Improving A Bayesian Procedure to Detect Breakpoints in Time Series Data

Dainia Higgins, Nathaniel Wilson, Madison Ell, Sarah Klingbeil
Mentor: Jeff Liebner

Coppin State University, Fullerton College, Southern Adventist University

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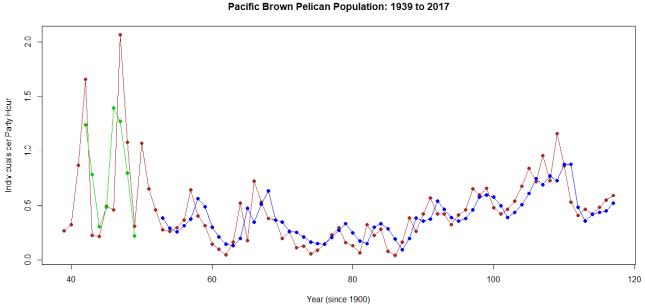
1. The Project Overview
2. Our Progress
3. Our Goals

Remember breakpoints?

- Breakpoints are changes in time series data, be they big or small
- Our goal of the REU is to find breakpoints using Bayesian adaptive models.



How do we find Breakpoints?

- We use Bai Perron to search for all possible locations for a breakpoint.
- Then we start with an initial amount of breaks and propose new ones by utilizing the birth, death, and movement steps.
- Our current algorithm currently is able to simulate birth and death steps

What are birth and death steps?

- **Birth** A birth step randomly proposes a breakpoint at an available location. Hence, the probability of a specific birth step b is defined as
$$b = b_k \times \frac{1}{A}$$
where b_k is the combined probabilities of performing a birth step, and A is the number of available locations.
- **Death** A death step randomly chooses an existing breakpoint and proposes a set without that chosen breakpoint. Similarly, the probability of a specific death step d is defined as
$$d = d_k \times \frac{1}{K}$$
where d_k is the combined probabilities of performing a death step and K is the number of given breakpoints.

What is the Metropolis-Hastings algorithm?

Metropolis-Hastings : Markov Chain Monte Carlo method that is used to gain a sequence of random samples from a probability distribution $P(x)$ when direct sampling is difficult.

1. Let $f(x)$ be a proportional distribution to $P(x)$. Select a state x_t and $q(x|x_{t-1})$ where q is the proposal density of x
2. For each iteration t draw $x_{new} \sim q(x_{new}|x_{t-1})$
3. Compute ratio $r = \frac{f(x_{new})q(x_{t-1}|x_{new})}{f(x_t)q(x_{new}|x_{t-1}))}$
4. If $r \geq 1, x_{t+1} = x_{new}$
5. If $r < 1$,

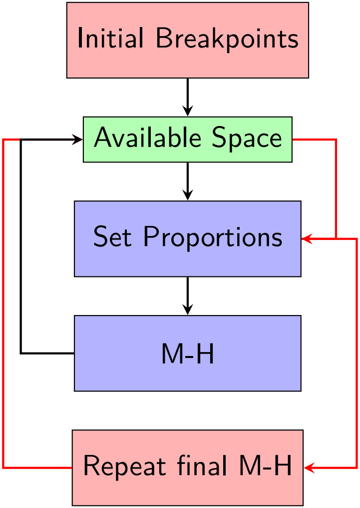
$$x_{t+1} = \begin{cases} x_{new} & \text{with probability } r, \\ x_{t-1} & \text{with probability } 1 - r \end{cases}$$

How does BAAR use the Metropolis-Hastings algorithm?

$$ratio \approx \exp\left(\frac{-\Delta BIC}{2}\right)\frac{\pi(K_n)}{\pi(K_o)}\frac{\pi(\tau_n|K_n)}{\pi(\tau_o|K_o)}\frac{q(\tau_o K_o|\tau_n K_n)}{q(\tau_n K_n|\tau_o K_o)}$$

where π represents the posterior densities and q the proposal densities, τ the location of breakpoints, and K the number of breakpoints.

Visual Diagram of BAAR



- **Initial Breakpoints**: Find Initial Breakpoints
- **Available Space**: calculate spaces available for new breakpoints and the number of ends
- **Set Proportions**: Set birth, death and move proportions
- **M-H**: Burn-in M-H Algorithm
- **Final M-H**: Repeat for final Metropolis-Hasting

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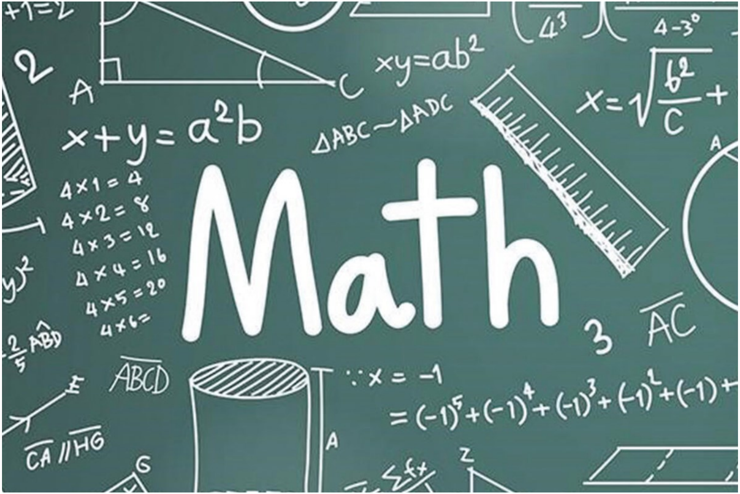
Math corrections



