Improving Bayesian Procedures to Detect Breakpoints in Time Series Data

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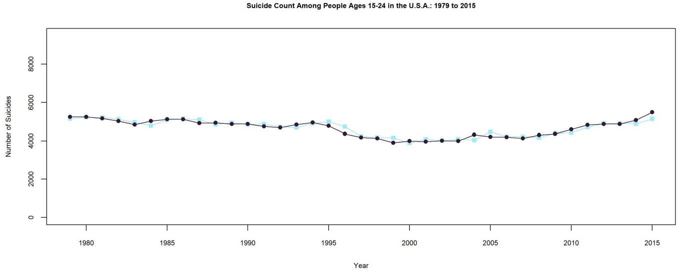
1. Project Overview
2. Progress and Results
3. Conclusion and Further Research

What is a breakpoint?

Definition

Breakpoints (also known as change points or structural breaks) are points in which time series data changes.



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4. **Bayesian** approach: the parameter is random and the data is fixed. With this method, you apply prior information to the data that is being explored. The posterior is the result.
   - **Bayes' theorem**

$$f(\theta|x) = \frac{f(x|\theta)\pi(\theta)}{\pi(x)}$$

What is the Bai-Perron test?

Definition

The **Bai-Perron test** is a general algorithm to find an optimal breakpoint set.

1. a frequentist approach
2. checks almost every single location for a breakpoint and returns the optimal set
3. requires a user to specify the number of breakpoints

What are some common types of time series models?

Auto-regressive (AR) model: each output value depends linearly upon previous values and an independent error term

AR(1)

$x_t = \phi x_{t-1} + \epsilon_t$

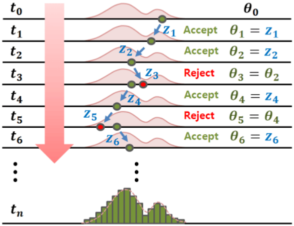
Moving average (MA) model: output value depends linearly upon previous error terms

MA(1)

$x_t = \epsilon_t + \theta \epsilon_{t-1}$

What is Markov Chain Monte Carlo (MCMC)?

1. A class of algorithms for Bayesian sampling from a probability distribution
2. Generates and records a random sample sequence from one sample to another where each proposed sample is accepted or rejected by the algorithm; this process is repeated until a stationary distribution sample is found



The Project

Bayesian Adaptive Auto-Regression (BAAR)

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- A new breakpoint set is proposed at each step of the MCMC
  - birth, death, and move

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- **Metropolis-Hastings** (a MCMC method for receiving a sequence of random samples from the probability distribution when direct sampling is difficult) ratio determines the set's acceptance
  - Acceptance Ratio

$$ratio \approx \exp\left(\frac{-\Delta BIC}{2}\right)\frac{\pi(K_n)}{\pi(K_o)}\frac{\pi(\tau_n|K_n)}{\pi(\tau_o|K_o)}\frac{q(\tau_o K_o|\tau_n K_n)}{q(\tau_n K_n|\tau_o K_o)}$$

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- Hence, a distribution of possible breakpoints locations can be obtained

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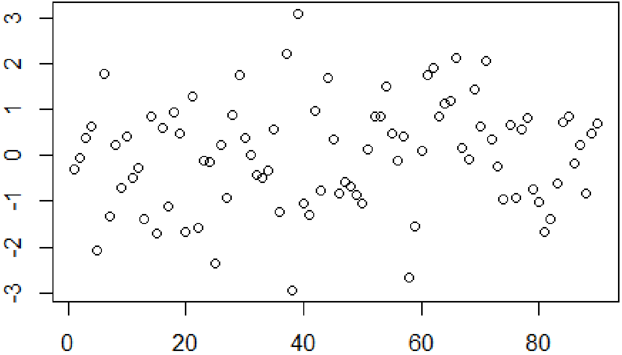
- Uses a Metropolis-Hastings procedure within a Gibbs sampler procedure to generate new coefficients
- Can handle seasonal/cyclical datasets
- Improves the Bayesian adaptive algorithm compatibility with various kinds of datasets

Stress Test

Example

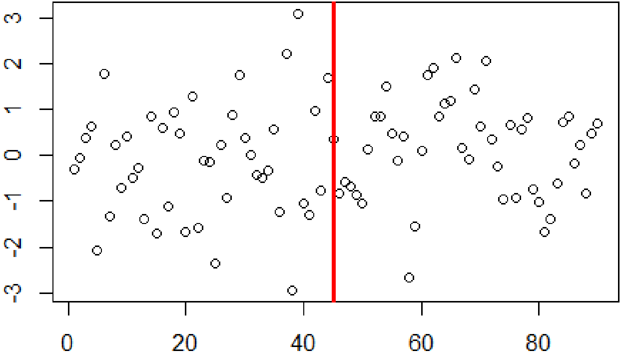
Where do you think a breakpoint is located?



