftarrow \alpha_i + (\Lambda - \mathrm{P})$ ▷ Update violator's alpha value
    $b \leftarrow b + (\Lambda - \mathrm{P})\beta$ ▷ Update bias term
  end if
  $i \leftarrow i + 1$ ▷ Get new sample
  if $i = n$ **then** ▷ If the sample index exceeds the number of samples
    $i = 0$ ▷ Reset sample index
  end if
  $o_i \leftarrow \boldsymbol{K}(\boldsymbol{x}_i,\, \boldsymbol{x}_{\boldsymbol{S}})\, \boldsymbol{\alpha}_{\boldsymbol{S}} + b$ ▷ Calculate the new sample's output value
end for

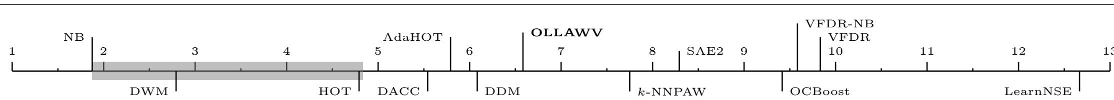
Accuracy (%) for Data Stream Classifiers

Dataset	OLLAWV	HOT	AdaHOT	NB	k-NNPAW	DDM	VFDR	VFDR-NB	SAE2	LearnNSE	DWM	DACC	OCBoost
CovType	**90.28**	85.34	86.22	60.04	87.89	59.56	60.32	75.58	76.21	69.97	71.96	61.70	71.29
Census	93.76	94.70	**94.74**	87.02	93.65	91.85	93.65	84.06	90.13	84.14	91.40	90.37	93.47
Shuttle	**99.67**	98.18	98.52	90.02	99.26	98.49	88.40	96.06	90.24	93.79	89.91	92.03	74.21
RBFNoDrift	**94.21**	92.94	92.96	71.99	93.75	92.48	77.53	81.71	89.16	70.28	70.43	65.01	92.08
LEDNoDrift	73.83	73.85	73.84	**73.94**	65.83	73.64	41.16	73.75	67.60	67.84	71.15	48.27	17.44
HyperplaneSlow	**90.09**	82.10	82.42	77.69	84.03	81.57	68.88	85.19	82.67	86.20	88.06	80.66	85.78
HyperplaneFaster	**89.51**	82.72	85.34	77.23	84.27	84.33	78.63	85.18	83.01	86.50	86.76	81.15	87.80
RBFGradualRecurring	98.41	94.63	94.44	58.33	**98.43**	93.47	60.08	86.16	88.48	72.87	74.96	61.91	49.62
RBFBlips	**99.07**	95.67	95.60	60.83	98.94	94.92	66.90	88.35	89.04	77.53	79.98	68.27	47.46
WaveformGenerator	83.94	82.99	**84.14**	80.41	80.13	83.61	63.88	75.84	80.18	80.19	78.39	73.59	55.05
STAGGERGeneratorF1	**100.0**	99.99	99.99	**100.0**	**100.0**	**100.0**	99.91	**100.0**	95.04	89.82	**100.0**	99.96	**100.0**
HyperplaneFaster02	**89.49**	82.77	85.36	77.25	84.27	87.64	78.89	85.11	83.04	86.51	86.76	81.23	87.59
RBFGradualRecurringv2	**97.18**	93.29	93.00	57.47	95.73	93.19	57.96	80.71	84.45	62.41	63.79	49.42	48.88
MixedGeneratorBT	98.00	99.11	**99.32**	91.93	97.67	99.11	83.12	93.28	93.16	90.91	91.19	89.16	98.98
MixedGeneratorBF	98.03	99.18	**99.36**	92.04	97.59	99.20	89.96	94.30	93.41	90.76	91.46	88.61	98.94
RandomRBFGeneratorC4A25	**99.12**	97.43	97.14	81.59	98.69	96.89	72.33	89.03	90.61	78.23	79.67	63.02	52.16
RandomRBFGeneratorC4A50	**99.74**	99.16	99.14	91.90	99.17	98.99	80.63	95.63	92.00	86.03	90.45	73.78	50.66
SineGeneratorF1BF	97.79	**99.75**	99.73	93.55	95.54	99.66	94.83	95.90	94.51	92.55	93.30	92.20	99.51
SineGeneratoF2BF	97.79	**99.75**	99.73	93.55	95.41	99.66	95.26	96.10	94.57	92.55	93.34	92.20	99.48
STAGGERGeneratorF1BF	**100.0**	99.99	99.99	**100.0**	**100.0**	**100.0**	99.91	**100.0**	95.04	89.82	**100.0**	99.96	**100.0**
STAGGERGeneratorF2BF	**100.0**	99.98	99.98	**100.0**	**100.0**	99.98	99.87	**100.0**	95.02	44.41	**100.0**	**100.0**	0.61
HyperplaneFasterAN5	**89.29**	82.69	85.24	87.38	84.19	88.74	79.14	84.89	82.92	86.41	86.67	81.12	87.38
SEASuddenAN0	87.80	84.92	85.18	88.23	87.22	**88.97**	81.56	85.17	85.11	85.77	86.93	83.73	88.23
SEASuddenAN05	87.53	84.57	84.82	87.53	86.96	**88.28**	81.55	85.11	84.57	85.63	86.54	83.53	87.53
Average	**93.94**	91.90	92.34	82.50	92.03	91.43	78.93	88.21	87.51	81.30	85.55	79.20	73.92
Rank	**2.52**	5.42	4.54	8.19	4.8542	4.63	10.96	6.77	8.46	9.04	7.27	10.90	7.46