Math Corrections

Acceptance Ratio

$$ratio \approx \exp\left(\frac{-\Delta BIC}{2}\right)\frac{\pi(K_n)}{\pi(K_o)}\frac{\pi(\tau_n|K_n)}{\pi(\tau_o|K_o)}\frac{q(\tau_o K_o|\tau_n K_n)}{q(\tau_n K_n|\tau_o K_o)}$$

The previous REU group derived the acceptance ratio a similar fashion to Kass and Wasserman (1995). Their derivation also assumed that the prior was continuously uniform and constant. However, in our model, breakpoints are discrete and limited by available locations with probability mass function being conditional on the number of breaks.

Math Corrections

Hence, using a stars and bars technique to limit breakpoints to occur discreetly and in available locations, we can derive our priors in the following manner. In the case of a birth step,

$$\pi(\tau_n|k_n)\pi(K_n)q(\tau_o, K_o|\tau_n, K_n) = \frac{d \cdot Poisson(K_n|\lambda) \cdot (K+1)!}{\Pi_{i=1}^{K+1}(n - 3p - (K+1)(2p) + i)}$$

$$\pi(\tau_o|k_o)\pi(K_o)q(\tau_n, K_n|\tau_o, K_o) = \frac{b \cdot Poisson(K_o|\lambda) \cdot K!}{\Pi_{i=1}^{K}(n - 3p - (K)(2p) + i)}.$$

where K is the number of breaks, p is the model degree, n is the data and b and d is the probability of a specific birth and death step respectively.

Math Corrections

Similarly, in the case of a death step,

$$\pi(\tau_n|K_n)\pi(K_n)q(\tau_o, K_o|\tau_n, K_n) = \frac{b \cdot Poisson(K_n|\lambda) \cdot (K-1)!}{\Pi_{i=1}^{K-1}(n - 3p - (K-1)(2p) + i)}$$

$$\pi(\tau_o|K_o)\pi(K_o)q(\tau_n, K_n|\tau_o, K_o) = \frac{d \cdot Poisson(K_o|\lambda) \cdot K!}{\Pi_{i=1}^{K}(n - 3p - K(2p) + i)}$$

Where λ has a default value of one. This means that we are assuming there is one breakpoint, the user can change this value based on preferences.

Coding Progress

Coding Progress

- Cleaned up syntax for existing BAAR algorithms
- New birth/death step procedures now reflected in both original scripts
- Began work on Bayesian adaptive moving-average (BAMA) algorithm

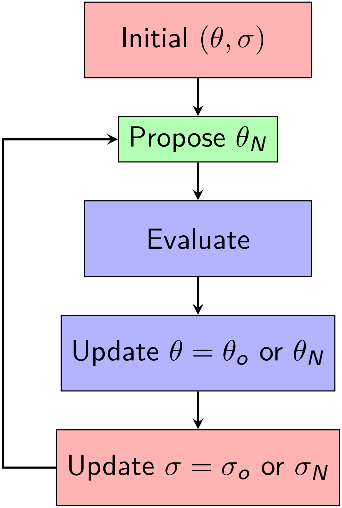
Why is BAAR alone not enough?

- On its own, the existing BAAR method is limited in scope
- Autoregressive (AR) model is not ideal for seasonal/cyclical datasets
- Goal is to improve the Bayesian adaptive algorithm compatibility with various kinds of datasets
- We want to pave the way for future work on autoregressive moving-average (ARMA) and autoregressive integrated moving-average (ARIMA) model frameworks

What is Bayesian Adaptive Moving-Average (BAMA)?

- A variant of the Bayesian adaptive algorithm that identifies breakpoints in datasets fit with the moving-average model
- Similar to the BAAR algorithms in breakpoint placement and birth, death, and move proportions
- Unique in log likelihood, coefficient derivations and fit of data

Progress on BAMA



Moving average (MA) model: output value depends linearly upon previous error terms

$$MA(1) = x_t = \epsilon_t + \theta\epsilon_{t-1}$$

where $\epsilon_t \sim N(0, \sigma)$

Our Goals

- Make progress building BAMA code
- Derive math for needed to find breakpoints for MA
- Code BAMA fit and then work on breakpoints
- Stress-testing all algorithms and codes

References

Kass, R.E. and Wasserman, L., (1995). *A reference Bayesian test for nested hypotheses and its relationship to the Schwarz criterion*. Journal of the american statistical association, 90(431), pp.928-934.

Any Questions?