Stress Test

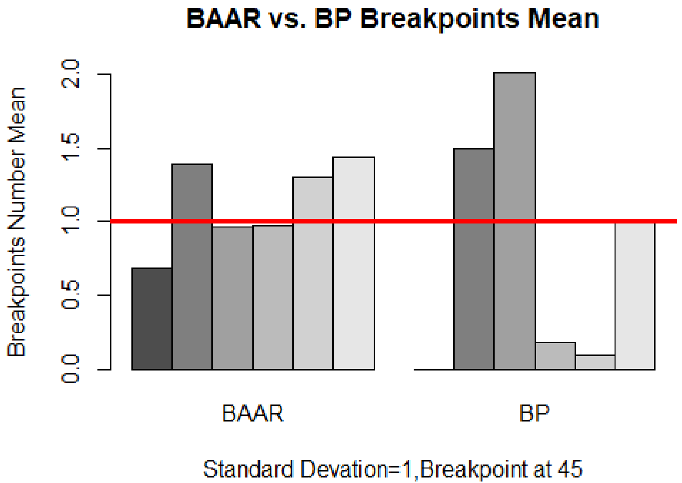
Example

How did you do?



Stress Test

Simulation results



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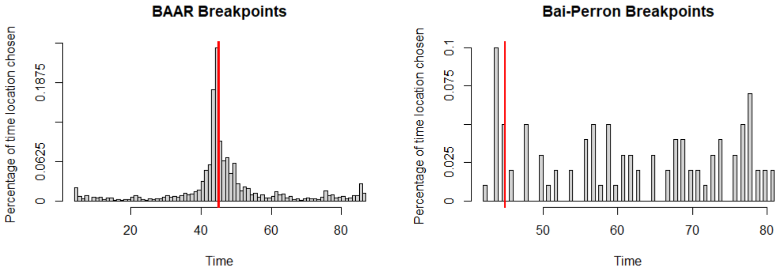
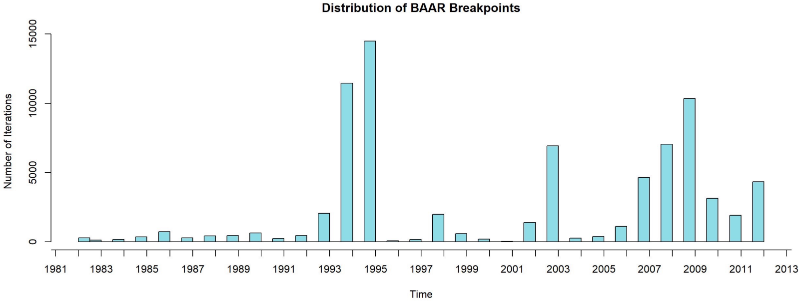


Case Study: Suicide Among People Ages 15-24 in the U.S.A.



Distribution of breakpoint locations in suicide data among 15-24 year olds in the United States of America, showing a 99% probability that a single breakpoint exists between 1994 and 1996, a 47% probability that a second breakpoint exists between 2002 and 2003, and a 73% probability that a third breakpoint exists between 2007 and 2010.

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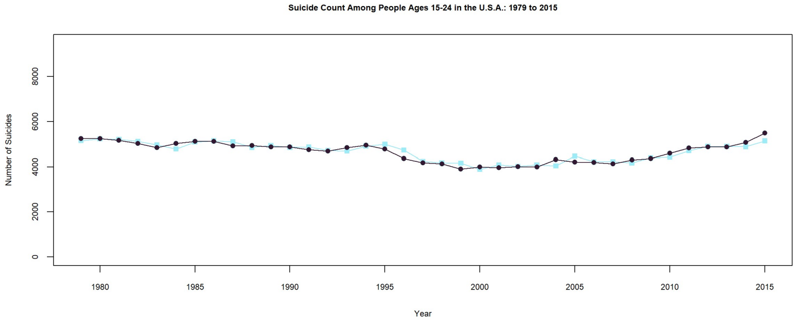

Suicide counts among people ages 15 to 24 in the United States of America: navy blue circles are true values; light blue squares are fitted values from a single AR(2).

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Conclusion

- BAAR correctly identifies breakpoints with greater accuracy than existing algorithms (i.e. Bai-Perron)
- BAMA still a work in progress, in part due to COVID outbreak

Further Research

- Improve starting points of the Bayesian adaptive algorithms
- Expand BAAR and BAMA techniques to more complicated time series analysis models

References

Bai, J. and Perron, P., (2003). *Computation and analysis of multiple structural change models*. Journal of applied econometrics, 18(1), pp.1-22.

Any Questions?