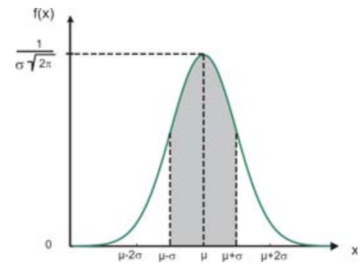
Normal (i.e., Gaussian) Distribution Function

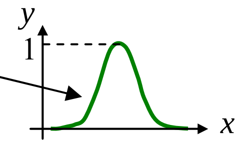
1-Dimensional Gaussian function, D=1

$$\mathcal{N}(x|\mu,\sigma^2) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}$$



This is a scaling factor only, making area below the curve = 1

Without it, Gaussian function looks as this.
Now, area is not 1 below the curve,
and the curve doesn't represent
probability density function, but it is still **the Bell-Shaped** curve.



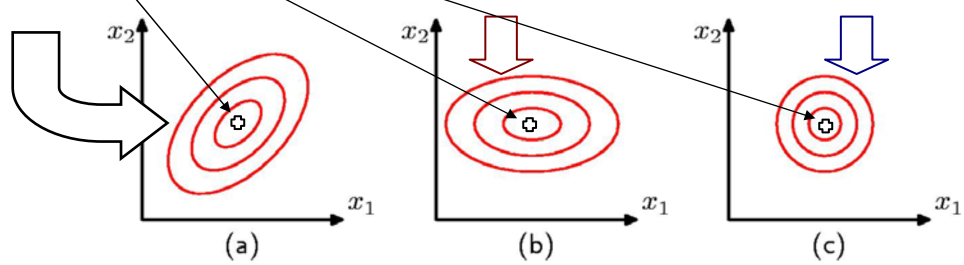
2 and D - Dimensional Gaussian function, for a 2-Dim aka bivariate D=2

$$\mathcal{N}(\mathbf{x}|\boldsymbol{\mu},\boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{D/2}}\frac{1}{|\boldsymbol{\Sigma}|^{1/2}} \exp\left\{\underbrace{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}_{\text{This is a scalar for any D}}\right\}$$

$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$, ρ is the correlation coefficient between x_1 and x_2. For ρ=0 $\Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$,

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \qquad \Sigma^{-1} = \begin{bmatrix} \frac{1}{\sigma_1^2} & \\ 0 & \frac{1}{\sigma_2^2} \end{bmatrix}, \qquad \Sigma^{-1} = \begin{bmatrix} \frac{1}{\sigma^2} & \\ 0 & \frac{1}{\sigma^2} \end{bmatrix},$$

$\sigma_1 \neq \sigma_2$, $\sigma_{12} = \sigma_{21} \neq 0$ $\quad$ $\sigma_1 \neq \sigma_2$, $\sigma_1 > \sigma_2$, $\sigma_{12} = \sigma_{21} = 0$ $\quad$ $\sigma_1 = \sigma_2$, $\sigma_{12} = \sigma_{21} = 0$



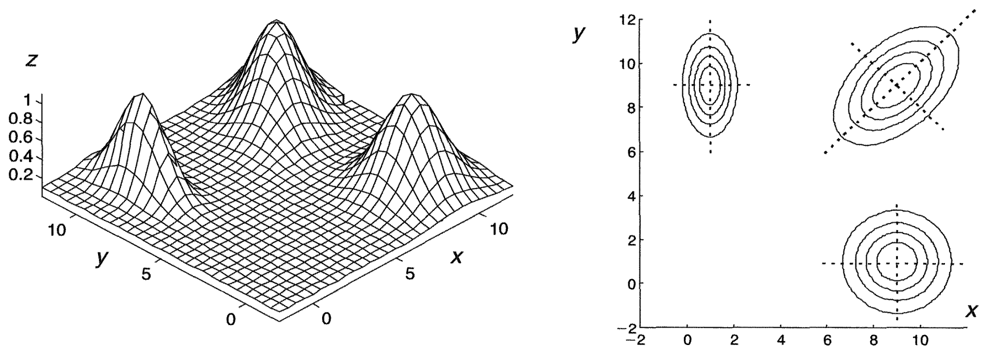


Figure 5.14
Left, three different normalized Gaussian basis functions. *Right*, their corresponding level curves or contours. The first RBF with a covariance matrix $\Sigma = [2.25 \quad 0; 0 \quad 2.25]$, $(\sigma_x = \sigma_y = 1.5)$, is placed at the center $(9, 1)$. The second Gaussian is nonradial, with a covariance matrix $\Sigma = [0.5625 \quad 0; 0 \quad 2.25]$, $(\sigma_x = 0.75\ \sigma_y = 1.5)$ and with a center at $(1, 9)$. The third one is also nonradial, centered at $(9, 9)$, with correlated inputs $(\rho = 0.5)$ and with a covariance matrix $\Sigma = [2.25 \quad 1.125; 1.125 \quad 2.25]$, $(\sigma_x = \sigma_y = 1.5)$.