Training Time (seconds) for Data Stream Classifiers

Dataset	OLLAWV	HOT	AdaHOT	NB	k-NNPAW	DDM	VFDR	VFDR-NB	SAE2	LearnNSE	DWM	DACC	OCBoost
CovType	0.0451	0.0765	0.0909	**0.0009**	0.0387	0.0577	0.0371	0.0445	0.1121	6.2806	0.0419	0.0357	0.0984
Census	0.0168	0.1040	0.1162	**0.0007**	0.0386	0.0345	0.0735	0.0458	0.0363	0.9368	0.0156	0.0175	0.0543
Shuttle	0.0103	0.0276	0.0298	**0.0004**	0.0386	0.0069	0.0108	0.0069	0.0217	0.2692	0.0104	0.0186	0.0581
RBFNoDrift	0.0329	0.0174	0.0247	**0.0002**	0.0383	0.0828	0.4267	0.3172	0.0491	3.4029	0.0104	0.0137	0.0402
LEDNoDrift	0.0697	0.0373	0.0649	**0.0003**	0.0389	0.0245	0.0097	0.0082	0.0694	5.4734	0.0175	0.0290	0.0547
HyperplaneSlow	0.0353	0.0094	0.0142	**0.0002**	0.0389	0.1405	0.1871	0.3006	0.0473	3.3734	0.0091	0.0142	0.0389
HyperplaneFaster	0.0263	0.0295	0.0416	**0.0002**	0.0384	0.1231	0.3016	0.3784	0.0424	3.2991	0.0098	0.0143	0.0376
RBFGradualRecurring	0.0456	0.0316	0.0369	**0.0003**	0.0389	0.0391	1.1284	0.9295	0.1004	11.7980	0.0335	0.0474	0.1070
RBFBlips	0.0396	0.0243	0.0299	**0.0003**	0.0384	0.0278	0.7587	0.8342	0.0948	11.9934	0.0342	0.0473	0.1050
WaveformGenerator	0.0394	0.0751	0.0965	**0.0006**	0.0390	0.1847	0.9670	0.9271	0.1715	17.9675	0.0511	0.0743	0.1318
STAGGERGeneratorF1	0.0021	0.0006	0.0007	**0.0001**	0.0389	0.0005	0.0013	0.0007	0.0018	0.5517	0.0003	0.0020	0.0113
HyperplaneFaster02	0.0268	0.0302	0.0465	**0.0002**	0.0388	0.0260	0.3192	0.3010	0.0456	3.2893	0.0095	0.0141	0.0372
RBFGradualRecurringv2	0.0545	0.0169	0.0203	**0.0003**	0.0383	0.0859	0.7622	0.7634	0.0552	12.0529	0.0367	0.0464	0.1053
MixedGeneratorBT	0.0205	0.0027	0.0026	**0.0001**	0.0336	0.0071	1.7640	1.5534	0.0086	0.9761	0.0024	0.0052	0.0204
MixedGeneratorBF	0.0299	0.0024	0.0024	**0.0001**	0.0334	0.0065	1.1035	1.0924	0.0093	0.8971	0.0024	0.0058	0.0209
RandomRBFGeneratorC4A25	0.0200	0.0441	0.0407	**0.0003**	0.0358	0.0730	1.7831	1.6927	0.1245	13.9876	0.0395	0.0719	0.1580
RandomRBFGeneratorC4A50	0.0157	0.0333	0.0337	**0.0007**	0.0350	0.0909	2.8802	2.5559	0.1917	27.8456	0.0752	0.1432	0.2974
SineGeneratorF1BF	0.0122	0.0056	0.0056	**0.0001**	0.0334	0.0073	1.9301	2.9169	0.0127	1.3768	0.0034	0.0071	0.0235
SineGeneratoF2BF	0.0121	0.0051	0.0053	**0.0001**	0.0314	0.0077	5.0598	5.9477	0.0117	1.3996	0.0027	0.0074	0.0236
STAGGERGeneratorF1BF	0.0021	0.0006	0.0006	**0.0001**	0.0323	0.0005	0.0006	0.0005	0.0015	0.6175	0.0003	0.0022	0.0143
STAGGERGeneratorF2BF	0.0022	0.0010	0.0009	**0.0001**	0.0340	0.0006	0.0005	0.0005	0.0017	0.6752	0.0003	0.0020	0.0139
HyperplaneFasterAN5	0.0264	0.0200	0.0200	0.0440	0.0269	0.0115	0.2298	0.2417	0.0312	2.5014	**0.0080**	0.0147	0.0740
SEASuddenAN0	0.0208	0.0051	0.0052	0.0268	0.0312	0.0099	0.0370	0.0373	0.0176	1.3545	**0.0032**	0.0059	0.0335
SEASuddenAN05	0.0284	0.0055	0.0055	0.0257	0.0309	0.0095	0.0321	0.0334	0.0167	1.3365	**0.0032**	0.0061	0.0326
Average	0.0264	0.0252	0.0307	**0.0043**	0.0359	0.0441	0.8252	0.8721	0.0531	5.5690	0.0175	0.0269	0.0663
Rank	6.5833	4.7917	5.7917	**1.8750**	7.7500	6.0833	9.8333	9.5833	8.2917	12.6667	2.7917	5.5417	9.4167

