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Problem 1

a)

line 2: initialising a result value = 0 takes O(1) time
line 3: loops through all values inside A, which is n elements perfoming the actions within the loop all take O(1) time
inside that loop O(1) time if statement
add the element to the result O(1) time

returning the result is O(1) time

total time = O(1) + O(n)
total time = O(n)

b)
$T(n) = T(n/2) + O(1)$

$a = 1$
$b = 2$
$f(n) = O(1)$

first calculate $n^{\log_b a}$ to find if it is > than f(n) = to f(n) or < than f(n), and thus find the case
$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

MASTER THEORUM

Case 1: $f(n) < n^{\log_b a}$ AKA $f(n) = O(n^{\log_b a - \epsilon})$ for $\epsilon > 0$
$T(n) = \Theta(n^{\log_b a})$

Case 2: $f(n) = n^{\log_b a} \cdot \log^k n$ AKA $f(n) = \Theta(n^{\log_b a} \cdot \log^k n)$ for $k \geq 0$
$T(n) = \Theta(n^{\log_b a} \cdot \log^{k+1}(n))$

Case 3: $f(n) > n^{\log_b a}$ AKA $f(n) = \Omega(n^{\log_b a + \epsilon})$ and $a \cdot f(n/b) \leq \delta f(n)$ for $\epsilon > 0$ and $\delta < 1$ then
$T(n) = \Theta(f(n))$

We have case 2 if $k = 0$

which means:

$T(n) = O(n^{\log_b a} \cdot \log^{k+1}(n))$
$T(n) = O(1 \cdot \log^{0+1}(n))$

$T(n) = O(\log^1(n))$
$T(n) = O(\log(n))$

c)
this is an attempt at a greedy algorithm. that allways picks the greatest value rather than taking the actual size of the game into account

given a set of (fi, si) fi = the fun of the game and si = the space the game occupies.

Say the size of the shelf is 10
f = {10, 9, 8}
s = {10, 5, 5}

the algorithm would take the value 10. and would have an overall fun of 10.

Where the actual greatest fun value would be 9 + 8, 17.

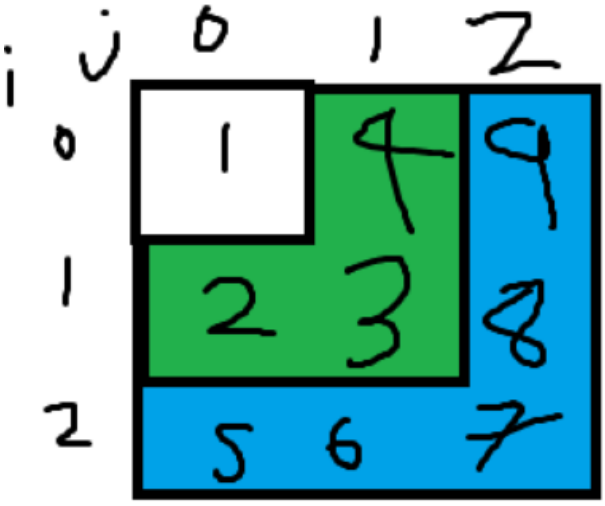
Problem 2

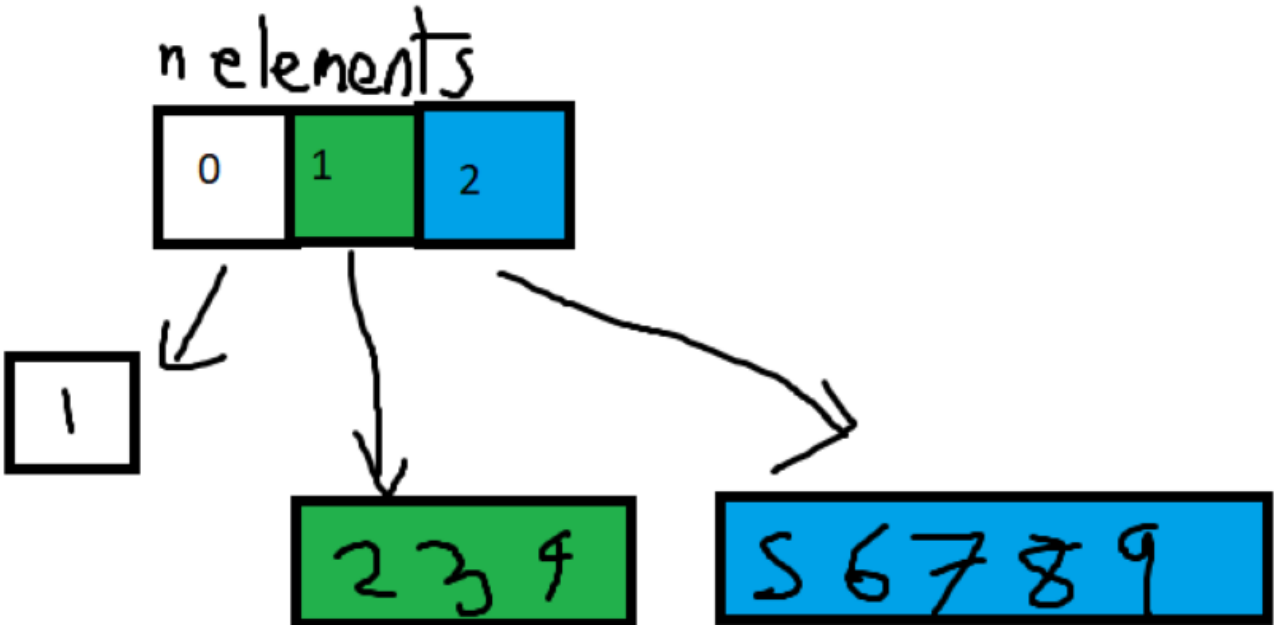
a) and b)

