CMCS Exam Project
Implementation of Macroscopic and Microscopic Traffic Models

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Traffic Models

Macroscopic:

Microscopic:

Macroscopic Traffic Models

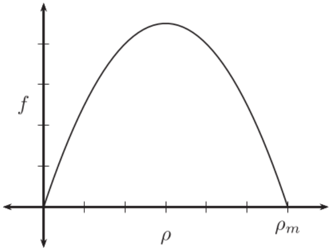
- Macroscopic traffic models aim at simulating the evolution of the density of roads in a road network, given some initial flow conditions.
- Example of a PDE model: Lighthill-Whitham model

$$\frac{\partial}{\partial t}\rho(t,x) + \frac{\partial}{\partial x}f(t,x) = 0$$

  where $\rho(t,x)$ is the density of the road at position x and time t, and $f(x,t)$ is the *flux*, defined as $f = \rho \cdot v$ where v is the traffic speed.

Greenshield's model for flux

A common assumption made in traffic modeling is that flux is solely a function of the density of the road. The relationship between these is called the *fundamental diagram*.



A common choice is Greenshield's model:

$$f(\rho) = v_f \rho \left(1 - \frac{\rho}{\rho_m}\right)$$

Where v_f is freeflow speed and ρ_m is maximum density.

Modeling traffic in a network

We provide:

- A PDE modeling the density on each road:

$$\frac{\partial}{\partial t}\rho_i(t,x) + \frac{\partial}{\partial x} f(\rho_i(t,x)) = 0$$

- Law of conservation of flux (Kirchoff's law):

$$\sum_{i \in in(n)} f(\rho_i(b_i,t)) = \sum_{i \in out(n)} f(\rho_i(a_i,t))$$

  Where each road is modeled as an interval $[a_i, b_i]$.

- A matrix for each intersection, describing the percentage of drivers coming from a certain road i who choose to drive into any of the outgoing roads j.

ODE Model

Agarwal et al. propose an ODE approximation of the PDE model. For each road of length l, the density varies as follows:

$$\frac{d\rho(t)}{dt} = \frac{f_{in}(t) - f_{out}(t)}{l}$$

Where f_{out} is computed using the Greenshield model:

$$f_{out}(t) = v_f \rho(t) \left(1 - \frac{\rho(t)}{\rho_m}\right)$$

Additionally, we simplify the model by assuming that each road i has a probability α_i of being chosen at an intersection.

Microscopic Model

A microscopic traffic model aims to capture the behaviour of the individual cars based on the interaction with other cars.

In our model, the car will accelerate or decelerate depending on the behaviour of the car in front of it (its *leader*).

To implement this choice, we employ a *car following model*.

GHR Car Following Model

The model takes three parameters p_1, p_2, p_3:

$$a = p_1 \cdot v^{p_2} \cdot \left(\frac{\Delta v}{(\Delta x)^{p_3}} \right)$$

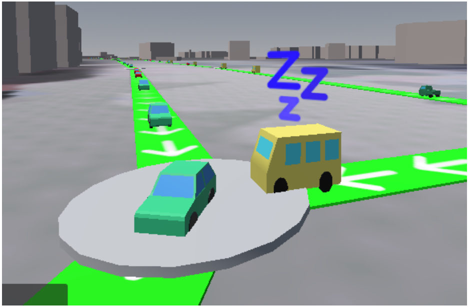
Where Δv is the difference in speed between the leader and the follower, and Δx is the distance between the two.

According to a book by Olstam and Tapani, by setting $p_2 = 0$ and $p_3 = 2$ we obtain a model that agrees with Greenshield's model.

p_1 is called *driver sensitivity*, and in a more realistic simulation every driver has its own. We set it to 1.

Giving Way

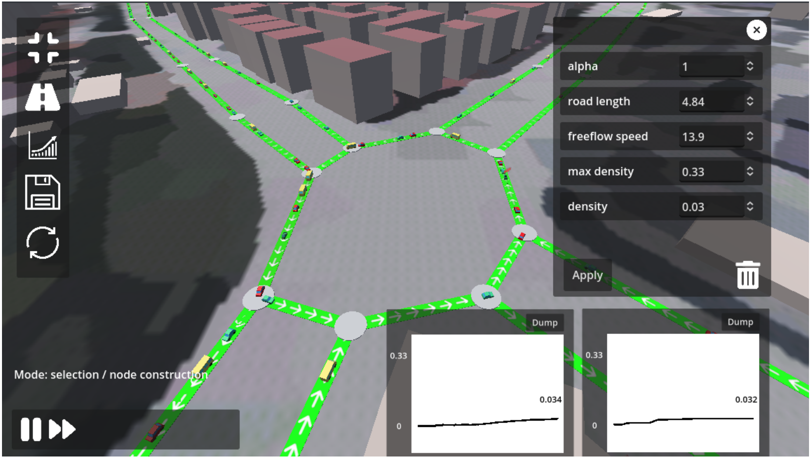
In order to avoid collisions at intersections, there needs to be a mechanism for giving way to other vehicles.



The model uses a FCFS synchronization mechanism: the first car to arrive is the first to pass.

Implementation

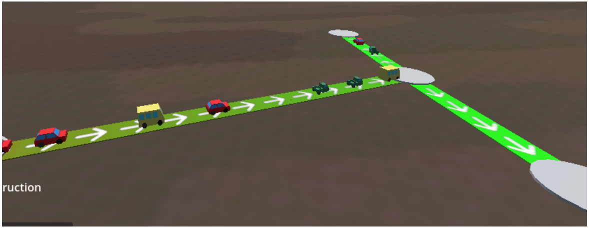
Made using the Godot engine.



Implementation: ODE Model

At each time frame, the variation in density for each road is computed according to the ODE model.

In order to better approximate the original PDE model, each road is divided into shorter roads, connected through invisible nodes.



The colors of a road represents its density. Green means zero density, red means max density.

Implementation: Microscopic model

The GHR model, as is, is not very suitable for modeling urban traffic. If two cars are very far apart on the same road, the follower will, in practice, act as if there was no car in front of it.

Some models use different *regimes* (e.g. free driving, car following, emergency) to adapt to different situations.

The model used in the simulator has two regimes:

- When the two cars are at least 50m apart, the GHR model is used to keep them from colliding.
- When it is higher, the cars are free to accelerate up to freeflow speed.

Differences between the models

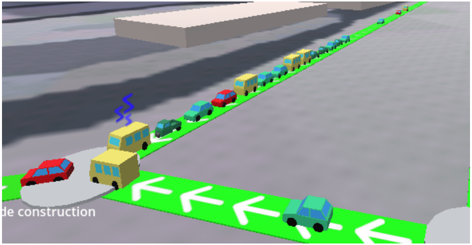
- In most situations, the agents of the microscopic model tend to travel at freeflow speed in both regimes. This leads to a mismatch between the two models, as the density according to the microscopic model will often be closer to:

$$\rho = f / v_f$$

  This effect has been slightly mitigated through randomization of the car-spawn time and initial speed.

Differences between the models

- In some cases, the agents may form queues at intersections by giving way, which could lead to the formation of waves that propagate backwards along the road.



This behaviour is not modeled by the microscopic model, hence this is another source of divergence.