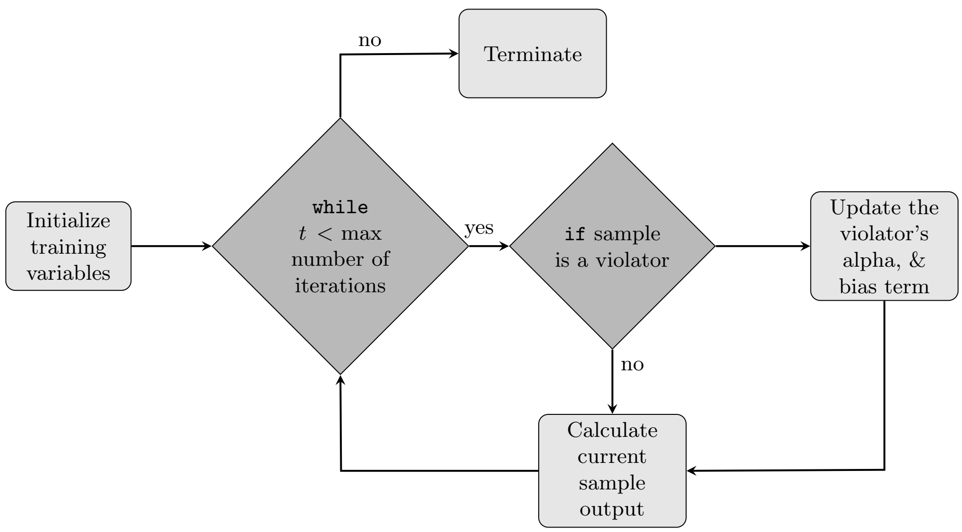


Algorithms

Algorithm	Description
HOT	Hoeffding Option Tree
AdaHOT	Adaptive Hoeffding Option Tree
NB	Naïve Bayes
k-NNPAW	k-NN with Probabilistic Adaptive Windows
DDM	Drift Detection Method with HOT
VFDR	Very Fast Decision Rules
VFDR-NB	VFDR with Naïve Bayes
SAE2	Social Adaptive Ensemble 2
Learn.NSE	Learn++ for Non-Stationary Environments
DWM	Dynamic Weighted Majority
DACC	Dynamic Adaptation to Concept Changes
OCBoost	Online Coordinate Boosting

Base Streamed Datasets & Generators

Dataset	# Samples	# Attributes	# Classes
Static			
Shuttle	57,999	10	7
Census	299,284	42	2
CovType	581,012	55	7
Generators			
RandomRBFGenerator	1,000,000	10	2
LEDGenerator	1,000,000	2	10
HyperplaneGenerator	1,000,000	10	2
WaveformGenerator	1,000,000	40	3
STAGGERGenerator	1,000,000	3	2
MixedGenerator	1,000,000	4	2
SineGenerator	1,000,000	2	2
SEAGenerator	1,000,000	2	2


Terminate
no
Initialize training variables
while $t <$ max number of iterations
yes
if sample is a violator
Update the violator's alpha, & bias term
no
Calculate current sample output