{(D, E), (C,F), (E,F), (C,B), (A, D)}

1. (D, E)
2. (C,F)
3. (E,F)
4. (C,B)
5. (A, D)

Problem 3





create would simply allocate space for the 1×1 element

set/get. i, j. to get the index,
if i > j youd have to add the index. to get it

```
def get(i, j):
        # accessing the row (which colour/layer you are trying to take
elements from)

        if i >= j:
                return matrix[i][j]
        if j > i:
                #because when you try to access the element, it "wraps
around" and thus you need to skip over the initial row elements to get to
the one you want
                return matrix[j][j + i]
```

then to expand the array, you would copy all of the elements from the pointer array into a new array with n + 1 spaces.

then in that newest space, allocate memory the size of nx2 + 1.

set the new n to be n + 1

b)

- create(): creates a 1 × 1 matrix where a1,1 = 0.
- set/get(i, j): set or get the value of the entry ai,j.
- increase-size: If the current size of the matrix is n × n, increase it to n +1× n +1 such that the new entries are set of 0. In other words, A becomes A ′ such that a ′ i,j = ai,j if 1 ≤ i, j ≤ n, and a ′ i,j = 0 otherwise.

CLAIM: this dynamic matrix will maintain the invariant, that if x = $a[i][j]$ then after increasing the size, x will still be $a[i][j]$

when increase size occurs, it creates a new array of length n + 1. and transfer all the other array pointers to this new array. this ensures that nothing in the previous arrays have been changed.

say if we had an array of size n x n. in i,j we had the value x.
the pointer array will look like

[p0,p1,p2 ..., p(n - 2), p(n - 1)]

each pointer p value pointing to an array of size k*2 + 1. and x would be inside some p[i][j] or p[j][i+ j]

increasing the size will change the pointer array to
[p0,p1,p2 ..., p(n - 2), p(n - 1), pn]

meaning that the x value will still be as it was before

c)
allocating a single space should take O(1) time

set/getting is an if else statement with O(1)operations inside, so it would take O(1) time

to create a new pointer array, that holds the addresses to the other arrays, it would take O(1) time

then to move all the pointers to that new array you would need to loop through the old length n, taking O(n) time

to create a new array that will take O(1) time

to get that pointer to the new array and put it in the pointers list, it will take O(1)

total time = 5O(1) + O(n)
total time = O(n)

Problem 4

a)
$T \cap S$ are the edges they have in common.
S/T are the edges in S that are not in T

T and S will have the same number of edges, if it is a minimum spanning tree they will both have the MINIMUM edges avaliable
take the T/S to see which edges in T are not in S. the differing edges in T but not in S.

then you can add an edge from S/T into T.
then take an edge from T/S and remove it from T, that the new edge was connected to continue doing this until you get S

common_edges = take the common values of S and T

```
def swapping_trees(S, T):
        differing_S = take the S/T
        differing_T = take the T/S
        swapping_sequence = empty list {}

        add S to the swapping sequence
        for every value in differing_S:
                //swap the edges
                s_edge = differing_S.pop()
                t_edge = differing_T.pop()

                R = T (make a copy of T and save that as R)

                add s_edge to R
                remove t_edge from R (if the edge was part of a cycle)
                add R to swapping_sequence

        return swapping_sequence
```

b)

- for the algorithm to be correct, every value inside swapping sequence must be a tree
    - this can be proven by defining a tree then induction
    - by adding an edge, it means that the tree is no longer a minimum spanning tree, and because every vertice is connected, that means that there must

be another route to a vertex that has already been connected in the tree. This breaks the cycle property that minimum spanning trees have, meaning we are able to remove an edge, that is part of a newly created cycle.
    - To save time, you remove the edge that isnt a part of S/T, this restores the cycle and cut edge property to the Rth tree
    - By doing this every time you ensure a minimum spanning tree in each value of R

- value Ri and Ri+1 must be one swap away from each other
    - swapping is to remove and add a new value to the trees. This is done in every iteration
- the value at the beginning must be T, and the end result of swapping sequence must be S
    - if all edges from S/T are added
    - and everything from T/S is removed
    - what you have left is S

c) Analyze the time complexity of your algorithm.

taking the S/T would take O(n^2) time to loop through all the edges of S
and check if they are in T
similarly, taking T/S would also take O(n^2) time
initialising the swapping sequence value would take O(1) time
adding S to the swapping sequence is also O(1) time

looping through the differing values could potentially take O(n) time is all values in S are not in T.
poping takes O(1)
poping takes O(1)
making a deep copy would take O(n)
adding an edge to R would take O(1)
removing an edge would need to loop through and take it out of R taking O(n)
adding r would be O(1)

total time = O(n^2) + O(n^2) + O(n) *(O(n)) + O(n) O(1) + O(1))*
total time = 2O(n^2) + O(n) (2O(n))
total time = 2O(n^2) + 2O(n^2)
total time = 4O(n^2)
total time = O(n^2)